An Attempt at a Unified Theory of Elementary Particles

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Abstract

The introduction of non-local interaction between local fields is ~~a~~ likely to be an important step toward the construction of a consistent field theory free from divergence difficulties. However, a further step in the same direction seems to be necessary in order to approach nearer to a unified theory of elementary particles. The concept of non-local field is introduced for this purpose. A non-local field describes relativistically a system which is elementary in the sense that it could no longer be decomposed into more elementary constituents, but was so substantial that it contains implicitly a great variety of particles ~~s~~ with different masses, spins and other intrinsic properties.(1) For instance, a non-local scalar field is defined as a scalar function depending on two sets of space-time parameters and can be written as

$$(x'_\mu|\varphi|x''_\mu) \equiv \varphi(X_\mu, r_\mu)$$

where

$$X_\mu = \frac{x'_\mu + x''_\mu}{2}, \qquad r_\mu = x'_\mu - x''_\mu$$

The most general equation for the free field is of the form

$$F\left(\frac{\partial}{\partial X_\mu}, r_\mu, \frac{\partial}{\partial r_\mu}\right)\varphi(X_\mu, r_\mu) = 0 \qquad (1)$$

where the operator F is a certain function of $\frac{\partial}{\partial X_\mu}$, r_μ and $\frac{\partial}{\partial r_\mu}$ which is invariant under any inhomogeneous Lorentz transformation. If we assume

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that F is linear in $\frac{\partial^2}{\partial X_\mu \partial X_\mu}$ and separable, i. e.

$$F \equiv -\frac{\partial^2}{\partial X_\mu \partial X_\mu} + F^{(r)}\left(r_\mu r_\mu, \frac{\partial^2}{\partial r_\mu \partial r_\mu}, r_\mu \frac{\partial}{\partial r_\mu}\right), \tag{2}$$

then we have eigen-solutions of the form $\varphi \equiv u(X)\chi(r)$, where u and χ satisfy

$$\left(\frac{\partial^2}{\partial X_\mu \partial X_\mu} - \mu\right)u(X) = (F^{(r)} - \mu)\chi(r) = 0, \tag{3}$$

μ being the separation constant. Thus, the masses of the free particles, which are associated with the non-local field φ, are given ~~by~~ as the eigenvalues of the square root of the operator $F^{(r)}$ which characterizes the so-to-speak internal structure of the elementary non-local system. One can choose the operator $F^{(r)}$ such that the eigenvalues $\sqrt{\mu_n} \equiv m_n$ are all positive and discrete. In that case, one can expand an arbitrary non-local field into series of the corresponding internal eigenfunctions $\chi_n(r)$:

$$\varphi(X, r) = \sum_n u_n(X)\chi_n(r) \tag{4}$$

Now, when a non-local scalar field φ interacts with a local spinor field $\psi(x_\mu)$, for instance, one can reduce the problem to that of the interaction between the spinor field ψ and the infinitely many local Boson fields, which are defined respectively by $u_1(x_\mu), u_2(x_\mu), \cdots$. The field equations become

$$\left(\frac{\partial^2}{\partial x''_\mu \partial x''_\mu} - m_n^2\right)u_n(x''_\mu) = \int \Phi_n(x', x'', x''')\bar{\psi}(x')\psi(x''')dx'dx''' \tag{5}$$

$$\left(\gamma_\mu \frac{\partial}{\partial x'_\mu} + M\right)\psi(x') = -\sum_n \int \Phi_n(x', x'', x''')u_n(x'')\psi(x''')dx''dx''' \tag{6}$$

where

$$\Phi_n(x', x'', x''') = g\,\tilde{\chi}_n(x' - x''')\,\delta\!\left(\frac{x' + x'''}{2} - x''\right) \tag{7}$$

and M is the mass of the spinor particle. If we compare these equations with the field equations in the case of non-local interaction between local fields, we notice that the internal eigenfunction $\chi_n(r)$ characterizes the form function for the particle with the mass m_n. The essential difference between the theory of non-local field and that of non-local interaction is that, in the former case, we have to take into account simultaneously all the particles with different masses $m_1, m_2, \ldots$ ~~m_n~~ which are derived from an eigenvalue problem. Furthermore, the form function for each of these particles is uniquely determined ~~by~~ by the same eigenvalue problem. In the above example, in which we started from a non-local scalar or pseudoscalar field, all these particles have integer spins. On the contrary, particles with half integer spins would be obtained, if we would have started from a non-local spinor field.

In this connection, ~~Watanabe suggest~~ it is to be remarked that Watanabe suggested recently a possible relation between the non-local Boson field and ~~the~~ de Broglie's fusion theory.[2] Namely, a pair of spinor particle and anti-particle could be regarded as a non-local field which describes a ~~v~~ great variety of Bose particles.

In any case, the choice of the invariant operator F ~~is~~ remains to be arbitrary, ~~except for the requirement of invariance,~~ ~~with unless~~ until ~~we~~ a new principle for its determination ~~is~~ would be revealed. At the present stage, we should be satisfied with considering ~~the illustrations~~ simple examples

~~which which may serve for~~ in order to ~~see the~~ understand the general situation which we may face in non-local field theory. Thus, for the sake of illustration, ~~a~~ one can assume a very simple ~~for~~ form for F:

$$F \equiv -\frac{\partial^2}{\partial X_\mu \partial X_\mu} + \frac{\lambda^2}{2}\left(-\frac{\partial^2}{\partial r_\mu \partial r_\mu} + \frac{1}{\lambda^4} r_\mu r_\mu\right)^2 \qquad (8)$$

where λ is a small constant with the dimension of length. One may call this the four dimensional oscillator model for the structure of elementary particles which was considered first by Born[a] in connection with his idea of self-reciprocity[3]. One can easily see that the mass spectrum in this case is discrete and is given by

$$m_{n_1, n_2, n_3, n_0} = \frac{\sqrt{2}}{\lambda} |n_1 + n_2 + n_3 - n_0 + 1| \qquad (9)$$

where n_1, n_2, n_3, n_0 are zero or positive integers.[a] The main trouble with the four dimensional eigenvalue problem in general is the infinite degeneracy. ~~The~~ If we try to get rid of this difficulty, the theory ~~becomes~~ may well become more complicated.*

There are a number of points which are to be investigated in order to see whether a consistent theory could be constructed if we procede in this direction. One serious limitation of non-local theories is that we have to make use of the ~~weak~~ so far coupling approximation. ~~We are well~~ Although we ~~are well aware of the limita~~ doubt the validity of such an approximation in connection with the problems of mesons and nuclear forces, we cannot depart from it easily, simply because we ~~do~~ do not have as yet any thoroughly relativistic formulation of field theory which is free from the assumption of weak coupling.

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* In connection with the problem of the decomposition of a non-local field into irreducible part, ~~the~~ a rotator model for the structure of elementary particles was suggested by Hara.[4] A rigid sphere model as suggested by Nakano[5] indicates another possibility. A modification of the problem of the self-energy in the ordinary local field theory as proposed by Enatsu[6] could ~~be~~ also be regarded as a different way of approach to the problem of determination of the structure of elementary particles

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(1) H. Yukawa, Phys. Rev. 91 (1953), in press.
~~(2) O. Hara and T. Marumori~~
(2) S. Watanabe, Phys. Rev. 91 (1953) in press.
(3) M. Born, Rev. Mod. Phys. 21 (1949), 463.
(4) O. Hara and T. Marumori, Prog. Theor. Phys. 9 (1953), 559.