W. D. Schulz

Projection operator

$$\varphi(X, r) = e^{ikX} \chi_k(\cancel{\rho} r)$$

$$\chi_k(r) = \chi_{n_1}(\rho_1)\chi_{n_2}(\rho_2)\chi_{n_3}(\rho_3)\chi_{n_0}(\rho_0)$$

$$\rho_\mu = \sum_\nu a_{\mu\nu}^{(k)} r_\nu$$

$a_{\mu\nu}(k)$ is coefficients of Lorentz transformation from the system in which k is k to the ~~sy~~ rest system in which $k \equiv (0, 0, 0, \pm\kappa)$ where $\kappa = \sqrt{k_\mu k_\mu}$

The important thing is that the rest system is defined uniquelly except for the spacial rotation.

(Only exception is $k_\mu k_\mu = 0$)

Thus,

$$\varphi(X, r) \to \varphi(X', r') = e^{ik'X'} \chi_{k'}(r')$$

$$\chi_{k'}(r') = \sum_{n_1' \cdots n_0'} \chi_{n_1'}(\rho_1')\chi_{n_2}(\rho_2')\chi_{n_3}(\rho_3') \times \chi_{n_0}(\rho_0')$$

where

$$n_1' + n_2' + n_3' + n_0' \leq n_1 + n_2 + n_3 + n_0$$

and since $n_1 + n_2 + n_3 - n_0$ is invariant, the highest order of $n = n_1 + n_2 + n_3$ and

n_0 must be invariant.

Thus, one can define an invariant projection operator ~~which is~~

$$P_{N,N_0}$$

such that it is a projection to a subspace of φ with

$$n \leq N, \quad n_0 \leq N_0 .$$

The most crucial question is whether it is possible to restrict the interaction ~~all the way through~~ such that ~~the~~ φ is inside the subspace all the way through.
Also is it possible that k in φ ~~can~~ ~~be~~ restricted to time-like vectors?
The first question ~~could~~ be answered favorably, perhaps, because, if the interaction contains $P_{N,N_0}\varphi$, ~~there would~~ the subspace outside will ~~have~~ no chance to interfere with the subspace inside.
As for the second question, one can ~~also~~ introduce ~~the~~ another projection operator P_{κ_0} such that the condition $k_\mu k_\mu < -\kappa_0^2$ is not violated.

Sch 2.

One feature of the theory, which makes us feel uneasy, is that the particles with 0-mass such as photons and possibly neutrinos could not be fitted into the picture easily. Only possibility, which is conceivable so far, is that these zero mass particles are to be described by ordinary local fields ~~as~~ just as before.

Projection Operators

W. D. Schulz, March

$$\varphi(X, r') = \sum_{n_1 n_2 n_3 n_0} C_{n_1 n_2 n_3 n_0}\, e^{ik'_\mu X'_\mu} H_{n_1}(r'_1) H_{n_2}(r'_2) H_{n_3}(r'_3)$$
$$\times H_{n_0}(r'_0) \exp\{-\tfrac{1}{2}(r'^2_1 + r'^2_2 + r'^2_3 + r'^2_0)\}$$

$k'_1 = k'_2 = k'_3 = 0 \qquad k'_0 = \kappa.$

$$\rightarrow \varphi(X, r) = \sum C_{n_1 n_2 n_3 n_0}\, e^{ik_\mu X_\mu} H_{n_1}(r'_1) H_{n_2}(r'_2) H_{n_3}(r'_3)$$
$$\times H_{n_0}(r'_0) \exp\{-\tfrac{1}{2}(r_\mu r_\mu + 2\frac{(k_\mu r_\mu)^2}{k_\mu k_\mu}\}$$

$\cancel{\Rightarrow} \sum$

Projection operator

$$P\varphi = \sum_{\substack{n_1, n_2, n_3 \\ \cancel{n_0}}} \cancel{C_{n_1 n_2 n_3 n_0}}\, C_{n_1 n_2 n_3 0}\, e^{ik_\mu X_\mu}$$
$$\times H_{n_1}(r'_1) H_{n_2}(r'_2) H_{n_3}(r'_3) \cancel{H_{n_0}(r'_0)}$$
$$\times \exp\{ \qquad , \qquad , \qquad \}$$

~~For arbitrary value of k_μ.~~

An arbitrary function of X, r can be expanded into Fourier series or integral first with respect to X and then each Fourier coef. or Fourier transform $\cancel{A}(k, r)$ can further be expanded into series of complete set of orthogonal functions

$$H_{n_1}(r'_1) H_{n_2}(r'_2) H_{n_3}(r'_3) H_{n_0}(r'_0)$$
$$\cancel{\exp\{-\tfrac{1}{2}(r_\mu r_\mu + 2} \exp(-\tfrac{1}{2}(r'^2_1 + \cdots + r'^2_0)$$

where $(r_1' r_2' r_3' r_0')$ is the Lorentz transform of $(r_1\ r_2\ r_3\ r_0)$ ~~and in the~~ in such a way that new coordinate system,

$k_1' = k_2' = k_3' = 0$ for a space-like k_μ

and $k_0' = 0$ for a time-like k_μ.

(For a k_μ with $k_\mu k_\mu = 0$ on the light cone, this procedure fails.)

Then, the projection operator is an invariant operator:

The interaction term for Lagrangian density

$$L_{int} = g\, P\varphi(X, r) \sum_\alpha \bar{\psi}_\alpha\left(X+\frac{r}{2}\right)\psi_\alpha\left(X-\frac{r}{2}\right).$$

$$\iint L_{int}(X, r)\, dX\, dr$$

$$= g \int \sum_{n_1 n_2 n_3} C_{n_1 n_2 n_3 n_0}\, e^{ik_\mu X_\mu}\, dX \int H_{n_1}(r_1') H_{n_2}(r_2') H_{n_3}(r_3') \exp(\ ) \times \sum_\alpha \bar{\psi}_\alpha\left(X+\frac{r}{2}\right)\psi_\alpha\left(X-\frac{r}{2}\right) dr$$

$$= g \int \sum_{n_1 \cdots n_0} C_{n_1 n_2 n_3 n_0}\, e^{ik_\mu X_\mu}\, \underbrace{(H_{n_1} \cdots H_{n_0} \exp)}_{P(k_\mu)} \cdot \sum_\alpha \bar{\psi}_\alpha\left(X+\frac{r}{2}\right)\psi_\alpha\left(X-\frac{r}{2}\right) dX\, dr$$

$$= g \int \varphi(X, r) \cdot P \sum \bar{\psi}_\alpha\left(X+\frac{r}{2}\right)\psi_\alpha\left(X-\frac{r}{2}\right) \times dX\, dr$$

P.O.2.

Projection operator which is independent of k_μ.

Spherical-Hyperbolic coordinates.

Restriction to invariant functions?

~~Non~~ Quadratic integrability?

Katz

Born, R.M.P.

k_μ: time-like

$$H_{n_1}(r_1^{(k)}) H_{n_2}(r_2^{(k)}) H_{n_3}(r_3^{(k)}) H_{n_0}(r_0^{(k)}) \exp \left\{ -\frac{1}{2}\left(r_\mu r_\mu - \frac{2(k_\mu r_\mu)^2}{k_\mu k_\mu} \right) \right\}$$

$k_1^{(k)} = k_2^{(k)} = k_3^{(k)} = 0, \quad k_0^{(k)} = \pm \kappa, \quad k_\mu k_\mu = -\kappa^2,$

$$= -\frac{1}{2}\left(r_1^{(k)2} + r_2^{(k)2} + r_3^{(k)2} + r_0^{(k)2} \right)$$

invariant

$n_1 + n_2 + n_3 - n_0$: invariant

$n_1 + n_2 + n_3 + n_0$: invariant

(being the highest degree of the polynomial, which is invariant under linear transformation)

$n_1 + n_2 + n_3$: invariant

n_0 : invariant

P: Invariant projection operator.

k_μ: space-like

$$H_{n_1}(r_1)\,H_{n_2}(r_2)\,H_{n_3}(r_3)\,H_{n_0}(r_0)$$

$$\exp -\frac{1}{2}\left(-r_\mu r_\mu + \frac{2(k_\mu r_\mu)^2}{k_\mu k_\mu}\right)$$

$$\parallel$$

$$(\quad) = -\frac{1}{2}\left(r_1^{(k)^2} + r_2^{(k)^2} + r_3^{(k)^2} + r_0^{(k)^2}\right)$$

$$k_1^{(k)} = \pm K, \qquad k_2^{(k)} = k_3^{(k)} = k_0$$

$$k_1^{(k)} = k_2^{(k)} = k_3^{(k)} = \frac{K}{\sqrt{3}} \qquad (x \pm \quad)$$

$$(x+y+z)^2$$