Manuscript of a lecture to be delivered at the Third Meeting of Nobel Prize Winners in Lindau, Bodensee, on July 2, 1953

Attempts at a Unified Theory of Elementary Particles

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Introduction

Modern atomic theory has been trying to draw as complete a picture of the material world as possible in terms of as few elementary constituents as possible. It seemed that we came closer to the goal in 1932 than ever before, when the neutron was discovered. The electrons, protons and neutrons turned out to be the only constituents of ordinary substances, while the photons were associated with the electromagnetic field. The positron was discovered in the same year, but this was welcome as the confirmation of the already successful theory of electron by Dirac. On the other hand, however, it was clear even at that time that the picture was not yet complete. There were two outstanding problems: the beta-decay and the nuclear forces. The success of Fermi's theory of beta-decay lead us to ~~t~~ accept the existence of the neutrino which had been postulated by Pauli. A relativistic field theory of nuclear forces lead us further to another new elementary particle. The duality of field and particle seemed to presuppose the existence of the mesons which were to be associated with the nuclear force field. One type of mesons, i. e. the mu-meson, ~~which~~ was discovered by Anderson and Neddermeyer in 1937, but later turned out to have very little, if any, to do with the nuclear forces. Instead, the pi-meson which was discovered by Powell in 1947 is the one that is responsible for a part, at least, of nuclear forces.

This appeared already to be a little too complicated to be accepted as something final, but ~~was~~ turned out later to be merely the beginning of further complications, as discussed by Prof. Powell the day before yesterday, Since 1947, a great variety

Prof. Powell ~~ascribed the~~ as a matter of fact responsibility to me, but ~~as~~ I merely needed one type of mesons, whereas Prof. Powell discovered a great number of extra particles which I did not want.

of unstable particles were discovered in cosmic rays one after another. Some of them were created artificially by high energy acceralators. It seems that more and more new particles are disclosed as we go further and further in search for the high energy region. at any rate It seems that we are in an open world in the sense that a ~~likely to be~~ small number of elementary particles which have been familiar to us are not the sole elementary constituents of our world, but are ~~more likely to be~~ merely the more stable members of a large family of elementary particles. Of course, there is still room for the argument that most of the newly discovered unstable particles are not elementary, but are compound systems which consist of two or more elementary particles in the true sense. However, irrespective of whether ~~even if~~ we took such a conservative view of the present situation of the theory of elementary particles, or a more radical view of the accepting a great ~~number~~ variety of ~~elementary~~ particles as ~~such~~, as elementary systems, we could not help asking ourselves the question; "What is the elementary particle?"

Reexamination of the Concept of Elementary Particle

At first ~~first~~ sight, there is ~~no~~ little difficulty in defining the elementary particle precisely in mathematical terms. In relativistic quantum mechanics, which was established by 1930 chiefly by Dirac, Heisenberg and Pauli, the duality of wave and corpuscle is best represented by the concept of quantized field. It is the totality of infinitely many operators $\psi_\alpha(x_\mu)$, where x_μ is a set of space-time parameters and α is an index discriminating the components of a quantity such as a vector or a spinor, which transforms linearly under Lorentz transformations. Let us call it a local field in order to distinguish it from a non-local field which will be discussed later on. Now, an elementary particle could be defined as that which is associated with an irreducible local field. A field $\psi_\alpha(x_\mu)$ is said to be irreducible, if it can no longer be decomposed into parts, each of which transforms linearly by itself under Lorentz transformations. In this way,

In classical physics, a field is defined as a function or functions $\psi_\alpha(x, y, z, t)$, or the totality of infinitely many quantities $\psi_\alpha(x_\mu)$ .... In q.m. it is replaced by the

the spin of the elementary particle is defined: For instance, the scalar or pseudoscalar field with only one component is associated with the particles with spin zero, whereas the spinor field is associated with those with spin 1/2. The commutation relations between the quantized field ~~quantities~~ operators determin the statistics of the corresponding assembly of particles. One of the most attractive features of quantum theory of fields was that it ~~####~~ enabled us to deduce the wellknown relation between spin and statistics: The particles with zero or integer spin obey Bose-Einstein statistics, while those with half integer spin obey Fermi-Dirac statistics. We take it for granted, furthermore, that each type of elementary particle has its unique mass m. The difficulty of the present field theories arises in this connection. Suppose that the free field satisfies the second order wave equation

$$\left(\frac{\partial^2}{\partial x_\mu \partial x_\mu} - \kappa^2\right)\psi_\alpha(x_\mu) = 0 \qquad (1)$$

as usual, where κ is a constant with the dimension of reciprocal length. Then, of course, the mass of the associated particles is uniquely defined by $m = \frac{\kappa\hbar}{c}$ as long as the particles are completely free. However, in the present field theories, one can find no apriori reason for choosing one value or another for the constant κ or m. Therefore, what one does is just to equate m with the observed mass of the particle in question. However, this again is objectionable, simply because the particle in question is observable for the very reason that it is not free, but interacts with other particles.

Thus, the problem of mass of an elementary particle cannot be separated from the problem of interaction between quantized fields. In the usual local field theory, we assume the local interaction between local fields. For instance, the effect of another field $\varphi_\beta(x_\mu)$ on the field $\psi_\alpha(x_\mu)$ could be described by adding certain terms ~~ø/~~ to the left hand side of (1), which are functions of and φ_β, ψ_α at the same space-time point x_μ. If we introduce such an interaction, the mass of the particle which is associated with the field $\psi_\alpha(x_\mu)$ is altered by an amount which is c^{-2} times the self-energy due to the interaction. Unfortunately, the self-energies of particles turned out to be infinite or, at least, indefinite in the known simple cases of local field theories with local interaction. This difficulty was known already ~~known~~ in 1930, when quantum electrodynamics was established by Heisenberg and Pauli. As a matter of fact, a part, at least, of this pathological character of quantum theory of fields was inherited from its predecesser, i. e. classical electrodynamics. One must admit that the precise definition of the mass of an elementary particle is impossible, unless one may be able to get rid of the infinite self-energy somehow.

Mixed Field Theories

The socalled mixed field theory, which was proposed by Pais and Sakata, is of great interest in this connection. Let us take the familiar case of the electron interacting with the electromagnetic field. As I said, the self-energy of the electron due to the electromagnetic field produced by the electron itself becomes infinite. However, if we assume further that the electron interacts at the same time with another field of appropriate kind in an appropriate manner, we may hope that the self-energy due to the latter interaction just counterbalances the electromagnetic self-energy of the electron so that the resultant self-energy becomes finite. This is actually the case, if we choose as the second field a scalar

In classical electrodynamics, the energy of the electromagnetic field around a point ~~charged~~ charge was known to be infinite, so that the mass of the point electron ~~must~~ was to be infinite.

field, with which neutral spin zero particles with the rest mass of the order of meson masses are associated and which interact with the electron as strongly as the electromagnetic field. Moreover, if we extend the same idea to the case of the proton, we obtain the correct sign and correct order of magnitude for the difference of masses of the proton and the neutron. This seemed to give rise to a new hope of constructing a consistent field theory which was free from the pathological divergence difficulties, by assuming the coexistence of a number of fields, known and unknown, in such a way that the self energies of all those particles which were associated with these fields became finite on account of mutual compensation. Such an attempt was successful to some extent, but there is little hope in arriving at the complete removal of all divergences as long as we stick to the local field theories with local interactions. Namely, the divergence which is related to the socalled vacuum polarization in quantum electrodynamics cannot be removed by the assumption of coexistence of various charged particles with different spins. In spite of this, however, the idea of mutual compensation is significant in indicating that the coexistence of various fields and particles is not accidental, but one may be able to find cogent reason for it.

It is to be mentioned in passing that recent development in quantum electrodynamics by Tomonaga and Schwinger was remarkable in ~~reducing~~ reproducing all experimental results so far known unambiguously and with great accuracy, but this was possible only after replacing the theoretically infinite masses and electric charge by the observed finite masses and charge. Complete justification for this renormalization cannot be found in the theoretical framework itself.

Local Fields with Non-Local Interaction

In connection with the procedure of renormalization, various types of local interaction between local fields can be divided into two classes. The first class includes all interactions which are renormalizable. The familiar interaction between the charged particle with spin 1/2 and the electromagnetic field is called renormalizable because the renormalization of the masses of the charged particle and photon and of the electric charge is sufficient for deriving finite results for all other observable quantities. The scalar or pseudoscalar interaction between the scalar or pseudoscalar meson field and the nucleon,~~,~~ which is familiar in meson theory of nuclear forces, is also renormalizable. There are a few other interactions which belong to the first class. However, most of other interactions such as those between electric and magnetic dipoles ~~[illegible] and the electric field~~ and the electromagnetic field or the pseudovector interaction between the pseudoscalar meson field and the nucleon belong to the second class, because the divergences appearing in these cases cannot be removed by applying the renormalization procedure finite number of times.(1)

In this connection, one may raise the question: Is it possible to describe atomic and nuclear phenomena in terms of renormalizable interactions alone ? The answer is very likely to be negative. The interaction between the electron-neutrino field and the nucleon field in Fermi's theory of beta-decay is known to be a linear combination of five types of interactions, in general. Among them, the tensor interaction which is not renormalizable is indispensable for accounting for a number of experimental results. Even if we accept the view that the beta-decay is not an elementary process, but can be decomposed into two stages in which creation and annihilation of a virtual meson of an unknown kind take place, still we need unrenormalizable interactions. Now, if the interaction between fields is not renormalizable in the ordinary sense, it amounts to the same thing to say that the procedure of renormalization necessitates the introduction of higher and higher derivatives of field quantities in the interaction without end. An interaction which involves derivatives of arbitrary order of field quantities is equivalent to a non-local interaction, i. e. an interaction which refers to two or more field quantities at different space-time points. In other words, the introduction of a non-local interaction in field theories can be regarded as a revival of the theory of action at distance which was thought to be contradictory to the notion of field itself in classical physics. However, in quantum theory of fields, this may not be so, because ~~the notion of~~ the quantized field ~~seems to be more flexible,~~ contains already (2) field and particle ~~being~~ as two aspects of the same physical object.

Let us consider, for example, the case of a non-local interaction between the scalar (or pseudoscalar) meson field ~~$\varphi(x_\mu)$~~ $u(x_\mu)$ and the nucleon field $\psi_\alpha(x_\mu)$. The field equations can be written, in general, in the form

$$\left(\frac{\partial^2}{\partial x''_\mu \partial x''_\mu} - \kappa^2\right) u(x'') = \int \sum_{\alpha,\beta} \bar{\psi}_\alpha(x')\, \Phi_{\alpha\beta}(x', x'', x''')\, \psi_\beta(x''')\, dx' dx''' \qquad (2)$$

$$\sum_\beta \left(\gamma_\mu \frac{\partial}{\partial x'_\mu} + M\right)_{\alpha\beta} \psi_\beta(x') = -\int \sum_\beta \Phi_{\alpha\beta}(x', x'', x''')\, u(x'')\, \psi_\beta(x''')\, dx'' dx''' \qquad (3)$$

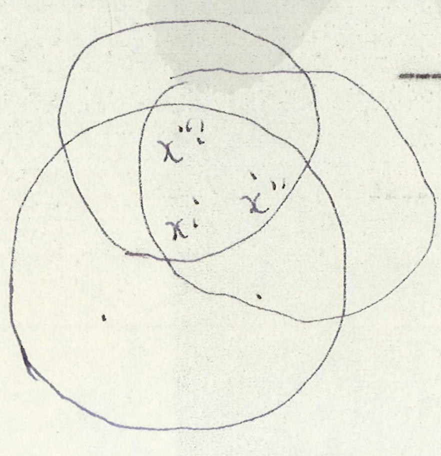
where $\varkappa$, M are masses of the meson and nucleon in units of $\hbar/c$ and γ_μ are Dirac matrices. $\Phi_{\alpha\beta}(x', x'', x''')$ is a matrix with four rows and columns, each matrix element being a function of three space-time points x', x'', x'''. The most general non-local interaction as characterized by arbitrary three-point functions $\Phi_{\alpha\beta}(x', x'', x''')$ reduces to familiar local scalar coulpling, if $u(x_\mu)$ is a scalar field and

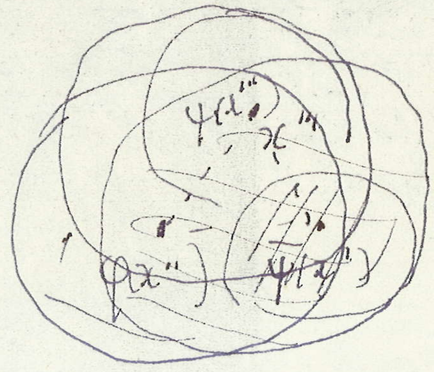
$$\Phi_{\alpha\beta}(x', x'', x''') = g\,\delta_{\alpha\beta}\,\delta(x'-x'')\,\delta(x''-x''')$$

where g is the coupling constant. Similarly, it reduces to local pseudoscalar coupling, if $u(x_\mu)$ is a pseudoscalar field and

$$\Phi_{\alpha\beta}(x', x'', x''') = g(\gamma_5)_{\alpha\beta}\,\delta(x'-x'')\,\delta(x''-x''')$$

The quantization of the fields can be carried out as usual. However, an essential departure from the local interaction theory is inevitable on account of the absence of Schrödinger equation as such for the whole system in non-local interaction theory. The role of Schrödinger equation was to determine uniquely the Schrödinger function or the probability amplitude at any time instant t in terms of the function at the immediate past instant $t-dt$. This was so in the usual field theory, because the Hamiltonian H(t) for the whole system depended only on the field quantities at the instant t. If we once introduce a non-local interaction in a relativistically invariant manner, we can no longer have the Hamiltonian which satisfies the above requirement. We really do not know what would be the substitute for the Schrödinger equation, much less about any final formulation of non-local theories.





We know, however, that there is a formulation of ordinary field theory which seems to be suited for the extension to non-local theories. Namely, one can define <u>S-matrix</u>, which characterizes the statistical relation between the possible results of experiments at a remote future ($t \to +\infty$) and the given results of experiments at a remote past ($t \to -\infty$), in terms of Schrödinger functions at $t = +\infty$ and $t = -\infty$ in quantum mechanics. Heisenberg pointed out that the S-matrix ~~may~~ might well remain to be significant in future theories of elementary particles, while the Schrödinger function itself ~~may~~ might be removed from the picture. In fact, the field equations in non-local interaction theory (2) and (3) can be integrated directly by using the same method of successive approximation as in ordinary field theory, which enables us to construct S-matrix as a series in powers of the coupling constant. The trouble with local field theories with local interactions was that each term of power series for S-matrix was infinite because of infinite self-energies and some other infinite quantities. Recently, Møller and Kristensen have shown that, if we chose the <u>form function</u> $\Phi_{\alpha\beta}(x', x'', x''')$ in non-local interaction theory suitably, the self-energies of both the meson and nucleon become finite, at least, in the first approximation. In other words, the masses of these particles could be renormalized without getting into trouble of divergence. This gives us a new impetus to proceed further in this direction.

Non-Local Fields

The introduction of non-local interaction between local fields was the first step toward the solution of the problem of masses of elementary particles. However, another step is to be taken, if we want to approach nearer to a unified theory of elementary particles. The concept of a non-local field was introduced (3) in order to describe relativistically a system which was elementary in the sense that it could no longer be decomposed into more elementary constituents, but was so substantial, nevertheless, as to be able to contain implicitly a great variety of particles with different masses, spins and other intrinsic properties. For instance, a non-local scalar field is defined as a scalar function depending on two sets x'_μ, x''_μ of space-time parameters and can be written as

$$(x'_\mu|\varphi|x''_\mu) \equiv \varphi(X_\mu, r_\mu)$$

where

$$X_\mu = \frac{x'_\mu + x''_\mu}{2}, \qquad r_\mu = x'_\mu - x''_\mu.$$

The free field equation is supposed to have a general form

$$F\left(\frac{\partial}{\partial X_\mu}, r_\mu, \frac{\partial}{\partial r_\mu}\right)\varphi(X_\mu, r_\mu) = 0, \tag{4}$$

where the operator F is a certain function of $\frac{\partial}{\partial X_\mu}$, r_μ and $\frac{\partial}{\partial r_\mu}$ which is invariant under any inhomogeneous Lorentz transformation. In particular, if we assume that F is linear in $\frac{\partial^2}{\partial X_\mu \partial X_\mu}$ and separable, i. e.

$$F \equiv -\frac{\partial^2}{\partial X_\mu \partial X_\mu} + F^{(r)}\left(r_\mu r_\mu, \frac{\partial^2}{\partial r_\mu \partial r_\mu}, r_\mu \frac{\partial}{\partial r_\mu}\right), \tag{5}$$

then we have eigen-solutions of the form $\varphi \equiv u(X)\chi(r)$, where u and χ satisfy

$$\left(\frac{\partial^2}{\partial X_\mu \partial X_\mu} - \mu\right)u(X) = (F^{(r)} - \mu)\chi(r) = 0, \tag{6}$$

μ being the separation constant. Thus, the masses of the free particles, which are associated with the non-local field φ, are given as the eigenvalues of the square root of the operator $F^{(r)}$ which characterizes the so-to-speak internal structure of the elementary non-local system. If one chooses the operator $F^{(r)}$ such that the eigenvalues $\sqrt{\mu_n} \equiv m_n$ are all positive and discrete, one can expand an arbitrary non-local field φ into series of internal eigenfunctions $\chi_n(r)$

$$\varphi(X, r) = \sum_n u_n(X)\chi_n(r) \tag{7}$$

Now, when the non-local scalar field $(x'|\varphi|x'')$ interacts with a local spinor field $\psi_\alpha(x_\mu)$, for instance, the field equations become

$$\left(-\frac{\partial^2}{\partial X_\mu \partial X_\mu} + F^{(r)}\right)\varphi(X, r) = -g\sum_\alpha \bar{\psi}_\alpha\left(X+\frac{r}{2}\right)\psi_\alpha\left(X-\frac{r}{2}\right) \tag{8}$$

$$\left(\gamma_\mu \frac{\partial}{\partial x'_\mu} + M\right)\psi(x') = -g\int \psi(x'')(x''|\varphi|x')\,dx'' \tag{9}$$

We insert (7) in (8), multiply both hand sides by the complex conjugate of $\chi_n(r)$ and integrate over the four dimensional r-space, provided that $\chi_n(r)$ is quadratically integrable and, therefore, is normalized. The result is

$$\left(\frac{\partial^2}{\partial x''_\mu \partial x''_\mu} - m_n^2\right)u_n(x''_\mu) = \int \Phi_n(x', x'', x''')\sum_\alpha \bar{\psi}_\alpha(x')\psi_\alpha(x''')\,dx'\,dx''' \tag{10}$$

where

$$\Phi_n(x', x'', x''') \equiv g\tilde{\chi}_n(x'-x''')\,\delta\left(\frac{x'+x'''}{2} - x''\right) \tag{11}$$

Similarly, we obtain from (7) the equation

$$\left(\gamma_\mu \frac{\partial}{\partial x'_\mu} + M\right)\psi(x') = -\sum_n \int \Phi_n(x', x'', x''')\,u_n(x'')\,\psi(x''')\,dx''\,dx''' \tag{12}$$

If we compare them with the field equation (2) and (3) in the case of non-local interaction between local scalar and spinor fields, we notice that the internal eigenfunction $\chi_n(r)$ characterizes the form function for the particle with the mass m_n. The essential difference between the theory of non-local field and that of non-local interaction is that, in the former case, we have to take into account simultaneously all the particles with different masses m_n which are derived from an eigenvalue problem. Furthermore, the form function for each of these particles is uniquely determined by the same eigenvalue problem.

The above general considerations can be illustrated by assuming a very simple form for F:

$$F \equiv -\frac{\partial^2}{\partial X_\mu \partial X_\mu} + \frac{\lambda^2}{2}\left(-\frac{\partial^2}{\partial r_\mu \partial r_\mu} + \frac{1}{\lambda^4} r_\mu r_\mu\right)^2 \qquad (13)$$

where λ is a small constant with the dimension of length. One may call this the four dimensional oscillator model for the elementary particle which was considered first by Born(4) in connection with his idea of self-reciprocity. However, our model differs from his in that the internal structure of the particle appears explicitly in our case in connection with the non-localizability of the field itself. One can easily see that the mass spectrum for our case is discrete and is given by

$$m_{n_1,n_2,n_3,n_0} = \frac{\sqrt{2}}{\lambda}\,|n_1+n_2+n_3-n_0+1| \qquad (14)$$

where n_1, n_2, n_3, n_0 are zero or positive integers.

The main trouble with the four dimensional eigenvalue problems is the infinite degeneracy. The theory ~~will be necessarily~~ may well become more complicated, if we try to get rid of this difficulty. In any case, what has been discussed in the above is just the beginning of ~~a ///////~~ an attempt, which may happen to lead us to a possible formulation of a unified theory of elementary particles, if we ~~were~~ are very lucky.

In conclusion, it is to be remarked that there are a number of important points which were not discussed at all. One is the validity of the weak coupling approximations in the theories of elementary particles. We are well aware of the limitations of such approximations in connection with the problem of nuclear forces, but we cannot depart from it easily, simply because we do not have yet any thoroughly relativistic formulation of field theory which is free from the assumptions of weak coupling.

charge independence, isotopic spin
other possible selection rules.

Foot-notes

(1) Relations between mixed field theory and non-local interaction theory were discussed by A. Pais and G. Uhlenbeck extensively (Phys. Rev. 79 (1950), 145). As for the classification of local interactions according to their renormalizability or unrenormalizability, refer to S. Sakata, H. Umezawa and S. Kamefuchi, Prog. Theor. Phys. 7 (1952), 377; H. Umezawa, Prog. Theor. Phys. 7 (1952), 551.

(2) Theory of local fields with non-local interactions were discussed by many authors. Most recent of these papers are C. Bloch, Dansk. Videns. Sels. 27 (1952), Nr. 8; P. Kristensen and C. Møller, ibid. Nr. 7.

(3) H. Yukawa, Phys. Rev. 77 (1950), 219; 80 (1950), 1047; 91 (1953), in press.