$$x'_\mu = a_{\mu\nu} x_\nu \qquad \underline{\underline{x_0 = cT}}, \quad \underline{\underline{a_{\mu\nu}: \text{real}}}$$

$$g_{\mu\nu} x'_\mu x'_\nu = g_{\kappa\lambda} x_\lambda x_\nu$$

$$g_{\mu\nu} a_{\mu\kappa} a_{\nu\lambda} x_\kappa x_\lambda = g_{\kappa\lambda}$$

$$a^*_{\kappa\mu} = a_{\mu\kappa}$$

$$a^* g a = g. \qquad a^{-1} = g^{-1} a^* g$$

$$g^2 = 1 \qquad g^{-1} = g \qquad a^{-1} = g a^* g$$

$$g a^{-1} = a^* g$$

<u>scalar</u> $\varphi'(x'_\mu) = \varphi(x_\mu)$

$$\varphi(x_\mu) = b_k \exp(i k g x)$$

$$\varphi'(x'_\mu) = b'_{k'} \exp(i k' g x')$$

$$= b'_{k'} \exp(i k' g a x)$$

$$\begin{cases} k' g a = k g \\ \underline{\underline{b'_{k'}}} = \underline{\underline{b_k}} \end{cases} \qquad k' = k g a^{-1} g = k a^* = a k \qquad \underline{\underline{b'_{\cancel{ka^*}\, ak}}} = \underline{\underline{b_k}}$$

$\varphi(x^{(1)})$ — $\varphi'(x^{(1)'})$

$\varphi(x^{(2)})$ — $\varphi'(x^{(2)'})$

point to point correspondence in space-time world

b_k — $b'_{k'}$ in momentum-energy world

functional form ~~changes~~ in general
as the result of ~~Fr~~ coord. transformation

$$\cancel{\varphi(x_\mu) = b_{\alpha} \cos(i k g x)}$$
$$\cancel{\varphi'(x'_\mu) = b'_{\alpha} \cos(i k' g x')}$$

mapping on itself of a four-dimensional continuum (x_μ)

$$x' = a x \rightarrow Q' = A Q$$

→ ~~~~ similar mapping on itself of an n-dimensional continuum $(Q_1, Q_2 \cdots Q_n)$

invariance　　there exists g such that

$$a^* g a = g.$$

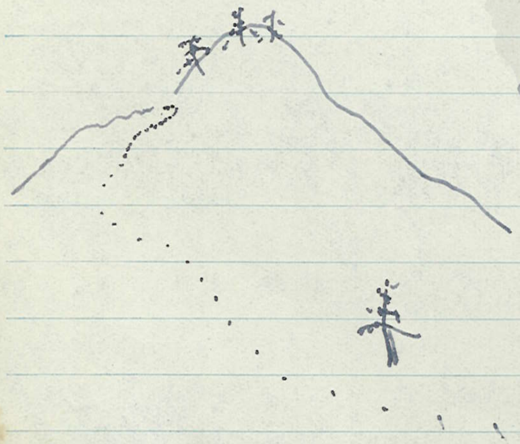
for ~~all~~ any a (real Lorentz transformations) ~~belonging to whole group of~~

$$A^* G A = G.$$

g: time reversal　$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$

4-dimensional repres. with

$$G = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} = \rho_3$$



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Infinite
Representation of Lorentz group [3]

$\varphi_1(x), \varphi_2(x), \cdots\cdots$: complete system of linearly indep. fns

$\varphi_j(x') = \varphi'_j(x) = \sum A_{jk}\varphi_k(x)$

$\varphi_l(x'') = \sum B_{lk}\varphi_k(x') = \sum B_{lk} A_{kj}\varphi_j(x)$

$a \to A \qquad b \to B$

$ba \to BA.$

$a^* g a \to A^* G A$

$G_{jk} = \varepsilon_j \delta_{jk}$

$\varepsilon_j = \pm 1$ according as $\varphi_j(x)$ is even or odd with respect to time.

$\varphi_j \propto e^{i\omega_j t} \to e^{-i\omega_j t}$

$A^* G A = G.$

$$\sum_j \varepsilon_j \varphi_j^*(x')\varphi_j(x') = \sum_j \varepsilon_j \varphi_j^*(x)\varphi_j(x)$$

$$\sum_j A^*_{lj}\varepsilon_j A_{jk} = \varepsilon_l \delta_{lk}$$

$\varphi(x) = \sum_j c_j \varphi_j(x)$

$\varphi'(x') = \sum c'_j \varphi_j(x) = \sum c_j \varphi_j(x')$

$= \sum_{jk} c_j A^T_{kj}\varphi_k(x)$

$c'_k = \sum_j A^T_{kj} c_j$

$$\rho(x,y) = \sum_j \varepsilon_j \varphi_j^*(x) \varphi_j(y)$$

$$\rho'(x',y') = \sum \varepsilon_j \varphi_j^*(x') \varphi_j(y')$$

$$\rho(x,y) = \rho(x',y')$$

$$\rho(x,y) = \rho(xgx,\ xgy,\ ygy)$$

In particular, if

$$\rho(x,y) = \rho(x-y)$$

then $\rho(x,y) = \rho((x-y)g(x-y))$

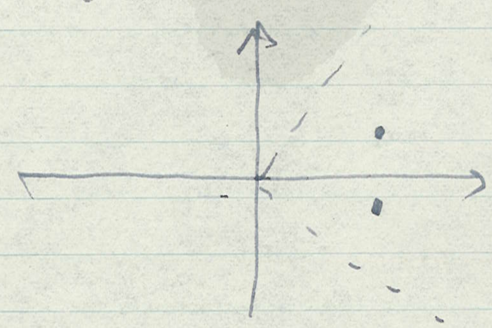
On the other hand

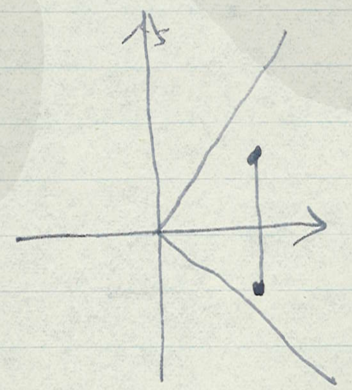
$$\sum_j \varphi_j^*(x)\varphi_j(y) = \delta^4(x-y)$$

$$\sum_{m,j} u_m^*(\vec{r}) \cancel{\varphi(\vec{r})} e^{-i\omega_j t} u_m(\vec{r}') e^{-i\omega_j t'}$$

$$= \delta^3(\vec{r}-\vec{r}') \sum_j e^{-i\omega_j (t+t')}$$

$$= \delta^3(\vec{r}-\vec{r}')\, \delta(t+t')$$





Normalization

$$\int \varepsilon_j \varphi^*_j(x') \varphi_j(x') d^4x'$$

$$= \varepsilon_j \quad A^*_{lj} A_{jk} \underbrace{\int \varphi^*_l(x) \varphi_k(x) d^4x}_{\varepsilon_{lk} \, C_{kl}}$$

$$C_{jj} = \varepsilon_j (A C A^*)_{jj}$$

$$C_{kj} = \int \varphi^*_j(x') \varphi_k(x') d^4x'$$

$$= \sum A^*_{lj} A_{km} \int \varphi^*_l(x) \varphi_m(x) d^4x$$

$$= \quad A^*_{lj} A_{km} C_{ml}$$

$$C = A C A^*$$

$\underline{G = 1}$ $\underline{C = 1}$

$\varphi^* \varphi$