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# On a Unified Theory of Elementary Particles. I.

(1) March, 1955

By Hideki Yukawa

recent,

## 1. Introduction

Present status of the theory of elementary particles is very unsatisfactory in various respects. <sup>above</sup> ~~the~~ discovery of a great variety of <sup>new</sup> ~~unstable~~ <sup>unstable</sup> particles in cosmic rays and in nuclear events indicates

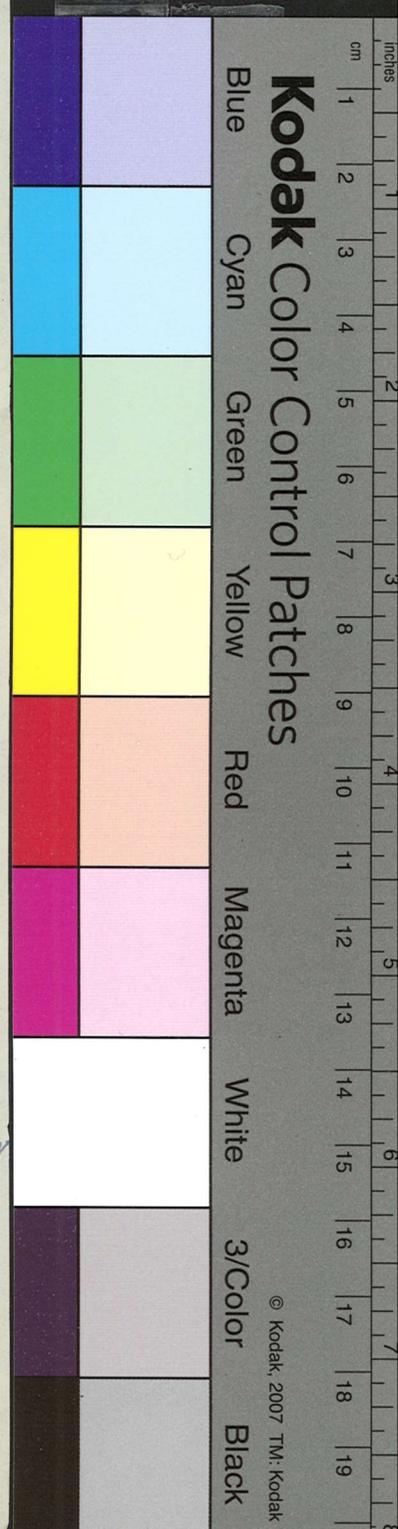
that the ~~existing~~ <sup>artificially produced</sup> theory is of little help as long as it remains at the stage of dealing with each kind of particle separately, <sup>and as such,</sup> ~~without~~ <sup>deeper</sup> insight into the systematic interrelations between these particles which would lead us to the systematics of elementary particles as a whole is certainly in need.

~~If we confine our attention to~~ as a matter of fact, the interrelations between various particles <sup>revealed to a certain</sup> ~~have been dealt with~~ <sup>extent</sup> in the <sup>current</sup> ~~existing~~ theory by considering appropriate interactions between

particles. However, such interactions <sup>have been</sup> ~~were~~ again introduced ad hoc, just as new particles <sup>have been</sup> ~~were~~ introduced into

the theory, ~~wherever~~ <sup>one by one after another</sup> without any regard to possible underlying general principles in sight. Furthermore,

~~On the other hand~~ such interactions were usually treated as small perturbations, ~~which might be so that the effects of~~



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drastic  
did not seem to (2)  
these interactions ~~could not be~~ could not  
be so ~~far reaching~~ as to cause give rise  
from a small number of particle to a  
great variety of parti elementary and  
compound particles, if we started from  
a small number of elementary particles.

On the in connection with this, there  
were another such <sup>undertakings</sup> attempts, were inevitably  
accompanied by the appearance of  
the divergence difficulties, which  
destroyed the consistency of the theory  
as a whole.

Under these circumstances, we <sup>may</sup> ~~should~~  
admit that, ~~to~~ at least, two ways are open.  
One is to make the current theory free  
from the restriction that the interactions  
between elementary particles are regarded  
as small perturbations. The In other words,  
~~is~~ the essential non-linearity of field  
equations for elementary particles is to  
be <sup>taken seriously</sup> investigated. In fact, it has been  
generally recognized that the interaction  
between a nucleon and a pion was so  
strong that the usual perturbation  
method ~~fails~~ should give place to  
other methods such as <sup>stronger</sup> intermediate coupling  
method and Tamm-Dancoff method.  
However, it is not at all clear whether any  
of such methods can hardly answer  
the question of construction of a consistent  
theory of whole family of elementary particles

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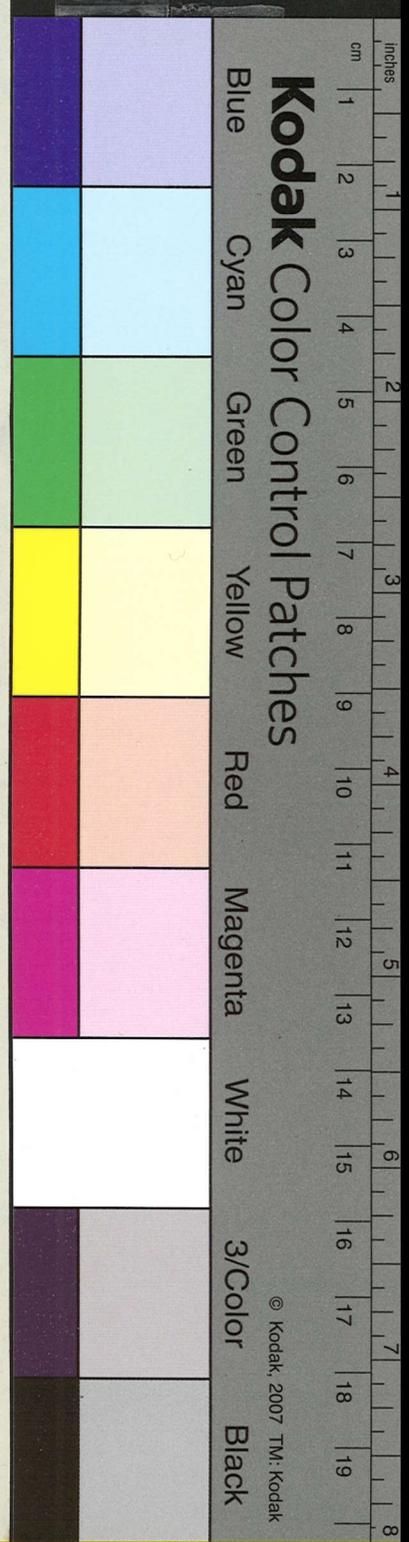
\* ~~a serious restriction~~ ~~the causality condition~~ ~~very seriously~~ ~~in this connection~~ ~~is very seriously used later on.~~

\* The some of the divergence difficulties could be removed by assuming suitable non-local interactions. However, ~~the problem of causality condition becomes in this case~~ ~~me should take~~ ~~we make clear~~ (3) role played unless ~~it is incorporated into~~ the significance of such a method in the whole scheme of relativistic quantum theory of fields. (1)

Recently Hiesenberg developed his idea of unification that the whole family of elementary particles could be contained in a single non-linear spinor field equation as various stationary solutions. (2) Although his basic idea is very attractive, it is not at all easy to see whether the non-linearity of field equations alone is sufficient for responsible for the appearance in Nature of a great variety of particles. It is even more difficult to see whether the divergences are dealt with when the low non-linear fields are quantized. (3)

The other direction is way is to introduce, generally speaking, new degrees of freedom, which is useful for classifying particles, in addition to those which are directly connected with the transformation properties of the <sup>local</sup> field quantities' behavior under Lorentz transformations in the ordinarily space-time world. Now the new degrees of freedom could be introduced ~~either~~ by supposing quantized fields in a world which has

In this connection, one may try to generalize the interaction between quantized fields so as to include non-local interactions. ~~It was shown by Moller and Kristensen that~~ ~~in fact~~ \*



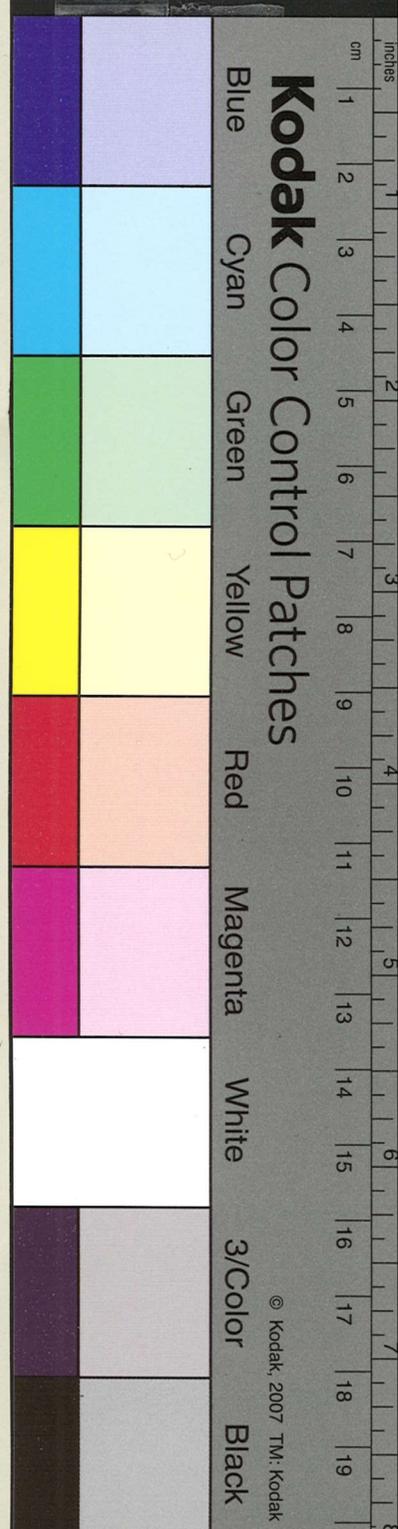
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to the ordinary  
four dimensions  
(4) added

the dimensions larger than 4. Pais introduced  
~~three or four more dimensions which are supposed~~  
~~to correspond to~~ interpreted as themselves  
constitute an isotopic spin space <sup>(5)</sup> the  
advantage of such a procedure is that <sup>of</sup>  
~~one can~~ introducing new degrees of freedom  
keeping close connection with  
is that one can introduce new degrees  
of freedom step by step by keeping  
close connection with the superficial  
knowledge of new particles. As for the  
problem of divergence, however, one can not  
expect ~~little~~ much from such a  
procedure, because the newly as long as  
the newly added dimensions are independent  
of the original four dimensions, because  
the root of divergence lies in the space-time  
world itself.

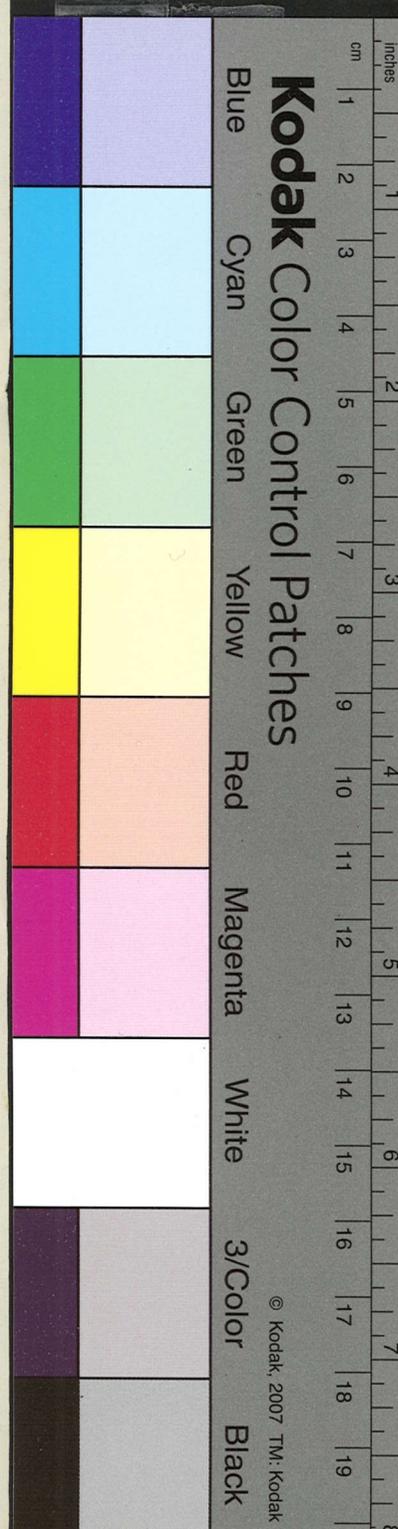
Now, in order to Unfortunately, it is very  
little known whether as to how to establish  
a mutual relation between the original  
four dimensions and the newly added ones.  
However, there is one particular case, in which  
the relation is very simple and definite, namely  
the added dimensions by themselves constitute  
another 4-dimensional world which has the  
same structure as the ordinary space world.  
except for the following point: <sup>the newly added</sup>  
degrees of freedom should correspond to so-to-speak  
internal degrees of freedom of elementary  
each



in a certain sense

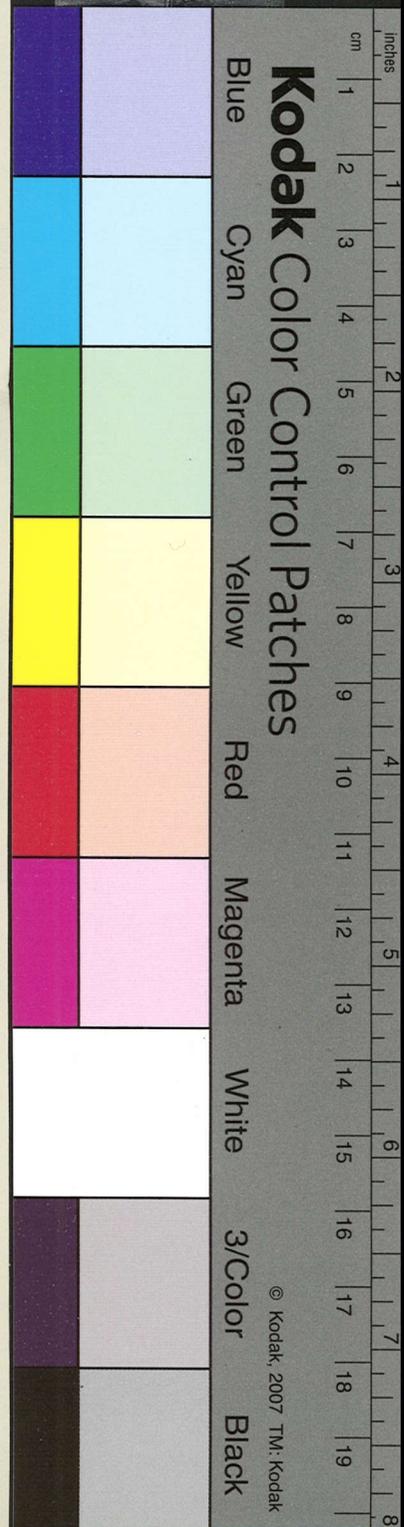
(5)

particles rather than the translational degrees of freedom of ~~the~~ its motion as a whole, the newly constructed 4-dimensional world the translation of the origin in the new 4-dim. world has ~~no~~ does not have a simple meaning contrary to the original space-time world. Thus, the coordinates in this new world can be regarded as ~~those~~ relative ~~to~~ relative coordinates, while the original four coordinates are regarded as those of the center of the particle. If we assume further that these two sets of coordinates transform simultaneously and similarly under Lorentz transformations, ~~we have~~ we can construct a new type of relativistic field theory of particles ~~in~~ with internal structure. ~~without an~~ As an example, the present author tried to formulate such a theory by introducing the concept of "local field." In this way, ~~it was hoped that~~ a convergent unified ~~could~~ field theory of elementary particles might be attempted. ~~the construction of~~ ~~the~~ ~~simplest, conceivable~~ ~~the~~ ~~situation~~ ~~was~~ ~~not~~ ~~at~~ ~~all~~ ~~theory~~ ~~of~~ ~~this~~ ~~type~~ ~~constructed~~ turned out to be, either ~~so~~ from the serious difficulty of infinite degeneracy ~~inherent~~ ~~in~~ ~~the~~ which was intimately related to the non-compactness



(6)  
of Lorentz group. In order to avoid  
this defect, it seemed necessary to introduce  
a coupling bet

↳ A recipe for avoiding this difficulty  
was to introduce a coupling between  
internal and external motions of a particle.  
However, this, in turn, brought about  
a complication which made it very  
difficult to deal with the actual case  
of interaction between particles.  
Under these circumstances,



$$S = \sum_{n=0}^{\infty} \frac{(-ig)^n}{n!} \int \dots \int H(x^{(1)}) \frac{\delta(x^{(1)}, x^{(2)})}{\theta(x^{(1)}, x^{(2)})} H(x^{(2)}) \theta(x^{(2)}, x^{(3)}) \dots H(x^{(n)}) dx^{(1)} \dots dx^{(n)} \quad (1)$$

$$\begin{aligned} i \frac{\partial S}{\partial g} &= \sum_{n=0}^{\infty} (-ig)^{n-1} \int \dots \int H^{(1)} \theta^{(1,2)} H^{(2)} \theta^{(2,3)} \dots H^{(n)} dx^{(1)} \dots dx^{(n)} \\ &= \sum_{n'=0}^{\infty} (-ig)^{n'} \int \dots \int H^{(1)} dx^{(1)} \dots H^{(n')} \theta^{(1,2)} \dots H^{(n')} dx^{(1)} \dots dx^{(n')} \end{aligned}$$

- ∴
- 1 > 2 > ... > n  $\quad \uparrow [H^{(n)}, H^{(2)}] \quad (1)$
  - 1 < 2 > ... > n = 2 > 1 > 3 ... > n  $\quad (2)$
  - + 2 > 3 > 1 > ... > n  $\quad (3)$
  - + ...
  - + 2 > 3 > ... > n > 1 + ...
  - + 2 > 3 > ... > n > 1  $\quad (n)$

n times  $\int \dots \int H^{(1)} \theta^{(1,2)} \dots H^{(n-1)} \theta^{(n-1,n)} dx^{(1)} \dots dx^{(n)}$

$$i \frac{\partial S}{\partial g} = \int H dx \cdot S$$

$$i \frac{\partial S}{\partial g} = \bar{I} \cdot S \quad \bar{I} : \text{interaction representation}$$

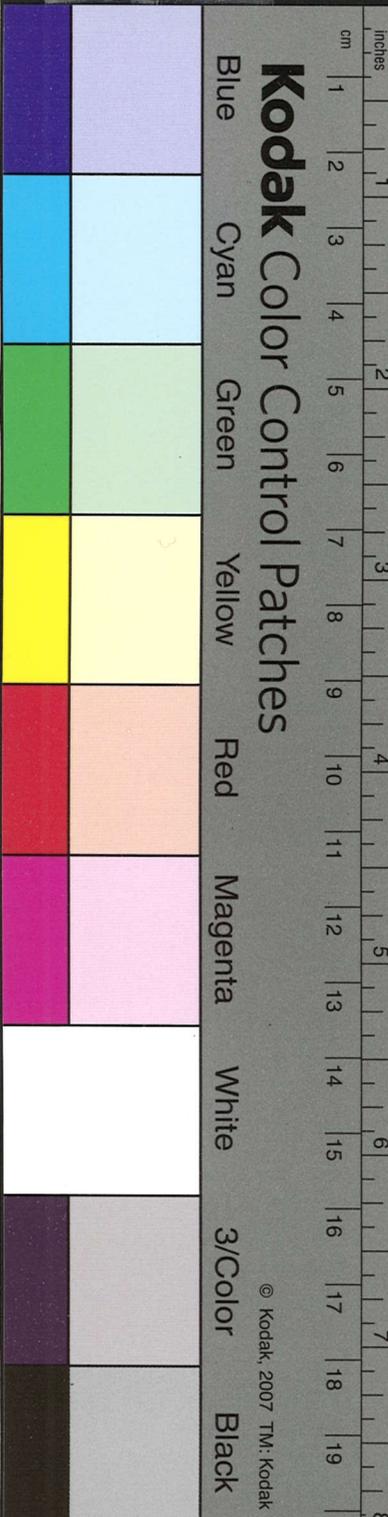
one solution:

$$S = \exp(i \bar{I}) = \sum_n \frac{1}{n!} (ig \bar{I})^n$$

$$i \frac{\partial S}{\partial g} = \bar{I} S \rightarrow$$

$$i \frac{\partial S_{mn}}{\partial g} = \sum_l \bar{I}_{ml} S_{ln}$$

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(2)

boundary condition  
 $g=0: S_{mn} = \delta_{mn}$

$$S_{mn} = \delta_{mn} + g S_{mn}^{(1)} + g^2 S_{mn}^{(2)} + \dots$$

$$i(n+1) S_{mn}^{(n+1)} = \sum_l \bar{I}_{ml} S_{ln}^{(n)}$$

$$i S_{mn}^{(1)} = \bar{I}_{mn}$$

$$2i S_{mn}^{(2)} = (-i) \sum_l \bar{I}_{ml} \bar{I}_{ln}$$

$$S_{mn}^{(2)} = -\frac{1}{2} \sum_l \bar{I}_{ml} \bar{I}_{ln}$$

$$S_{mn} = \delta_{mn} + g \bar{I}_{mn} + \frac{1}{2} g^2 (\bar{I}^2)_{mn} + \dots$$

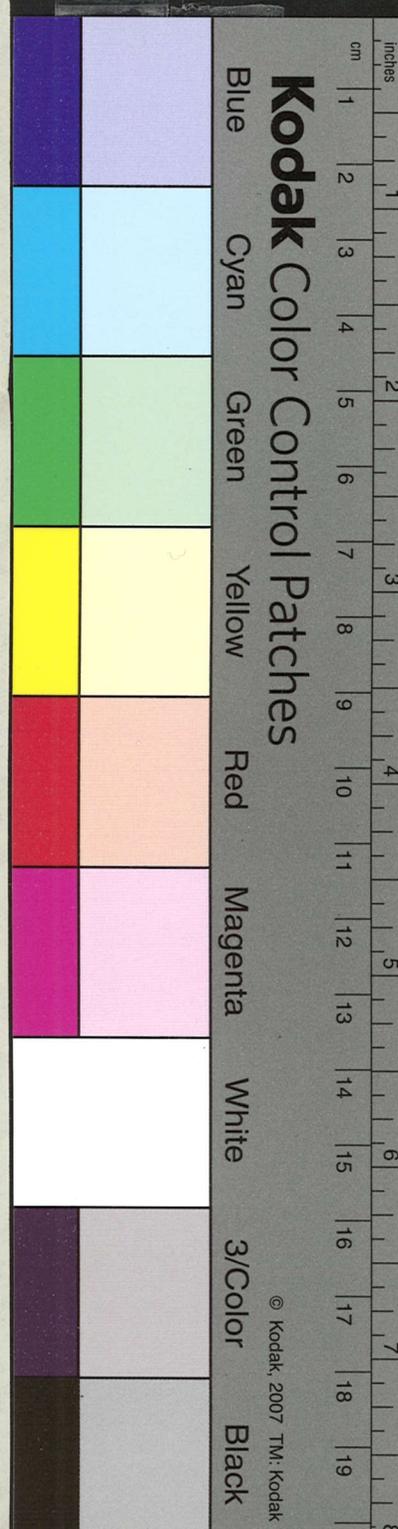
$$\int \dots \int P(H^{(1)}, \dots, H^{(n)}) dx^{(1)} \dots dx^{(n)}$$

$$= n \int H^{(1)} dx^{(1)} \int \dots \int P(H^{(2)}, \dots, H^{(n)}) dx^{(2)} \dots dx^{(n)}$$

$$i \frac{\partial S}{\partial g} = \sum_{n=1}^{\infty} n (-ig)^{n-1} \int \dots \int$$

$$i \log S = \sum_{n=0}^{\infty} n (-ig)^n \int \dots \int$$

$$= S + (S-1)$$



$$i\hbar \frac{\partial \Psi}{\partial t} = H \Psi$$

$$i\hbar H_g \Psi_g$$

$$\Psi = U \Psi_0$$

$$i\hbar \frac{\partial U}{\partial t} \Psi_0 = H U \Psi_0$$

$$i\hbar \frac{\partial U}{\partial t} = H U$$

$$g(t) : \quad \frac{\partial}{\partial t} = g'(t) \frac{\partial}{\partial g}$$

$$i g'(t) \frac{\partial \Psi}{\partial g} = H \Psi$$

$$i \frac{\partial \Psi}{\partial g} = \frac{g I}{g'} \Psi$$

$$g' = 0$$

$$\int H dt = \int H / g' dg$$

