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On a Unified Theory of Elementary Particles. I.

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1. Introduction

Present recent status of the theory of elementary particles is very unsatisfactory in various respects. ~~Above~~ On the ~~all~~ one hand, the discovery of a great variety of unstable new particles in cosmic rays and in nuclear artificially produced events indicates that the ~~exi~~ theory is of little help as long as it remains at the stage of dealing with each kind of particle separately, and as such, ~~without~~ ~~An~~ deeper insight into the ~~systematic~~ interrelations between these particles which would lead us to the systematics of elementary particles as a whole is certainly in need. ~~If we confine our attention to~~ As a matter of fact, the interrelations between various particles ~~were~~ have been revealed to a certain extent ~~dealt with~~ in the ~~existing~~ current theory by considering appropriate interactions between particles. However, such interactions ~~were~~ have been again introduced ad hoc, just as new particles ~~were~~ have been introduced into the theory, ~~whenever it~~ one ~~by one~~ after another, without any ~~regard to~~ possible underlying general ~~principles~~ ~~so far as~~ in sight. Furthermore, ~~On the other hand~~ such interactions were usually treated as small perturbations, ~~Which might be~~ so that the effects of

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these interactions ~~could not be a could not~~ did not seem to be so ~~farreaching~~ drastic as to ~~cause~~ give rise ~~from a small number of particle~~ to a great variety of ~~parti~~ elementary and compound particles, if we started from a small number of elementary particles.

~~On the~~ In connection with this, ~~there were another~~ such ~~attempts~~ undertakings were inevitably accompanied by the appearance of the divergence difficulties, which destroyed the consistency of the theory as a whole.

Under these circumstances, we ~~should~~ may admit that, ~~to~~ at least, two aveneys are open. One is to make the current theory free from the restriction that the interactions between elementary particles are regarded as small perturbations. ~~The~~ In other words, ~~we~~ the essential non-linearity of field equations for elementary particles is to be ~~investigated~~ taken seriously. In fact, it has been generally recognized that the interaction between a nucleon and a pion was so strong that the usual perturbation method ~~fails~~ should give place to other methods such as strong or intermediate coupling method and Tamm-Dancoff method. However, ~~it is not at all clear whether~~ any of ~~such~~ methods can hardly answer the question of construction of a consistent theory of whole family of elementary particles

** ~~a serious restriction~~ as will be discussed later on.
the causality condition very seriously

~~In this connection~~

* the some of the divergence difficulties could be removed by assuming suitable non-local interactions.(4) However ~~the problem of causality condition becomes in this case~~ ** one ~~should~~ take

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unless we make clear ~~it is incorporated into~~ the (3) role played ~~the significance~~ by ~~of~~ such a method in the whole scheme of relativistic quantum theory of fields.(1) Recently Heisenberg developed his idea ~~of unification~~ that the whole family of elementary particles could be contained in a single non-linear spinor field equation as its various stationary solutions,(2) ~~of~~ Although his basic idea is very attractive, it is not at all easy to see whether the non-linearity of field equations alone is ~~sufficient~~ ~~for~~ responsible for the appearance in Nature of a great variety of particles. It is even more difficult to see ~~whether~~ how the divergences ~~are~~ dealt with when the non-linear fields are quantized.(3).

The other ~~direction is~~ way is to introduce, generally speaking, new degrees of freedom, which is useful for classifying particles,* in addition to those which are directly connected with the ~~transformation properties~~ of the local field quantities behavior under Lorentz transformations in the ordinally space-time world. Now the new degrees of freedom could be introduced ~~either~~ by supposing quantized fields in a world which has

In this connection, one may try to generalize the interaction between quantized fields so as to include non-local interactions. In fact, it was shown by Møller and Kristensen that *

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the dimensions larger than 4 (to the ordinary four dimensions).(4) Pais ~~introduced~~ added ~~three~~ or four more dimensions which, ~~one supposed to correspond to interpreted as~~ by themselves constitute an isotopic spin space.(5) The ~~advantage of such a procedure is that of~~ Obviously ~~one can~~ introducing new degrees of freedom ~~keeping close connection with~~ advantage of such a procedure is that one can introduce new degrees of freedom step by step by keeping close connection with the empirical knowledge ~~of~~ of new particles. As for the problem of divergence, however, one can not expect ~~little from~~ much from such a procedure ~~because the newly~~ as long as the new~~ly added~~ dimensions are independent of the original four added dimensions, because the root of divergences lies in the space-time world itself.

~~Now, in order to~~ Unfortunately, ~~it is~~ very little is known ~~whether~~ as to how to establish a mutual relation between the original four dimensions and the newly added ones. However, there is one particular case, in which the relation is very simple and definite. Namely the added dimensions by themselves constitute another 4-dimensional world which has the same structure as the ordinally ~~space~~ world. ~~except for except for the following point~~ except for the following point: Since the new added ~~degrees of freedom~~ dimensions should correspond to so-to-speak internal degrees of freedom ~~each~~ of each elementary in a certain sense

particles rather than the ~~translational~~ degrees of freedom of ~~the mo~~ its motion as a whole, ~~the newly constructed 4-dimensional world~~ the translation of the origin in the new 4-dim. world ~~has no~~ does not have a simple meaning contrary to the original space-time world. ~~Thus,~~ the coordinates in this new world can be regarded as ~~those relative to~~ relative coordinates, while the original four coordinates are regarded as those of the center of the particle. If we assume further that these two sets of coordinates transform simultaneously and similarly under homogeneous Lorentz transformation, ~~we have~~ we can construct ~~a new type of~~ relativistic field theory of particles ~~in~~ with internal structure. ~~without an~~ As an example, the present author tried to formulate such a theory by introducing the concept of non-local field.( ) In this way, ~~it was~~ hoped ~~that~~, ~~the~~ a convergent unified called field theory of elementary particles might be ~~attempted~~ expected. the construction of simplest conceivable

Unfortunately, ~~the situation was not at all~~ theory of this type ~~thus constructed~~ turned out to ~~be~~ suffer, ~~either~~ ~~so~~ from the serious difficulty of infinite degeneracy ~~inherent in the~~ which was intimately related to the non-compactness of Lorentz group. ~~In order to avoid this defect, it seemed necessary to introduce a coupling bet~~

A recepe for avoiding this difficulty was to introduce a coupling between internal and external motions of a particle. However, this, in turn, brought about a complication which made it very difficult to deal with the ~~actual~~ case of interaction between particles. Under these circumstances,

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(1)

$$S = \sum_{n=0}^{\infty} (-ig)^n \int\cdots\int H(x^{(1)}) \underbrace{\ }_{\theta(x^{(1)},x^{(2)})} H(x^{(2)})\,\theta(x^{(2)},x^{(3)}) \cdots H(x^{(n)})\, dx^{(1)}\cdots dx^{(n)}$$

$$i\frac{\partial S}{\partial g} = \sum_{n=0}^{\infty} (-ig)^{n-1}(n) \int\cdots\int H^{(1)}\theta^{(12)} H^{(2)}\theta^{(23)}\cdots H^{(n)}\, dx^{(1)}\cdots dx^{(n)}$$

$n' = n-1$

$$= \sum_{n'=0}^{\infty} (-ig)^{n'} \int\cdots\int H^{(1)} dx^{(1)} \cdots H^{(2)}\theta^{(23)}\cdots H^{(n)} dx^{(2)}\cdots dx^{(n)}$$

$\therefore$ $1>2>\cdots>n$ $\quad + [H^{(1)}, H^{(2)}]$ (1)

$1<2>\cdots>n = 2>1>3\cdots>n$ (2)

$+\, 2>3>1>\cdots>n$ (3) $+[\ ]+\cdots$

$+\cdots\cdots\cdots$ $+\cdots$

$+\, 2>3>\cdots>1>n + \cdots$

$+\, 2>3>\cdots>n>1$ (n) $+\cdots$

n times $\cdot \int\cdots\int H^{(2)}\theta^{(23)}\cdots H^{(n-1)} dx^{(2)}\cdots dx^{(n)} \; dx^{(1)}$

$\int H^{(1)} dx^{(1)}$ $\quad (H^{(1)})\theta^{(12)}$

$$i\frac{\partial S}{\partial g} = \int H\, d^4x \cdot S$$

$$i\frac{\partial S}{\partial g} = \bar{I}\cdot S \qquad \bar{I}: \text{interaction representation}$$

one solution;

$$S = \exp(ig\bar{I}) = \sum_n \frac{1}{n!}(ig\bar{I})^n$$

$$i\frac{\partial S}{\partial g} = \bar{I} S \rightarrow$$

$$i\frac{\partial S_{mn}}{\partial g} = \sum_l \bar{I}_{ml} S_{ln}$$

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boundary condition

$$g=0: \quad S_{mn} = \delta_{mn}$$

$$S_{mn} = \delta_{mn} + g S^{(1)}_{mn} + g^2 S^{(2)}_{mn} + \cdots$$

$$i(\nu+1) S^{(\nu+1)}_{mn} = \sum_l \bar{I}_{ml} S^{(\nu)}_{ln}$$

$$i S^{(1)}_{mn} = \bar{I}_{mn}$$

$$2i S^{(2)}_{mn} = (-i) \sum_l \bar{I}_{ml} \bar{I}_{ln}$$

$$S^{(2)}_{mn} = -\frac{1}{2} \sum_l \bar{I}_{ml} \bar{I}_{ln}$$

$$S_{mn} = \delta_{mn} + g \bar{I}_{mn} + \frac{1}{2} g^2 (\bar{I}^2)_{mn} + \cdots$$

$$\int \cdots \int P(H^{(1)}, \cdots H^{(n)})\, dx^{(1)} \cdots dx^{(n)}$$

$$= n \int H^{(1)} dx^{(1)} \int \cdots \int P(H^{(2)} \cdots H^{(n)})\, dx^{(2)} \cdots dx^{(n)}$$

$$i \frac{\partial S}{\partial g} = \sum_{n=1}^{\infty} n (-ig)^{n-1} \int \cdots \int$$

$$\cancel{\log S} \qquad (-ig)^n$$

$$g \frac{\partial S}{\partial g} = \sum_{n=0}^{\infty} n (-ig)^n \int \cdots \int$$

$$= S + (S-1)$$

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$$i\frac{\partial\Psi}{\partial t} = H\Psi \qquad i\partial \qquad H_g\,\Psi_g$$

$$\Psi = U\Psi_0$$

$$i\frac{\partial U}{\partial t}\Psi_0 = HU\Psi_0$$

$$i\frac{\partial U}{\partial t} = HU$$

$$g(t): \qquad \frac{\partial}{\partial t} = g'(t)\frac{\partial}{\partial g}$$

$$ig'(t)\frac{\partial\Psi}{\partial g} = H\Psi.$$

$$i\frac{\partial\Psi}{\partial g} = \frac{gI}{g'}\Psi \qquad g' = 0$$

$$\int H\,dt = \int H/_{g'}\,dg$$

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