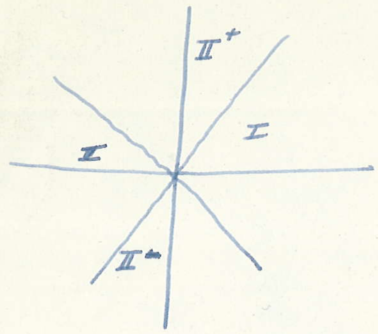
We divide the space time into four regions given below



In region I introduce

$$X^0 = R\sinh\chi \qquad 0 \le R \le \infty$$
$$X^1 = R\sin\theta\cos\varphi\cosh\chi \qquad -\infty \le \chi \le \infty$$
$$X^2 = R\sin\theta\sin\varphi\cosh\chi \qquad 0 \le \theta \le \pi$$
$$X^3 = R\cos\theta\cosh\chi \qquad 0 \le \varphi \le 2\pi$$

$$X_\mu X^\mu = -X^{0^2} + X^{1^2} + X^{2^2} + X^{3^2} = R^2 \ge 0$$

In region $II^\pm$ introduce

$$X^0 = R\cosh\chi$$
$$X^1 = R\sin\theta\cos\varphi\sinh\chi$$
$$X^2 = R\,\cancel{\sin\theta\sin\varphi}\,\sinh\chi$$
$$X^3 = R\cos\theta\,\sinh\chi$$

II^+: $0 \le R \le \infty$, $0 \le \chi \le \infty$

II^-: $-\infty \le R \le 0$, $-\infty \le \chi \le 0$

$0 \le \theta \le \pi$

$0 \le \varphi \le 2\pi$

$$X_\mu X^\mu = -(X^{1^2} + X^{2^2} + X^{3^2}) + X^{0^2} = R^2 \ge 0$$

Assume for internal operator $\frac{\partial^2}{\partial X_\mu \partial X^\mu} - X_\mu X^\mu$

In region $II^\pm$ this becomes since

$$g_{\mu\nu} = \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{vmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{vmatrix} \end{array}$$

$$\frac{1}{R^3}\frac{\partial}{\partial R}\left(R^3\frac{\partial f}{\partial R}\right) - \frac{1}{R^2\sinh^2\chi\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) - \frac{1}{R^2\sinh^2\chi\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}$$
$$- \frac{1}{R^2\sinh^2\chi}\frac{\partial}{\partial\chi}\left(\sinh^2\chi\frac{\partial f}{\partial\chi}\right) - R^2 f + \mu^2 f = 0$$

set $f = f_1(R) f_2(\chi) f_3(\theta) f_4(\varphi)$ and separate variables

$$\frac{d^2 f_4}{d\varphi^2} = -m^2 f_4$$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f_3}{\partial\theta}\right) - \frac{m^2 f_3}{\sin^2\theta} + l(l+1) f_3 = 0$$

$$\frac{1}{\sinh^2\chi}\frac{\partial}{\partial\chi}\left(\sinh^2\chi\frac{\partial f_2}{\partial\chi}\right) - \frac{l(l+1) f_2}{\sinh^2\chi} - k(k+2) f_2 = 0$$

$$\frac{1}{R^3}\frac{\partial}{\partial R}\left(R^3\frac{\partial}{\partial R}f_1\right) - \frac{k(k+2)}{R^2}f_1 - R^2 f + \mu^2 f = 0$$

The solns are

$f_4 = e^{im\varphi}$ $\qquad m = 0, \pm 1, \pm 2 \ldots$

$f_3 = P_l^m(\cos\theta)$ $\qquad l = 0, 1, 2 \ldots$ $\qquad |m| \leq l$

set $u = \operatorname{ctnh} \chi$ $\qquad$ then $|u| \geq 1$ for $0 \leq |\chi| \leq \infty$

$G = G(u) = \sinh \chi \cdot f_2$

$$(1-u^2)G'' - 2uG' + G\left\{l(l+1) - \frac{(k+1)^2}{1-u^2}\right\} = 0$$

and so $G(u) = P_l^{k+1}(u)$

$f_2 = \dfrac{P_l^{k+1}(\operatorname{ctnh}\chi)}{\sinh \chi}$ but no good for u ranges ≥ 1

$\therefore$ take $f_2 = \dfrac{Q_l^{k+1}(\operatorname{ctnh}\chi)}{\sinh\chi}$

also possibly $k = -1$

$k = 0, 1, 2 \ldots\ldots$

~~$k+1 \leq l$~~

~~This cuts out $l = 0$~~

to be square integrable need investigate.

$f_1 = e^{-\frac{R^2}{2}} R^k L_{n+1}^{k+1}(R^2)$ $\qquad$ Laguerre func

$n = 0, 1 \ldots\ldots$, $\quad n \geq k$

$\mu^2 = 4n - 2k + 4$

The f_2 are unfortunately undefinable for $|u| = 1$ or $|\chi| = \infty$ or on light cone from inside

For region II

$$g_{\mu\nu} = \left| \begin{matrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{matrix} \right|$$

$$\frac{1}{R^3}\frac{\partial}{\partial R}\left(R^3 \frac{\partial f}{\partial R}\right) + \frac{1}{R^2 \sin\theta \cosh^2\chi}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial f}{\partial\theta}\right) + \frac{1}{R^2\sin^2\theta\cosh^2\chi}\frac{\partial^2 f}{\partial\varphi^2}$$
$$- \frac{1}{R^2\cosh^2\chi}\frac{\partial}{\partial\chi}\left(\cosh^2\chi\frac{\partial f}{\partial\chi}\right) - R^2 f + \mu^2 f = 0$$

separate variables

$$\frac{d^2 f_4}{d\varphi^2} = -m^2 f_4$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{df_3}{d\theta}\right) - \frac{m^2}{\sin^2\theta} f_3 + l(l+1) f_3 = 0$$

$$\frac{1}{\cosh^2\chi}\frac{\partial}{\partial\chi}\left(\cosh^2\chi \frac{\partial f_2}{\partial\chi}\right) + \frac{l(l+1)f_2}{\cosh^2\chi} - h(h+2)f_2 = 0$$

$$\frac{1}{R^3}\frac{\partial}{\partial R}\left(R^3\frac{\partial f_1}{\partial R}\right) - \frac{h(h+2)f_1}{R^2} - R^2 f_1 + \mu^2 f_1 = 0$$

The solns are

$f_4 = e^{im\varphi}$ $\qquad m = 0, \pm1, \pm2 \cdots$

$f_3 = P_l^m(\cos\theta)$ $\qquad l = 0, 1, 2 \cdots \qquad |m| \le l$

set $u = \tanh\chi$ $\qquad$ then $|u| \le 1$

$G(u) = \cosh\chi \; f_2$

$$(1-u^2)G'' - 2uG' + \left\{l(l+1) - \frac{(h+1)^2}{1-u^2}\right\}G = 0$$

solns

$$f_2 = \frac{P_l^{h+1}(\tanh\chi)}{\cosh\chi} \qquad h = 0, 1, \ldots \qquad (h+1) \le l$$

to be square integrable
need omit $h = -1$
hence $l \neq 0$.

$$f_1 = e^{-R^2/2} R^k L_{n+1}^{k+1}(R^2) \qquad n = 1, 2 \cdots \quad n \geq k$$

$$\mu^2 = 4n - 2k + 4$$

Hence in both cases I and II$^{\pm}$ the solns are the same form in R, θ, φ; for the χ dependence there is a difference. The solns are

I $\quad e^{im\varphi} P_l^m(\cos\theta) \dfrac{P_l^{k+1}(\tanh\chi)}{\cosh\chi} e^{-\frac{R^2}{2}} R^k L_{n+1}^{k+1}(R^2)$

$|m| = 0, 1, 2 \cdots \quad |m| \leq l$

$l = 1, 2 \cdots \quad k+1 \leq l$

$k = 0, 1 \cdots \quad k \leq n$

$n = 0, 1 \cdots \quad \mu^2 = 4n - 2k + 4$

$-1 \leq \tanh\chi \leq +1$

$+\infty < \cosh\chi < +\infty$

II $\quad e^{im\varphi} P_l^m(\cos\theta) \dfrac{Q_l^{k+1}(\cancel{\coth}\chi)}{\sinh\chi} e^{-\frac{R^2}{2}} R^k L_{n+1}^{k+1}(R^2)$

$|m| = 0, 1, 2 \cdots \quad |m| \leq l$

$l = 0, 1, 2 \cdots$ I don't think $k+1 \leq l$ true

$k = -1, 0, 1 \cdots \quad k \leq n$

$n = 0, 1, 2 \cdots$

$\mu^2 = 4n - 2k + 4.$

$0 \leq \chi \leq \infty$

$0 \leq \sinh\chi \leq \infty$

$\chi = 0$

$Q_l^{k+1}(\infty) = 0$

$\sinh\chi = 0$

$\coth\chi = 1 \quad Q_l^{k+1}(1) = \infty$