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Define a Base Function Set

We have for the free field solutions of

$$F \phi_n = 0$$

$$\phi_n = e^{i \mathbb{k} \cdot \mathbb{X} - i k_0^{(s)} X_0} \prod_i \chi_{n_i} (a_{ij}^{(\mathbb{k})} r_j)$$

$$k_0^{(s)} = + \sqrt{\mathbb{k}^2 + \mu_s} \qquad s = n_1 + n_2 + n_3 - n_0$$

For the ϕ_n; k_0 is not arbitiary and thus we only have a complete set for $\mathbb{X}$ and r. Want to remove the restriction in both the exponent and in the $a_{ij}^{(\mathbb{k})}$ matrix.

$$a_{ij}^{(\mathbb{k})} = \left(\begin{array}{ccc|c} 1 + \frac{k_1^2}{\mathbb{K}^2} & \frac{k_1 k_2}{\mathbb{K}^2} & \frac{k_1 k_3}{\mathbb{K}^2} & \frac{i k_1}{\sqrt{\mu_s}} \\ \frac{k_1 k_2}{\mathbb{K}^2} & 1 + \frac{k_2^2}{\mathbb{K}^2} & \frac{k_2 k_3}{\mathbb{K}^2} & \frac{i k_2}{\sqrt{\mu_s}} \\ \frac{k_1 k_3}{\mathbb{K}^2} & \frac{k_2 k_3}{\mathbb{K}^2} & 1 + \frac{k_3^2}{\mathbb{K}^2} & \frac{i k_3}{\sqrt{\mu_s}} \\ \hline \frac{-i k_1}{\sqrt{\mu_s}} & \frac{-i k_2}{\sqrt{\mu_s}} & \frac{-i k_3}{\sqrt{\mu_s}} & \frac{\sqrt{\mathbb{k}^2 + \mu_s}}{\sqrt{\mu_s}} \end{array} \right)$$

$$\mathbb{K}^2 = \sqrt{\sqrt{\mu_s} \left(\sqrt{\mu_s} + \sqrt{\mathbb{k}^2 + \mu_s} \right)} \qquad \left(K = \sqrt{\mu_s} \right)$$

$$v_x = \frac{k_1 c}{\sqrt{\mathbb{k}^2 + \mu_s}} \qquad v_y = \frac{k_2 c}{\sqrt{\mathbb{k}^2 + \mu_s}} \qquad v_z = \frac{k_3 c}{\sqrt{\mathbb{k}^2 + \mu_s}}$$

Make the substitution in the $a_{ij}^{(k)}$ matrix as follows.

$$\left.\begin{array}{l} \sqrt{\mu_s} = +\sqrt{-k^2} \\ \sqrt{\mathbf{k}^2+\mu_s} = +k_0 \end{array}\right\} \text{for } k \text{ futurelike}$$

The previous $a_{ij}^{(\mathbf{k})}$ matrix is a transformation to the rest frame for a futurelike vector only. For a pastlike vector we want the matrix

$$b_{ij}^{(\mathbf{k})} = a_{ij}^{(-\mathbf{k})}$$

Thus in the $a_{ij}^{(\mathbf{k})}$ matrix make the substitutions

$$\left.\begin{array}{l} \sqrt{\mu_s} = +\sqrt{-k^2} \\ \sqrt{\mathbf{k}^2+\mu_s} = -k_0 \\ \mathbf{k} = -\mathbf{k} \end{array}\right\} \text{for } k \text{ pastlike}$$

Then making use of sign factor, $\dfrac{k_0}{|k_0|}$, we can write the one matrix for k timelike as follows,

$$a_{ij}^{(k)} = \left[\begin{array}{ccc|c} & & & \frac{k_1 k_4}{|k_0|\sqrt{-k^2}} \\ & \text{Same} & & \frac{k_2 k_4}{|k_0|\sqrt{-k^2}} \\ & & & \frac{k_3 k_4}{|k_0|\sqrt{-k^2}} \\ \hline \frac{-k_1 k_4}{|k_0|\sqrt{-k^2}} & \frac{-k_2 k_4}{|k_0|\sqrt{-k^2}} & \frac{-k_3 k_4}{|k_0|\sqrt{-k^2}} & \frac{-k_4^2}{|k_0|\sqrt{-k^2}} \end{array}\right]$$

$k^2 < 0$

(Note that this has the property

$$a_{ij}^{(-k)} = a_{ij}^{(k)}\text{)}$$

$$\underline{K}^2 = \sqrt{-k^2}\left(\sqrt{-k^2} + |k_0|\right)$$

This takes a vector $k_j = (k_1\ k_2\ k_3\ k_4)$ into $k_i^{(0)} = \left(0, 0, 0, i\frac{k_0}{|k_0|}\sqrt{-k^2}\right)$ with the relations $k_i^0 = a_{ij}^{(k)} k_j$ and $k_j = a_{ij}^{(k)} k_i^0$

We now have a set of functions

$$\varphi_{sq}^{(k)}(x, r) = e^{ikx} \prod_i \chi_{n_i}(a_{ij}^{(k)} r_j)$$

which, if we set $a_{ij}^{(k)} = \delta_{ij}$ for $k^2 \geq 0$, is a complete set for x, r.

Transformation Properties

The $\varphi^{(k)}_{sq}(x, r)$ will always appear in integrals over k. Essentially we are interested in how these different integrals over k transform with respect to the indicies $s\,q$. Thus when we apply a lorentz transform to $\varphi^{(k)}_{sq}(x, r)$, we will transform , x, r and k as vectors.

We have most generally

$$s = m - n_0 \qquad m = n_1 + n_2 + n_3$$

where s = a invariant independent of k. Thus for

$$\begin{cases} \underline{k^2 > 0} \longrightarrow k^2, s \text{ are invariant lables} \\ \\ \underline{k^2 = 0} \longrightarrow k^2, \dfrac{k_0}{|k_0|}, s \text{ are invariant lables} \end{cases}$$

For $k^2 < 0$ we must examine the form of the $\varphi^{(k)}_{sq}$ more closely.

$$\chi_{n_i}(r_i^0) = h_{n_i}(r_i^0)\, e^{-\frac{r_i^{0\,2}}{2\lambda^2}}$$

$$r_i^0 = a^{(k)}_{ij}\, r_j$$

$$\prod_i \chi_{n_i}(r_i^\circ) = \prod_i h_{n_i}(r_i^\circ)\, e^{-\frac{1}{2\lambda^2}\left(r_1^{\circ 2}+r_2^{\circ 2}+r_3^{\circ 2}+r_0^{\circ 2}\right)}$$

$$= \prod_i h_{n_i}(r_i^\circ)\, e^{-\frac{1}{2\lambda^2}\left(r_\mu^2+2r_0^{\circ 2}\right)}$$

$$r_0^\circ = -i\, a_{ij}^{(k)} r_j = \frac{-k_0\, k_j r_j}{|k_0|\sqrt{-k^2}} = \text{form invariant}$$

$$r_i^\circ = k_i \left[r_i + \frac{\mathbf{k}\cdot\mathbf{r}}{\mathbf{k}^2} + \frac{k_4 r_4}{|k_0|\sqrt{-k^2}} \right] \neq \text{form invariant}$$
$i = 1, 2, 3$

Thus

$$\prod_i \chi_{n_i}(r_i^\circ) = \prod_{i=1}^{3} h_{n_i}(r_i)\; \underbrace{h_{n_0}\left(\frac{-k_0 k_j r_j}{|k_0|\sqrt{-k^2}}\right) e^{-\frac{1}{2\lambda^2}\left(r_\mu^2+\frac{2(kr)^2}{-k^2}\right)}}_{\text{form invariant part}}$$

Thus the exponent and the n_0 polynomial transform into themselves. We thus have n_0 a invariant lable. Since

$$m = s + n_0$$

we also ~~have~~ have m a invariant lable since both s and n_0 are. Thus for

$$\left\{ \underline{k^2 < 0} \rightarrow k^2, s, m, n_0, \frac{k_0}{|k_0|} \quad \text{are invariant lables} \right.$$

Invariant Projection Operator

If we define

$$\mathrm{P}\,\varphi^{(k)}_{sq}(x,r) = \begin{cases} 0 \\ \varphi^{(k)}_{sq}(x,r) \end{cases}$$ depending on value of the invariant lables

$$\left[\underline{k^2<0},\ k^2,\ \frac{k_0}{|k_0|},\ m,\ n_0\right],\ \left[\underline{k^2=0},\ \frac{k_0}{|k_0|},\ s\right],\ \left[\underline{k^2>0},\ k^2,\ s\right]$$

Then the P ~~[illegible]~~ operator performs a invariant subdivision of a arbitray function $f(x,r)$.

$$f'(x',r') = f(x\ r)$$

then $$P_{(x'r')}f'(x',r') = P_{(xr)}f(x,r)$$

Thus we can use this operator in the Lagrangian giving the equations of motion.

Take

$$\delta\bar{L}_{m.e.} = \int\{-F\,\phi(\varsigma)\}\,\delta\phi(\varsigma)\,d\varsigma \qquad \varsigma \sim x, r$$

$$\bar{L}_{d.} = -\int\bar{\Psi}(x)\left(\gamma_\mu\frac{\partial\Psi(x)}{\partial x_\mu} + M\Psi(x)\right)dx$$

$$\bar{L}_{int} = -g\int\bar{\Psi}(x')\left(\mathrm{P}\,\phi(\varsigma)\right)\Psi(x''')\,F(x',\varsigma,x''')\,dx'\,d\varsigma\,dx'''$$

$\delta \bar{L} = 0$ gives

$$F\,\phi(\mathfrak{z}) + g\, P^{c.}_{\mathfrak{z}} \int \bar{\Psi}(x')\,\Psi(x''')\,F(x',\mathfrak{z},x''')\,dx'\,dx''' = 0$$

$$\gamma_\mu \frac{\partial \Psi(x)}{\partial x_\mu} + m\,\Psi(x) + g\int \left(P\,\phi(\mathfrak{z})\right)\Psi(x''')\,F(x,\mathfrak{z},x''')\,d\mathfrak{z}\,dx''' = 0$$

$$-\gamma_\mu^T \frac{\partial \bar{\Psi}(x)}{\partial x_\mu} + m\,\bar{\Psi}(x) + g\int \bar{\Psi}(x')\left(P\,\phi(\mathfrak{z})\right)F(x',\mathfrak{z},x)\,dx'\,d\mathfrak{z} = 0$$

($F(x',\mathfrak{z},x''') = \delta\left(x'-(X+\frac{r}{2})\right)\delta\left(x'''-(X-\frac{r}{2})\right)$ for previous choice of $\bar{L}_{int}$)

These give us the integral equations

$$\phi(\mathfrak{z}) = P_{\mathfrak{z}}\,\phi^{in}(\mathfrak{z}) + g\int \left(P_{\mathfrak{z}''}\,G_+(\mathfrak{z},\mathfrak{z}'')\right)\bar{\Psi}(x')\,\Psi(x''')\,F(x',\mathfrak{z}'',x''')\,dx'\,d\mathfrak{z}''\,dx'''$$

$$\Psi(x) = \Psi^{(in)}(x) + g\int S_+(x-x')\left(P\,\phi(\mathfrak{z}'')\right)\Psi(x''')\,F(x',\mathfrak{z}'',x''')\,dx'\,d\mathfrak{z}''\,dx'''$$

$$\bar{\Psi}(x) = \bar{\Psi}^{(in)}(x) + g\int \bar{S}_+(x-x')\,\bar{\Psi}(x')\left(P\,\phi(\mathfrak{z}'')\right)F(x',\mathfrak{z}'',x''')\,dx'\,d\mathfrak{z}''\,dx'''$$

Choosing $P_{\mathfrak{z}}\,\phi^{(in)}(\mathfrak{z})$ instead of $\phi^{(in)}(\mathfrak{z})$. Since

$$P_{\mathfrak{z}''}\,G_+(\mathfrak{z},\mathfrak{z}'') = P^{c.}_{\mathfrak{z}}\,G_+(\mathfrak{z},\mathfrak{z}'') = P_{\mathfrak{z}}\,G_+(\mathfrak{z},\mathfrak{z}'')$$

($P^{c.} = P$ if P does not depend on $\frac{k_0}{|k_0|}$, and we will assume it does not. See comment on the bottom of page 5)

$$\phi(\mathfrak{z}) = P_{\mathfrak{z}}\left\{\phi^{(in)}(\mathfrak{z}) + g\int G_+(\mathfrak{z}\,\mathfrak{z}'')\,\bar{\Psi}(x')\,\Psi(x''')\,F(x',\mathfrak{z}'',x''')\,dx'\,d\mathfrak{z}''\,dx'''\right\}$$

and ∴

$$\phi(\varsigma) = \mathrm{P}\,\phi(\varsigma) \quad ;$$

~~All~~ Using the notation

$$\mathrm{P}\, f(\varsigma) = f_{mn_0}(\varsigma)$$

$$\xi \sim x', \varsigma, x'''$$

$$d\xi \sim dx'\, d\varsigma\, dx'''$$

we have then

$$\phi_{mn_0}(\varsigma) = \phi^{(in)}_{mn_0}(\varsigma) + g\int G_{mn_0+}(\varsigma, \varsigma_1)\,\overline{\psi}(x_1')\,\psi(x_1''')\,F(\xi_1)\,d\xi_1$$

$$\psi(x) = \psi^{in}(x) + g\int S_+(x - x_1')\,\phi_{mn_0}(\varsigma_1)\,\psi(x_1''')\,F(\xi_1)\,d\xi_1$$

$$\overline{\psi}(x) = \overline{\psi}^{in}(x) + g\int \overline{S}_+(x - x_1''')\,\overline{\psi}(x_1')\,\phi_{mn_0}(\varsigma_1)\,F(\xi_1)\,d\xi_1$$

Using adv. potentials we get similarly

$$\phi_{mn_0}(\varsigma) = \phi^{out}_{mn_0}(\varsigma) + g\int G_{mn_0-}(\varsigma\,\varsigma_1)\,\overline{\psi}(x_1')\,\psi(x_1''')\,F(\xi_1)\,d\xi_1$$

$$\psi(x) = \psi^{out}(x) + g\int S_-(x - x_1')\,\phi_{mn_0}(\varsigma_1)\,\psi(x_1''')\,F(\xi_1)\,d\xi_1$$

$$\overline{\psi}(x) = \overline{\psi}^{out}(x) + g\int \overline{S}_-(x - x_1''')\,\overline{\psi}(x_1')\,\phi_{mn_0}(\varsigma_1)\,F(\xi_1)\,d\xi_1$$

except that here $\phi^{out} = \mathrm{P}\,\phi^{out}$ is deduced from $\mathrm{P}\,\phi = \phi$ and the above equation for ϕ.

Define $G(\xi, \xi_1) = G_-(\xi\xi_1) - G_+(\xi\xi_1)$

for the time being. (investigate later), then

$$G_{mn_0}(\xi, \xi_1) = G_{mn_0-}(\xi\xi_1) - G_{mn_0-}(\xi\xi_1)$$

and we can write

$$\phi^{out}_{mn_0}(\xi) = \phi^{(in)}_{mn_0}(\xi) - g\int G_{mn_0}(\xi\xi_1)\,\bar\Psi(x_1')\,\Psi(x_1'')\,F(\xi_1)\,d\xi_1,$$

$$\Psi^{out}(x) = \Psi^{in}(x) - g\int S(x-x_1')\,\phi_{mn_0}(\xi_1)\,\Psi(x_1''')\,F(\xi_1)\,d\xi_1,$$

$$\bar\Psi^{out}(x) = \bar\Psi^{in}(x) - g\int \bar S(x-x_1''')\,\bar\Psi(x_1')\,\phi_{mn_0}(\xi_1)\,F(\xi_1)\,d\xi_1,$$

so we have a set of equations of exactly the same form as Blocks except that have a mn_0 subscript on all ϕ's & G's (and also ξ has 8 variable instead of 4). Investigate now the properties of G.

Comment: $\left(\varphi^{(k)}_{lq}\right)^{c.} = e^{-ikx}\,\chi_{lq}(kr) = e^{-ikx}\,\chi_{lq}(-k, r) = \varphi^{(-k)}_{lq}$

Thus taking complex conjugate is equivalent to changing the sign on k to $(-k)$. This will only affect the invariant lable $\frac{k_0}{|k_0|}$. Thus if P does not depend on $\frac{k_0}{|k_0|}$ we have $\underline{P^{c.} = P}$.

Non-local Greens Function

$$\varphi_{sqk} = \frac{e^{ikx}}{(2\pi)^2} \prod_i \chi_{n_i}(r_i)$$

$$F\,\varphi_{sqk} = (k^2+\mu_s)\,\varphi_{sq}$$

$$\therefore \quad G_\pm(x-x', r, r') = -\sum\!\!\int \frac{\varphi_{sqk}(x r)\,\varphi_{sqk}(x' r')}{k^2+\mu_s}\,dk$$

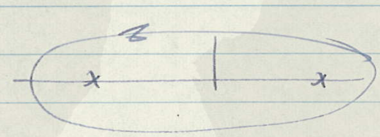
$$G(x-x', r, r') = \frac{-1}{(2\pi)^4} \sum\!\!\int \frac{e^{ik(x-x')}}{k^2+\mu_s}\,dk\;\chi_{sq}(r)\,\chi_{sq}(r')$$

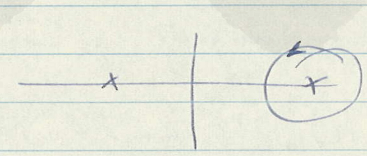
$$= \sum_s \Delta(x-x')\,\Lambda_s(r, r')$$

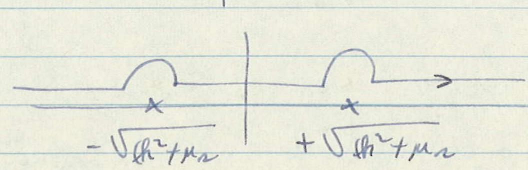
$$\Delta(x-x') = \frac{-1}{(2\pi)^4} \int \frac{e^{ik(x-x')}}{k^2+\mu_s}\,dk$$

$$\Lambda_s(r, r') = \sum_q \chi_{sq}(r)\,\chi_{sq}(r')$$

then

$\Delta(x) =$ 

$\Delta^{(+)}(x) =$ 

$\Delta_+(x) =$ 

= retarded

etc.

Define $G_\pm(x, r, r') = \sum_s \Delta_\pm(x)\, \Lambda_s(r, r')$; etc.

then

$$G_+(x, r, r') = \begin{cases} -G & x_0 < 0 \\ 0 & x_0 > 0 \end{cases}$$

etc. as desired.

Now lets find G in a form suitable for operation on by the P operator. do k_0 integration.

$$\Delta = \frac{i}{(2\pi)^3} \int \left\{ \frac{e^{i\mathbf{k}\cdot\mathbf{x} - ik_0^{(s)} x_0}}{2k_0^{(s)}} - \frac{e^{i\mathbf{k}\cdot\mathbf{x} + ik_0^{(s)} x_0}}{2k_0^{(s)}} \right\} d\mathbf{k}$$

$$k_0^{(s)} = +\sqrt{\mathbf{k}^2 + \mu_s}$$

$$\delta(k^2 + \mu_s) = \frac{1}{2\sqrt{\mathbf{k}^2 + \mu_s}} \left\{ \delta\left(k_0 - \sqrt{\mathbf{k}^2 + \mu_s}\right) + \delta\left(k_0 + \sqrt{\mathbf{k}^2 + \mu_s}\right) \right\}$$

$$\boxed{\begin{gathered} \Delta(x) = \frac{i}{(2\pi)^3} \int \delta(k^2 + \mu_s)\, \varepsilon(k)\, e^{ikx}\, dk \\ \varepsilon(k) = \frac{k_0}{|k_0|} \end{gathered}}$$

Consider $\Lambda_s(r, r')$, how does it transform?

$$\chi_{sq}(r^0) = \prod_i \chi_{n_i}(a_{ij} r_j) = \sum_{q'} C_{q'q}\, \chi_{sq'}(r)$$

since s is an invariant. $C_{q'q}$ is an orthogonal matrix

$$\sum_q C_{q'q}\, C_{q''q} = \delta_{q'q''}$$

$$\sum_q C_{qq'}\, C_{qq''} = \delta_{q'q''}$$

thus

$$\sum_q \chi_{sq}(kr)\,\chi_{sq}(kr') = \sum_q \prod_i \chi_{n_i}(a_{ij}^{(k)} r_j) \prod_{i'} \chi_{n_{i'}}(a_{i'j'}^{(k)} r_{j'}') =$$

$$= \sum_{qq'q''} C_{q'q}^{(k)}\, C_{q''q}^{(k)}\, \chi_{sq'}(r)\,\chi_{sq''}(r') =$$

$$= \sum_{q'} \chi_{sq'}(r)\,\chi_{sq'}(r') = \Lambda_s(r, r')$$

$$\therefore\ \Lambda_s(r,r') = \sum_q \chi_{sq}(kr)\,\chi_{sq}(kr') =$$

$=$ function independent of k. $\therefore$

$$G(x,r,r') = \sum_s \frac{i}{(2\pi)^3} \int \delta(k^2+\mu_s)\,\varepsilon(k)\, e^{ikx}\, \Lambda_s(r,r')\, dk$$

$$= \frac{i}{(2\pi)^3} \int \sum \delta(k^2+\mu_s)\,\varepsilon(k)\, e^{ikx}\, \chi_{sq}(kr)\,\chi_{sq}(kr')\, dk$$

and we have $G(x,r,r')$ in the desired form.

$$\theta(x) = \begin{cases} 0 & x_0 > 0 \\ 1 & x_0 < 0 \end{cases}$$

$$G_{mn_0+} = -\theta(x)\, G_{mn_0} = \begin{cases} -G_{mn_0} & x_0 < 0 \\ 0 & x_0 > 0 \end{cases}$$

etc.

Find $G^{(+)}$ and $G^{(-)}$

$$G(x, r, r') = \frac{i}{(2\pi)^3} \sum \int dk \left[\frac{e^{ik\cdot x - ik_0^{(n)} x_0}}{2k_0^{(n)}} \chi_n(kr)\chi_n(kr') - \frac{e^{ik\cdot x + ik_0^{(n)} x_0}}{2k_0^{(n)}} \chi_n(k, -k_0^{(n)}, r)\chi_n(k, -k_0^{(n)}, r') \right]$$

$$k_0^{(n)} = +\sqrt{k^2 + \mu_n}$$

$$= G^{(+)}(x, r, r') + G^{(-)}(x, r, r')$$

$$G^{(+)}(x, r, r') = \frac{i}{2(2\pi)^3} \sum \int \frac{e^{ikx}}{k_0^{(n)}} \chi_n(kr)\chi_n(kr')\, dk$$

Change the variable of integration from k to $-k$ in $G^{(-)}$ and use $\chi_n(-k, r) = \chi_n(k, r)$. This gives.

$$G^{(-)}(x, r, r') = \frac{-i}{2(2\pi)^3} \sum \int \frac{e^{-ikx}}{k_0^{(n)}} \chi_n(kr)\chi_n(kr')\, dk$$

Commutation Relation

$$\phi^{free}(xr) = \sqrt{\frac{\hbar c}{2(2\pi)^3}} \sum \int \frac{dk}{\sqrt{k_0^{(n)}}} \left(a_{kn} e^{ikx} + a_{kn}^* e^{-ikx} \right) \chi_n(kr)$$

$$[a_{kn}, a^*_{k'n'}] = \delta(k - k')\,\delta_{nn'}$$

$$[\phi^{(f)}(x, r), \phi^{f}(x'r')] = \frac{\hbar c}{2(2\pi)^3} \sum_{nn'} \int \frac{dk\, dk'}{\sqrt{k_0^{(n)} k_0'^{(n')}}} [a_{kn}, a^*_{k'n'}]\, e^{ikx - ik'x'} \chi_n(kr)\chi_{n'}(k'r')$$

$$+ \frac{\hbar c}{2(2\pi)^3} \sum_{nn'} \int \frac{dk\, dk'}{\sqrt{k_0^{(n)} k_0'^{(n')}}} [a^*_{kn}, a_{k'n'}]\, e^{-ikx + ik'x'} \chi_n(kr)\chi_{n'}(k'r')$$

$$= \frac{\hbar c}{2(2\pi)^3} \int\!\!\sum \frac{dk}{k_0^{(a)}} e^{ik(x-x')} \chi_n(kr)\chi_n(kr') - \frac{\hbar c}{2(2\pi)^3} \int\!\!\sum \frac{dk}{k_0^{(a)}} e^{-ik(x-x')} \chi_n(kr)\chi_n(kr')$$

$$= -i\hbar c\left[G^{(+)}(x-x', r, r') + G^{(-)}(x-x', r, r') \right]$$

$$\left[\phi^b(x,r), \phi^b(x',r')\right] = -i\hbar c\, G(x-x', r, r')$$

$$\left[P_{xr}\phi^b(x,r), P_{x'r'}\phi^b(x'r')\right] = P_{xr}P_{x'r'}\left[\phi^b(x,r), \phi^b(x'r')\right]$$

$$= P_{xr}P_{x'r'}\left(-i\hbar c\, G(x-x', r, r')\right)$$

$$\therefore \left[\phi^b_{mm_0}(x,r), \phi^b_{mm_0}(x',r')\right] = -i\hbar c\, G_{mm_0}(x-x', r, r')$$

as desired. . . . . .

$$G_{mm_0}(x-x', r, r') = \frac{i}{(2\pi)^3}\int \delta(k^2+\mu_{m-m_0})\,\varepsilon(k)\, e^{ik(x-x')} \sum_q \chi_{mm_0q}(kr)\chi_{mm_0q}(kr')$$

since

$$\sum_q \chi_{mm_0q}(k^0r^0)\chi_{mm_0q}(k^0r^{0\prime}) =$$

$$= \sum_q \chi_{mm_0q}(kr)\chi_{mm_0q}(kr')$$

where $k_i^0 = a_{ij}k_j$, etc., a_{ij} = Lorentz transform;

then the above $G_{mm_0}(x-x', r, r')$ is form invariant

as desired.