Reference No.

Name

Subject

Date when Commenced

Date when Finished

Ref. No. of Previous File

Ref. No. of Succeeding File

物理学演習

電磁気演習
1934

木曜午後一時より

電磁気学演習（毎週一回、一年間 4→10回位）
昭和十年
十一年（火曜）五月廿一日開始。
十二年（月曜）十月廿五日開始 ①

(A) Image の方法，

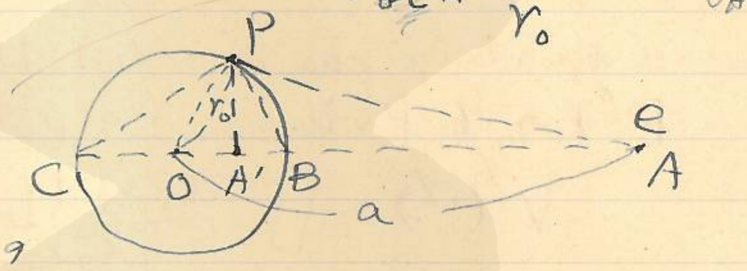
ある surface が equipotential になる様な charge distribution を求めておいて，後でその surface を conductor でおきかへると，外部の charge によって conductor に induce された charge が求まる．

最も簡単な場合は conducting plane と point charge の場合で，之は既知であった．

次に sphere の場合を考へてみる．

A点に point charge があるとすると，

$\cancel{a} = \frac{OA'}{r_0}$ 故に $OA\cdot OA'=r_0^2$.



$$\cancel{\frac{A'B}{AB}=\frac{A'C}{AC}}=\frac{r_0}{a}$$

を満足する点 A' ~~を~~ A ~~の~~ image と名付ける．この点に $-e\frac{r_0}{a}$ なる charge をおくと sphere が equipotential になる．

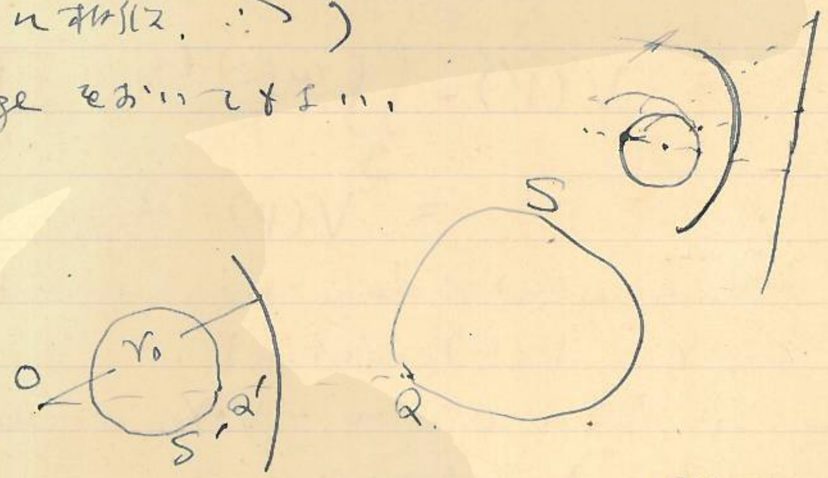
$$\left(\because \frac{A'P}{AP}=\cancel{\frac{A'B}{AB}}=\frac{r_0}{a}, \quad \therefore \frac{e}{AP}+\frac{-er_0}{a}\cdot\frac{1}{A'P}=0.\right.$$

△OAP は △OPA' に相似．∴)

尚 O 点に任意の point charge をおいてもよい．

(B) Inversion の方法，

空間に一つの図形，例へば closed surface S が与へられてゐるとする．O を中心として半径 r_0 なる補助の sphere を考へ，之に関する S の各点の image をつくると，それが又一つの（図形 例へば）closed surface S' になる．之を S の sphere に関する inversion といふ．



この場合，circle，直線の inversion は又 circle，直線である．

今 ~~S なる surf.~~ が conductor とし，~~之が其の~~ S' 上に ω(Q) なる surface density で charge が distribute してゐるとおく。S 上の任意点 P の potential は

$$V(P) = \iint_S \frac{\omega(Q)\, df_Q}{\overline{PQ}}$$

で与へられる。

その代りに S'上の Q'点（Q の image）に

$$\omega'(Q') = \omega(Q)\left(\frac{\overline{OQ}}{r_0}\right)^3 = \omega(Q)\left(\frac{r_0}{\overline{OQ'}}\right)^3$$

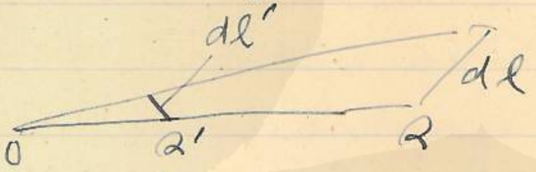
なる surface density で charge が distribute してゐる場合を考へると、S'上の ~~P'~~ P'点の（P の image）の potential は

$$V'(P') = \iint_{S'} \frac{\omega'(Q')\, df'_{Q'}}{\overline{P'Q'}}$$

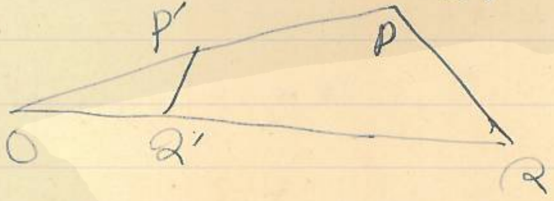
しかるに

$$df'_{Q'} = \left(\frac{r_0}{\overline{OQ}}\right)^4 df_Q$$

$$\frac{1}{\overline{P'Q'}} = \frac{\overline{OP}\cdot\overline{OQ}}{r_0^2}\cdot\frac{1}{\overline{PQ}}$$



$$\frac{dl'}{dl} = \frac{OQ'}{OQ} = \left(\frac{x_0}{OQ}\right)^2$$



$$\therefore\ V'(P') = \iint_S \frac{\omega(Q)\, df_Q}{\overline{PQ}}\cdot\frac{\overline{OP}}{r_0} = V(P)\cdot\frac{r_0}{\overline{OP'}}$$

しかるに ω(Q) なる charge distribution によって S が equipotential になってゐるとすると　$V(P) = \text{const} = V$.

そこで更に O 点に $-V r_0$ なる charge があったとすると、S'上の凡ての点の potential が 0 になる。

即ち、S なる conductor に於ける charge distribution を知れば、それからして、O点に point charge があった時に S' 上に induce される charge の distribution がわかる。（earth した conductor）

即ち、最初に O と S' とが与へられ、O点の charge が e であるとすると、

$e = -V r_0$ を満足する V, r_0 を適当にとり、Sを決め、S上で potential が V に等しい様に ~~solution~~ を density distribution $\omega(\alpha)$ を求め、それから、上の方法で $\omega'(\alpha')$ を出せば、之が induced charge density になる。之から field の模様もわかる。

(C) Two Dimensional Problems

この場合には Laplace の equation は

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0 \qquad (*)$$

or $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 V}{\partial\theta^2}=0$$

origin に point singularity (三次元なら line singularity) のある時の solution は

$$\varphi = C - 2\sigma\log r$$

$$-\frac{1}{4\pi}\cdot 2\pi r\frac{\partial\varphi}{\partial r} = \sigma,$$

(*) の general solution は

$$\varphi = f(x+iy) + g(x-iy)$$

$f(x+iy) = u + iv = w$ とすると

u, v 夫々 (*) の solution.

而して、u=const. v=const. は互に orthogonal である。

$$\because\ \frac{\partial u}{\partial x}=\frac{\partial v}{\partial y},\quad \frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x} \qquad \therefore\ \frac{\partial u}{\partial x}\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\cdot\frac{\partial v}{\partial y}=0.$$

u=const. を equipot とすると、v=const は lines of force になる。

Examples. $z = x+iy$　$w = u+iv$.

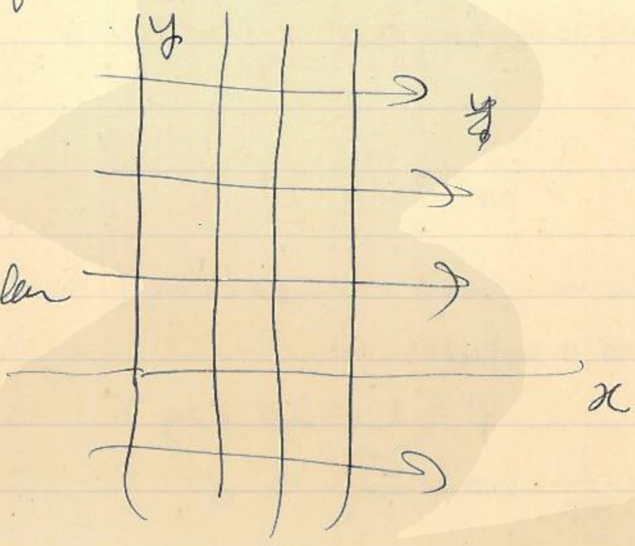
$w = z^n$.　$z = re^{i\theta}$.　$w = r^n e^{in\theta}$.

$u = r^n \cos^n\theta$　$v = r^n \sin n\theta$.

$n=1$:　$u = x$　∴ $x=$ const or equipotential surface

$v = y$　$y=$ const or lines of force

これは ~~unif~~ y-方向の uniform field を表わす.

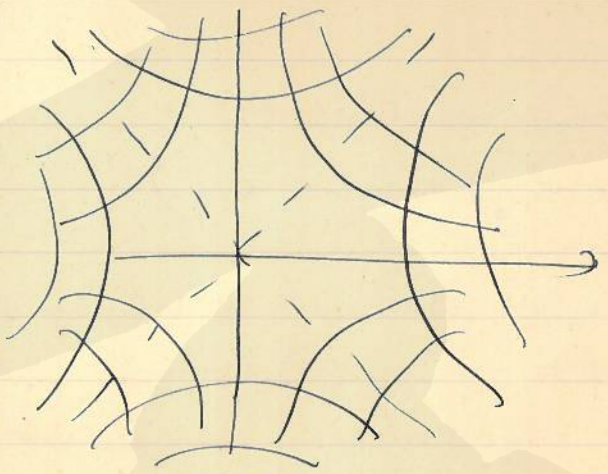


$n=2$;　$u = x^2-y^2$

$v = 2xy$.

これは equipotential は rectangular hyperbola. lines of force も之れと直角な hyperbola 之は互に直交する二つの conducting plane の附近の電場を表わす.

又 coaxial な二つの rectangular hyperbola の間の field を表す.



$n=\frac{1}{2}$;　$\cos^2\frac{\theta}{2} = \frac{1}{2}(\cos\theta+1)$

$\sin^2\frac{\theta}{2} = \frac{1}{2}(1-\cos\theta)$

$$u = \frac{r^{\frac{1}{2}}}{2}\left(1+\frac{x}{r}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left\{\sqrt{x^2+y^2}+x\right\}^{\frac{1}{2}} = \text{const}$$

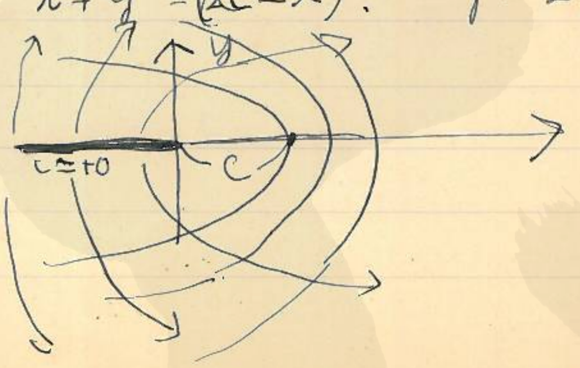
$$v = \frac{1}{2}\left\{\sqrt{x^2+y^2}-x\right\}^{\frac{1}{2}} = \text{const}$$

$\cancel{x^2+y^2}$

$y^2 = 2c'(c'+x)$

$c'=0$, ~~[illegible]~~

$x^2+y^2 = (\cancel{2x}c-x)^2$.　$y^2 = \cancel{2x} = 2c(c-x)$



$n=-1$:　$u+iv=\frac{x-iy}{x^2+y^2}.=\frac{\cos\theta-i\sin\theta}{r}.$

$u=\frac{\cos\theta}{r}=\text{const}=\frac{x}{x^2+y^2}$

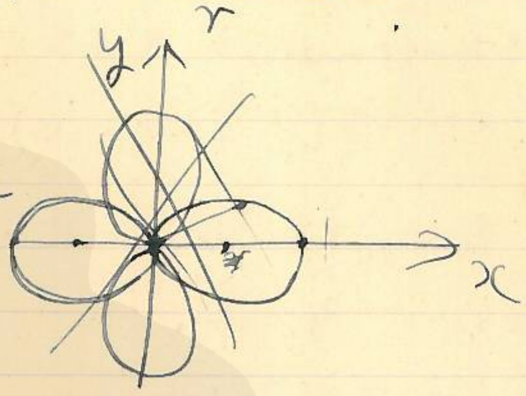
$v=\frac{-\sin\theta}{r}=\text{const}=\frac{-y}{x^2+y^2}$

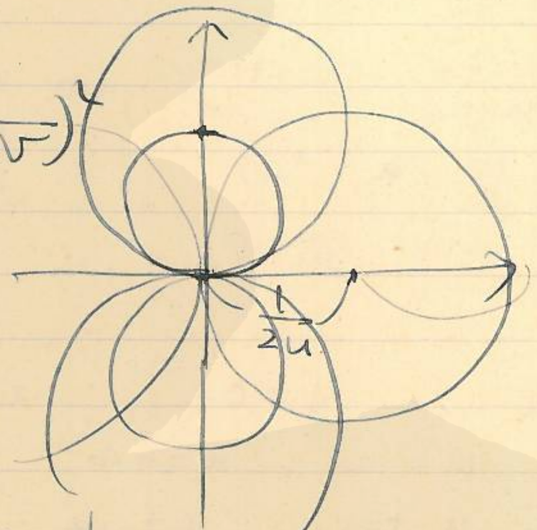
$\cancel{(x-\frac{u}{2})^2+y}^2$

$\cancel{\text{比x}}(x-\frac{1}{2u})^2+y^2=(\frac{1}{2u})^2$

$\cancel{(x+\frac{1}{2v})^2}.$

$x^2+(y+\frac{1}{2v})^2=(\frac{1}{2v})^2$

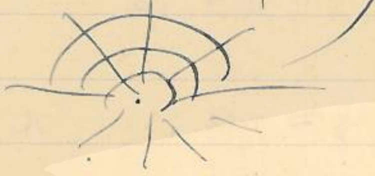




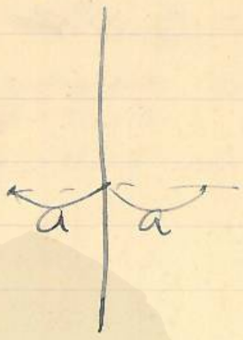
~~ある大きさの~~
これ２つの cylinder が ならんでいる時の potential field をあらわす。又は dipole (line source) のある場合.

$w=\log z.$

$u=\log r$　　　　$v=\theta,$



$w=\log\frac{z-a}{z+a}.$



$$z = \frac{a}{\pi}(e^{w} - w + i\pi).$$

此 transformation を考へる.

$$\left.\begin{aligned} x &= \frac{a}{\pi}(e^{u}\cos v - u) \\ y &= \frac{a}{\pi}(e^{u}\sin v - v + \pi) \end{aligned}\right\}$$

$v = 0$: $y = a$.

$x = \frac{a}{\pi}(e^{u} - u)$.

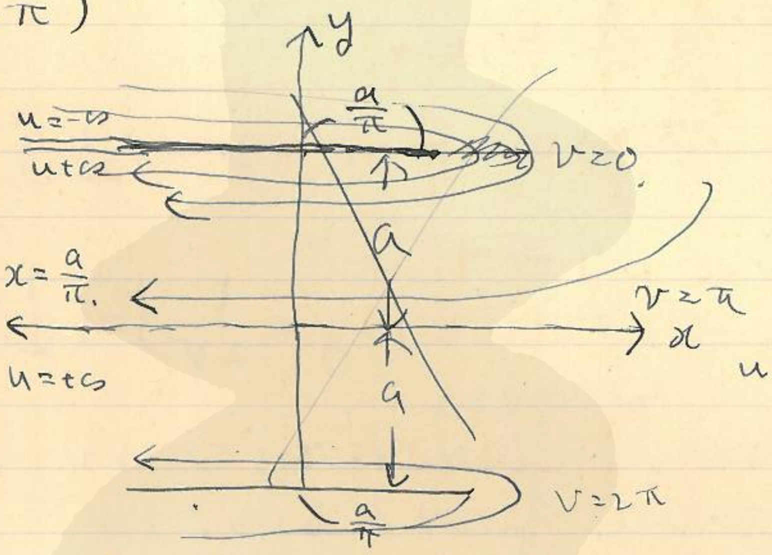
$u = -\infty$: $x = +\infty$

$u = +\infty$: $x = +\infty$

$\frac{dx}{du} = \frac{a}{\pi}(e^{u} - 1)$

$u = 0$: $\frac{dx}{du} = 0$.

$u = 0$: $x = \frac{a}{\pi}$.



$v = \pi$: $y = 0$. $x = -\frac{a}{\pi}(e^{u} + u)$

$u = -\infty$: $x = +\infty$

$u = +\infty$: $x = -\infty$.

$v = 2\pi$: $y = -a$. $x = \frac{a}{\pi}(e^{u} - u)$.

(~~$v = -\pi$~~ $y =$ ~~$\frac{a}{\pi}$~~ $2a$.

($u \to +\infty$ for ~~$v \neq 0$ or π or 2π~~, $0 < v < \frac{\pi}{2}$:

$u \to -\infty$ for $0 < v < \frac{\pi}{2}$:

$x = +\infty$
$y = +\infty$
$x = \mp\infty$
$y = +$ finite

$\frac{\pi}{2} < v < \pi$ $x = -\infty$
$y = +\infty$

$u = 0$ $x = \frac{a}{\pi}\cos v$
$y = \frac{a}{\pi}(\sin v - v + \pi)$

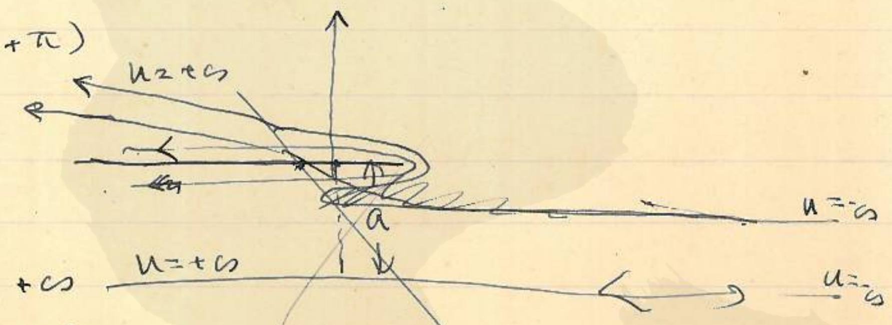
$v = \frac{\pi}{2}$: $x = \frac{-au}{\pi}$

$y = \frac{a}{\pi}(e^{u} + \frac{\pi}{2})$

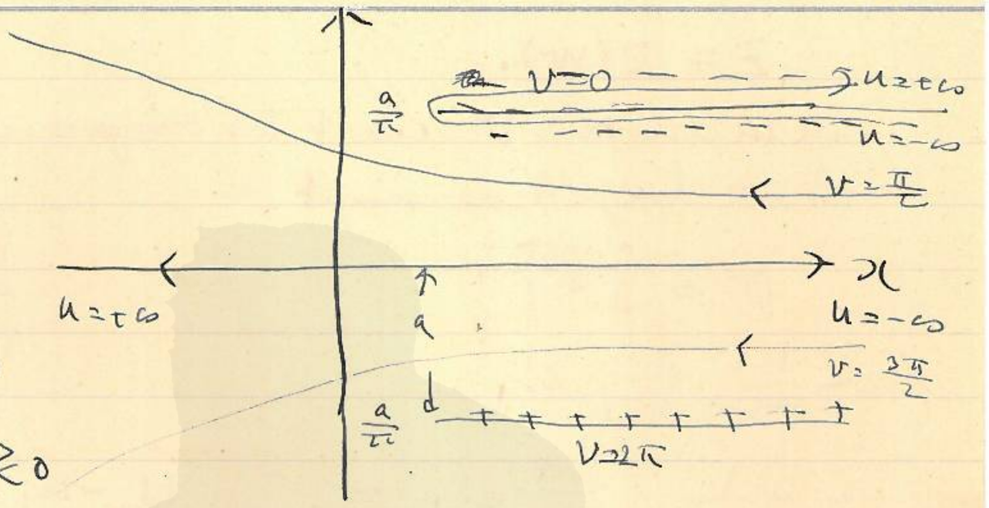
$u = -\infty$: $y = \frac{a}{2}$, $x = +\infty$

$u = +\infty$: $y = \infty$, $x = -\infty$

$u = 0$: $y = \frac{a}{\pi} + \frac{a}{2}$, $x = 0$



$e^{u} = \frac{\pi}{2}$ $u = \log\frac{\pi}{2}$.



v=0 は equipotential surface で charge の surface density は

$$4\pi\sigma = \mp\frac{\partial v}{\partial y}\Big|_{v=0} \qquad v=\pi \quad \text{for } u \gtrless 0$$

$$= \mp\frac{1}{\frac{\partial y}{\partial v}\Big|_{v=0}} \quad \text{for } u \gtrless 0$$

$$= \begin{cases} -\frac{\pi}{a}\frac{1}{(e^u-1)} & \text{for } u>0 \qquad \sim 0 \quad \text{for } u \gg 0 \\ +\frac{\pi}{a}\frac{1}{(e^u-1)} & \text{for } u<0 \qquad \sim -\frac{\pi}{a} \quad \text{for } u \ll 0 \end{cases}$$

$v=0$: ~~$y=a$~~

$\therefore \; \sigma \cong 0 \quad \text{for } u \gg 0$

$\cong -\frac{1}{4a} \quad \text{for } u \ll 0 .$

unit length あたりの charge は $\qquad \frac{\partial v}{\partial y} = \frac{\partial u}{\partial x}$

$$e_1 = \int_{\text{下面}} \sigma\, dx = \frac{+1}{4\pi}\int_0^{x_1} \frac{\partial u}{\partial x}dx = +\frac{1}{4\pi}\,u_1(x_1)$$

u_1 ~~$< a_1 = a$~~ $\qquad e^{u_1} - u_1 = \frac{\pi x_1}{a}$

$$e_2 = \int_{\text{上面}} \sigma\, dx = -\frac{1}{4\pi}\int_0^{x_1} \frac{\partial u}{\partial x}dx = -\frac{1}{4\pi}\,u_2(x_1)$$

$u_2 > 0 \qquad e^{u_2} - u_2 = \frac{\pi x_1}{a}$,

~~$\frac{x_1}{a} \gg e^{u_1} - u_1$~~ ~~$u_1 = \log\frac{\pi x_1}{a} + \delta$~~. $\; u_1 = -\frac{\pi x_1}{a} + \delta_1$,

$\frac{x_1}{a} \gg 1 \qquad u_2 = \log\frac{\pi x_1}{a} + \delta_2$,

$$\therefore \; e = e_1 + e_2 = \frac{1}{4\pi}\left\{-\frac{\pi x_1}{a} + \delta_1 - \log\frac{\pi x_1}{a} - \delta_2\right\}.$$

$$= -\frac{1}{4a}\left(x_1 + \log\frac{\pi x_1}{a} + \cdots\right)$$

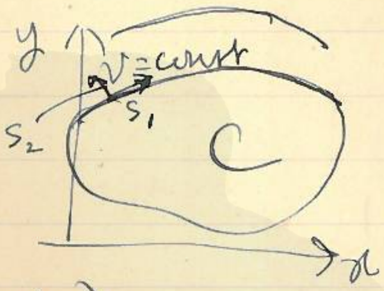
第三

z
x, y

$$Z = Z(W).$$

conductor が $v = \text{const}$ なる ~~surface~~ curve で与へられるとすると、その上の charge ~~の density~~ 量 は

~~$\sigma = \int \frac{\partial v}{\partial y} d$~~



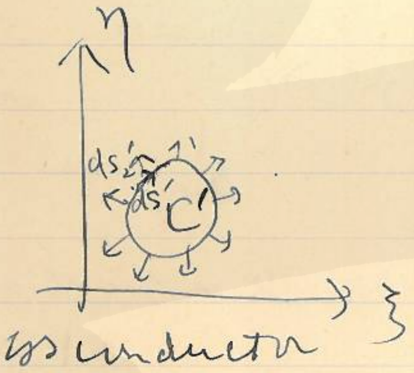
$$\sigma = -\frac{1}{4\pi}\int \frac{\partial v}{\partial s_2} ds_1$$

$$= -\frac{1}{4\pi}\int \frac{\partial u}{\partial s_1} ds_1 = -\frac{1}{4\pi}(u_2 - u_1)$$

之を conformal transformation によつて、z-plane から ζ-plane に transform すると、~~$w = w(\zeta)$~~, $\zeta = \zeta(z)$ $v = \text{const.}$ なる curve の形を変へることである）が、その時の charge density は

$$\sigma' = -\frac{1}{4\pi}\int \frac{\partial v}{\partial s_1'} ds_2'$$

$$= -\frac{1}{4\pi}\{u_2 - u_1]$$



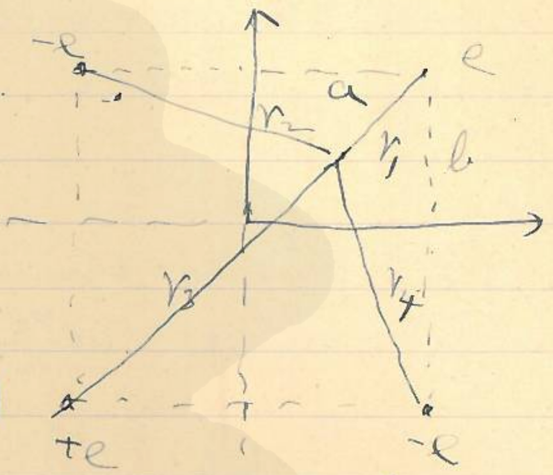
故に potential と total charge の値が ~~変化~~ 変らないことがわかる。故に C なる conductor と C' なる conductor の capacity は同じである。

(1) An infinite conducting wall stands vertically on a horizontal conducting plane. Find the charge induced on each plane by a point charge outside of them.

$$e' = -\frac{2e}{\pi} \operatorname{arctg} \frac{b}{a}$$

$$e'' = -\frac{2e}{\pi} \operatorname{arctg} \frac{a}{b}$$

$$e' + e'' = -e$$



$$\because \quad \varphi = e\left(\frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{r_3} - \frac{1}{r_4}\right)$$

$$w' = -\frac{1}{4\pi}\frac{\partial\varphi}{\partial x}\Big|_{x=0} = \frac{e}{4\pi}(-2a)\left[\{a^2+(y-b)^2+z^2\}^{-\frac{3}{2}} - \{a^2+(y+b)^2+z^2\}^{-\frac{3}{2}}\right]$$

$$w'' = -\frac{1}{4\pi}\frac{\partial\varphi}{\partial y}\Big|_{y=0} =$$

$$e' = \int_{z=-\infty}^{\infty}\int_{y=0}^{\infty} w'\,dy\,dz = -\frac{ae}{2\pi}\left[\int_0^\infty dy\left\{\frac{z}{\{a^2+(y-b)^2\}\{a^2+(y-b)^2+z^2\}^{\frac{1}{2}}}\right.\right.$$

$$\left.\left. - \quad '' \quad \right\}\Big|_{-\infty}^{+\infty}\right\}$$

$$= -\frac{ae}{2\pi}\left[\int_0^\infty dy \frac{2}{a^2+(y-b)^2} - \int_0^\infty \frac{2\,dy}{a^2+(y+b)^2}\right]$$

$$= -\frac{ae}{2\pi}\left[\frac{\pi}{a} - \frac{2}{a}\operatorname{arctg}\left(\frac{-b}{a}\right) - \frac{\pi}{a} + \frac{2}{a}\operatorname{arctg}\left(\frac{b}{a}\right)\right].$$

$$= -\frac{2e}{\cancel{2}\pi}\operatorname{arctg}\frac{b}{a},$$

$$e'' = \int_{z=-\infty}^{+\infty}\int_{x=0}^{\infty} w''\,d\cancel{y}x\,dz = -\frac{2e}{\pi}\operatorname{arctg}\frac{a}{b}.$$

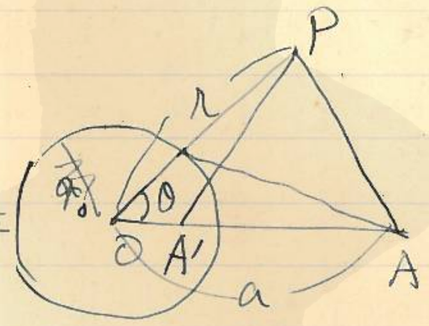
$$2\pi\int_0^\pi \frac{\sin\theta\, d\theta}{(a^2+r_0^2-2ar_0\cos\theta)^{\frac{3}{2}}} = \frac{4\pi r_0}{a^2-r_0^2}$$

(2) Find the ~~di~~ distribution of the surface charge induced on a sphere of radius r_0 by a point charge e at a distance a from the centre of the sphere, which is

i) insulated with no charge

or ii) at zero potential, .

a) Image



ii) $\overline{PA} = (a^2+r^2-2ar\cos\theta)^{\frac{1}{2}}$

$\overline{PA'} = (\frac{r_0^4}{a^2}+r^2-2r\frac{r_0^2}{a}\cos\theta)^{\frac{1}{2}}$

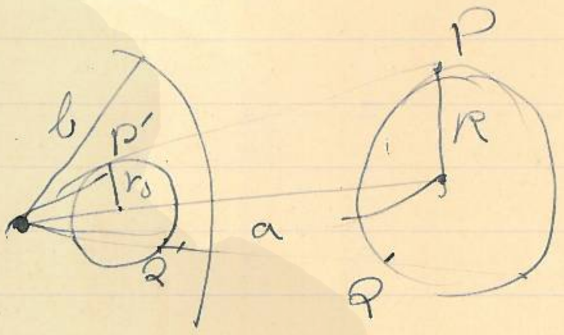
$$\varphi = \frac{e}{PA} - \frac{e r_0}{a\cdot PA'}$$

$$\omega = -\frac{1}{4\pi}\frac{\partial\varphi}{\partial r}\Big|_{r=r_0} = \frac{-e}{4\pi}\left\{\frac{r_0-a\cos\theta}{(PA)^3} - \frac{r_0}{a}\frac{r_0-\frac{r_0^2}{a}\cos\theta}{(PA')^3}\right\}$$

$$= -\frac{e}{4\pi}\frac{a^2-r_0^2}{\overline{PA}^3\cdot r_0} = -\frac{e}{4\pi}\frac{a^2-r_0^2}{(a^2+r_0^2-2ar_0\cos\theta)^{\frac{3}{2}}\, r_0}\dagger$$

i) $$\omega' = \omega + \frac{e r_0}{a}\frac{1}{4\pi r_0^2} = \omega + \frac{e}{4\pi a r_0}.$$

b) Inversion



ii) $$\omega'(Q') = \frac{b^3}{\overline{OQ'}^3}\,\omega(Q)$$

$$-V(P)\,b = e \quad a^2$$

$$V(P) = \frac{4\pi^2 R\,\omega(Q)}{R} = 4\pi R\omega$$

$$= -\frac{e}{b}.$$

$$\therefore\ \omega = \frac{-e}{4\pi R b}.$$

$$\frac{r_0}{R} = \frac{\overline{OP'}}{OP} = \frac{\overline{OP'}^2}{b^2} = \frac{a^2-r_0^2}{b^2} \qquad \omega = \frac{-e(a^2-r_0^2)}{4\pi r_0\, b^3}$$

$$\omega'(Q') = -\frac{e(a^2-r_0^2)}{4\pi r_0\,\overline{OQ'}^3};\quad \text{i) } \omega'' = \omega' + \frac{e}{4\pi a r_0}$$

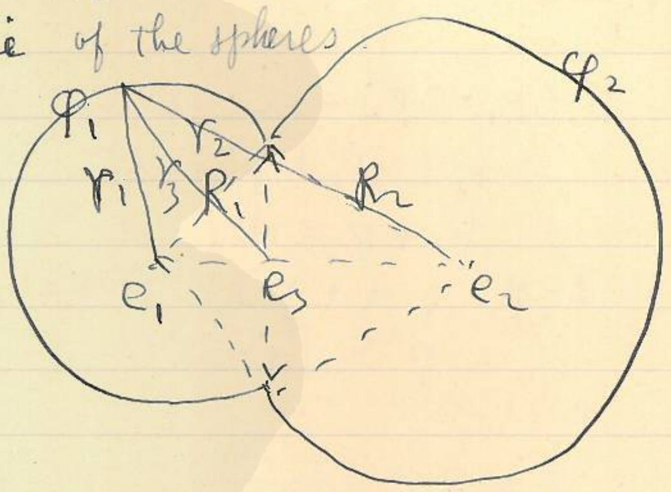
(3) Two spheres intersecting at right angle are charged in free space. Show that the charge density is minimum on the circle of intersection and maximum at the ends of the common diameter.

Show also that the capacity is given by

$$C = R_1 + R_2 - \frac{R_1 R_2}{\sqrt{R_1^2 + R_2^2}}$$

where R_1 and R_2 are the radii of the spheres

(i) Image a' the



$e_1 = VR_1$

$e_2 = VR_2$

$$e_3 = -\frac{\cancel{e_1 e_2}}{\cancel{V}} \frac{R_1}{\sqrt{R_1^2+R_2^2}}$$

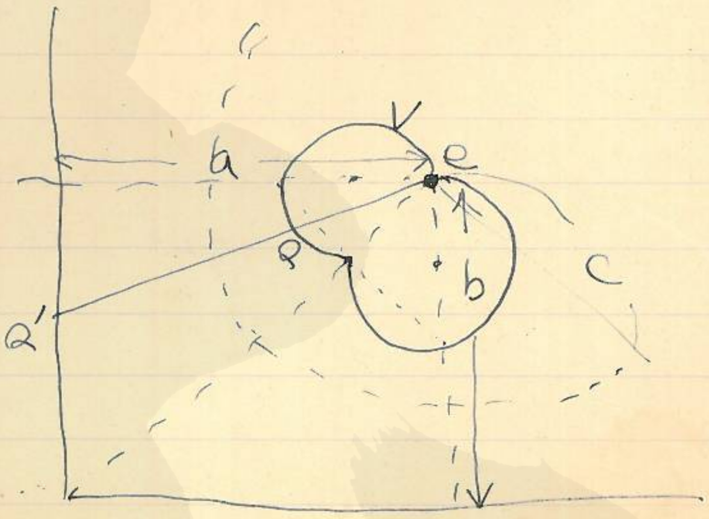
$$= -V \frac{R_1 R_2}{\sqrt{R_1^2+R_2^2}}.$$

$$\varphi_1 = V + \frac{e_2}{r_2} + \frac{e_3}{r_3}, \qquad e_1 + e_2 + e_3 = VC,$$

$$\frac{r_2}{r_3} = \frac{\sqrt{R_1^2+R_2^2}}{R_1} \qquad \frac{e_2}{e_3} = -\frac{\sqrt{R_1^2+R_2^2}}{R_1}$$

$$\sigma_1 = \frac{V}{4\pi R_1}\left\{1 - \left(\frac{R_2}{r_2}\right)^3\right\}$$

(ii) Inversion



$e = -VC.$

$$\omega(Q) = \omega(Q')\left(\frac{\overline{OQ'}}{c}\right)^3.$$

$$a = \frac{c^2}{2R_1}$$

$$b = \frac{c^2}{2R_2}.$$

$\omega'(Q') = .$

(4) Show that the capacity per unit length of two parallel cylinder, each with radius a, and with the equal charge of sign, is given by

$$C = \frac{1}{4\log\frac{d+\sqrt{d^2-4a^2}}{2a}},$$

where d is the distance between the axes of the cylinders.

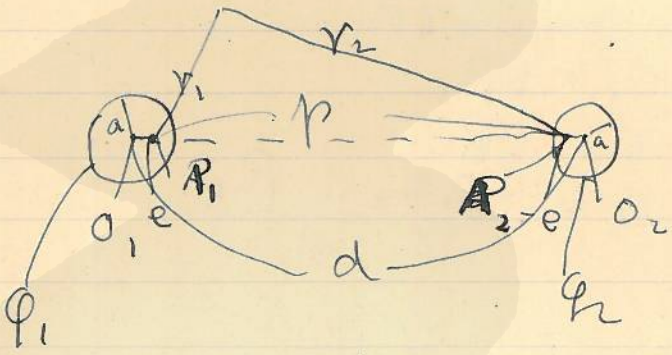
$$O_1P_1 \cdot O_1P_2 = a^2 = O_2P_1 \cdot O_2P_2$$

$$O_1P_1(O_1P_1 + p) = a^2.$$

$$2O_1P_1 + p = d.$$

$$(d-p)(d+p) = 4a^2$$

$$p^2 = \sqrt{d^2-4a^2}.$$



$$\varphi = 2e\log\frac{r_2}{r_1}.$$

$$O_1P_1 = \frac{d-p}{2}$$

$$\varphi_1 = 2e\log\frac{p \mp a + O_1P_1}{a - O_1P_1}$$

$$p \pm a + O_1P_1 = p \pm a + \frac{d}{2} \mp \frac{p}{2}$$

$$= \frac{1}{2}p \mp a + \frac{d}{2}$$

$$a - O_1P_1 = a - \frac{d}{2} + \frac{p}{2}$$

$$\varphi_2 = 2e\log\frac{a - O_1P_1}{p + a - O_1P_1}$$

$$\frac{\varphi_1}{\varphi_2} \quad C = \frac{e}{\varphi_1 - \varphi_2} = \frac{1}{4\log\frac{d-2a+p}{-d+2a+p}} = \frac{1}{4\log\frac{p+d}{2a}}.$$

$$p^2 - (d-2a)^2$$

$$= d^2 - 4a^2 - d^2 + 4ad \pm 4a^2$$

$$= 4a(d-2a).$$

$$\{p + d - 2a\}^2 = d^2 - 4a^2 + d^2$$

$$- 4ad + 4a^2$$

$$+ 2p(d-2a)$$

$$= (d-2a)\{2p + 2d\}$$

⑦

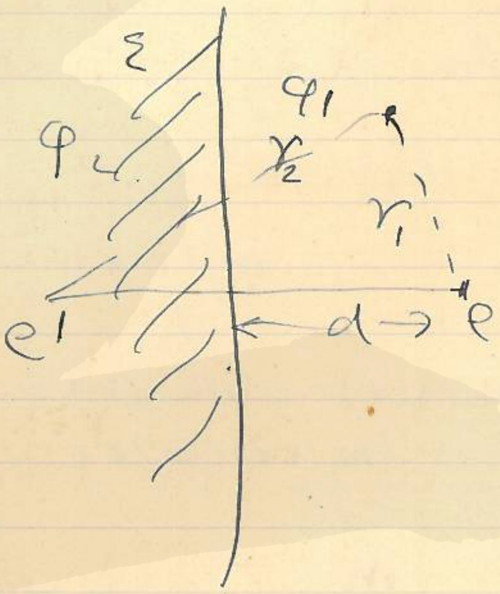
(5) An electrified particle with charge e is placed in front of an infinitely thick plate of dielectric constant ε. Show that the particle is urged towards the plate by a force

$$\frac{\varepsilon-1}{\varepsilon+1}\ \frac{e^2}{4d^2},$$

where d is the distance of the point from the plate.

$$\varphi_1 = \frac{e}{r_1} + \frac{e'}{r_2} \qquad \varphi_2 = \frac{e''}{r_1}$$

$$\varphi_2 = \frac{e''}{r_1}$$



$$\left.\frac{\partial\varphi_1}{\partial x}\right|_{x=0} = \frac{(e-e')d}{\sqrt{d^2+y^2+z^2}}$$

$$\left.\frac{\partial\varphi_2}{\partial x}\right|_{x=0} = \frac{e''d}{\sqrt{\ \ }}$$

$$e' = \frac{1-\varepsilon}{1+\varepsilon}e$$

$$\overline{W} = \frac{1}{2}\frac{ee'}{2d}.$$

e を除いたの field は $\overset{e}{e'}$ なる点の charge によるものであるから. e に働く力は ~~e点に stress~~ $\frac{ee'}{(2d)^2}$ である.

急いで (1957)

(6) Two small magnets of moments m, m', being at a distance r have their centres apart, make angles θ, θ' with the line joining them and an angle ε with each other. Show that the force on the first magnet in its own direction is

$$\frac{3mm'}{r^4}\left(5\cos^2\theta\cos\theta' - \cos\theta' - 2\cos\varepsilon\cos\theta\right).$$

Show that the couple about the line joining them which the magnet exert on each other is

$$\frac{mm'}{r^4}\, d\sin\varepsilon\,,$$

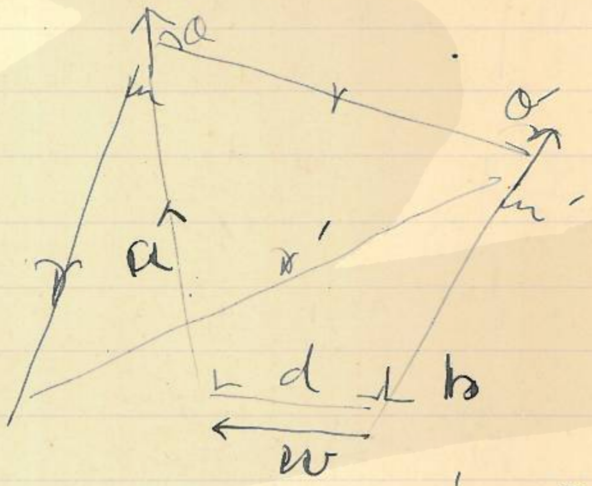
where d is the shortest distance between their axes ~~produces~~;

$$\varphi(\mathfrak{r}) = \mathfrak{m}'\,\mathrm{grad}'\frac{1}{|\mathfrak{r}-\mathfrak{r}'|}$$

$$= \frac{\mathfrak{m}'(\mathfrak{r}-\mathfrak{r}')}{r^3}.$$

$$\mathfrak{m}\mathfrak{H}(\mathfrak{r}) = -\mathfrak{m}\,\mathrm{grad}\,\varphi(\mathfrak{r})$$

$$= \frac{3\{\mathfrak{m}(\mathfrak{r}-\mathfrak{r}')\}\{\mathfrak{m}'(\mathfrak{r}-\mathfrak{r}')\}}{r^5} - \frac{\mathfrak{m}\mathfrak{m}'}{r^3}$$



$\mathfrak{r}'-\mathfrak{r} = \mathfrak{a}+\mathfrak{w}+\mathfrak{b}$

$$F = \frac{\mathfrak{m}}{m}\,\mathrm{grad}\{\mathfrak{m}\mathfrak{H}(\mathfrak{r})\}$$

$$= \frac{3m\{\mathfrak{m}'(\mathfrak{r}-\mathfrak{r}')}{r^5} + \frac{3\{\mathfrak{m}(\mathfrak{r}-\mathfrak{r}')\}(\mathfrak{m}\mathfrak{m}')}{m r^5}$$

$$- \frac{3\{\mathfrak{m}(\mathfrak{r}-\mathfrak{r}')\}\times 5\{\mathfrak{m}(\mathfrak{r}-\mathfrak{r}')\}}{m r^5} + \frac{3\{\mathfrak{m}\mathfrak{m}'\}\{\mathfrak{m}(\mathfrak{r}-\mathfrak{r}')\}}{m r^5}$$

$\mathfrak{m}'(\mathfrak{r}-\mathfrak{r}') = -m' r\cos\theta'$ $\qquad \mathfrak{m}\mathfrak{m}' = mm\cos\varepsilon$

$\mathfrak{m}(\mathfrak{r}-\mathfrak{r}') = mr\cos\theta$

$$\frac{\mathfrak{r}'-\mathfrak{r}}{r}[\mathfrak{m}\mathfrak{H}] = \frac{\mathfrak{r}'-\mathfrak{r}}{r}\left[\mathfrak{m},\left\{\frac{3(\mathfrak{r}-\mathfrak{r}')\{\mathfrak{m}'(\mathfrak{r}-\mathfrak{r}')\}}{r^5} - \frac{\mathfrak{m}'}{r^3}\right\}\right]$$

$$= \frac{\mathfrak{r}'-\mathfrak{r}}{r^4}[\mathfrak{m},\mathfrak{m}'] = \frac{\cancel{\mathfrak{r}'-\mathfrak{r}}\,\mathfrak{w}}{r^4}[\mathfrak{m},\mathfrak{m}'] = \frac{mm'}{r^4}\sin\varepsilon\cdot d$$

(7) Two parallel straight wires with circular cross section convey equal currents in opposite directions. Show that the magnetitude of the magnetic field of the point a distances r_1, r_2 respectively from the axes of the wires is given by

$$H = \frac{2Ja}{c r_1 r_2},$$

where a is the distance between the axes. Show also that the field H makes an angle

$$\theta = \theta_1 + \theta_2 - \frac{\pi}{2}$$

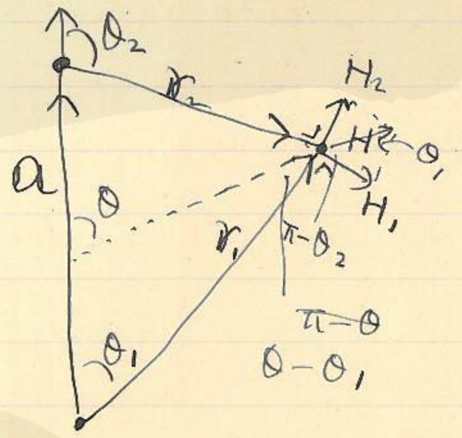
with the vector $\mathbf{a}$ perpendicular to the axes, where θ_1, θ_2 are the angles between $\mathbf{a}$ and the vectors r_1, r_2 respectively.

$$H_1 = \frac{2J}{c r_1}$$

$$H_2 = \frac{2J}{c r_2}$$

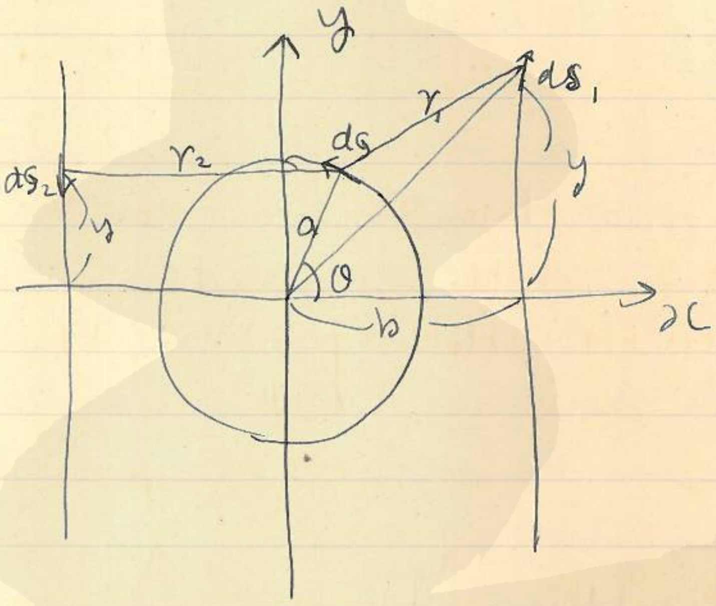
$$\frac{H_1}{r_2} = \frac{H_2}{r_1} = \frac{2J}{c r_1 r_2} = \frac{H}{a}$$

$$\theta = \theta_1 + \left\{ \begin{array}{l} \frac{\pi}{2} \cancel{+ \theta_2 = 2\pi} \\ \pi - \theta_2 = \pi \end{array} \right. \quad \left(+\frac{\pi}{2}\right)$$



(18) Prove that the coefficient of mutual induction between a pair of infinitely long straight wires and a circular one of radius a in the same plane and with its centre at a distance b (> a) from each of the straight wires is

$$8\pi\left(b-\sqrt{b^2-a^2}\right).$$



$$r_1=\sqrt{(b-a\cos\theta)^2+(y-a\sin\theta)^2}$$

$$r_2=\sqrt{(b+a\cos\theta)^2+(y-a\sin\theta)^2}$$

$$ds\,ds_1=a\,d\theta\,dy\cos\theta$$

$$ds\,ds_2=-a\,d\theta\,dy\cos\theta$$

$$I=\int_{\theta=0}^{2\pi}\int_{y=-\infty}^{+\infty}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)a\cos\theta\,d\theta\,dy$$

$$\lim_{y\to\infty}\int_{-y}^{+y}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)dy$$

$$=\lim_{y\to\infty}\log\left\{\frac{y-a\sin\theta+\sqrt{(y-a\sin\theta)^2+(b-a\cos\theta)^2}}{-y-a\sin\theta+\sqrt{(y+a\sin\theta)^2+(b-a\cos\theta)^2}}\right.$$

$$\left.\times\frac{-y-a\sin\theta+\sqrt{(y+a\sin\theta)^2+(b+a\cos\theta)^2}}{y-a\sin\theta+\sqrt{(y-a\sin\theta)^2+(b+a\cos\theta)^2}}\right\}$$

$$=2\log\frac{b+a\cos\theta}{b-a\cos\theta}$$

$$I=2a\int_0^{2\pi}\cos\theta\,d\theta\log\left(\frac{b+a\cos\theta}{b-a\cos\theta}\right)$$

$$\int_0^{2\pi}\cos\theta\,d\theta\log(b+a\cos\theta)=\log(b+a\cos\theta)\sin\theta\Big|_0^{2\pi}+\int_0^{2\pi}\frac{a\sin^2\theta}{b+a\cos\theta}d\theta$$

$$\therefore I=4a^2b\int_0^{2\pi}\frac{\sin^2\theta\,d\theta}{b^2-a^2\cos^2\theta}=16a^2b\int_0^{\frac{\pi}{2}}\frac{\sin^2\theta}{b^2-a^2\cos^2\theta}d\theta \qquad \theta'=2\theta$$

$$=8a^2b\int_0^{\pi}\frac{(1-\cos\theta')\,d\theta'}{(2b^2-a^2)-a^2\cos\theta'}=8\left\{b\int_0^{\pi}d\theta'-2b(b^2-a^2)\int_0^{\pi}\frac{d\theta'}{(2b^2-a^2)-a^2\cos\theta'}\right.$$

$$=8\pi b-16b(b^2-a^2)\frac{2}{2b\sqrt{b^2-a^2}}\tan^{-1}\left(\frac{2b\sqrt{b^2-a^2}\tan\theta'/2}{2b^2}\right)\Big|_0^{\pi}.$$

(9) A given current sent through a tangent galvanometer deflects the magnet through an angle θ. The plane of the coil is slowly rotated round the vertical axis through the centre of the magnet. Prove that if $\theta > \frac{\pi}{4}$, the magnet will describe complete revolution, but if $\theta < \frac{\pi}{4}$, the magnet will oscillate through an angle $\sin^{-1}(\tan\theta)$ on each side of the meridian.

$H = H_0 \tan\theta$ $\qquad$ $\tan\theta = \lambda$.

$\theta \gtreqless \frac{\pi}{4}$. $H \gtreqless H_0$ $\qquad \lambda \lesseqgtr 1$.

$\sqrt{H_0^2 + H^2 - 2H_0 H \sin\phi} \cdot \sin\chi = H\cos\phi$.

$$\sin\chi = \frac{\lambda\cos\phi}{\sqrt{1+\lambda^2-2\lambda\sin\phi}}.$$

$$\frac{d}{d\phi}(\sin\chi) = \lambda\left\{\frac{-\sin\phi}{\sqrt{\quad}^{\prime\prime}} + \frac{\lambda\cos^2\phi}{\sqrt{\quad}^{3}}\right\}$$

$$= \lambda\left\{\frac{\lambda-(1+\lambda^2)\sin\phi+\lambda\sin^2\phi}{\sqrt{\quad}}\right\}$$

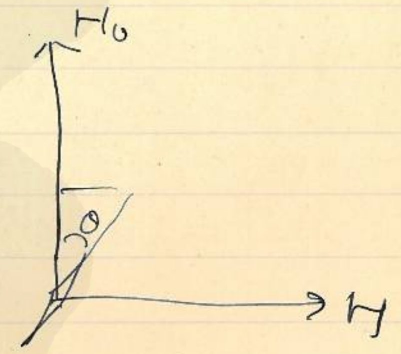
$= 0$ for $\sin\phi = \frac{1}{\lambda}$ or λ.

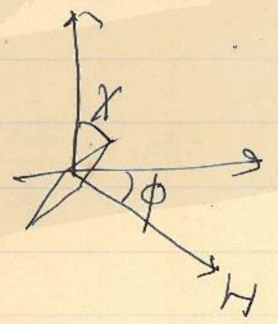
$\therefore$ $\theta < \frac{\pi}{4}$. $(\lambda < 1)$. $\sin\phi = \lambda$. $\sin\chi = \lambda$. $\chi = \sin^{-1}(\tan\theta)$.

$\theta > \frac{\pi}{4}$. $\sin\phi = \frac{1}{\lambda}$. $\sin\chi = 1$. $\chi = \frac{\pi}{2}$

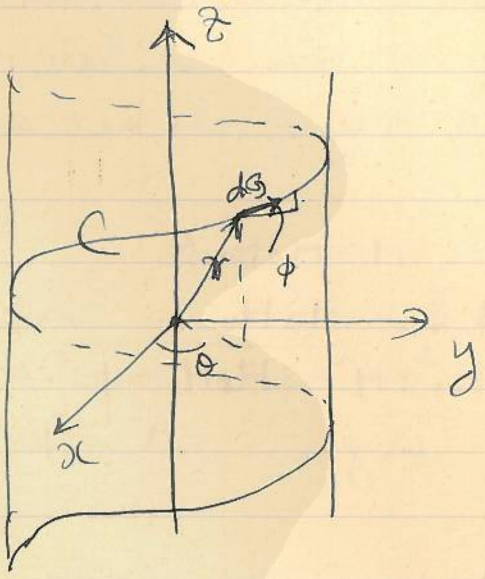
但しこの時には $\frac{d\chi}{d\phi} \neq 0$. 故に magnet は coil につれて

廻る.





(10) A wire is wound in a spiral of angle ϕ on the surface of an insulating cylinder of radius a, so that it makes n complete turns on the cylinder. A current J flows through the wire. Find the magnetic field along the axis at the centre of the cylinder.



$$\mathbf{H} = \int_C \frac{J[r, ds]}{c r^3}$$

$$H_z = \frac{J}{ac} \int_{-n\pi}^{+n\pi} \frac{a \cdot a \cancel{\phi}\, d\theta}{(a^2 + a^2\theta^2 \tan^2\phi)^{\frac{3}{2}}}$$

$$= \frac{J}{ac} \int_{-n\pi}^{+n\pi} \frac{d\theta}{(1+\theta^2 \tan^2\phi)^{\frac{3}{2}}}$$

$$= \frac{J}{ac} \frac{\theta}{(1+\theta^2\tan^2\phi)^{\frac{1}{2}}} \Big|_{n\pi}^{n\pi}$$

$$= \frac{2n\pi J}{ac(1+n^2\pi^2\tan^2\phi)^{\frac{1}{2}}}$$

$$\frac{1}{(\quad)^{\frac{1}{2}}} - \frac{\theta^2\tan^2\phi}{(\quad)^{\frac{3}{2}}}$$

(11) Two electrodes connected by an insulated wire are immersed in electrolyte extending to infinity with the conductivity σ. Show that the resistance of the electrolyte is equal to $\frac{1}{4\pi\sigma}$ the reciprocal of the electrostatic capacity of the electrodes, when the electrolyte does not exist. Thus, the resistance per unit length between two concentric cylindrical electrodes of radii r_1, r_2 is given by

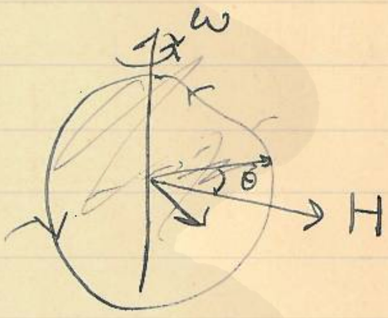
$$R = \frac{1}{2\pi\sigma} \log \frac{r_2}{r_1}.$$

$$J = -\sigma \iint \frac{\partial \varphi}{\partial n} df$$

$$\left(e = -\frac{1}{4\pi} \iint \frac{\partial \varphi}{\partial n} df \right)$$

$$R = \frac{\varphi_1 - \varphi_2}{-\sigma \iint \frac{\partial \varphi}{\partial n} df} = \frac{1}{4\pi\sigma} \frac{\varphi_1 - \varphi_2}{e} = \frac{1}{4\pi\sigma\, C}$$

(12) A coil is rotated with constant angular velocity ω about an axis in its plane in a uniform field of force perpendicular to the axis of rotation. Find the current in the coil at any time and show that it is greatest when the plane of the coil makes an angle $\tan^{-1}\left(\frac{L\omega}{R}\right)$ with the line of magnetic force.



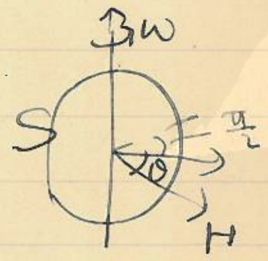
$$\int B_n\, df = HS\cos\theta$$

$$\Phi = -\frac{HS}{\cancel{c}}\frac{d(\cos\theta)}{dt}$$

$\theta = \omega t$,

$$= -\frac{HS\omega}{\cancel{c}}\sin\omega t$$

$$L\frac{dJ}{dt} + JR = -HS\omega \sin\omega t$$



$$J = J_0\sin(\omega t + \delta)$$

$$= J_0\sin(\theta + \delta^*)$$

$$\delta = \tan^{-1}\left(\frac{-L\omega}{R}\right) = -\tan^{-1}\left(\frac{L\omega}{R}\right)$$

$$\theta + \delta = \pm\frac{\pi}{2}$$

~~$\pm\frac{\pi}{2} - \theta = -\tan^{-1}\frac{L\omega}{R}$~~

$$\theta \pm \frac{\pi}{2} = \tan^{-1}\frac{L\omega}{R}$$

(c1)

(13) In a circuit with resistance R_0, induction L and constant electromotive force V, a steady current V/R_0 flows up the moment. After $t=0$ the resistance R increases according to the expression

$$R = R_0 \frac{\tau}{\tau - t},$$

which becomes infinity at $t=\tau$.
Find the current which flows during the interval $(0, \tau)$, and show that the electromotive force induced by self induction becomes infinite, if τ is smaller than L/R_0

$$JR = V - L\frac{dJ}{dt}$$

$$R = R_0 \frac{\tau}{\tau - t}$$

$$\frac{dJ}{dt} = \frac{V}{L} - \frac{R_0}{L}\tau\frac{J}{\tau - t}.$$

$V=0$ の general solution は　$J = J_0(\tau - t)^{\alpha}$　とおく $< \tau$.
$\alpha = R_0\tau/L$　　$J_0 = J_0(\tau - t)^{\frac{R_0\tau}{L}}$

V あるときの ~~electromotive force~~ particular solution は
$$J = C(\tau - t).$$
とおく$<\tau$.　$$-C = \frac{V}{L} - \frac{R_0\tau C}{L}$$
$$\therefore C = \frac{V}{R_0\tau - E}$$

∴ General solution は　$J = \frac{V}{R_0\tau - L}(\tau - t) + J_0(\tau - t)^{\frac{R_0\tau}{L}}$

$\frac{L}{R_0} = \vartheta$: $J = \frac{V}{R_0}\frac{\tau - t}{\tau - \vartheta} + J_0(\tau - t)^{\frac{\tau}{\vartheta}}$

$$t=0 ; \quad \frac{V}{R_0} = \frac{V}{R_0}\frac{\tau}{\tau-\vartheta} + J_0\tau^{\frac{\tau}{\vartheta}}.$$

$$J_0 = \tau^{-\frac{\tau}{\vartheta}}\frac{V}{R_0}\left(1-\frac{\tau}{\tau-\vartheta}\right) = \tau^{-\frac{\tau}{\vartheta}}\frac{V}{R_0}\frac{-\vartheta}{\tau-\vartheta}$$

$$\therefore\ J = \frac{V}{R_0}\frac{1}{\tau-\vartheta}\left\{\tau-t-\vartheta\left(\frac{\tau-t}{\tau}\right)^{\frac{\tau}{\vartheta}}\right\}$$

$$= \frac{V}{R_0}\frac{\tau-t}{\tau-\vartheta}\left\{1-\frac{\vartheta}{\tau}\left(\frac{\tau-t}{\tau}\right)^{\frac{\tau}{\vartheta}-1}\right\}$$

~~$t=\tau ; \quad J=$~~

induced el. motive force $-L\frac{dJ}{dt} = -\frac{LV}{R_0}\frac{1}{\tau-\vartheta}\left\{-1+\left(\frac{\tau-t}{\tau}\right)^{\frac{\tau}{\vartheta}-1}\right\}$

$\tau<\vartheta \qquad = \infty \quad$ for $t=\tau$

$\tau>\vartheta \qquad = -\frac{V\vartheta}{\tau-\vartheta} \quad$ for $t=\tau$.

$\tau<\vartheta$ 　$t=\tau$ では spark が飛んで resistance を急に減ず
ることになる.

(14) Two circuits with resistances R_1, R_2 and self inductions lie near each other, the coeffecient of mutual induction being L_{12}. Show that the total quantity of electricity that traverses one of them, when an electromotive force V is switched into the other, is $\frac{V L_{12}}{R_1 R_2}$.

$$J_1 = \frac{V_1}{R_1} + C_1 e^{-\lambda t} + C_1' e^{-\lambda' t}$$

$$J_2 = C_2 e^{-\lambda t} + C_2' e^{-\lambda' t}$$

$t=0$; $J_1 = J_2 = 0$ $\qquad \frac{V_1}{R_1} + C_1 + C_1' = 0$

$$C_2 \pm C_2' = 0$$

$$e_2 = -\int_0^\infty I_2 dt = -C_2 \int_0^\infty (e^{-\lambda t} - e^{-\lambda' t}) dt$$

$$= -C_2 \left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

$$\frac{V_1}{R_1} + \frac{-L_{12}\lambda + R_2}{L_{12}\lambda} C_2 - \frac{-L_2\lambda' + R_2}{L_{12}\lambda'} C_2 = 0$$

√.

$$-L_{12}\lambda C_1 + (-L_2\lambda + R_2) C_2 = 0$$

$$-L_{12}\lambda' C_1' + (-L_2\lambda' + R_2) C_2' = 0$$

$$\therefore \frac{R_2}{L_{12}} \left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right) C_2 = \frac{V_1}{R_1}$$

$$\therefore e_2 = \frac{V_1 L_{12}}{R_1 R_2}$$

(15) In a circuit with resistance R, induction L and capacity C, R being smaller than $2\sqrt{\frac{L}{C}}$, a constant electromotive force V is applied at an instant. Show that the condensor is charged to maximum potential

$$V\left(1+e^{-\frac{\lambda\pi}{\omega}}\right)$$

after a time $\frac{\pi}{\omega}$, where

$$\omega = \sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2} \qquad \lambda = \frac{2R}{L}$$

$$R < 2\sqrt{\frac{L}{C}} , \qquad e = a e^{-\lambda t}\sin(\omega t+\delta) \mp VC$$

$$J = a\{\lambda\sin(\omega t+\delta) - \omega\cos(\omega t+\delta)\}$$

$$t=0 \quad e=0. \qquad a\sin\delta - VC = 0$$

$$J=0 \qquad \lambda\sin\delta - \omega\cos\delta = 0$$

$$a=\frac{VC}{\sin\delta} \qquad \tan\delta = \frac{\omega}{\lambda}.$$

$$e = -VC\left(1-e^{-\lambda t}\left\{\frac{\lambda}{\omega}\sin\omega t+\cos\omega t\right\}\right)$$

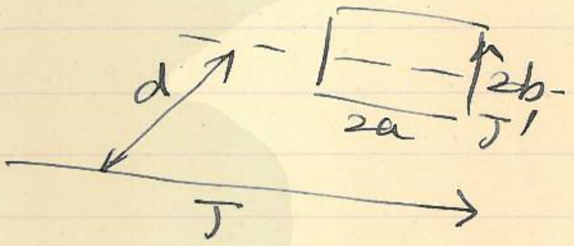
$$-\frac{de}{dt} = J = 0 ; \quad \tan(\omega t+\delta) = \frac{\omega}{\lambda} = \tan\delta .$$

$$\omega t = n\pi ; \quad e = -VC\left(1-(-1)^n e^{-\frac{n\lambda\pi}{\omega}}\right)$$

$$n=1 : \quad e = -VC\left(1+e^{-\frac{\lambda\pi}{\omega}}\right)$$

講釈.

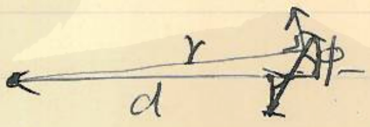
(16) A current J flows in a rectangular circuit whose sides are of lengths 2a, 2b and the circuit is free to rotate about and axis through its centre parallel to the sides of length 2a. Another current J′ flows ~~≠~~ in a long straight wire $\parallel$el to the axis and at a distance d from it. Find the couple of force required to keep the plane of the rectangle at an angle ϕ with the plane through ~~this~~ axis and the straight current.



magnetic energy

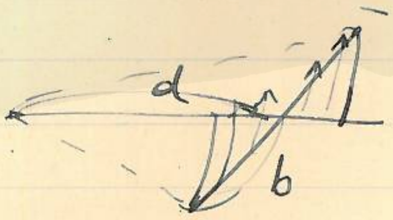
$$\overline{W}_m = \frac{1}{8\pi}\iiint H^2 dv$$

$$= \frac{J'}{\cancel{\pi}}\iint H\,df$$



$$H = \frac{2J}{r}$$

$$\iint H\,df = 2J\cdot 2a\int_{r_1}^{r_2}\frac{dr}{r}$$



$$= 2aJ\log\frac{r_2^2}{r_1^2}$$

$$r_1^2 = b^2+d^2-2bd\cos\phi$$

$$r_2^2 = b^2+d^2+2bd\cos\phi.$$

$$\overline{W}_m = \frac{2aJJ'}{\cancel{4\pi}}\log\frac{b^2+d^2+2bd\cos\phi}{b^2+d^2-2bd\cos\phi}.$$

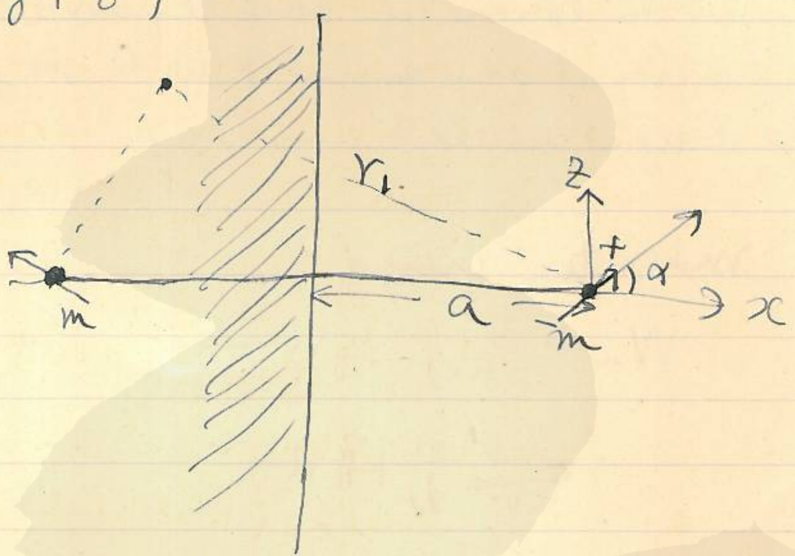
$$-\frac{\partial V}{\partial\phi} = \frac{\partial\overline{W}_m}{\partial\phi} = 2aJJ'\left\{\frac{-2bd\sin\phi}{b^2+d^2+2bd\cos\phi}\right.$$

$$\left.-\frac{2bd\sin\phi}{b^2+d^2-2bd\cos\phi}\right\} = \frac{8JJ'abd\sin\phi(b^2+d^2)}{b^4+d^4-2b^2d^2\cos^2\phi.2\phi}.$$

補部、

(17) The whole of the space on the negative side of the yz-plane is filled with soft iron, and a magnetic of moment m at the point (a, 0, 0) points in the direction (cos α, 0, sin α). Prove that the magnetic potential at the point inside the iron is

$$\frac{2m}{1+\mu}\;\frac{z\sin\alpha-(a-x)\cos\alpha}{\{(a-x)^2+y^2+z^2\}^{\frac{3}{2}}}$$



point change e があつた時の
~~ma~~ el. field は (問題 ある)
$x<0$ に於ては

$$\varphi_2=\frac{e''}{r_1}$$

故 $e''=\frac{2}{1+\varepsilon}$

故に 今の場の potential は

$$\frac{2m}{1+\mu}(m\,\mathrm{grad})_{\text{source}}\frac{1}{r_1}=-\frac{2m}{1+\mu}(m\,\mathrm{grad})\frac{1}{r_1}$$

$$r_1=\sqrt{(x-a)^2+y^2+z^2}$$

$$(m\,\mathrm{grad})\frac{1}{r_1}=-\frac{1}{2}\frac{2(x-a)\cos\alpha+z\sin\alpha}{r_1^3}$$

$$Z=\{R^2+(L\omega)^2\}\frac{R+i\{L\omega-\cancel{2}\,\omega C(R^2+L^2\omega^2)\}}{R^2+\omega^2(L-C(R^2+L^2\omega^2))^2}$$

$$=\frac{R}{1-2\omega^2LC+\omega^2C^2(R^2+L^2\omega^2)}$$

$$+\,i\,\frac{L\omega(1-\omega^2LC)-\omega^2R^2C}{1-2\omega^2LC+\omega^2C^2(R^2+L^2\omega^2)}$$

(18) i) In a circuit ~~with~~ with resistance R, induct~~ance~~ion L and capacity C ~~are~~ inserted in parallel. ~~A~~ alternating current of frequency $\frac{\omega}{2\pi}$ flows) Show that the impedance ~~has its~~ ~~becomes~~ takes its maximum value R, when $\omega = \frac{1}{\sqrt{LC}}$.

ii) ~~Show also that~~ An alternating current of frequency $\frac{\omega}{2\pi}$ flow in a circuit with R, L in series and C in parallel with R, L. Show that it is equivalent to a circuit with the resistance

$$\frac{R}{(1-\omega^2LC)^2+(\omega RC)^2}$$

and the reactance

$$\frac{\omega L(1-\omega^2LC)-\omega R^2C}{(1-\omega^2LC)^2+(\omega RC)^2}.$$

i) $$\frac{1}{Z} = \left\{\frac{1}{R} - \frac{1}{\frac{i}{\omega C}} + \frac{1}{iL\omega}\right\}$$

$$Z = \frac{\frac{1}{R} - i\left(\omega C - \frac{1}{L\omega}\right)}{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{L\omega}\right)^2}.$$

Impedance $$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{L\omega}\right)^2}}$$

ii) $$\frac{1}{Z} = \frac{1}{R+iL\omega} - \frac{1}{\frac{i}{\omega C}} = \frac{R - iL\omega + i\omega C(R^2+L^2\omega^2)}{R^2+(L\omega)^2}$$

$$Z = \frac{1}{R^2 + \cancel{L\omega + \omega}\,\omega^2(L^2 - R^4C^2 + L^4}$$

(19) The current through a submarine cable leaks to the sea at every point owing to the imperfection of insulation. Let the resistance of the cable per unit length be r and that of the leakage be R.
Show that the potential distribution along the cable is expressed by

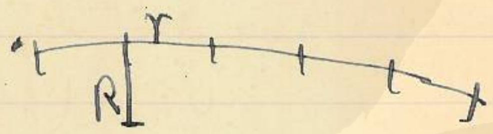
$$\varphi = A \cosh \sqrt{\frac{r}{R}}\, x + B \sinh \sqrt{\frac{r}{R}}\, x,$$

where x is the distance from a fixed point along the cable.

(i) 講義の limiting case

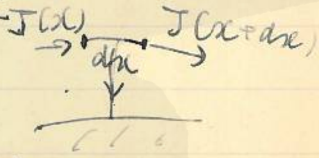
$$V = A \cosh k\alpha + B \sinh k\alpha$$

$$k\alpha = k\left(1 + \frac{r}{2R}\right).$$



unit length の resistance が R なら unit length からの leak の current は $\frac{V}{R}$、dx の間では $\frac{V dx}{R}$.

(ii) $$J(x+dx) - J(x) = -\frac{V dx}{R}$$



$$\frac{dJ}{dx} = -\frac{V}{R}$$

$$J r\, dx = -dV. \qquad \frac{dV}{dx} = -J r$$

$$\frac{d^2V}{dx^2} = \frac{r}{R} V.$$

$$V = A \cosh \sqrt{\frac{r}{R}}\, x + B \sinh \sqrt{\frac{r}{R}}\, x,$$

考え

(20) A charged particle with charge e and rest mass m passes perpendicular to the uniform magnetic field of intensity H and is deflected to form an orbit of radius of curvature ρ.

Show that the velocity of the particle is given by

$$v = \frac{H\rho c}{\sqrt{(H\rho)^2+(\frac{mc^2}{e})^2}}$$

and the kinetic energy is given by ~~eV, where~~

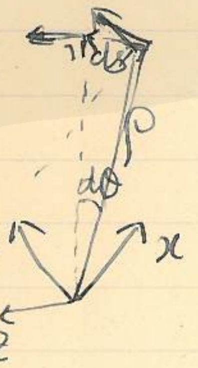
$$\cancel{eV=}$$

$$T = \sqrt{(e\cancel{\xi}H\rho)^2 + \cancel{mc^2}\, mc^2} - mc^2,$$

$$\rho = \left|\frac{ds}{d\theta}\right| = \left|\frac{\cancel{v}\frac{ds}{dt}}{\frac{d\theta}{dt}}\right|. \qquad \frac{ds}{dt} = v.$$

$$\frac{d\theta}{dt} = \frac{1}{v^2}(\dot v_x v_y - \dot v_y v_x)$$

$$= \frac{v^3}{|\dot v_x v_y - \dot v_y v_x|}$$



$$\dot v_x = \frac{e}{m_0 c}\sqrt{1-\frac{v^2}{c^2}}\cdot v_y H \Big) \qquad \rho = \frac{mc}{eH}\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\dot v_y = \frac{-e}{mc}\sqrt{1-\frac{v^2}{c^2}}\,.$$

~~$\Big(\because\ v_x \frac{d}{dt}\frac{mv}{\sqrt{1-\frac{v^2}{c}}}$~~

$$\Big(\because\ \frac{d}{dt}\left(\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}\right) = -\frac{e}{c}[vH] \qquad v\frac{d}{dt}\left(\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}\right) = 0.$$

$$\cancel{\frac{d}{dt}\left(\frac{mv^2}{\sqrt{1-\frac{v^2}{c^2}}}\right) =} \frac{\frac{d}{dt}\left(\frac{1}{2}mv^2\right)}{\sqrt{\ \ ''}} + \cancel{v}\frac{mv^2\cdot\frac{1}{c^2}\frac{d(v^2)}{dt}}{\sqrt{1-\frac{v^2}{c}}^3} = 0.$$

$$\frac{dv^2}{dt} = 0 \Big)$$

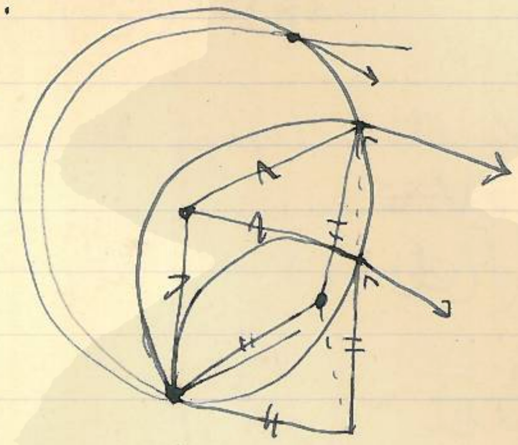
$$\therefore\ \frac{v}{c} = \frac{H\rho}{\sqrt{(H\rho)^2+(\frac{mc^3}{e})^2}}, \qquad \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 = $$

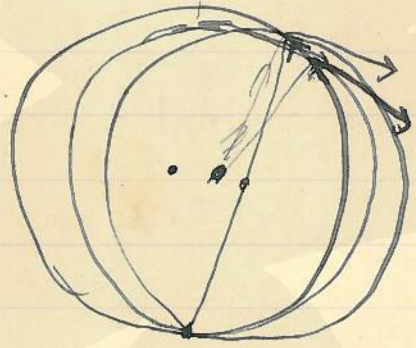
(21) An electron source is placed at a point P on the periphery of the circle of radius r_0 in which a uniform magnetic field H is applied perpendicular to the plane of the circle.

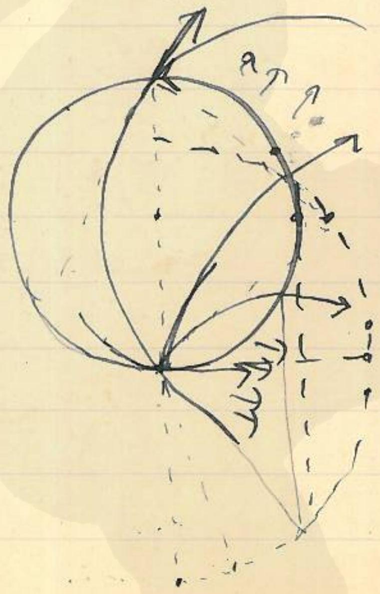
Show that the electrons, which has a certain velocity parallel to the plane of the ~~coil~~ circle are deflected all to the same direction.

$$\rho = \frac{mc}{eH}\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= r_0$$







through the centre of the hole, z^2

If we take the cylindrical coordinates (r, θ, z) with the z-axis perpendicular to the plate

⑯

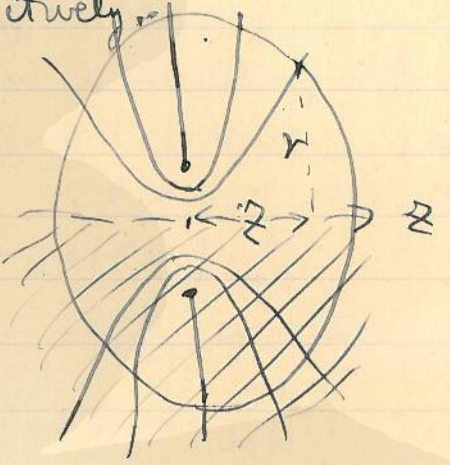
(22) ~~Find the~~ An infinitely thin conducting plate has a circular hole of ~~radius~~ unit radius. ~~Show that~~ the ~~electrostatic~~ potential at any point (r, θ, z) is given by

$$\varphi = -\frac{E_1+E_2}{2} z - \frac{E_1-E_2}{\pi} z \left(\arctan\mu - \frac{1}{\mu}\right),$$

~~when with~~

where $$\mu^2 = \frac{(r^2+z^2-1) \pm \sqrt{(r^2+z^2-1)^2+4z^2}}{2}$$

with $\mu \gtrless 0$ for $z \gtrless 0$ and E_1, E_2 are the uniform field in the z-direction for $z \gtrless 0$ respectively.



Elliptic Coordinate.

$$\left.\begin{aligned} &\cdot(\mu^2+1) \quad \frac{r^2}{\mu^2+1} + \frac{z^2}{\mu^2} = 1. \\ &-(1-\nu^2) \quad \frac{r^2}{1-\nu^2} - \frac{z^2}{\nu^2} = 1. \end{aligned}\right\}$$

$\nu = 0$: $z = 0$. $r \geq 1$.

~~$\left(\because \left(\frac{\nu^2}{1-\nu^2} + \frac{\mu^2}{1+\mu^2}\right) r^2 = \nu^2+\mu^2 \right.$~~

~~$r^2 = \frac{\mu^2+\nu^2}{\mu^2 + \frac{(1+\mu^2)\nu^2}{1-\nu^2}} (1+\mu^2) \geq 1$ for $\nu = 0)$~~

$\{(\mu^2+1)\nu^2+\mu^2(1-\nu^2)\}z^2$
$= \{(\mu^2+1)$ ~~[illegible]~~ $\nu^2(1-\nu^2)\}\mu^2\nu^2$
$= (\mu^2+\nu^2)\mu^2\nu^2$

$z > 0$; $\mu \geq 0$. ~~$\nu \geq 1$~~ ; $-1 \leq \nu \leq 1$.

~~$z < 0$; $\mu < 0$.~~ μ^2, ν^2 を与えれば r^2, z^2 の一つの値が決まる.

$\because$ $r = \sqrt{\mu^2+1}\sqrt{1-\nu^2}$ $z^2 = \mu^2\nu^2$; $z = \mu\nu$

$1 \geq \nu \geq 0$ $(r \geq 0)$; $\mu > 0$ $z > 0$; $\mu < 0$ $z < 0$.

$$dr = \frac{\mu\sqrt{1-\nu^2}\,d\mu}{\sqrt{\mu^2+1}} - \frac{\nu\sqrt{\mu^2+1}\,d\nu}{\sqrt{1-\nu^2}}$$

$$dz = \nu\, d\mu + \mu\, d\nu.$$

$$dr^2 + dz^2 = \frac{\mu^2+\nu^2}{1+\mu^2} d\mu^2 + \frac{\mu^2+\nu^2}{1-\nu^2} d\nu^2$$

$$\Delta\varphi = \frac{1}{\mu^2+\nu^2}\left\{ \frac{\partial}{\partial\mu}(\mu^2+1)\frac{\partial\varphi}{\partial\mu} + \frac{\partial}{\partial\nu}(1-\nu^2)\frac{\partial\varphi}{\partial\nu} \right\}$$
$$= 0$$

$$\varphi = M(\mu)N(\nu)$$

$$\frac{\partial}{\partial\mu}\left\{(\mu^2+1)\frac{\partial M}{\partial\mu}\right\} = \lambda M.$$

$$\frac{\partial}{\partial\nu}\left\{(\nu^2-1)\frac{\partial N}{\partial\nu}\right\} = \lambda N.$$

$z=\pm\infty$ には　$\mu=\pm\infty$ が対応。（この場合 ν≠1 であるから ν も infinite）（$\nu=1$ には　$r=0$ が対応）そこでは field は z-方向に向い potential は $z=\mu\nu$ に比例する筈である。　$N\propto\nu$.

$$\frac{d}{d\mu}\left\{(\mu^2+1)\frac{dM}{d\mu}\right\} = 2M.$$

この式の particular solution は　$M=\mu$.

しかし．これは ~~[illegible]~~ ~~ν=0 or equipo~~ 等の面上の solution ではない．

$M=C\mu$ とおくと，

$$\frac{d}{d\mu}\left\{(\mu^2+1)C+\mu(\mu^2+1)\frac{dC}{d\mu}\right\} = 2C\mu.$$

$$2(\mu^2+1)\frac{dC}{d\mu}+2\mu^2\frac{dC}{d\mu}+\mu(\mu^2+1)\frac{d^2C}{d\mu^2}=0$$

$$\frac{dC}{d\mu} = \exp\left\{-\int\frac{2(2\mu^2+1)}{\mu(\mu^2+1)}d\mu\right\}$$

$$= \exp\left\{-\int\left(\frac{2}{\mu}+\frac{2\mu}{\mu^2+1}\right)d\mu\right\}$$

$$= C\,\cancel{\exp}\left\{-2\log\mu - \cancel{2}\log(\mu^2+1)\right\}$$

$$\cancel{= C\left(\frac{1}{\mu^2}\frac{1}{(\mu^2+1)}\right)}$$

$$= \frac{C}{\mu^2(\mu^2+1)} = C\left\{\frac{1}{\mu^2}-\frac{1}{\mu^2+1}\right\}$$

$$c = C\left(\frac{1}{\mu}-\arctan\mu\right)+C'$$

$$\varphi = A\mu\nu + B\nu(\mu\arctan\mu - 1)\ \cancel{+C}$$

$$= Az + Bz\left(\arctan\mu-\frac{1}{\mu}\right)$$

$z\to+\infty$ ~~$z\to\infty$~~: $\mu\to+\infty$　　$\varphi\to\{A+B(\frac{\pi}{2})\}z = -E_1 z$

~~$z\to\infty$~~ $z\to-\infty$: $\mu\to-\infty$　　$\varphi\to\{A-B(\frac{\pi}{2})\}z = -E_2 z$　$\left(\begin{matrix}\nu=0\\ \varphi=0\end{matrix}\right)$

$z\to-\infty$　　$A=-\frac{1}{2}(E_1+E_2)$,　$B=-\frac{1}{\pi}(E_1-E_2)$

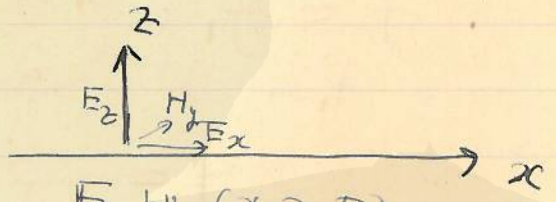
(23) One side of a plane is vacuum and the other side is a conductor with the conductivity σ, dielectric constant ε and permeability μ. Show that the ~~electron~~ velocity of propagation of the electromagnetic wave of frequency ν along the plane is given approximately by

$$c\left(1+\frac{\varepsilon\mu\;\cancel{\mu}^2+1-)\frac{\mu^2}{4}}{8\sigma^2}\nu^2\right)$$

if $\frac{\sigma}{\nu}$ is large compared with $\cancel{\varepsilon\mu}$, Show also that the amplitude of the wave decreases to e^{-1}, when it propagates a distance

$$\frac{2c\sigma}{4\pi\mu\nu^2}.$$

$$\left.\begin{aligned}
&\times\ \frac{\partial E_y}{\partial z}=-\frac{\mu}{c}\frac{\partial H_x}{\partial t}\\
&\circ\ \frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}=-\frac{\mu}{c}\frac{\partial H_y}{\partial t}\\
&\times\ \frac{\partial E_y}{\partial x}=-\frac{\mu}{c}\frac{\partial H_z}{\partial t}\\
&\times\ \frac{\partial H_x}{\partial x}+\frac{\partial H_z}{\partial z}=0
\end{aligned}\right\}$$



$\mathbb{E}, \mathbb{H}\ (x, z, t)$

$\circ\ (E_x, E_z, H_y)$

$\times\ (E_y, H_x, H_z)$

$$\left.\begin{aligned}
&\circ\ -\frac{\partial H_y}{\partial z}=\frac{\varepsilon}{c}\frac{\partial E_x}{\partial t}+\frac{4\pi\sigma}{c}E_x\\
&\times\ -\frac{\partial H_z}{\partial x}+\frac{\partial H_x}{\partial z}=\frac{\varepsilon}{c}\frac{\partial E_y}{\partial t}+\frac{4\pi\sigma}{c}E_y\\
&\circ\ \frac{\partial H_y}{\partial x}=\frac{\varepsilon}{c}\frac{\partial E_z}{\partial t}+\frac{4\pi\sigma}{c}E_z\\
&\circ\ \frac{\partial E_x}{\partial x}+\frac{\partial E_z}{\partial z}=0
\end{aligned}\right\}$$

$E_x = f(z)e^{\cancel{i(\omega t-kx)}\ i(kx-\omega t)}$

$E_z = g(z)e$ 〃

$H_y = h(z)e$ 〃

$$\left.\begin{aligned}
&\frac{df}{dz}=+ikg+\frac{i\mu\omega}{c}h\\
&\frac{dg}{dz}=-ikf\\
&\frac{dh}{dz}=\left(\frac{\varepsilon\omega i}{c}-\frac{4\pi\sigma}{c}\right)f\\
&-ikh=\left(\frac{\varepsilon\omega i}{c}\pm\frac{4\pi\sigma}{c}\right)g.
\end{aligned}\right\}\ z<0.
\qquad
\left.\begin{aligned}
&\frac{df}{dz}=+ikg+\frac{i\omega}{c}h\\
&\frac{dg}{dz}=-ikf\\
&\frac{dh}{dz}=+\frac{\omega i}{c}f\\
&h=-\frac{\omega}{kc}g.
\end{aligned}\right\}\ z>0.$$

$A = \pm\sqrt{\frac{\sqrt{}+a}{2}}$

$(A+Bi)^2 = (a+bi)$　$B = \pm\sqrt{\frac{\sqrt{}-a}{2}}$

$A^2 - B^2 = a$　　$b>0$: $\begin{smallmatrix}A\\B\end{smallmatrix}$ + 　$\begin{smallmatrix}A\\B\end{smallmatrix}$ −

$2AB = b.$　　$b<0$: $\begin{smallmatrix}A+\\B-\end{smallmatrix}$　$\begin{smallmatrix}-\\+\end{smallmatrix}$

$\sqrt{a^2+b^2} = A^2 + B^2$

$$\left.\begin{aligned} A^2 &= \frac{a+\sqrt{a^2+b^2}}{2} \\ B^2 &= \frac{\sqrt{a^2+b^2}-a}{2} \end{aligned}\right\}$$

~~$z>0.$~~ ~~$f = e^{-\sqrt{k^2-\frac{\omega^2}{c^2}}z}$, $g = -ik f/\sqrt{k^2-\frac{\omega^2}{c^2}}$, $h = +\frac{i\omega}{c} f/\sqrt{k^2-\frac{\omega^2}{c^2}}$~~

~~$z<0.$~~ $z>0$:

$$\left.\begin{aligned} f &= f_0 e^{\lambda_0 z} \\ g &= \frac{-ik}{\lambda_0} f_0 e^{\lambda_0 z} \\ h &= \frac{i\omega}{c\lambda_0} f_0 e^{\lambda_0 z} \end{aligned}\right\}$$

λ_0 : real part negative

$\lambda_0^2 = k^2 - \frac{\omega^2}{c^2}$

$z<0.$

$$f'' = \left(k^2 - \frac{\varepsilon\mu\omega^2}{c^2} - i\frac{4\pi\sigma\mu\omega}{c^2}\right) f$$

$$f = f_1 e^{\lambda_1 z}$$

$$\lambda_1^2 = k^2 - \frac{\varepsilon\mu\omega^2}{c^2} - i\frac{4\pi\sigma\mu\omega}{c^2},$$

λ_1 : real part ~~negative~~ positive.

$$g = \frac{-ik}{\lambda_1} f_1 e^{\lambda_1 z}$$

$$h = \left(\frac{\varepsilon\omega i}{c} - \frac{4\pi\sigma}{c}\right)\frac{f_1}{\lambda_1} e^{\lambda_1 z},$$

$z=0$:　E_x, H_y : conti.

$f_0 = f_1.$　　$\frac{i\omega}{c\lambda_0} f_0 = \left(\frac{\varepsilon\omega i}{c} - \frac{4\pi\sigma}{c}\right)\frac{f_1}{\lambda_1},$

$$\frac{i\omega}{c\lambda_0} = \left(\frac{\varepsilon\omega i}{c} - \frac{4\pi\sigma}{c}\right)\frac{1}{\lambda_1}$$

$$-\frac{\omega^2}{c^2}\left(k^2 - \frac{\varepsilon\mu\omega^2}{c^2} - i\frac{4\pi\sigma\mu\omega}{c^2}\right) = \left(\frac{\varepsilon\omega i}{c} - \frac{4\pi\sigma}{c}\right)^2\left(k^2 - \frac{\omega^2}{c^2}\right)$$

~~$k^2 =$~~

$$\{(4\pi\sigma - \varepsilon\omega i)^2 + \omega^2\} k^2 = \frac{\omega^2}{c^2}\{(4\pi\sigma - \varepsilon\omega i)^2 + \varepsilon\mu\omega^2 + i\,4\pi\sigma\mu\omega\}$$

~~$= \frac{\omega^2}{c^2}\{(4\pi\sigma)^2$~~

$\sigma \gg \varepsilon\omega.$　~~$k^2 \cong \frac{\omega^2}{c^2}\left\{1 - \frac{i\,4\pi\sigma\mu\omega - \varepsilon\mu\omega^2}{(4\pi\sigma - \varepsilon\omega i)^2}\right\}$~~

~~$= \frac{\omega^2}{c^2}\left\{1 - i\frac{4\pi\sigma\mu\omega}{(4\pi\sigma)^2}\left\{1 + \frac{\varepsilon\omega i}{2\pi\sigma}\right\}\right\}$~~

$16\pi^2 \cdot 4\pi\sigma^2$　　~~$= \frac{\omega^2}{c^2}\left\{1 + \frac{\varepsilon\mu\omega^2}{8\pi^2\sigma^2} - i\frac{\mu\omega}{4\pi\sigma}\right\}$~~

$$(1+a+bi)$$

$$(1+A+Bi)^2 = (1+A)^2 - B^2 + 2ABi.$$
$$(1+2A)$$

⑱

$\sigma \gg \omega$;

$$k^2 \simeq \frac{\omega^2}{c^2}\,\frac{1+\dfrac{\varepsilon\mu\omega^2+i4\pi\sigma\mu\omega}{(4\pi\sigma)^2\left\{1-\frac{\varepsilon\omega i}{4\pi\sigma}\right\}^2}}{1+\dfrac{\omega^2}{(4\pi\sigma)^2\left\{1-\frac{\varepsilon\omega i}{4\pi\sigma}\right\}^2}}$$

$$\cong \frac{\omega^2}{c^2}\left\{1+\frac{\varepsilon\mu\omega^2}{(4\pi\sigma)^2}+i\frac{\mu\omega}{(4\pi\sigma)}\left\{1+\frac{2\varepsilon\omega i}{(4\pi\sigma)}\right\}\right\}$$

$$\times\left\{1 - \frac{\omega^2}{(4\pi\sigma)^2}\right\}$$

$$\cong \frac{\omega^2}{c^2}\left\{1-\frac{\varepsilon\mu\omega^2}{(4\pi\sigma)^2}-\frac{\omega^2}{(4\pi\sigma)^2}+i\frac{\mu\omega}{4\pi\sigma}\right\}$$

$$k = \pm\frac{\omega}{c}\left\{1-\frac{1}{2}\frac{\varepsilon\mu\omega^2+\omega^2-\frac{\mu^2}{4}}{(4\pi\sigma)^2}+\frac{\mu\omega}{8\pi\sigma}i\right\}$$

∴ prop. velocity は,

$$\frac{\omega}{R(k)} = c\left\{1+\frac{1}{2}\frac{\left(\varepsilon\mu+1-\frac{\mu^2}{4}\right)}{(4\pi\sigma)^2}\omega^2\right\}$$

$$= c\left\{1+\frac{\left(\varepsilon\mu+1-\frac{\mu^2}{4}\right)}{8\sigma^2}\nu^2\right\}$$

[illegible]
e^{-1} に damp するのは,

$$\left(\frac{\mu\omega^2}{8\pi\sigma c}\right)^{-1} = \frac{2c\sigma}{\pi\mu\nu^2} = \frac{2\sigma}{\pi\mu\nu}\cdot\lambda$$

なる距離である.

$$\lambda_1^2 \cong k^2 - \frac{\omega^2}{c^2}(1-\varepsilon\mu) - i\frac{4\pi\sigma\mu\omega}{c^2}.$$

$$\lambda_1 \cong \pm\left\{\frac{\sqrt{2\pi\sigma\mu\omega}}{c} - i\frac{\sqrt{2\pi\sigma\mu\omega}}{c}\right\}$$

故に $z<0$ に於ては $\dfrac{c}{\sqrt{2\pi\sigma\mu\omega}} = \dfrac{c}{2\pi\sqrt{\sigma\mu\nu}}$ 位の深さで amplitude が e^{-1} になる.

$$\left(\frac{2c\sigma}{\pi\mu\nu^2}\Big/\frac{c}{2\pi\sqrt{\sigma\mu\nu}} \cong \left(\frac{\sigma}{\nu}\right)^{\frac{3}{2}} \gg 1\right)$$

$$\frac{\partial\rho}{\partial t} = \mathrm{div}\, I = \sigma\, \mathrm{div}\, E$$

$$\rho = \mathrm{div}\, E$$

$$\frac{\partial \rho}{\partial t} = \sigma\rho$$

$z > 0$ に於いては、

$$\lambda_0^2 \cong \frac{\omega^2}{c^2}\left(-\frac{\varepsilon\mu\omega^2}{(4\pi\sigma)^2} - \frac{\omega^2}{(4\pi\sigma)^2} + \frac{i\mu\omega}{4\pi\sigma}\right)$$

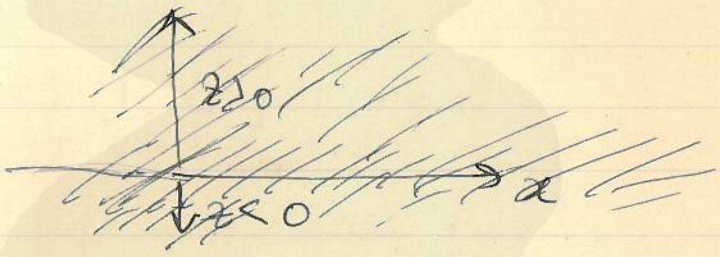
$$\lambda_0 \cong \pm\sqrt{\frac{\mu\omega}{4\pi\sigma}}\cdot\frac{\omega}{c}\{1+i\}$$

故に $z>0$ に於いて、amplitude が e^{-1} になる距離は

$$\sqrt{\frac{8\pi\sigma}{\mu\omega}}\cdot\frac{c}{\omega} = \sqrt{\frac{\sigma}{\mu\nu}}\cdot\frac{c}{\pi\nu}$$

$$\frac{\sqrt{\frac{\sigma}{\mu\nu}}\cdot\frac{c}{\pi\nu}}{\frac{2c\sigma}{\pi\mu\nu^2}} = \frac{1}{2}\sqrt{\frac{\mu\nu}{\sigma}} \ll 1$$

$z>0$, $g, h \gg f$.



σ 小:

$$+\frac{\omega^2}{c^2}\left(k^2 - \frac{\varepsilon\mu\omega^2}{c^2}\right) = +\left(\frac{\varepsilon\omega}{c}\right)^2\left(k^2 - \frac{\omega^2}{c^2}\right)$$

$$(\varepsilon^2-1)\, k^2 = \varepsilon\mu(\varepsilon-\mu)\frac{\omega^2}{c^2}$$

$$\left(\frac{\omega}{k}\right)^2 \cong c^2\frac{\varepsilon^2-1}{\varepsilon(\varepsilon-\mu)}.$$

$\mu=1$: $\cong c^2\frac{(\varepsilon+1)}{\varepsilon}. > c^2$

$$k = \frac{\sqrt{\varepsilon}}{\sqrt{\varepsilon+1}}\left(\frac{\omega}{c}\right)$$

$$\varepsilon^2\left(k^2 - \frac{\omega^2}{c^2}\right)$$

$$\left(k^2 - \frac{\varepsilon\mu\omega^2}{c^2}\right) =$$

σ dielectrics の boundary に沿ふ wave の propag.

$\mu=1$;

$$\lambda_0^2 = \frac{\varepsilon(\varepsilon-\mu)}{\varepsilon^2-1} - \frac{1}{\varepsilon+1}\left(\frac{\omega}{c}\right)^2$$

~~ε=1~~. $\lambda_0 = \pm\frac{i}{\sqrt{\varepsilon+1}}\left(\frac{\omega}{c}\right).$

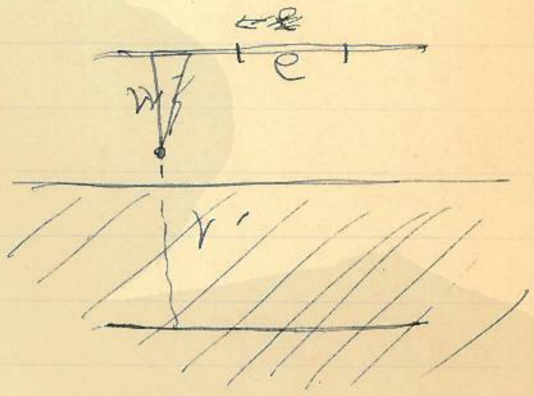
$$\lambda_1^2 = \left\{\frac{\varepsilon}{\varepsilon+1} - \varepsilon\right\}\left(\frac{\omega}{c}\right)^2 = \frac{-\varepsilon^2}{\varepsilon+1}\left(\frac{\omega}{c}\right)^2$$

$$\lambda_1 = \pm\frac{i\varepsilon}{\sqrt{\varepsilon+1}}\left(\frac{\omega}{c}\right).$$

即ち之は dielectric に沿ふて進む。之に加へて refracted 及 reflect される波もある筈。

(4) Show that the capacity per unit length of the infinitely long wire of circular cross section of radius ~~b~~, at height h parallel to the surface of the ~~stretched~~ placed earth, is given by

$$\frac{1}{2\log\frac{2h}{a}}.$$



$$\varphi = 2e\log\frac{r'}{r}$$

$r=a \quad r'=2h$

$$\varphi = 2e\log\frac{2h}{a}$$

$$-\int\frac{1}{4\pi}\frac{\partial\varphi}{\partial r}r\,d\theta = \frac{1}{2}\cdot 2e = e.$$

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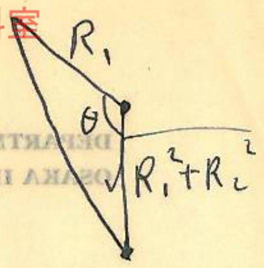
(5) Two parrallel plates, each of area S, are insulated by two layers of dielectric constants ~~ε,~~ ε_1, ε_2. The thickness of each layer is a and b respectively.
~~Find the capacity~~ Show that the capacity of this condensor is given by

$$C = \frac{S}{4\pi\left(\frac{a}{\varepsilon_1} + \frac{b}{\varepsilon_2}\right)}$$

$$\varphi = \cancel{\frac{e}{\varepsilon_1} E a + \frac{e}{\varepsilon_2}} E b = E a + E' b$$
$$= 4\pi e\left(\frac{a}{\varepsilon_1} + \frac{b}{\varepsilon_2}\right)$$
$$\angle\ E = 4\pi e/\varepsilon_1$$

$$\frac{VR_2^2}{2} \quad \frac{1}{\sqrt{''}} \left\{ \frac{\sqrt{R_1 + \sqrt{''}} - R_2}{\sqrt{R_1 + \sqrt{''}}} \right\}$$

$$\cdot\ \frac{2\pi V}{4\pi R_1}\iint^{\pi}\left\{1-\left(\frac{R_2}{r_2}\right)^3\right\} R_1^2\,\cancel{d\theta}\,\sin\theta\, d\theta$$



$$\cos\theta = \frac{R_1}{\sqrt{R_1^2+R_2^2}}$$

$$= \frac{\cancel{2\pi V}}{}\ \frac{V}{2R_1}\int_{-1}^{\frac{R_1}{\sqrt{R_1^2+R_2^2}}} \cancel{dx}\left(1-\frac{R_2^3}{\sqrt{2R_1^2+R_2^2-2R_1\sqrt{R_1^2+R_2^2}\cdot x}^{\,3}}\right)dx$$

$$= \frac{VR_1\cancel{V}}{2\ \cancel{2R_1}}\left\{\frac{R_1}{\sqrt{R_1^2+R_2^2}} + \cancel{R_1}\,1\right\}$$

$$\cancel{+}\ -\ \frac{VR_2^3 R_1}{2\cancel{R_1}}\left\{\frac{1}{\sqrt{2R_1^2+R_2^2-2R_1\sqrt{R_1^2+R_2^2}\cdot x}\cdot \cancel{R_1}\sqrt{R_1^2+R_2^2}}\right\}_{-1}^{\frac{R_1}{\sqrt{''}}}$$

$$= \frac{\cancel{\pi}VR_2^3\cancel{R_1}}{2\cancel{R_1}\sqrt{R_1^2+R_2^2}}\left\{\frac{1}{\cancel{R_1}R_2\cancel{\sqrt{R_1^2+R_2^2}}} - \frac{1}{\cancel{R_1\sqrt{R_1^2}}\ R_1+\sqrt{R_1^2+R_2^2}}\right\}$$

$$= \quad\Big\}\ \frac{1}{R_2} - \Big\{\ \frac{1}{R_1}$$

$$\frac{V}{2}R_1 + \frac{V}{2}\sqrt{R_1^2+R_2^2}\cdot - \frac{VR_2^3}{2R_1\sqrt{R_1^2+R_2^2}\,(R_1+\sqrt{''})}$$

$$+\ \frac{V}{2}R_2 + \frac{V}{2}\sqrt{''} - \frac{VR_1^3}{2R_2\sqrt{''}\,(R_2+\sqrt{''})}$$

$$= \quad '' \quad '' \quad - \frac{V}{2R_1R_2\sqrt{''}\ \sqrt{R_1+\sqrt{''}}\cdot\sqrt{R_2+\sqrt{''}}}\left\{R_2^4(R_2+\sqrt{''})+R_1^4(R_1+\sqrt{''})\right)$$

(13) a) Mit welcher Kraft (pro cm^2) ziehen sich die Belegungen eines Plattenkondensators an bei 1000 Volt Spannung ($300^{\text{Volt}} = 1$ E.S.E.) und 1 mm Plattenabstand?

b) Wie gross ist die Kraft, wenn man den Kondensator nach erfolgter Aufladung von der Batterie (1000) trennt und mit Petroleum ($\varepsilon =$ 2,0) füllt?

c) Wie gross ist die Kraft, wenn man den Kondensator zuerst mit Petroleum füllt und dann ladet?

$$2\pi w^2 = \frac{E^2}{8\pi} = \frac{1\cancel{333}^2}{8\pi}\left(\frac{100}{3}\right)^2 = 44{,}3 \text{ dyn/cm}^2 = K$$

$$\cancel{\frac{E^2}{8\pi}}\ \frac{2\pi w^2}{\varepsilon} = \frac{1}{16\pi}\left(\frac{100}{3}\right)^2 = \frac{K}{2}$$

$$\frac{2\pi w^2}{\varepsilon} = \cancel{\frac{D^2}{\varepsilon\cdot 8\pi} =}\ \frac{\varepsilon E^2}{8\pi} = \frac{1}{4\pi}\left(\frac{100}{3}\right)^2 = 2K.$$

~~Problems in Two Dimensions~~

~~Conformal Representation~~

(14) (a) Wie groß ist die Kraft, die auf die Platten des in der vorigen Aufgabe gegebenen Plattenkondensators wirkt, wenn man den Kondensator nach erfolgter Aufladung von der Batterie trennt und ~~f~~ eine 1mm dicke, aber die Belegungen gerade noch nicht berührende Paraffinplatte ($\varepsilon = 2{,}0$) einschiebt?

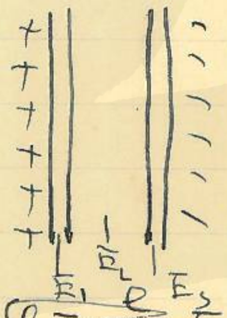
b) Wie groß ist die Kraft, wenn man die Paraffinplatte zuerst einschiebt und dann ladet?

$$2\pi w^2 = K,$$

$$2\pi w^2 = \frac{1}{8\pi}\left(\frac{\varepsilon V}{d}\right)^2$$

$$= \frac{\varepsilon^2}{8\pi} E^2$$

$$= 4K,$$



$$\varphi = \frac{e}{\varepsilon_1} E$$

$$E_1 = 4\pi w$$

$$E_2 = \frac{4\pi w}{\varepsilon}$$

$$E_3 = 4\pi w$$

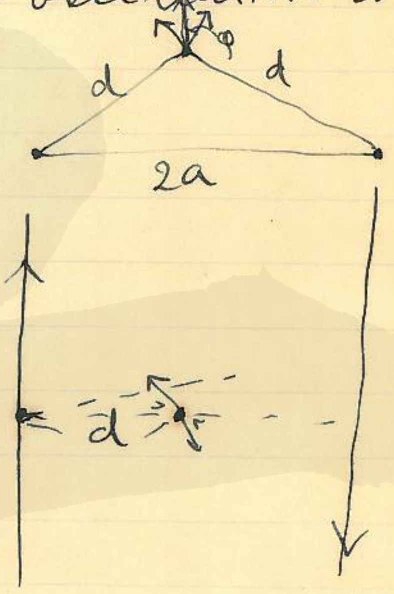
$$V = \frac{4\pi w}{\varepsilon} d$$

十月廿七日

(5) Two parallel straight infinite wires convey equal currents of strength J in opposite directions, their distance apart being $2a$. A ~~magne~~ small magnet of strength μ and moment of inertia A is free to turn about a pivot at its centre, distance d from each of the wires.

Show that the period of a small oscillation is that of a pendulum of length l

$$l = \frac{cAgd^2}{4aJ\mu}$$



$$H = \frac{2J}{cd^2}\cdot 2a$$

$$\theta = \frac{\pi}{2}$$

$$\cancel{K} = \cancel{H\mu}\ [\mu H]$$

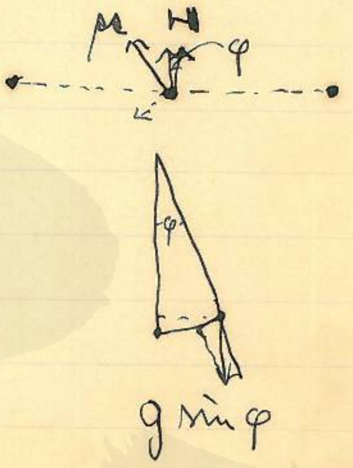
$$K\cancel{F} = \mu H \sin\cancel{\theta}\varphi$$

$$= \frac{2J\cdot 2a}{cd^2}\cancel{H}\mu \sin\varphi = K_0\sin\varphi$$

$$\cancel{I}\,A\cdot\ddot{\varphi} = -K_0\sin\varphi\ \cancel{\frac{2J\cdot 2a}{cd^2}\mu\sin\varphi}$$

$$l\cdot\ddot{\varphi} = -g\sin\varphi.$$

$$l = \frac{A}{K_0}g = \frac{Acd^2\cancel{J}g}{4aJ\cdot\mu}$$



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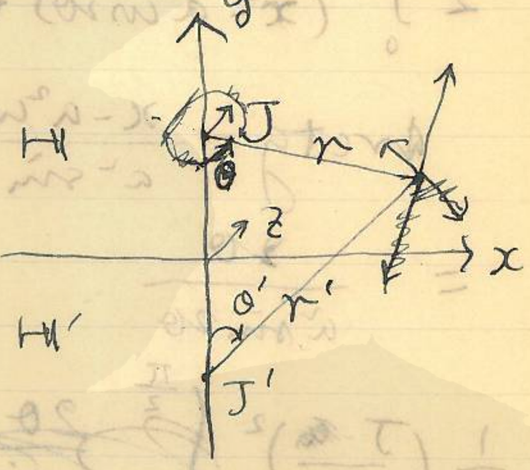
(17) An infinitely long straight wire conveys a current and lies in front of and parallel to an infinite block of soft iron bounded by a plane face. Find the magnetic potential at all points, and the force which tends to displace the wire.

$$H_x = \frac{2J}{cr}\cos\theta - \frac{2J}{cr'}\cos\theta'$$

$$H_y = -\frac{2J}{cr}\sin\theta - \frac{2J'}{cr'}\sin\theta'$$

$$H_x' = \frac{2J''}{cr}\cos\theta$$

$$H_y' = -\frac{2J''}{cr}\sin\theta \quad ,,$$



$y=0$; $H_x = H_x'$ $\qquad H_y = \mu H_y'$

$r = r'$, $\theta = \theta'$; $J - J' = J''$

$-J - J' = -\mu J''$

$$J'' = \frac{2J}{\mu+1} \qquad J' = \frac{\mu-1}{\mu+1}$$

$\mu \gg 1$; $J'' = \frac{2J}{\mu}$, $J' = J$.

これが field eq を満足することは.

$\text{~~curl~~}\oint H\,ds = 0$ or J.

$\iint \mathfrak{B}\,df = 0$

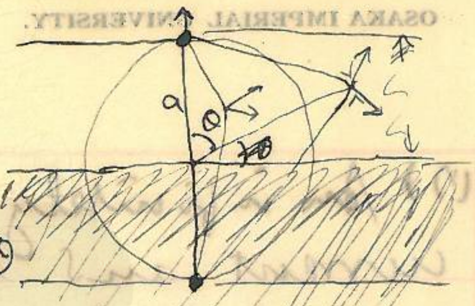
からわかる.

per unit length

$$\frac{1}{8\pi}\iiint H_3^2\,dV = \frac{1}{8\pi}\left(\frac{2J\,2a}{c}\right)^2 \iint_{0\ -\frac{\pi}{2}}^{\infty\ \frac{\pi}{2}} \frac{r\,dr\,d\theta}{r_1^2 r_2^2}$$

$\int H_1 \cdot H_2\, dV$

+ curl



$$r_1^2 r_2^2 = (r^2+a^2-2ar\cos\theta) \times (r^2+a^2+2ar\cos\theta)$$

$$= r^4+a^4-2a^2r^2\cos 2\theta$$

$$= (r^2-a^2\cos 2\theta)^2 + a^4\sin^2 2\theta$$

$$\cancel{r^2 = x \quad \frac{1}{2}\int_0^\infty \frac{dx}{\sqrt{x^2+a^4-2a^2x\cos\theta}}}$$

$$\frac{1}{2}\int_0^\infty \frac{dx}{(x-a^2\cos 2\theta)^2+a^4\sin^2 2\theta} = \frac{1}{2}\,\text{arctg}\frac{1}{a^2\sin 2\theta}$$

$$\left.\text{arctg}\,\frac{x-a^2\cos 2\theta}{a^2\sin 2\theta}\right|_0^\infty = \frac{1}{2a^2\sin 2\theta}\left\{\frac{\pi}{2}+\left(\frac{\pi}{2}-2\theta\right)\right\}$$

$$= \frac{2\theta}{a^2\sin 2\theta}$$

$$\frac{1}{\pi}\left(\frac{J}{c}\right)^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2\theta\, d(2\theta)}{\sin 2\theta}$$ " : indep of a

---

$$\frac{\mu}{8\pi}\iiint H'^2\,dV = \frac{\mu}{8\pi}\cdot\left(\frac{2J}{\mu c}\right)^2 \iint_{0\ -\frac{\pi}{2}}^{\infty\ \frac{\pi}{2}} \frac{r\,dr\,d\theta}{r^2+a^2+2ar\cos\theta}$$

$$\int_0^\infty \frac{d\theta}{a+b\cos\theta} = \frac{1}{\sqrt{a-b}}\,\text{arctg}\left\{\sqrt{a-b}\;\text{tg}\frac{\theta}{2}\right\}$$

$a = r^2+a^2$

$b = 2ar$

$\sqrt{\frac{a-b}{a+b}} = \frac{r-a}{r+a}$

$$\cancel{= \frac{\mu}{8\pi}\left(\frac{2J}{\mu c}\right)^2 \int_0^\infty r\,dr \cdot \frac{1}{r^2-a^2}\,\text{arctg}\left.\left\{\frac{r-a}{r+a}\,\text{tg}\frac{\theta}{2}\right\}\right|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}$$

$$\cancel{= \frac{1}{\mu\pi}\left(\frac{J}{c}\right)^2 \int_0^\infty \frac{r\,dr}{r^2-a^2}\,\text{arctg}\,\frac{r-a}{r+a}}$$

$$= \text{const}\,(\infty + \frac{J^2}{2\mu c^2}\log a$$

(12) ~~In a circuit with~~ Eine Luftdrosselspule von 0,3 Henry Selbstinduktivität und 20 Ohm Wirkwiderstand wird an eine Wechselspannung ~~, deren Effektivwert~~ ~~2~~ von 220 Volt effektiv bei 50 Hertz angeschlossen. Welche Wärmemenge (in cal) wird minutlich in der Spule entwickelt?

Spule

$$\frac{J_0^2R}{2} = \frac{V_0^2}{2}\cdot\frac{R}{R^2+\left(\frac{L\omega}{c^2}\right)^2}$$

$$= (220)^2\cdot\frac{20}{20^2+(2\pi\cdot\{0.3\times 50)^2}$$

~~watt Joule~~
~~Watt.~~ Joule/sec

$$\frac{V_0}{\sqrt{2}} = 220\ \text{volt}$$

$$\cancel{= 1549.\ \text{joule/sec}}$$

$$\cancel{= \frac{1550\times 60}{4.2}\ \text{cal}}/\text{min}$$

$$= (\quad)\times\frac{60}{4.2}\ \text{cal/min}$$

$$= 1490^{2}\ \text{cal/min}$$

E.S.U.	Practical Unit
V	$cV\cdot 10^{-8}$ Volt
R	$c^2R\cdot 10^{-9}$ ohm
L	$L\cdot 10^{-9}$ Henry

~~$(cV\cdot 10^{-8})^2$~~ ~~$c^2R\cdot 109$~~

~~c^2~~

$$\left(\frac{10^8\cdot V}{c}\right)^2\cdot\frac{\frac{10^9R}{c^2}}{\left(\frac{10^9R}{c^2}\right)^2+\left(\frac{10^9\cdot L\omega}{c^2}\right)^2}\ \text{erg/sec} = \frac{V^2R}{R^2+L^2\omega^2}\cdot 10^7\ \text{erg}/\text{sec}$$

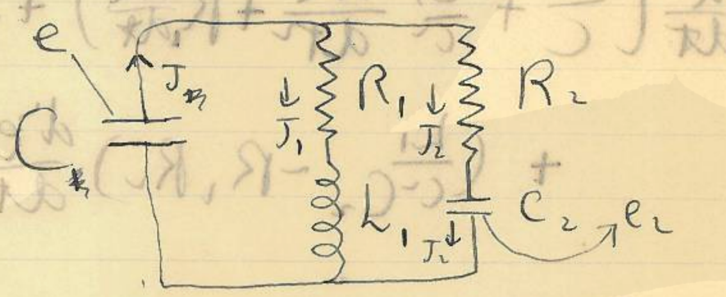
Joule ~~Joule~~

(13) A condensor of capacity C_1 is discharged through two circuits, one of resistance R_1 and self induction L_1, and the other of resistance R_2 and ~~self induction~~ containing a condensor of capacity C_2. Prove that if e is the charge on the first condensor at any time,

$$\frac{L_1}{c^2}R_2\frac{d^3e}{dt^3}+\left(\frac{L_1}{c^2C}+\frac{L_1}{cC_2}+R_1R_2\right)\frac{d^2e}{dt^2}$$
$$+\left(\frac{R}{C}+\frac{R_1}{C_2}+\frac{R_2}{C}\right)\frac{de}{dt}+\frac{e}{CC_2}=0.$$

$$-\frac{de}{dt}=J_1+J_2$$

$$-\frac{de_2}{dt}=J_2$$



~~$\frac{e}{C}=(J_1+J_2)\,J_1R_1+L_1$~~

$$\frac{e}{C}-L_1/_{c^2}\frac{dJ_1}{dt}=J_1R_1\rightarrow\frac{e}{C}+\frac{L_1}{c^2}\left(\frac{d^2e}{dt^2}-\frac{d^2e_2}{dt^2}\right)+R_1\left(\frac{de}{dt}-\frac{de_2}{dt}\right)=0$$

$$\frac{e}{C}+\quad e_2/C_2=J_2R_2\rightarrow\frac{e}{C}+\frac{e_2}{C_2}+R_2\frac{de_2}{dt}=0$$

$$\frac{1}{C_2}\quad\frac{e}{C}+\frac{L_1}{c^2}\frac{d^2e}{dt^2}+R_1\frac{de}{dt}-\frac{L_1}{c^2}\frac{d^2e_2}{dt}-R_1\frac{de_2}{dt}=0$$

$$R_1\quad\frac{1}{C}\frac{de}{dt}+R_2\frac{d^2e_2}{dt}+\frac{1}{C_2}\frac{de_2}{dt}=0$$

$$\frac{1}{C_2}\left(\frac{e}{C}+\frac{L_1}{c^2}\frac{d^2e}{dt^2}+R_1\frac{de}{dt}\right)\cancel{-\frac{L_2}{c^2C_2}\frac{d^2e_2}{dt^2}}+\frac{R_1}{C}\frac{de}{dt}$$
$$-\left(\frac{L_1}{c^2C_2}-R_1R_2\right)\frac{d^2e_2}{dt^2}=0$$

$$R_2\frac{d}{dt}\left(\frac{e}{C}+\frac{L_1}{c^2}\frac{d^2e}{dt^2}+R_1\frac{de}{dt}\right)-\frac{L_1}{c^2}\frac{d^3e_2}{dt^3}-R_2\frac{d^2e_2}{dt^2}=0$$

$$\frac{L_1}{c^2}\;\frac{1}{C}\frac{d^2e}{dt^2}\qquad+R_2\frac{d^3e_2}{dt^2}+\frac{1}{C_2}\frac{d^2e_2}{dt^2}=0$$

$$R_2\frac{d}{dt}\left(\frac{e}{C}+\frac{L_1}{c^2}\frac{d^2e}{dt^2}+R_1\frac{de}{dt}\right)+\frac{L_1}{c^2C}\frac{d^2e}{dt^2}$$
$$+\left(\frac{L_1}{c^2C_2}-R_1R_2\right)\frac{d^2e_2}{dt^2}=0$$

$$\therefore\quad\frac{L_1R_2}{c^2}\frac{d^3e}{dt^3}+\left(\frac{L_1}{c^2C}+\frac{L_1}{c^2C_2}+\cancel{\frac{L_1R_1}{c^2}}R_1R_2\right)\frac{d^2e}{dt^2}$$
$$+\left(\frac{R_1}{C}+\frac{R_1}{C_2}+\frac{R_2}{C}\right)\frac{de}{dt}+\frac{1}{CC_2}e=0$$

~~[illegible] n $R_1=0$~~.

(14) Two conductors ABD, ACD are arranged in multiple arc. Their resistance are R_1, R_2 and their coefficients of self- and ~~multip~~ mutual-induction are L_1c^2, L_2c^2 and $L_{12}c^2$. Prove that when placed in series with leads conveying a current of frequency ω, the two circuits produce the same effect as a single circuit whose coefficient of self-induction is

$$\frac{L_2R_1^2+L_1R_2^2+2L_{12}R_1R_2+\omega^2(L_1L_2-L_{12}^2)(L_1+L_2-2L_{12})}{(L_1+L_2-2L_{12})^2\omega^2+(R_1+R_2)^2}$$

and whose resistance is

$$\frac{R_1R_2(R_1+R_2)+\omega^2\{R_1(L_2-L_{12})^2+R_2(L_{12}-L_1)^2\}}{(L_1+L_2-2L_{12})^2\omega^2+(R_1+R_2)^2}$$

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(2)は cut off

~~直流~~ constant voltage を apply した瞬間 L, C, R circuit に流れる電流.

(14) ~~問~~ A condenser of capacity C, ~~which has been raised to potential V~~ is discharged through a wire of resistance R and soefficient of self induction L. Show that the ~~total amount of heat generated in the wire~~ is current becomes maximum after a time $t = \frac{\cancel{2}L}{c^2} \log \frac{R+\sqrt{R^2-\frac{4L}{c^2C}}}{R-\sqrt{\ \prime\prime}} \Big/ \sqrt{R^2-\frac{4L}{c^2C}}$, ~~and its value is~~ ~~if~~ R is greater than $\frac{2}{c}\sqrt{\frac{L}{C}}$.

$$e = \cancel{\frac{V}{C} e^{-bt}} \quad a_1 e^{b_1 t} + a_2 e^{b_2 t}$$

$$b_{\frac{1}{2}} = \frac{-R \pm \sqrt{R^2 - \frac{4L}{c^2C}}}{2\frac{L}{c^2}}$$

$$J = -a_1 b_1 e^{b_1 t} - a_2 b_2 e^{-b_2 t}$$

$t=0$; $\quad a_1 + a_2 = \cancel{\frac{1}{C}} VC$

$$a_1 b_1 + a_2 b_2 = 0$$

$$\lambda_1 = \frac{R - \sqrt{R^2 - \frac{4L}{c^2C}}}{2\frac{L}{c^2}}$$

$R^2 \gg \frac{4L}{c^2C}$

$b_{\frac{1}{2}} = -\lambda_{\frac{1}{2}}$ $\quad$ ~~$e = -$~~ $\frac{a_1}{\lambda_2} = \frac{-a_2}{\lambda_1} = \frac{a_1+a_2}{\lambda_2-\lambda_1} = \frac{VC}{\lambda_2-\lambda_1}$

$$J = \frac{\lambda_1\lambda_2 VC}{\lambda_2-\lambda_1}\left(\cancel{\lambda_1} e^{-\lambda_1 t} - \cancel{\lambda_2} e^{-\lambda_2 t}\right)$$

$$\frac{dJ}{dt} = -\lambda_1^{\cancel{2}} e^{-\lambda_1 t} + \lambda_2^{\cancel{2}} e^{-\lambda_2 t} = 0.$$

$$e^{(\lambda_2-\lambda_1)t} = \frac{\lambda_{\cancel{2}}^2}{\lambda_1^{\cancel{2}}} \qquad \cancel{2}t = \frac{\cancel{2}\log\frac{\lambda_2}{\lambda_1}}{\lambda_2-\lambda_1}$$

$$t = \frac{\cancel{2}(\log\lambda_2 - \log\lambda_1)}{\lambda_2-\lambda_1}$$

$$= \frac{\cancel{2}L}{c^2} \frac{\cancel{2}\cancel{2}\log\frac{R+\sqrt{R^2-\frac{4L}{c^2C}}}{R-\sqrt{R^2-\frac{4L}{c^2C}}}}{\sqrt{R^2-\frac{4L}{c^2C}}}$$

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(16) In a circuit ~~with R, L, C~~ with very small resistance R, ~~and frequency of the proper oscillation~~ $\cancel{\omega =} \frac{\omega}{2\pi} = \frac{c}{2\pi}\sqrt{\frac{1}{LC}}$, an alternating electromotive force of frequency equal to that of the proper oscillation $\frac{\omega}{2\pi} = \frac{c}{2\pi}\sqrt{\frac{1}{LC}}$ is applied. ~~Show that~~ at time 0.
Show that the current and the potential of the condenser ~~varie~~ ~~change~~ oscillate (with the ~~same~~ frequency $\frac{\omega}{2\pi}$), ~~and their amplitu~~ and their amplitudes ~~increase~~ are ~~wi~~ ~~according to the with time~~ ~~like~~ proportional to

$$1 - e^{\cancel{-\lambda t}\ \cancel{-\frac{\lambda t}{4}}\ -\lambda t}$$

where $\lambda = \frac{\cancel{4}Rc^2}{2\cancel{3}L}$.

$$V = V_0 \sin(\omega t + \delta_0)$$

$$e = a e^{-\lambda t} \sin(\omega t + \delta) + a' \sin(\omega t + \delta')$$

$$J = -a\omega e^{-\lambda t} \cos(\omega t + \delta) \mp a'\omega \cancel{\sin}(\omega t + \delta')$$

$\cancel{\cos}\,\delta'^{\,1/3}$ $\quad 0 = a \sin\delta + a' \sin\delta'$

$+\sin\delta'$ $\quad 0 = -a \cos\delta \mp a' \cancel{\cos}\,\delta'$

$$a' \sin(\delta' - \delta) = 0.$$

$$\cancel{\delta' - \delta =} \quad \delta - \delta' = n\pi \qquad \delta - \delta'$$

$$a = \pm a'.$$

$$\sin\delta = \sin\delta' = 0 \quad 0$$

$$\frac{1}{Z} = \frac{R + i\omega\left\{ C\left(R^2+\left(\frac{L\omega}{c^2}\right)^2\right) - \frac{L\omega}{c^2}\right\}}{R^2 + \left(\frac{L\omega}{c^2}\right)^2}$$

$$Z = \frac{\left\{R^2+\left(\frac{L\omega}{c^2}\right)^2\right\}\left\{R + \frac{i\omega}{c^2}\left\{L - c^2C(\quad)\right\}\right\}}{R^2+\omega^2\{\qquad\}^2}$$

δ

$$\omega\frac{L}{\cancel{R}c^2}$$

$$(1-\omega^2LC)^2+(\omega RC)^2$$

$$-C\left\{R^2+\frac{L^2\omega^2}{RC^4}\right.$$

$$= 1-2\omega^2LC + \omega^4L^2C^2$$

$$= \qquad -\omega R^2C$$

$$+\omega^2R^2C^2$$

$$\frac{\omega L}{c^2}\left(1-\frac{\omega^2LC}{c^2}\right)$$

$$R^2+\omega^2\frac{L^2}{c^4} - 2C\{\quad\}\frac{\omega^2L}{c^2}$$

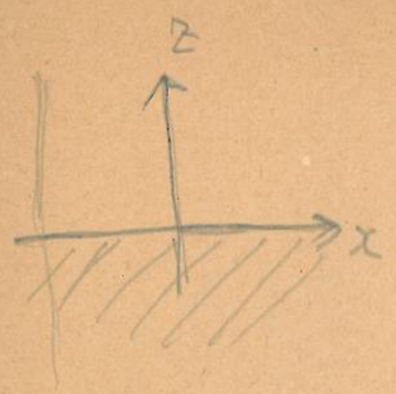
$$\frac{\omega^2C^2}{R}$$

$$+\omega^2\{\qquad\}^2$$

$$= 1 - 2\frac{\omega^2CL}{c^2} + \frac{\omega^4L^2C^2}{c^4}$$

$$-i\frac{1}{\omega C}$$

$$= \frac{R + \omega^2\underline{R^2C^2}}{\left(1-\frac{\omega^2LC}{c^2}\right)^2+(\omega RC)^2}$$



$\mathrm{curl}\,\mathbb{E} = -\frac{\mu}{c}\frac{\partial \mathbb{H}}{\partial t}$

$\mathrm{div}\,\mathbb{H} = 0$

$\mathrm{curl}\,\mathbb{H} = \frac{\varepsilon}{c}\frac{\partial \mathbb{E}}{\partial t} + \frac{4\pi\sigma}{c}\mathbb{E}$

$\mathbb{E}(x, z, t)$ $\qquad \mathrm{div}\,\mathbb{E} = 0$

$\otimes\ -\frac{\partial E_y}{\partial z} = -\frac{\mu}{c}\frac{\partial H_x}{\partial t}$ — $\bigcirc\ \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} = \frac{\mu}{c}\frac{\partial H_y}{\partial t}$

$\times\ \frac{\partial E_y}{\partial x} = -\frac{\mu}{c}\frac{\partial H_z}{\partial t}$

$\times\ \frac{\partial H_x}{\partial x} + \frac{\partial H_z}{\partial z} = 0$

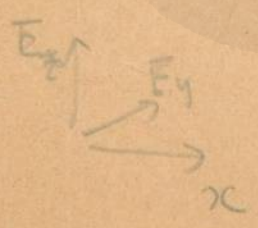
$\bigcirc\ \frac{\partial H_y}{\partial z} = \frac{\varepsilon}{c}\frac{\partial E_x}{\partial t} + \frac{4\pi\sigma}{c}E_x$

$\times\ \frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} = +\frac{\varepsilon}{c}\frac{\partial E_y}{\partial t} + \frac{4\pi\sigma}{c}E_y$

$\bigcirc\ \frac{\partial H_y}{\partial x} = \frac{\varepsilon}{c}\frac{\partial E_z}{\partial t} + \frac{4\pi\sigma}{c}E_z$

$\bigcirc\ E_x, E_z, H_y$ $\qquad \bigcirc\ \frac{\partial E_x}{\partial x} + \frac{\partial E_z}{\partial z} = 0$

$\times\ E_y, H_x, H_z$



$$\frac{\partial E_z}{\partial x}-\frac{\partial E_x}{\partial z}=-\frac{\mu}{c}\frac{\partial H_y}{\partial t}$$

$$\frac{\partial H_y}{\partial z}=\frac{\varepsilon}{c}\frac{\partial E_x}{\partial t}+\frac{4\pi\sigma}{c}E_x$$

$$\frac{\partial H_y}{\partial x}=\frac{\varepsilon}{c}\frac{\partial E_z}{\partial t}+\frac{4\pi\sigma}{c}E_z$$

$$\frac{\partial E_x}{\partial x}+\frac{\partial E_z}{\partial z}=0.$$

$$E_x=f(z)\,e^{i(\omega t-k_x x)}$$

$$E_z=g(z)\quad{}''$$

$$H_y=h(z)\quad{}''$$

$$+\frac{df}{dz}=-ik_x g(z)+\frac{i\mu\omega}{c}h(z)$$

$$\frac{dh}{dz}=\frac{\varepsilon\omega i}{c}f+\frac{4\pi\sigma}{c}f$$

$$-ik_x h=\frac{\varepsilon\omega i}{c}g+\frac{4\pi\sigma}{c}g$$

$$\frac{dg}{dz}=ik_x f.$$

$$f(z)=f\cdot e^{i\lambda z}\quad \text{etc}$$

$$\lambda f=ikg-\frac{i\mu\omega}{c}h$$

$$\lambda h=\left(\frac{4\pi\sigma}{c}+\frac{\varepsilon\omega i}{c}\right)f$$

$$\lambda g=ikf$$

$$-ikh=\cancel{\frac{\varepsilon\omega i}{c}g+4\pi\sigma}\left(\frac{4\pi\sigma}{c}+\frac{\varepsilon\omega i}{c}\right)g$$

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$$\lambda^2 \cancel{-2ik} + k^2 + \frac{i\mu\omega}{c}\left(\frac{4\pi\sigma}{c} + \frac{\varepsilon\omega i}{c}\right) = 0$$

$$\lambda = \left(\frac{\varepsilon\mu\omega^2}{c^2} - k^2 - \frac{4\pi\sigma\mu\omega i}{c^2}\right)^{\frac{1}{2}}$$

$z>0$: $\lambda = \left(\frac{\omega^2}{c^2} - k^2\right)^{\frac{1}{2}}$

$z = +\infty$ $f, g, h \to 0$.

Real part of λ : negative $= -\sqrt{\frac{\omega^2}{c^2} - k^2}$ $(= \lambda_1$

$$E_x = \lambda_1 C_1 \quad e^{\lambda z} \quad e^{i(\omega t - kx)}$$

$$E_z = ik C_1$$

$$H_y = \left(\frac{4\pi\sigma}{c} + \frac{\varepsilon\omega i}{c}\right) C_1 = \frac{\omega}{c} i C_1$$

$z<0$: $\lambda = \left(\frac{\varepsilon\mu\omega^2}{c^2} - k^2 - \quad\right)^{\frac{1}{2}}$

real part positive $= \lambda_2$

$E_x = \lambda_2 C_2$ etc

$$\left.\begin{aligned} &\lambda_1 C_1 = \lambda_2 C_2 \\ &\frac{\omega i}{c} C_1 = \left(\frac{4\pi\sigma}{c} + \frac{\varepsilon\omega i}{c}\right) C_2 \end{aligned}\right\}$$

$$\underbrace{\frac{\varepsilon\mu\omega^2}{c^2} - \frac{4\pi\sigma\mu\omega i}{c^2}}_{= k_2^2}$$

$$\frac{\frac{\omega^2}{c^2} - k^2}{\frac{\omega^2}{c^2} k^2} = -\frac{\frac{\varepsilon\mu\omega^2}{c^2} - k^2 - \frac{4\pi\sigma\mu\omega i}{c^2}}{(-i)^2\left(\frac{4\pi\sigma}{c} + \frac{\varepsilon\omega i}{c}\right)^2}$$

$$1 - \frac{k^2 c^2}{\omega^2} = \left(\frac{\mu\omega}{c}\right)^2 \Big/ \left(\frac{\varepsilon\mu\omega^2}{c^2} - \frac{4\pi\sigma\mu\omega i}{c^2}\right) \left\{1 - \frac{k^2}{(\quad)}\right\}$$

$$\frac{k_1^2-k^2}{k_1^2} = \frac{\mu^2 k_1^2}{k_2^2}\left(1-\frac{k^2}{k_2^2}\right)$$

$$\parallel$$

$$1-\frac{k^2}{k_1^2}$$

$$k^2\left(\frac{1}{k_1^2}-\frac{\mu^2 k_1^2}{k_2^4}\right) = 1-\frac{\mu^2 k_1^2}{k_2^2}$$

$$\frac{k_1^2}{k^2} = \frac{1-\frac{\mu^2 k_1^4}{k_2^4}}{1-\frac{\mu^2 k_1^2}{k_2^2}}$$

$$u^2 = \frac{\omega^2 c^2}{k^2} = c^2\frac{1-\alpha^2/\mu^2}{1-\mu\alpha}$$

$$\alpha = \mu\left(\frac{k_1}{k_2}\right)^2 = \cancel{\alpha}\frac{\mu\left(\frac{\omega}{c}\right)^2}{\frac{\varepsilon\mu\omega^2}{c^2}-\frac{4\pi\sigma\mu\omega i}{c^2}}$$

$$= \cancel{\alpha}\frac{1}{\varepsilon-\frac{4\pi\sigma}{\omega}i} \cancel{=} \frac{1}{\varepsilon-\frac{2\sigma}{\nu}i}$$

$$\varepsilon+\frac{2\sigma}{\nu}$$

$$\left(\varepsilon^2+\left(\frac{2\sigma}{\nu}\right)^2\right)^{\frac{1}{2}}$$

$$u^2 = c^2\xcancel{\frac{1-\frac{\left(\varepsilon+\frac{2\sigma}{\nu}i\right)^2}{\left(\varepsilon^2+\left(\frac{2\sigma}{\nu}\right)^2\right)^2}}{1-\varepsilon\mu+\frac{2\mu\sigma}{\nu}i}} = c^2\frac{\varepsilon^2}{\left\{\varepsilon^2+\left(\frac{2\sigma}{\nu}\right)^2\right\}^2}$$

$$\left\{(1-\varepsilon\mu)^2+\left(\frac{2\mu\sigma}{\nu}\right)^2\right\}$$

$$k = k_1\left(\frac{1-\mu\alpha}{1-\alpha^2}\right)^{\frac{1}{2}}$$

$$\frac{1-\dfrac{\mu}{\varepsilon-\frac{2\sigma}{\nu}i}}{1-\dfrac{1}{(\varepsilon-\frac{2\sigma}{\nu}i)^2}} = \frac{\{(\varepsilon-\mu)-\frac{2\sigma}{\nu}i\}(\varepsilon-\frac{2\sigma}{\nu}i)}{\varepsilon^2-1-(\frac{2\sigma}{\nu})^2-\frac{4\varepsilon\sigma}{\nu}i}$$

$$= \{(\varepsilon-\mu)-\tfrac{2\sigma}{\nu}i\}\{\varepsilon-\tfrac{2\sigma}{\nu}i\}$$

$$\times\frac{\varepsilon^2-1-(\frac{2\sigma}{\nu})^2+\frac{4\varepsilon\sigma}{\nu}i}{\{\varepsilon^2-1-(\frac{2\sigma}{\nu})^2\}^2+(\frac{4\varepsilon\sigma}{\nu})^2}$$

$$\frac{(\varepsilon^2-\varepsilon\mu-(\frac{2\sigma}{\nu})^2-(2\varepsilon-\mu)\frac{2\sigma}{\nu}i)}{\{\varepsilon^2-1-(\frac{2\sigma}{\nu})^2\}^2+(\frac{4\varepsilon\sigma}{\nu})^2}$$

$$\times(\varepsilon^2-1-(\tfrac{2\sigma}{\nu})^2+\tfrac{4\varepsilon\sigma}{\nu}i)$$

$=$

$\alpha \ll 1.$ $k = k_1\left(1-\frac{\mu\alpha}{2}\right)\cancel{\left(1+\frac{\alpha^2}{2}\right)}$

$$= k_1\left(1-\frac{\mu}{2(\varepsilon-\frac{2\sigma}{\nu}i)}\right)$$

$$= k_1\left\{1-\frac{\mu(\varepsilon+\frac{2\sigma}{\nu}i)}{2\{\varepsilon^2+(\frac{2\sigma}{\nu})^2\}}\right\}$$

$$u^2 = c^2 \cdot \frac{1 - \frac{1}{(\varepsilon - \frac{2\sigma}{\nu} i)^2}}{1 - \frac{\mu}{\varepsilon - \frac{2\sigma}{\nu} i}}$$

$$= c^2 \frac{\varepsilon - \frac{2\sigma}{\nu} i}{\varepsilon - \frac{2\sigma}{\nu} i - \mu} \cdot \frac{\varepsilon^2 - (\frac{2\sigma}{\nu})^2 - \frac{4\varepsilon\sigma}{\nu} i - 1}{(\varepsilon - \frac{2\sigma}{\nu} i)^2}$$

$$= c^2 \frac{\{(\varepsilon - \mu) + \frac{2\sigma}{\nu} i\}\{\varepsilon + \frac{2\sigma}{\nu} i\}}{\{(\varepsilon - \mu)^2 + (\frac{2\sigma}{\nu})^2\}\{\varepsilon^2 + (\frac{2\sigma}{\nu})^2\}}$$

$$\times \{\varepsilon^2 - 1 - (\frac{2\sigma}{\nu})^2 - \frac{4\varepsilon\sigma}{\nu} i\}$$

$$= c^2 \cdot \{$$

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$$k = k_1 (\pm) \left\{ \frac{2\{\varepsilon^2 + (\frac{2\sigma}{\nu})^2\} \; [illegible] \; \frac{2\mu\sigma}{\nu} i}{2\{\varepsilon^2 + (\frac{2\sigma}{\nu})^2\}} \right\}$$

$$\frac{\sigma}{\nu} \gg \varepsilon, \mu.$$

$-ikx$: ⊕

propagation velocity is

$$\frac{k_1}{k} \cdot c \cong c \frac{2\{\varepsilon^2 + (\frac{2\sigma}{\nu})^2\}}{2\{\varepsilon^2 + (\frac{2\sigma}{\nu})^2\} - \varepsilon\mu}$$

$$= c\left\{1 + \frac{\varepsilon\mu}{\cancel{2}(\frac{\cancel{8}\sigma}{\nu})^2}\right\}$$

$$= \underline{\underline{c\left\{1 + \frac{\varepsilon\mu\nu^2}{8\sigma^2}\right\}}}$$

$$\cancel{\frac{2\pi\nu}{2c\nu}} \cdot \frac{\frac{\mu\sigma}{\nu}}{\{\varepsilon^2 + (\frac{2\sigma}{\nu})^2} \simeq \frac{2\pi\nu}{c} \cdot \cancel{}\frac{\mu\nu}{\sigma}$$

$$\simeq 2\pi \frac{\mu\nu^2}{c\sigma\cancel{}}$$

$$2\pi \frac{\cancel{T}}{\cancel{T}} \frac{1}{T^2} \frac{T}{\cancel{}}$$

1) Wechselwiderstand (Sommerfeld ‥)

with the velocity

An infinite conductor ~~is~~ bounded by a plane

(17) One side of a plane is vacuum and the other side is a conductor. show that An electromagnetic wave of frequency $\omega/2\pi$ propagates along the plane. Show that the ~~feet~~ amplitude of the ~~field~~ wave also decreases in the conductor to e^{-1} at a distance

$$d = \underline{\qquad\qquad}$$

from the surface.

Show that the velocity of propagation of the electromagnetic wave of frequency ν is given approximately by

$$c\left(1+\frac{\varepsilon\mu\nu^2}{8\sigma^2}\right),$$

if $\frac{\sigma}{\nu}$ is large compared with ε, μ. Show also that the amplitude of the wave decreases to e^{-1}, when it propagates a distance

$$\boxed{\frac{c\sigma}{2\pi\mu\nu^2}},$$

$\sigma = \frac{I}{E}$

$e\frac{L}{T}\cdot\frac{1}{L^3}$

$$\sigma^2 = \frac{e^2\frac{L^2}{T^2}\frac{1}{L^3}}{M^2\frac{1}{LT^2}} = \frac{M\cdot L^2}{M\cdot\frac{L}{T^2}}\ \frac{1}{T^2L^2}$$

$e^2 = L\,M\,L^2/T^2$

$E^2L^3 = ML^2/T^2$

$$\sigma = \frac{ML^3/T^2}{ML} = \frac{M/T^2}{L} = \frac{1}{L}$$

(17) ~~An infinite conductor with plane boundary~~

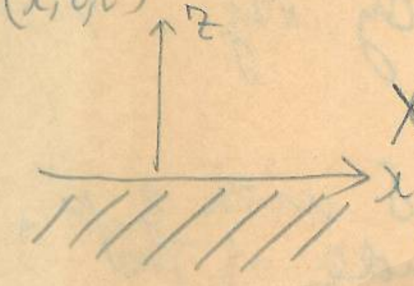
One side of a plane is vacuum and the other side is a conductor, whose ~~dielectric constant~~ permeability and conductivity are ~~ε~~, μ and σ respectively. An electromagnetic wave of frequency ν propagate ~~along~~ the plane, to in a direction rel to Show that the velocity of propagation is given ~~by~~ approximately by

$$\cancel{c\left(1+\left(\frac{\nu}{2\sigma}\right)^2\right)} \quad c\left(1+\left(\frac{\varepsilon\mu\nu^2}{8\sigma^2}\right)\right) = c\left(1+\cancel{\left(\frac{\omega}{4\pi\sigma}\right)^2}\right)$$

provided that the σ/ν is large compared with ε, μ. Show also that the amplitude of the wave decreases to e^{-1} while it propagates a distance

$$\frac{2c\sigma}{\cancel{2}\pi\mu\nu^2} = \frac{8\pi c\sigma}{\mu\omega^2}$$

(x, z, t)



$$\times\ \frac{\partial E_y}{\partial z} = -\frac{\mu}{c}\frac{\partial H_x}{\partial t} \qquad \circ\ \frac{\partial H_y}{\partial z} = \frac{\varepsilon}{c}\frac{\partial E_x}{\partial t} + \frac{4\pi\sigma}{c}E_x$$

$$\circ\ \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} = -\frac{\mu}{c}\frac{\partial H_y}{\partial t} \quad \Big| \quad \times\ \frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} = \frac{\varepsilon}{c}\frac{\partial E_y}{\partial t} + \frac{4\pi\sigma}{c}E_y$$

(Ex Ez Hy)

$$\times\ \frac{\partial E_y}{\partial x} = -\frac{\mu}{c}\frac{\partial H_z}{\partial t} \qquad \circ\ \frac{\partial H_y}{\partial x} = \frac{\varepsilon}{c}\frac{\partial E_z}{\partial t} + \frac{4\pi\sigma}{c}E_z$$

(Ey Hx Hz)

$$\times\ \frac{\partial H_x}{\partial x} + \frac{\partial H_z}{\partial z} = 0 \qquad \circ\ \frac{\partial E_x}{\partial x} + \frac{\partial E_z}{\partial z} = 0$$

$$\cancel{\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{}}$$

$$E_x = f(z)\, e^{i(\omega t - kx)}$$
$$E_z = g(z) \quad ''$$
$$H_y = h(z) \quad ''$$

$$\frac{dh}{dz} = \frac{\varepsilon\omega i}{c}f + \frac{4\pi\sigma}{c}f \qquad -ikh = \left(\frac{\varepsilon\omega}{c}i + \frac{4\pi\sigma}{c}\right)g$$

$$\frac{df}{dz} = -ikg + \frac{i\mu\omega}{c}h \qquad \frac{dg}{dz} = ikf$$

which the direction of
magnetic vector
~~parallel to the~~ p

$$k_2 = \frac{\omega\varepsilon}{c} - \frac{4\pi\sigma}{c} i$$

$$f(z) = f\, e^{\lambda z} \text{ etc}$$

$$\lambda f = -ikg + \frac{i\mu\omega}{c} h \qquad -ikh = \left(\frac{4\pi\sigma}{c} + \frac{\varepsilon\omega i}{c}\right) g$$

$$\lambda h = \left(\frac{4\pi\sigma}{c} + \frac{\varepsilon\omega i}{c}\right) f \qquad \lambda g = ikf$$

$$\lambda^2 = \frac{i\mu\omega}{c}\left(\frac{4\pi\sigma}{c} + \frac{\varepsilon\omega i}{c}\right) + k^2$$

$$=$$

$z > 0$; $\lambda = \left(k^2 - \frac{\omega^2}{c^2}\right)^{\frac{1}{2}} = \lambda_1$ d

$$E_x = \lambda_1 C_1 e^{\lambda z} e^{i(\omega t - kx)}$$

$$f'' = k^2 f - \frac{\omega^2}{c^2} f$$

$$f = e^{\pm\sqrt{k^2 - \frac{\omega^2}{c^2}}\, z}$$

$$E_z = ikC_1 \quad ,,$$

$$c > \frac{\omega}{k}$$

$$H_y = \frac{\omega i}{c} C_1 \quad ,, \qquad = i\cancel{k}\, g = \frac{ik}{\pm\sqrt{k^2 - \frac{\omega^2}{c^2}}} f$$

$$\frac{i\mu\omega}{c}\left(\frac{4\pi\sigma}{c} + \frac{\varepsilon\omega i}{c}\right) \qquad h = \frac{-\frac{\omega i}{c} k_2^2}{\pm\sqrt{k^2 - \frac{\omega^2}{c^2}}} f$$

$z < 0$; $\lambda = \left(k^2 + \frac{\varepsilon\mu\omega^2}{c^2} - \frac{4\pi\mu\sigma\omega}{c^2} i\right)^{\frac{1}{2}}$

$$= \lambda_2 \text{ (real part positive)}$$

$$E_x = \lambda_2 C_2 \quad e^{\lambda z} e^{i(\omega t - kx)} \qquad \frac{dg}{dz} =$$

$$E_z = ikC_2 \quad ,,$$

$$f'' = +\left(k^2 - \frac{\omega^2}{c^2}\right) f$$

$$H_y = \left(\frac{4\pi\sigma}{c} + \frac{\varepsilon\omega i}{c}\right) C_2 \quad ,, \quad = ik_2^2 C_2$$

$$\lambda_1 C_1 = \lambda_2 C_2 \qquad g = \frac{f}{\sqrt{k^2 - \frac{\omega^2}{c^2}}}$$

$$\frac{\omega i}{c} C_1 = \left(\frac{4\pi\sigma}{c} + \frac{\varepsilon\omega i}{c}\right) C_2 \qquad \frac{f}{\sqrt{\;,,}}$$

$$\mu k_1^2 C_1 = k_2^2 C_2$$

$$\lambda_2^2 = k^2 - \cancel{\frac{\mu\omega}{c}} k_2^2 \quad \cancel{k_2 - \mu k_1 k_2}$$

$$\lambda_1^2 = k^2 - k_1$$

$$\frac{k^2-k_1^2}{\mu^2 k_1^4} = \left(\frac{k^2 - \mu k_1 k_2}{k_2^{4}}\right)^2$$

$$\cancel{(k_2^2-k_1^2)k^2 = k_1^2 k_2 (k_2-\mu k_1),}$$

$$\cancel{k = k_1\left(\frac{k_2(k_2-\mu k_1)}{k_2^2-k_1^2}\right)}$$

$$1-\frac{k^2}{k_1^2} = \mu\left(1-\frac{k^2}{k^2}\right)$$

$$1-\frac{k^2}{k_1^2} = \frac{\mu^2 k_1^2}{k_2^2}\left(1-\frac{k^2}{k_2^2}\right)$$

$$\frac{1}{k^2} = \frac{1}{k_1^2}\,\frac{1-\frac{\mu^2 k_1^4}{k_2^4}}{1-\frac{\mu^2 k_1^2}{k_2^2}} \qquad \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2} = -\frac{1}{8}$$

$$k_1^2 = \left(\frac{\omega}{c}\right)^2$$

$$k_2^2 = -\frac{i\mu\omega}{c}\left(\frac{4\pi\sigma}{c}+\frac{\varepsilon\omega i}{c}\right)$$

$$\cong -i\,\cancel{\frac{4\pi\mu\sigma\omega}{c^2}}\,\frac{\mu\omega}{c}\,\frac{4\pi\sigma}{c}\left(1+\cancel{\frac{\varepsilon\omega i}{c^2}}\right) \quad \frac{4\pi\varepsilon\sigma\omega i}{} \quad + \frac{\varepsilon\omega i}{4\pi\sigma}$$

$$k^2 = \left(\frac{\omega}{c}\right)^2 \qquad -\frac{\mu^4 k_1^4}{8 k_2^4}$$

$$k \cong \frac{\omega}{c}\left(1-\frac{\mu^2 k_1^2}{2k_2^2}+\cancel{\frac{\mu^2 k_1^4}{2k_2^4}}\right)$$

$$\cong \frac{\omega}{c}\left(1 - \left(\frac{\mu\omega}{c}\right)^2 \frac{1}{2} \left(\frac{4\pi\mu\omega\sigma}{c^2}\right)^{-1}\left(1-\cancel{\frac{4\pi\varepsilon\sigma\omega i}{c^2}}\right) \quad \frac{\varepsilon\omega i}{4\pi\sigma}\right)$$

$$-\frac{1}{2}\left(\frac{\mu^2\omega^2}{c^2}\right)^2\cdot\left(\frac{\mu\omega 4\pi\sigma}{c^2}\right)^{-2}\left(1-2\frac{\varepsilon\omega i}{4\pi\sigma}\right)$$

$$\frac{\mu^3\omega^4}{2c\mu\sigma\omega} \qquad \frac{\mu\omega^2}{2c\sigma^2}$$

Sommerfeld

$$\frac{\varepsilon\mu^3\omega^4}{2\mu^2\sigma^2\omega^2} = \frac{\varepsilon\mu\omega^2}{2\sigma^2}$$

DATE

NO. 21

$$= \frac{4\varepsilon\mu\nu^2}{8\sigma^2}$$

$$\cong \frac{\omega}{c}\Big\{1 \mp \frac{\mu\omega}{4\pi\sigma}\bar{i}$$

$$~~- \mu\mu\left(\frac{\omega}{4\pi\sigma}\right)^2~~\Big\}$$

$$- \frac{\varepsilon\mu\omega^2}{2(4\pi\sigma)^2}\Big\}$$

$$-\frac{1}{2}\left(\frac{\omega}{4\pi\sigma}\right)^2 + \frac{\mu^2}{4}\left(\frac{\omega}{4\pi\sigma}\right)^2$$

$$= \frac{\omega}{c}\left(1 - \frac{\mu\omega}{8\pi\sigma}i - \frac{1}{2}\left(\frac{\omega}{4\pi\sigma}\right)^2\left\{\varepsilon\mu - \frac{\mu^2}{4} + \frac{1}{2}\right\}\right)$$

$$= -\frac{2\pi\mu\nu^2}{2c\sigma} + \frac{\omega}{c}\Big\{1 - \frac{\nu^2}{8\sigma^2}$$

$$\left(\varepsilon\mu - \frac{\mu^2}{4} + 1\right)\Big\}$$

$$k \cong k_1\left\{1 - \frac{\mu^2}{2}\frac{k_1^2}{k_2^2} - \frac{\mu^4}{8}\frac{k_1^4}{k_2^4} + \frac{\mu^2}{2}\frac{k_1^4}{k_2^4}\right\}$$

$$= \frac{\omega}{c}\Big\{1 - \frac{i}{2}\frac{\mu^2\omega^2}{c^2}\left(\frac{4\pi\mu\omega\sigma}{c^2}\right)^{-1}\left(1 - \frac{\varepsilon\omega i}{4\pi\sigma}\right)$$

$$- \frac{\mu^2\omega^4}{c^4}\left(1 - \frac{\mu^2}{4}\right)\left(\frac{4\pi\mu\omega\sigma}{c^2}\right)^{-2}\left(1 - 2\frac{\varepsilon\omega i}{4\pi\sigma}\right)\Big\}$$

$$= \frac{\omega}{c}\left[1 - i\frac{\mu\omega}{8\pi\sigma} - \frac{1}{2}\left(\frac{\omega}{4\pi\sigma}\right)^2\left\{\varepsilon\mu + 1 - \frac{\mu^2}{4}\right\}\right]$$

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

宿題

DATE Dec. 22.

NO. 22

An alternating current of frequency $10^7\,\mathrm{sec}^{-1}$ flows in a circular coil of radius 10 cm. and of ~~with~~ 5 turns, the maximum value of the current being 1 ampère.
Find the maximum value of the current induced in another circular coil of radius 5 cm. and of ~~with~~ 10 turns and of resistance 1 ohm at a distance 1 kilometer in the plane of the first coil, when the planes of the coils are inclined at ~~45°~~ an angle 45°.

$$H = \frac{1}{c^2 r}\cdot\frac{\omega^2 J S}{c}$$

$$J'R = \omega H S'/c \cdot \frac{1}{\sqrt{2}}$$

$$J' = \frac{\omega^3 J S S'}{c^4 r \sqrt{2}}\frac{1}{R}$$

$$= \frac{(2\pi)^3\ \cancel{(\pi\times 100)\times 5\times(2}\ \pi\times 10^2 \times 5^2\times\pi \times 5\times 10 \times 30}{(3\times 10^3)^3 \times 10^5\ \sqrt{2} \times 1}$$

$$= \frac{10^8}{50\times 10^{14}} = \cancel{\frac{6\times 10^{-6}}{2\times 10^{-6}}}\ \text{ampére}$$

$$= \frac{3\times 10^9 \times 3 \ldots}{05\times 10^{16}} = 2\times 10^{-6}$$

3×10^4

$2^3\cdot 5^{-3} = 10^3$

$\pi^4 = 10^2$

$3^3\sqrt{2} = 50$

(18) 電荷 ~~e~~ +or−e 静止質量 m_0 なる
高速度粒子が遠方から地球に近づいてくる場合、
地球を ~~磁~~ 磁能率 M なる二重極(dipole)とすると
速度が ~~磁気~~ $\frac{eM}{mcR^2}$ 以下で
又 ~~磁極~~ 磁気能率の ~~軸~~ 軸の方向における ~~component~~ 速 component の絶対
角運動量の が ~~一定~~ 与えられて
$2\left(\frac{eMmv}{c}\right)^{1/2}$
より大なる場合には 粒子は地表に到達し得ないことを証明せよ。 但し、R は地球の半径とす。

$$-\sin\theta = -\frac{2\gamma_1}{x\cos\lambda} + \frac{\cos\lambda}{x^2} \qquad x = 1.$$

$$\gamma_1 = \left(\frac{eM}{c}/mv\right)^{1/2}\gamma > 1.$$

$$-\sin\theta = -\frac{2\gamma_1}{\cos\lambda} + \cos\lambda$$

$$x_0 = \left(\frac{mvc}{eM}\right)^{1/2} R < 1.$$

$$v \cancel{\times} \frac{eM}{mcR^2} \qquad \beta < \frac{eM}{mc^2R^2}$$

$$\gamma = \frac{-Ac}{2eM} \qquad A > 2(eMm\beta)^{1/2}$$

$$A > \frac{2eM(mv)^{\frac{1}{2}}}{c\left(\frac{eM}{c}\right)^{\frac{1}{2}}} = 2\left(\frac{eM}{c}mv\right)^{1/2}$$

$r \cdot mv$

$\sqrt{m^2 r^4 t^{-2}}$

$e \cdot er$

$\cdot\, m r t^{-1}$

$m r^3 t^{-1}$

$m_0 = 0.9 \times 10^{-27}$ gr

$e = 4.77 \times 10^{-10}$ e.s.u.

$$\begin{array}{r} 59 \\ 810\overline{)477} \\ 4050 \\ \hline 720 \end{array}$$

5.9×10^{-4}

$$\begin{array}{r} 1.7 \times 10^{3} \\ 477\overline{)810} \\ 477 \\ \hline 3330 \\ 3339 \end{array}$$

$$\begin{array}{r} .51 \\ 477\overline{)243} \\ 2385 \\ \hline 450 \end{array}$$

$300 \times \frac{0.81 \times 10^{-8}}{4.77 \times 10^{-10}} = 5.1 \times 10^5$

electron

$V^{\text{volt}} = 5.1 \times 10^5 \left\{ \sqrt{1 + (\underbrace{H\rho}_{5.9 \times 10^{-4}})^2} - 1 \right\}$

$H = 10000$

$\rho = 10.$

$$\begin{array}{r} 58 \\ 51 \\ \hline 58 \\ 290 \\ \hline 30. \end{array}$$

59

$58 \times 5.1 \times 10^5$

3×10^7

又は kinetic energy を electron volts
であらはすこと

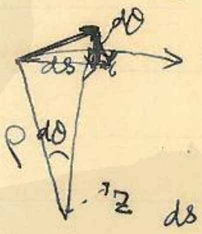
DATE

NO. 24

(1) Maxwell の張力について述べ，之より
電荷及び電流に働く力を求めること．

(3) 磁場に於ける
(1) 電荷 e，静止質量 m_0 なる粒子が H なる強
さの磁場に直角に横切る際の軌道の曲率半径
が ρ ~~なることを知つてなることを知つて~~ であるとすると．粒子の ~~速さは~~ 速さ
運動のエネルギー
~~或~~ 如何．

$$\rho = \frac{ds}{d\theta} = \cancel{\ }$$



$$ds = v\,dt$$

$$\cancel{d\theta = v_x \dot{v}_y - v_y \dot{v}_x}$$

$$\hat{v}\,d\theta = \frac{1}{v}(\dot{v}_x v_y - \dot{v}_y v_x)$$

$$\rho = \frac{v^3}{|\dot{v}_x \dot{v}_y - \dot{v}_y v_x|}$$

(nonrel. $\frac{v^2}{\rho} = \frac{e}{mc} H v \qquad \frac{v}{c} = \frac{H\rho}{\frac{mc^2}{e}}$)

$$\cancel{\frac{m_0 v_x}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$$\left.\begin{aligned} \dot{v}_x &= \frac{e}{m_0 c}\sqrt{1-\frac{v^2}{c^2}}\, v_y H \\ \dot{v}_y &= \frac{-e}{m_0 c}\sqrt{1-\frac{v^2}{c^2}}\, v_x H \end{aligned}\right\} \rho = \frac{m_0 c}{eH} \frac{v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{v}{c} = \frac{H\rho}{\sqrt{(H\rho)^2 + \left(\frac{m_0 c^2}{e}\right)^2}}$$

$$m_0 c^2\left(\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - 1\right) = m_0 c^2\left\{\frac{\sqrt{H^2\rho^2 + \left(\frac{m_0 c^2}{e}\right)^2}}{\frac{m_0 c^2}{e}} - 1\right\} = eV$$

$$V = \left\{\sqrt{H^2\rho^2 + \left(\frac{m_0 c^2}{e}\right)^2} - \frac{m_0 c^2}{e}\right\}$$

(32) 容量C，自感応~~係数~~L，電気抵抗Rを有する~~より成る~~回路に~~一定の~~ある外部的に与えた振動数の交流電圧を~~apply した時の~~かけた場合に流れる電流~~の~~ ~~有様を~~式を求めよ。

~~(共)~~

(3) (ii) 自由~~な荷電粒子~~電子による分散。電磁波が~~物体を~~ ~~通過する際、自由電子によって散乱された波の散乱に~~ ついては、~~電子中の~~稀薄なる気体中の電磁波が、単位体積内にN個の自由電子が存在する時伝播する電磁波の~~phase velocity~~ ~~dielectric constant~~は一の屈折率は大体

$$\frac{c \neq \varepsilon}{\sqrt{1 - N\frac{e^2\lambda^2}{\pi m c^2 \nu^2}}}$$ 倍

で与へられる~~結果~~ことを示せよ。但し、根号の中の第二項は1より小さいとする~~第一項に比べて十分小さいとして省略する~~。

$$m\ddot{x} = -eE = -e \sin(\cancel{2\pi}\,2\pi\nu t + \delta)$$

$$m\dot{x} = +\frac{e}{2\pi\nu}\cos(\quad) = \frac{e}{(2\pi\nu)^2}\frac{dE}{dt}$$

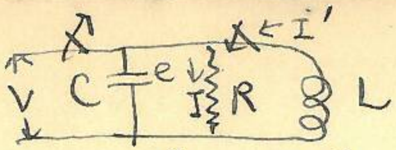
$$mx = \frac{e}{(2\pi\nu)^2}E$$

$$-eN\dot{x} = -\frac{N}{m}\left(\frac{e}{2\pi\nu}\right)^2\frac{dE}{dt}$$

$$\mathbf{P} = ex$$

$$4\pi . \quad \frac{e}{(2\pi\nu)^2}\frac{dE}{dt} .$$

(3)



~~$eC = IR$~~

$$V' = IR = -\frac{L}{c^2}\frac{dI'}{dt}$$

$$-\frac{de}{dt} = I - I'$$

$$\text{or } -C\cancel{\frac{1}{c}}\frac{dV'}{dt} = I - I'$$

$$-C\frac{dV'}{dt} = I - I' = -CR\cdot\cancel{I}\frac{dI}{dt},$$

~~$I + \frac{L}{c^2R}\,dI\,p$~~

$$CR\frac{d^2I}{dt^2} + \cancel{\frac{Rc^2}{L}}\frac{dI}{dt} + \frac{Rc^2}{L}I = 0.$$

$I = e^{\lambda t}$

$$\lambda = \frac{1}{2CR}\left\{\cancel{-\frac{Rc^2}{L}} - 1 \pm \sqrt{1 - \frac{4CR^2c^2}{L}}\right\}$$

$$R^2 \gg \frac{L}{2Cc^2} \qquad \frac{1}{2CR}\left\{-1 \pm i\frac{2\sqrt{C}R^2c}{\sqrt{L}}\right\}$$

$$= -\frac{1}{2CR} \pm i\frac{c}{\sqrt{LC}}$$

$t = 0$: ~~$I = \frac{V}{R}$~~

$e = \cancel{\frac{V}{C}}\,CV$

$V' = V.$

$I' = 0,$

~~$I = I_0$~~

$I = \ldots$

~~$I' =$~~

$V' = IR.$

$$I' = I + C\frac{dV'}{dt}$$

$=$

(2) 電磁偏重 ~~ぬ~~ は ~~如何~~ 於きて [illegible]

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

DATE Feb. 23, 1935
NO. 26

電磁気試験

九時半より正午迄

(1) Maxwellの張力につきて述べ、之より定常電磁場に於て電荷及び電流に働く力を誘導せよ.

~~(1) Maxwellの張力につきて述べ. 定常電流の流れて居る二つのcircuitの間に働く力を求めよ.~~

(2) 自己感応係数 L, 抵抗 R ~~の回路に~~ 容量 C なる回路に定電圧 V をかへた ~~[illegible]~~ 場合 ~~に流れる~~ の 電流 ~~の最大値をおもとめよ.~~ ~~を図示し、電圧を~~ について述べよ ~~変化~~

(3) 真空中に於て電荷 e, 静止質量 m_0 なる粒子が H なる強さの磁場を直角に横切る際の軌道の曲率半径 ~~が~~ ρ なり, ~~時~~ ~~その~~ 粒子の速度を求めよ.

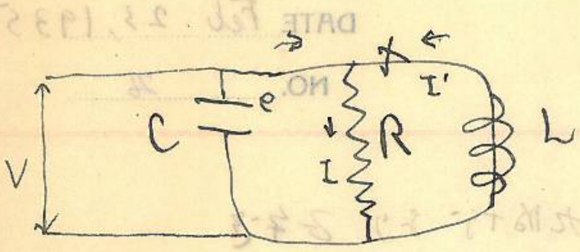
(4) ~~水素~~ 稀薄なガスの単位体積中に N 個の自由電子が存在する時. 振動数 ν なる電磁波の伝播速度は、~~大体~~、

$$\frac{c}{\sqrt{1-N\frac{e^2}{\pi m\nu^2}}}$$

で与へられることを示せよ. 但し ~~間~~ ⟹ $\left\{\begin{array}{l}m\text{ は電子の質量}\\ N<\frac{\pi m\nu^2}{e^2}\text{ とす}\end{array}\right.$

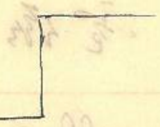
(2)

$I = I' = 0$



$$\frac{dI}{dt} = \frac{dI'}{dt} = 0$$

$$a_1 e^{-\lambda_1 t} + a_2 e^{-\lambda_2 t} =$$



$$eC = IR = -\frac{L}{c^2}\frac{dI'}{dt}$$

$$-\frac{de}{dt} = I - I'$$

$$C(I' - I) = \cancel{I} R\frac{dI}{dt}$$

$$R\frac{d^2I}{dt^2} + C\frac{dI}{dt} + \frac{RCc^2}{L}\cancel{dI}\, I = 0.$$

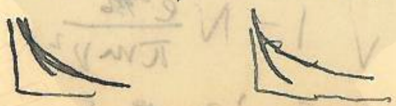
$$\lambda = \frac{-\cancel{R}C \pm \sqrt{C^2 - \frac{R^2Cc^2}{L}}}{2R} \qquad \frac{R^2c^2}{LC} \gg 1,$$

$$= \frac{C}{2R}\left(-1 \pm \sqrt{1 - \frac{R^2c^2}{LC}}\right)$$

$$\frac{RC}{\sqrt{LC}} < 1 \qquad \lambda = \frac{C}{2R}\left(-1 \pm \sqrt{1 -}\right.$$

$$R \to 0 \qquad \frac{C}{2R}\left(-1 \pm \left(1 - \frac{R^2c^2}{LC}\right)\right)$$

$$\lambda = \frac{Rc^2}{2L}, \quad -\frac{C}{R}$$



$$\frac{d}{dr}\left(r^{m}R^{n}\right)$$ 27

$$\frac{d}{dr}\left(m r^{m-1}R^{n}+r^{m}\frac{dR}{dr}\right)$$ 22

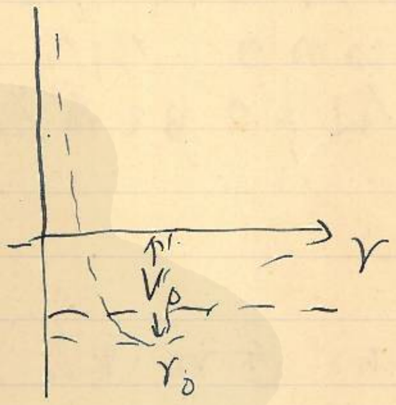
$V(r)$ が $r=r_0$ の点で min で、それに対しては急に増すとし、r_0から離れた所では

$E-\varepsilon_{trans}$ ~~著~~ $< V$

となる。正すでは、R の函数は exponential になる。故に eigenfunction は R が r_0 に近い所のみ大きな値をもって居る。その近では

~~$V(r-r_0)$~~ $= V_0 = V(r_0)$ とおき

$$V(r) = -V_0 + k(r-r_0)^2$$



なる parabola と assume すると

$$\frac{d^2}{dr^2}(rR) + \frac{2M_1M_2}{(M_1+M_2)\hbar^2}\{E-\varepsilon_{trans}+V_0 - k(r-r_0)^2\}(rR) - \frac{l(l+1)}{r^2}(rR)$$

今 $r-r_0 = x$　　$rR = y$.

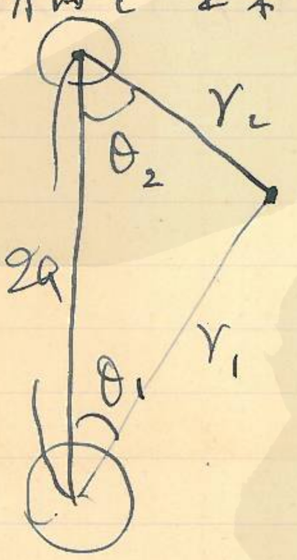
$$\left.\begin{aligned}\frac{2M_1M_2}{(M_1+M_2)\hbar^2}(E-\varepsilon_{trans}+V_0) &= \lambda\\ \frac{2M_1M_2}{(M_1+M_2)\hbar^2}k &= \kappa\end{aligned}\right\}$$

$$\frac{d^2y}{dx^2}+$$

(2) 比誘導度を無限大と見なし得る長さ l なる針金あり。一端は地絡し、他端に L_0 なる自己感応が接続しある。針金の固有振動の波長如何。但し、針金の自己感応係数は単位長さについて L とす。

(3) 一様な ~~運動~~ 速度で運動して居る電子による電磁場に就きて述べよ。

(4) 平行な二つの直線状な針金に~~並~~ 互に反対の方向に 同じ大いさの一様な電流~~が反対の方向に~~ 流れる場合の外部の任意点の磁場を求めよ。
切口は円にて、中心距離は 2a
大いさ、方向を表する式を図の文字にてあらはすこと



Current の問題

Jeans p 319

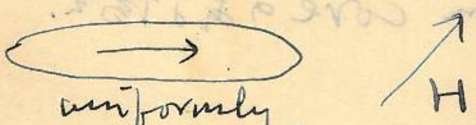
○ submarine cable imperfectly insulated

○ Planck : two cylinder の間の resistance

or cylindrical wire の間の rel (Jeans 351)

resistance

magnet の問題



○ straight wire X [illegible] field

1) field (Jeans 430)

2) moment of force

electrostatic problem ニ reduce シテ、 従ッテ これから ○ ○ 電極の間の 抵抗を求めること。 Jeans p. 350

四角い金属膜の resistance

~~current ニ於テ~~ ○ sphere ニ wire を通ジテ

大阪帝國大學理學部

uniform field を與へる方法, field の強さ.

① Galvanometer

② Coef. of Induction
Jean 448. ⑩ ⑲

③ ~~ferromagnetic substan~~
iron core のある場合.

○ Circle
○ Coaxial Circles
(Becker, 162 ---)

---

① mg. dipole の field 中の electron の motion,

② ionized gas 中の electro-magnetic wave の propagation

1

やめ 十一月七日. 木, 1時 − 3時. (やめ)

(9) Two parallel straight infinite wires ~~of~~ convey equal currents of strength J in opposite directions, their distance apart being 2a. Show that the magnetic field of a point at distances r_1, r_2 from the wires can be dirived from the vector potential of amount

$$\frac{2J}{c}\left|\log\frac{r_2}{r_1}\right|$$

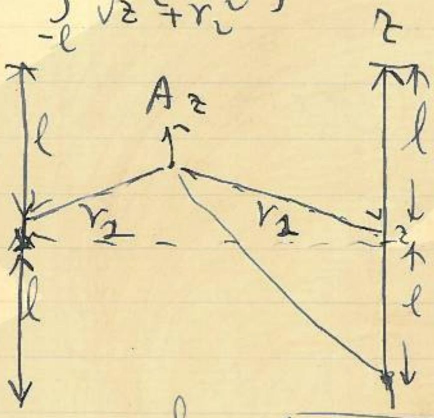
in the direction of the current farther to the point.

Show also that the lines of magnetic force are circles around each one of the wires.

$$A_z = \frac{J}{c}\left\{\int_{-l}^{+l}\frac{dz}{\sqrt{z^2+r_1^2}} - \int_{-l}^{+l}\frac{dz}{\sqrt{z^2+r_2^2}}\right\}$$

$x = \frac{z}{r_1}$ or $\frac{z}{r_2}$

$$= \frac{J}{c}\left\{\int_{-l/r_1}^{l/r_1}\frac{dx}{\sqrt{1+x^2}} - \int_{-l/r_2}^{l/r_2}\frac{dx}{\sqrt{1+x^2}}\right\}$$



$$= \frac{2J}{c}\left\{\int_{l/r_2}^{l/r_1}\frac{dx}{\sqrt{1+x^2}}\right\} = \frac{2J}{c}\log\frac{\frac{l}{r_1}+\sqrt{1+\frac{l^2}{r_1^2}}}{\frac{l}{r_2}+\sqrt{1+\frac{l^2}{r_2^2}}}$$

$l\to\infty$ $A_z = \frac{2J}{c}\log\frac{r_2}{r_1}$

$$H_x = \frac{\partial A_z}{\partial y} = \frac{2J}{c}\left\{\frac{y}{r_1^2} - \frac{y}{r_2^2}\right\}$$

$$H_y = -\frac{\partial A_z}{\partial x} = -\frac{2J}{c}\left\{\frac{x}{r_1^2} - \frac{x}{r_2^2}\right\}$$

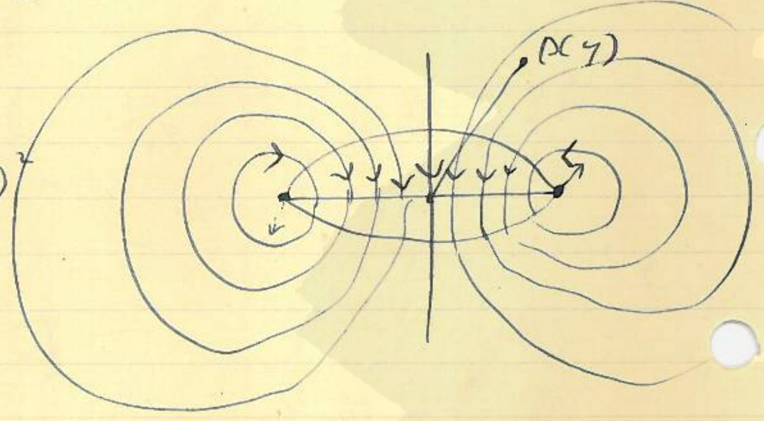
$$H_z = 0$$

~~grad A · H = 0.~~

$$\cancel{\frac{\partial A_z}{\partial x} H_x} \text{ or } \frac{\partial A_z}{\partial x} H_x + \frac{\partial A_z}{\partial y} H_y = 0.$$

故に. lines of force ~~と~~ A_z の grad の方向とは直角. 即ち. lines of force は
は $A_z = \text{const.}$ ~~即~~ 即ち
$r_2/r_1 = \cancel{A_z}\text{const.}$ 即ち circle 1 と 2 をすぎる
である.

$$H_x^2 + H_y^2 = \left(\frac{2J}{c}\right)^2 (x^2+y^2)\frac{(r_1^2-r_2^2)^2}{r_1^4 r_2^4}$$



2

( ) Show that

○ 宿題. ⑮ $\boxed{3}$

(i) A current J flows in a rectangular circuit whose sides are of lengths $2a$, $2b$ and the circuit is free to rotate about an axis through its centre parallel to the sides of length $2a$. Another current J' flows in a long straight wire parallel to the axis and at a distance d it from it. ~~Prov~~ Find the couple of force required to keep the plane of the rectangle at an angle ϕ with the plane through the axis and the straight current.

⑯

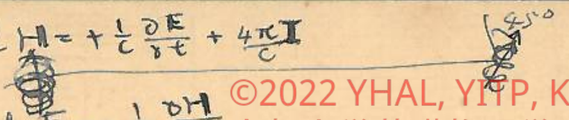
(ii) A wire is wound in a spiral of angle ϕ on the surface of an insulating cylinder of radius a, so that it makes n completen turns on the cylinder. A current J flows through the wire. Find the ~~resultant~~ ~~field~~ magnetic field along the ϕ axis at the centre of the cylinder.

$\mathbf{H} \to +\mathbf{E}^*$

$\mathbf{E} \to -\mathbf{H}$

$\mathbf{I} \to$ mag. current.

$\mathrm{curl}\,\mathbf{H} = +\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi \mathbf{I}}{c}$



$\mathrm{curl}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}$

(16) An alternating current of frequency $10^7 \sec^{-1}$ flows in a circular coil of radius 10 cm and of 5 turns, the maximum value of the current being 1 Ampere.

Find the maximum value of the current induced in another circular coil of radius 5 cm and of 10 turns and of ~~resistance~~ (impedance) 1 ohm at a distance 1 kilometer in the plane of the first coil, when the planes of the coils ~~are in~~ makes an angle 45°.

$$H = \frac{1}{c^2 r}\frac{\omega^2 J S}{c} \qquad 2\pi\nu = \omega$$

$$J' R = \frac{\omega H S'}{c}\frac{1}{\sqrt{2}}$$

$$\begin{array}{r} 27 \\ 1.4 \\ \hline 108 \\ 27 \end{array}$$

$\pi^4 = 10^2$

$$J' = \frac{\omega^3 J S S'}{c^4 r \sqrt{2}} \frac{1}{R} = 2.4 \times 10^{-6}\,\mathrm{amp}. \qquad 3^3 \sqrt{2} = 4$$

$$= \frac{(2\pi)^3 \times 10^2 \times 5^2 \times \pi \times 5 \times 10 \times 30}{(3\times 10^3)^3 \times 10^5 \sqrt{2}}$$

$$= \frac{3\times 10^9}{40 \times 10^{+14}} = \frac{3\times 10^9 \, \cancel{\times 10^8}}{40 \times 10^{+14}} = \quad \times 10^6$$

電磁気 問題

昭和十二年 一月 十四日

i) 円い穴を有する薄い金属板の帯電に依つて生ずる電場に就きて論ぜよ。
但し、金属板は非常に大きいとし、両側の遠方に於て電場は一般に異なるものとす。

ii) 平行に相対して置かれた同大の二つの円板状の磁極~~掛れ~~あり。~~一定方向~~ ~~間~~一定方向から一定~~の大いさ~~の速度で多数の電子がこの~~単~~間隙を通過する場合、磁場が適当の大いさを有する~~時~~ならば これらの電子は一点に集中されることを証せよ。
但し磁場は円周の内部では一様で、外部では0~~となる~~とする。又電子は~~始め~~円板の面に平行に通過するものとす。

iii) ~~[illegible]~~長さ l なる針金の一端に L_0 なる inductance ~~感應係数~~の ~~C_0 なる容量の蓄電器~~ あり、他端を地絡せる場合の~~電氣~~固有振動の~~周期~~波長を求めよ。但し針金は単位長さについて L なる~~容量を有する~~感應係数ものとす。

$\sin(d\theta)$ $$d\theta = \frac{v_x(t+dt)v_y(t) - v_y(t+dt)v_x(t)}{v^2}$$

$= \sin(\theta + d\theta)\cos\theta - \cos(\theta+d\theta)\sin\theta$

(17) A charged particle with charge e and rest mass m_0 passes perpendicular to the magnetic field of intensity H and is deflected to form an orbit of radius of curvature ρ.

Show that the velocity of the particle is given by

$$v = \frac{H\rho c}{\sqrt{(H\rho)^2 + \left(\frac{m_0c^2}{e}\right)^2}}$$

and the kinetic energy is given by eV, where

$$eV = e\sqrt{H^2\rho^2 + \left(\frac{m_0c^2}{e}\right)^2} - \frac{m_0c^2}{e}$$

$$\rho = \frac{ds}{d\theta} = v \Big/ \frac{d\theta}{dt}$$

$$\frac{ds}{dt} = v$$

$$\frac{d\theta}{dt} = \frac{1}{v^2}(\dot{v}_x v_y - \dot{v}_y v_x)$$

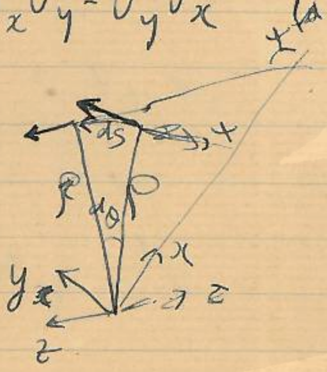
$$~~\frac{v^2}{\rho} = \frac{e}{mc} H v~~$$

$$\rho = \frac{v^3}{\dot{v}_x v_y - \dot{v}_y v_x}$$

$$\frac{dv}{dt} = 0.$$

$$\left.\begin{aligned}\dot{v}_x &= \frac{e}{m_0c}\sqrt{1-\frac{v^2}{c^2}}\, v_y H \\ \dot{v}_y &= \frac{-e}{m_0c}\sqrt{1-\frac{v^2}{c^2}}\, v_x H\end{aligned}\right\}$$

$$\rho = \frac{m_0c}{eH}\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}$$



(18) Show that the frequency ν' of the light emitted from a source, which is ~~approaching~~ atalving near or receding from the observer with the the velocity v is given by

$$\nu' = \nu \frac{\sqrt{1-\frac{v^2}{c^2}}}{1 \mp \frac{v}{c}},$$

where ν is the frequency observed when $v=0$.

Show also that it is given by

$$\nu' = \nu\sqrt{1-\frac{v^2}{c^2}}$$

when the light source is moving perpendicular to the direction of observation.

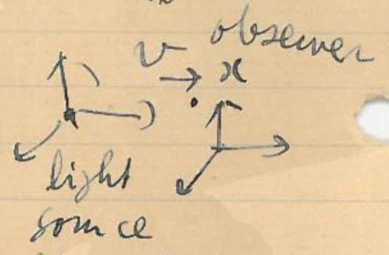
$$\nu^*\left(t^* - \frac{x^* n_x^* + y^* n_y^* + z^* n_z^*}{c}\right) = \nu\left(\frac{t' + \frac{v}{c^2}x'}{\sqrt{1-\beta^2}} - \frac{1}{c}\left\{\frac{x'+vt'}{\sqrt{1-\beta^2}} n_x + y' n_y + z' n_z\right\}\right)$$

$$= \nu'\left(t' - \frac{x' n_x' + y' n_y' + z' n_z'}{c}\right)$$

$$= \nu'\left(\frac{t^* - \frac{v}{c^2}x^*}{\sqrt{1-\beta^2}} - \frac{1}{c}\left\{\frac{x^* - vt^*}{\sqrt{1-\beta^2}} n_x' + y^* n_y' + z^* n_z'\right\}\right)$$

$$n_x'^2 + n_y'^2 + n_z'^2 = 1.$$

$$\nu^* = \nu' \frac{1+\beta n_x'}{\sqrt{1-\beta^2}}$$



1) $n_x' = 1$, $n_y' = n_z' = 0$

2) $n_x' = +1$, $n_y' = n_z' = 0$ 3) $n_x = 0$.

$\boxed{6}$

$$u\cos\theta = \cancel{u'}\, v_1$$

$$u\sin\theta = \frac{v_2'\sqrt{1-\beta^2}}{1+\cancel{\frac{v_2 v_1}{c^2}}}$$

$$v_2' = \frac{v_2}{\sqrt{1-\beta^2}}$$

(19)

Show that the apparent position of the fixed star is shifted by an angle $\sin^{-1}\frac{v}{c}$ toward the direction of the relative motion of the earth to it, provided that the direction of the observation is perpendicular to that of the relative velocity v.
(where v is the component of the velocity perpendicular to the direction of observation)

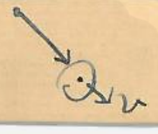
$$n_x' = \frac{-\beta + n_x^*}{1 \mp \beta n_x^*} = -\beta \qquad \left(\begin{array}{l} n_x^* = 0 \\ n_y^* = 1 \\ n_z^* = 0 \end{array} \right)$$

$$n_y' = \frac{n_y^*\sqrt{1-\beta^2}}{1 \mp \beta n_x^*} = \sqrt{1-\beta^2} \qquad \tan\alpha = \frac{-\beta}{\sqrt{1-\beta^2}}$$

$$n_z' = \frac{n_z^*\sqrt{1-\beta^2}}{1 \mp \beta n_x^*} = 0$$

$n_x' = 1$, $n_y' = 0$, $n_z' = 0$

$n_x' = 1 \qquad n_y = n_z = 0.$



$$\nu = \nu' \frac{1+\beta n_x'}{\sqrt{1-\beta^2}}$$

$$n_x' = \cancel{\frac{n_x - \beta}{1-\beta n_x}}$$

$$\nu n_x = \nu' \frac{\beta + n_x'}{\sqrt{1-\beta^2}}$$

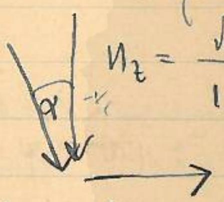
$$\nu n_y = \nu' n_y'$$

$$\nu n_z = \nu' n_z'$$

$$n_x = \frac{n_x'+\beta}{1+\beta n_x'}$$

$$n_y = \frac{\sqrt{1-\beta^2}\, n_y'}{\cancel{\sqrt{1-\beta^2}}\ 1+\beta n_x'}$$

$$n_z = \frac{\sqrt{1-\beta^2}\, n_z'}{1+\beta n_x'}$$



~~tan~~

$$\cos \cancel{\alpha} = n_x n_x' + n_y n_y' + n_z n_z'$$

$$= \frac{(1+\beta n_x') n_x = \beta + n_x'}{}$$

$$\nu\, n_x' = \frac{n_x - \beta}{1 \mp \beta n_x}$$

$$1+\beta n_x' = \frac{(1 \mp \beta n_x) + \beta (n_x - \beta)}{1-\beta n_x} = \frac{1-\beta^2}{1-\beta n_x}$$

$$1+\beta n_y$$

$$= \frac{n_x'(n_x'+\beta)}{1+\beta n_x'} + \frac{\sqrt{1-\beta^2}(n_y'^2 + n_z'^2)}{1+\beta n_x'}$$

$$= \frac{n_x'^2 + n_x'\beta + \sqrt{1-\beta^2}(1-n_x'^2)}{1+\beta n_x'}$$

$$1+\beta^2 n_x^2$$
$$-2\beta n_x$$
$$-n_x^2 - \beta^2$$

$$n. \quad = \frac{1-\beta n_x}{1-\beta^2} \cdot \left(\frac{(n_x-\beta)^2 + (1-\beta n_x)\overset{(n_x-\beta)}{\cancel{\beta}} + \sqrt{1-\beta^2}\{(1-\beta n_x)^2 - (n_x-\beta)^2\}}{(1-\beta n_x)^2} \right.$$

$$= \frac{1}{(1-\beta^2)(1-\beta n_x)} \Big\{ \overset{\cancel{n_x^2 - 2n_x\beta + \beta^2 + \beta}}{(n_x-\beta)(n_x - \beta + \beta - \beta^2 n_x)} + \sqrt{1-\beta^2}\{(1-\beta^2)(1-n_x^2)$$

$$\sin^2\alpha = \cancel{\beta\frac{\sin\omega}{1-\beta\cos\omega}} = \beta\sin\omega(1+\beta\cos\omega)$$

$$= \frac{1}{1-\beta n_x}\left\{ n_x^2 - \beta n_x + \sqrt{1-\beta^2}(1-n_x^2) \right\}$$

$n_x = \beta$ n_y ~~[illegible]~~ $n_z = 0$

$$\cos\alpha = \frac{}{\cancel{1-\beta}}\sqrt{1-\beta^2}$$

$$\underline{\underline{n_x' = -\beta.}}$$

$$\sin\alpha = \beta.$$

$n_x =$ ~~$\sin\omega$~~ $\cos\omega$

$$\cos\alpha = \frac{1}{1-\beta\cos\omega}\left\{\cos^2\omega - \beta\cos\omega + \sqrt{1-\beta^2}\sin^2\omega\right\}$$

$$\cong \frac{1}{1-\beta\cos\omega}\left\{1 - \frac{\beta^2}{2}\sin^2\omega - \beta\cos\omega\right\}$$

$$= \frac{1}{1-\beta\cos\omega}\left\{\left(1-\frac{\beta^2}{2}\right) + \frac{\beta^2}{2}\cos^2\omega - \beta\cos\omega\right\}$$

$$\cancel{= 1 - \frac{\frac{\beta^2}{2}\beta\sin^2\omega}{1+\beta\cos\omega}}$$

$$\sin\alpha \cong \frac{1}{1-\beta^2\cos^2\omega}\left\{(1-\beta\cos\omega)^2 - \frac{\left\{1-\frac{\beta^2}{2}+\frac{\beta^2}{2}\cos^2\omega - \beta\cos\omega\right\}^2}{\{1-\beta\cos\omega\}}\right\}$$

$$\boxed{\tan\alpha \cong \beta\sin\omega}$$

$$\cong \frac{1}{1-\beta\cos\omega}\left\{1 - 2\beta\cos\omega + \beta^2\cos^2\omega - 1 + \beta^2\sin^2\omega + 2\beta\cos\omega - \beta^2\cos^2\omega\right\} \cong \frac{\beta\sin\omega}{1-\beta\cos\omega}$$

(17) ~~Show that~~ Find the differential equation of the trajectory of an electron passing ~~through~~ near the axis of the ~~ele~~ electric lens and nearly parallel to it in the presence of the space charge of density ρ. Show ~~also~~ that

$$\sqrt{\Phi}\frac{d}{dz}\left(\sqrt{\Phi}\frac{dr}{dz}\right) = -Pr$$

$$P = \pi \cancel{r}^2 \rho + \frac{1}{4}\frac{d^2\Phi}{dz^2}$$

$$\because \quad v\frac{d}{dz}\left(v\frac{dr}{dz}\right) = -\frac{e}{m}E_r$$

$$2r\pi l E_r + r^2\pi l\frac{\partial E_r}{\partial z} = r^2\pi l\cdot 4\pi\cancel{e}\rho$$

この式の general solution は

$$r = c_a r_a(z) + c_\beta r_\beta(z)$$

緒言　~~Lorentz~~ Maxwell の理論と Lorentz の理論の間の関係を述べ(論じ)、相対律の必要な理由を述べる.

結論