Universal Timelike [vector] May 1953

C.O.1

Continuous-Invariant Integral Operators

$$I = \int \varphi(x'_\mu) K(x'_\mu, x''_\mu) \varphi(x''_\mu)\, dx'_\mu\, dx''_\mu$$

$$R^2 = r_\mu r_\mu + \frac{2(L_\mu r_\mu)^2}{\Lambda^2}$$ is invariant

with $\Lambda^2 = -L_\mu L_\mu$ and positive definite, if Λ is real or if L_μ is a time-like vector. One can choose L_μ is the total-energy-momentum of vector of the whole system.

$$U = \exp\left\{-\frac{1}{2}\left(r_\mu r_\mu + \frac{2(L_\mu r_\mu)^2}{\Lambda^2}\right)\right\} = e^{-\frac{R^2}{2}}$$

$$U_\mu \equiv \frac{\partial U}{\partial r_\mu} = -\left\{r_\mu + \frac{2(L_\nu r_\nu)}{\Lambda^2} L_\mu\right\}\cdot U$$

$$U_{\mu\mu} \equiv \frac{\partial^2 U}{\partial r_\mu \partial r_\mu} = \left[\left(r_\mu + \frac{2(L_\nu r_\nu)}{\Lambda^2} L_\mu\right)\left(r_\mu + \frac{2(L_\nu r_\nu)}{\Lambda^2} L_\mu\right) - \left(4 + \frac{2 L_\mu L_\mu}{\Lambda^2}\right)\right]\cdot U$$

$$= (R^2 - 2)\cdot U = \left(r_\mu r_\mu - \cancel{R^2}\, 2\right)\cdot U$$

$$\underbrace{R^2 - 2}_{-\frac{2(L_\mu r_\mu)^2}{\Lambda^2}}$$

$$I = \int \varphi(x'_\mu)\, U_{\mu\mu}(x'_\mu - x''_\mu)\, \varphi(x''_\mu)\, dx'_\mu\, dx''_\mu$$

$$\sim \int \left\{\varphi(x') \frac{\partial^2 \varphi(x')}{\partial x'_\mu \partial x'_\mu} + \cdots\right\} dx'_\mu$$

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[N24 220]

$$\prod_\mu \delta(x_\mu) \to U(r_\mu)$$

$$\cancel{\varphi(x'), \psi}$$

$$(\Box - \kappa^2)\bar{\Delta}_\kappa = -U(\cancel{x_\mu})$$

$$\bar{\Delta}_\kappa(x_\mu) = \int \frac{U_{k_\mu} e^{ik_\mu x_\mu}}{k_\mu k_\mu + \kappa^2}(dk_\mu)^4$$

$$\Delta_\kappa(x_\mu) = \int \varepsilon(k_\mu k_\mu)\,\delta(k_\mu k_\mu + \kappa^2)\,U_{k_\mu} e^{ik_\mu x_\mu}(dk_\mu)^4$$

$$\Big[k_\mu k_\mu + \alpha^2\Big(-\frac{\partial^2}{\partial r_\mu \partial r_\mu} + \frac{1}{\lambda^4} r_\mu r_\mu\Big)^2 + \beta^2\lambda^2\Big\{-\Big(L_\mu \frac{\partial}{\partial r_\mu}\Big)^2 + \frac{1}{\lambda^4}(L_\mu r_\mu)^2\Big\}\Big] \times \chi(r_\mu) = 0$$

$$\Big[-\kappa^2 + \frac{4\alpha^2}{\lambda^4}(n_1 + n_2 + n_3 - n_0 + 1)^2 + 2\beta^2 \frac{\Lambda^2}{\cancel{\ }}\Big(n_0 + \frac{1}{2}\Big)\Big]\chi(r_\mu) = 0$$

in the ~~rest~~ c. of m. system (for the whole system)

$$\kappa^2 = \frac{4\alpha^2}{\lambda^4}(n_1 + n_2 + n_3 - n_0 + 1)^2 + 2\beta^2\Lambda^2\Big(n_0 + \frac{1}{2}\Big)$$

C.O.2.

If $2\beta^2\Lambda^2 \ll \frac{4\alpha^2}{\lambda^4}$

$$\kappa^{2} \cong \frac{2\alpha}{\lambda^2}(\nu+1)^2$$

$$\nu = n_1+n_2+n_3-n_0$$

$$= \sqrt{\cancel{\frac{4\alpha^2}{\lambda^4}} + \cancel{\beta^2}\Lambda^2}$$

$$\kappa_0 \cong \cancel{\frac{2\alpha}{\lambda^2}} \quad n_0 = 1$$

$n_1 = n_2 = n_3 = \cancel{0}$

$\kappa_0 = \sqrt{3}\,\beta\Lambda.$

$2\beta^2\Lambda^2 \gg \frac{4\alpha^2}{\lambda^4}$

$$\kappa \cong \sqrt{2}\cdot\beta\Lambda\sqrt{\left(n_0+\frac{1}{2}\right)}$$

lowest state $\kappa_0 \cong \beta\Lambda$

with $\nu = -1$, $n_0 = 0$

lowest ~~state~~ mass:

$n_1+n_2+n_3-n_0+1 = 0$ or $n_0 = 0$

~~$n_0 = 0$, $n_1+n_2+n_3 =$~~

λ : universal length

L_μ : ~~intrinsic~~ Universal time-like vector

(universal mass Λ)

(intrinsic to the system)

Description of the whole system is simpler, if we stick to the rest system (of the centre of ~~gr~~ mass)

$$\left(\frac{\partial^2}{\partial X_\mu \partial X_\mu} - m_n^2\right) u_n(X)$$

$$= g \int \tilde{\chi}_n(r) \sum_\alpha \bar{\psi}_\alpha(X + \tfrac{1}{2}r)\, \psi_\alpha(X - \tfrac{1}{2}r)\, dr$$

$$\gamma_\mu \frac{\partial \psi(x')}{\partial x'_\mu} + M\psi(x')$$

$$= -g \sum_n \int \tilde{\chi}_n(x' - x'')\, u_n\left(\frac{x' + x''}{2}\right) \psi(x'')\, dx''$$

$$\Phi(x', x'', x''') \equiv g\, \tilde{\chi}_n(x' - x'')\, \delta\left(\frac{x' + x''}{2} - x'''\right)$$

$$m_n^2 = \frac{4\alpha^2}{\lambda^4}(\nu - n_0 + 1)^2 + 2\rho^2 \Lambda^2 (n_0 + \tfrac{1}{2})$$

$$\nu = n_1 + n_2 + n_3$$

Møller:

$$\Phi = (2\pi)^{-8} \int G(h, l) \exp i\left\{h\left(\frac{x' + x''}{2} - x'''\right) + l(x' - x'')\right\} dh\, dl$$

$$G(h, l) = \chi_n(l)$$

$$\int \exp -\tfrac{1}{2}\left(l_\mu l_\mu + \frac{2(k_\mu l_\mu)^2}{\Lambda^2}\right)$$

$$\cdot \exp i l_\mu r_\mu \cdot (d l_\mu)^4$$

$$\cancel{= \int \exp l_\mu (l_\mu}$$

$$\left(\frac{\partial^2}{\partial x''_\mu \partial x''_\mu} - m_n^2\right) u(x'')$$

$$= \int \Phi(x', x'', x''') \sum_\alpha \bar{\Psi}_\alpha(x') \Psi_\alpha(x''') dx' dx'''$$

$$\Phi_n(x', x'', x''') \equiv g\, \tilde{\chi}_n(x' - x''')\, \delta\left(\frac{x' + x'''}{2} - x''\right)$$

Kristensen. Møller. p. 22

$$\delta m_{nn'}^2 = -\frac{2g^2}{\pi}(2\pi)^{-2} \int dK\, W_{nn'}(K, -K-p')$$

$$\times \frac{(p'+K)K + M^2}{(p'+K)^2 + M^2}\, \delta(K^2 + M^2)$$

$$W_{nn'}(K, -K-p')$$

$$\Phi_n(x', x'', x''') = \int G_n(l' - l''')$$

$$\times\, e^{i(l'x' + l''x'' + l'''x''')}\, dl' dl'' dl'''$$

$$\cancel{G_n(l' - l''')}$$

$$\int e^{-i(l'x' + l''x'' + l'''x''')} \tilde{\chi}_n(x' - x''')\, \delta\left(\frac{x' + x'''}{2} - x''\right)$$

$$dx' dx'' dx'''$$

$$\left.\begin{aligned} \tfrac{1}{3}(x' + x'' + x''') &\equiv X \\ x' - x''' &= r \\ \frac{x' + x'''}{2} - x'' &= \rho \end{aligned}\right\}^{-1} \quad \begin{vmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 1 & -1 & 0 \\ \frac{1}{2} & \frac{1}{2} & -1 \end{vmatrix} = 1$$

$$\therefore \frac{1}{6}\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & -2 \end{vmatrix} = \frac{1}{6}\begin{vmatrix} 0 & 0 & 3 \\ 1 & -1 & 0 \\ 1 & 1 & -2 \end{vmatrix} = 1.$$

$$\cancel{x'} = \quad 3X + \cancel{4}\rho = \tfrac{3}{\cancel{2}}(x' + x''')$$

$$x' + x''' = \cancel{\tfrac{3}{2}}\,X + \tfrac{2}{3}\rho$$

$$x' - x''' = r$$

$$x' = \cancel{\tfrac{3}{4}}\,X + \frac{\rho}{3} + \frac{r}{2}$$

$$x''' = \cancel{\tfrac{3}{4}}\,X + \frac{\rho}{3} - \frac{r}{2}$$

$$x'' = \cancel{\tfrac{3}{4}}\,X - \rho/3$$

$$l'x' + l''x'' + l'''x''' = \cancel{3}(l' + l'' + l''')X$$

$$\cancel{+}\ (l' + l''' - l'')\rho/3 + (l' - l''')\,r/2$$

$$\int e^{-i(l'+l''+l''')X}\,dX$$

$$\times \int e^{-i(l'+l'''-l'')\rho/3}\,\delta(\rho)\,d\rho$$

$$\times \int \tilde{\chi}_n(r)\,e^{-i(l'-l''')r/2}\,dr$$

$$= \delta(l' + l'' + l''')\ \chi_n\left(\frac{l' - l'''}{2}\right)$$