— NOTE BOOK —

覚え書・梗概等

専門

I

April, 1957
~ Aug, 1957

湯川

SPARTA NOTE

CHAPTER

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(I) Parity

References

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/ 105 (1957), 1671

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T. D. Lee, Lecture at Seventh Rochester Conference 1957.

W. Pauli, N. Bohr etc

Ch. 12: Butsuri 12 (1957), Nr. 5. p. 181.

* C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes and R. P. Hudson, Phys. Rev. 105 (1957), 1413 (Feb. 15)

T R. L. Garwin, L. M. Lederman and M. Weinrich, ibid. 1415.

(I) 偶奇性の非保存 及びそれに関連する諸問題

Jan. ~ April, 1957

1. θ-τ-dilemma に関連して (104, 254;)
Lee-Yang (P. R. ~~Nov. 15~~, 1956)
は parity non-conservation を weak interaction について仮定する可能性を検討した。その後
~~1957 初頭~~ β-decay を引き起こす Fermi interaction に対して parity conservation が成立しているならば spin 方向をそろえた核から出てくる β-ray の angular distribution は spin 軸に垂直な平面に関する鏡映に関して対称でなくてはならぬことを示した。一般に
その理由は要するに、allowed transition の場合に、核スピン σ (axial vector) と電子の運動量 p (polar vector) の積 σp をふくむ項が transition matrix の中に、scalar term の他に現われないからである。
C. S. Wu 等実験は Co^{60} を 0.015°K の常磁性の塩の中に入れて磁場をかけて spin の方向をそろえ、磁場の方向を逆に

した場合に、一定方向に出る電子の
数が多いことを確かめ、Lee-Yang
の parity non-conservation を確
認した*（P. R.　　　1957）
これは1957年新年の大きな news であった。

2. Lee-Yang (　　　　), Landau
(　　　　), Salam (　　　　)
等は β-decay にあらわれた、この偶奇性
の非保存を中性微子自身の性質に帰着せし
めた。** neutrino の two component
theory (Weyl,　　　; Pauli, Handb.
24, 226, 1933) は今まで space reflection
に対して invariant でないとの理由で捨てら
れてきた。即ち

(1) $i\frac{\partial\varphi_\nu}{\partial t} = \sigma p\cdot\varphi_\nu$　　$\hbar=c=1$.

は proper Lorentz transf. に対しては不変で
あるが z-axis のまわりの角θだけの回転に対して

* 引続き Lederman 等は π → μ → e
decay の ~~μ に引続く~~ の過程にも
parity 非保存が認められた。これは 2.
で述べる 2成分中性微子の考えに
よって示唆されたものであった。

** T. D. Lee and C. N. Yang, Parity Non-cons.
and a Two-Component Theory of Neutrino
(P. R. 105 (1957), 1671)

$$\varphi_\nu \to \exp(-i\sigma_3\theta/2)\cdot\varphi_\nu$$

になる変換をする。従って σ が neutrino の spin であるといえる。運動量 p が一定の状態では energy は

$$H = \sigma p = |p|\cdot\sigma_p$$

$$\sigma_p = (\sigma p)/|p|$$

となる。energy が positive の ~~...~~ stationary state では σ_p が positive で $1/2$ の値をとる eigenstate, 即ち spin が momentum に平行。 negative energy state では ~~spin~~ σ_p が $-1/2$ で, spin が momentum と反平行。 anti-neutrino を negative energy state の hole と ~~考えると~~ 定義すると, spin と momentum は anti-parallel。つまり, neutrino は right-handed screw を表すとすれば, anti-neutrino は left-handed screw に対応する (screw on)

この考え方では

A. neutrino と anti-neutrino とは同一粒子でない。従って Majorana theory は不可能。

B. neutrino mass (physical mass についての問題) は zero。(mass term があらわれないが, これは physical neutrino は既に two

component theory の枠に入りきらぬ)。
C. space inversion P を施すと，spin と momentum の方向は逆になる。このような state は (1) の解の中には存在しない。従って ~~parity~~ theory は P に対して invariant ではない。
D. charge conjugation C に対しても invariant でない。何故かといえば C によって particle は anti-particle になるが spin と momentum の方向は変らない。そのような粒子は two-component theory の範囲内では許されない。*
E. Two component theory は CP に対しては invariant である。
Lüders-Pauli theorem (G. Lüders, Dan. Mat. Fys. Medd. 28 (1954) Nr. 5; W. Pauli, Niels Bohr, London, 1955) によれば，CPT に対する invariance は常に成立するはずだから，CP に対し inv. ならば T (weak time reversal) に対し

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* particle と anti-particle が全く入れ替った世界 ~~では，parity も違っている~~。へ移るような変換は，4-component theory では charge conjugation と呼ばれているものが含まれている。このような変換は two-component theory の枠からはみ出ている。

でも invariant な形。
実際に two-component theory では
そうなっている。

上記と同じ結論が普通の 4-component
theory から出せる。

$$\alpha \equiv \begin{pmatrix} \sigma & 0 \\ 0 & -\sigma \end{pmatrix} \qquad \beta \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\psi_\nu = \begin{pmatrix} \varphi \\ 0 \end{pmatrix}$$

$$\gamma \equiv -i\beta\alpha \qquad \gamma_4 \equiv \beta \qquad \gamma_5 \equiv \gamma_1\gamma_2\gamma_3\gamma_4 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\gamma_5\psi_\nu = -\psi_\nu, \qquad \psi_\nu = \frac{(1-\gamma_5)}{2}\psi_\nu$$

$$L_\nu = \psi_\nu^\dagger\gamma_4\left(\gamma_\mu\frac{\partial}{\partial x_\mu}\right)\psi_\nu$$

$$L_{int} = -H_{int}$$

$$= \sum -\frac{c_i}{2}(\psi_p^\dagger O_i \psi_n)(\psi_e^\dagger O_i \psi_\nu)$$

$$O_S = \gamma_4, \quad O_V = \gamma_4\gamma_\mu,$$

$$O_T = -\frac{1}{2\sqrt{2}}\, i\gamma_4(\gamma_\lambda\gamma_\mu - \gamma_\mu\gamma_\lambda)$$

$$O_A = -i\gamma_4\gamma_\mu\gamma_5$$

これは $n \to p + e + \bar{\nu}$ に対応している。
この他に $n \to p + e + \nu$ では

process にてψが二重になるような ~~formalism~~ interaction を同時に考えうるので。
($\psi_\nu' = (1+\gamma_5)/2\,\psi_\nu'$)

β-decay asymmetry

$$\beta = \mp\left(\frac{v_e}{c}\right)\frac{|C_T|^2 - |C_A|^2\,\frac{2Ze^2}{\hbar c p}\,\mathrm{Im}(C_A C_T^*)}{|C_T|^2 + |C_A|^2}$$

$- : n \to p + e + \bar{\nu}$
$+ : n \to p + e + \nu$

Wu 等の実験は Co^{60} の場合に

$$1 + \alpha\cos\theta$$

$$\alpha = \beta\frac{\langle J_z\rangle}{J} < 0$$

で $\beta < 0$ 即ち $n \to p + e + \bar{\nu}$ が正しいことを示している。

3. 次に π-μ-decay においては次の二つの場合が考えられる。

(A) $\pi^+ \to \mu^+ + \nu$ (μ^+ spin along $p_\mu = +1/2$)
$\pi^- \to \mu^- + \bar{\nu}$ (μ^- spin along $p_\mu = -1/2$)

(B) $\pi^+ \to \mu^+ + \bar{\nu}$ (μ^+ spin along $p_\mu = -1/2$)
$\pi^- \to \mu^- + \nu$ (μ^- spin along $p_\mu = +1/2$)

いずれにせよ μ は ν と共に完全に polarize

している.

μ-e-decay については

$$\mu^- \to e^- + \nu + \bar{\nu} \qquad (a)$$

$$\mu^- \to e^- + 2\nu \qquad (b)$$

$$\mu^- \to e^- + 2\bar{\nu} \qquad (c)$$

の3つの可能性がある

(a) $H_{int} = \sum_{i=V,A} f_i (\psi_e^+ O_i \psi_\mu)(\psi_\nu^+ O_i \psi_\nu)$

(∵ S, T, P ~~は ψ_ν, ψ_ν^+ に~~ は γ_5 と anti-commute するため, ψ_ν, ψ_ν^+ で書かきえる)

electron の分布は

$$dN = 2x^2[(3-2x) + \xi \cos\theta (1-2x)] \times dx\, d\Omega_e (4\pi)^{-1}$$

$$x = p_e/p_{e,max}$$

$$\xi = \{|f_V|^2 + |f_A|^2\}^{-1}\{f_V f_A^* + f_A f_V^*\}$$

unpolarized μ-meson に対しては

$$dN = 2x^2(3-2x)\,dx$$

∴ Michel parameter $\rho = 3/4$ になり 実験と両立し得る.

(b) 又は (c) の場合には $\rho = 0$ で実験と合わない.

Garwin, Lederman (loc. cit.) 等の主張によると、

$1 + a\cos\theta$

$a = -1/3$ で、定性的に (a) の理論の結論と一致する。

○ ソ連　合同原子核研 (モスコウ郊外) のシンクロファゾトロン完成, April 17, 1957, proton を百億ボルトに加速することに成功した.

① 二つの世界　April 19, 1957

parity non-conservationおよび
neutrinoの特異性に関連して

物質の世界と反物質の世界

同一の普遍的法則の下に、粒子と反粒子とが区別されないとした場合、即ち 核子と反核子、軽粒子と反軽粒子が（及び中間子、反中間子）区別された場合、核子と軽子からなる世界及び反核子と反軽子からなる世界が、それぞれ物質の世界及び反物質の世界として区別される。両者が会すれば 物質と反物質の~~相互~~作用が根源的な過程になる。逆に ~~エネ~~ エネルギーだけが非常より大~~きけ~~きければそこに物質と反物質の世界の二つが同時に作り出される可能性がある。

所で最近までは、普遍的な法則は左右対称性を保持するものと考えられていたのであるが、weak interactionにおける parity non-cons. あるいは neutrinoの特異性を考慮すると、普遍的法則は単に左と右、正と負とあり得ることになる。即ち普遍的法則中に

あらゆる物理法則の中，あるものの符号が~~正と負とで~~を逆にしても不変なものが，もう一つの物理法則として同等の資格を持つ。例えば物理法則にAとBとあり

	~~座標の~~右手系	左手系
Aにおける電気の符号	+	−
Bにおける電気の符号	−	+

になる関係になっていれば，Aの世界とBの世界は違った法則の支配する世界であるから，両者が現実世界に共存することはできない。しかし例えば両者を違った関係を持っていれば，必ずどこかに矛盾を生ずる。

中性微子自身の性質にすべてを帰着させようとする考え方でも，法則そのものはそのままにしておいて粒子と反粒子とを入れかえることではない。表の世界の中性微子と裏の世界の~~法則~~の中性微子（表の世界の反中性微子）とは同一粒子としても法則が違う。その違いは，表では右手系，裏では左手系~~の~~を取ることによってなくなる。

他の粒子，例えば電子の場合には，自由粒子としては，電子も反電子も同じ法則

に従う。電磁場との相互作用を考えると
違いがでてくる。
4成分中性粒子理論では ~~中~~ 粒子と反
粒子 ~~[illegible]~~ とは同一の法則に従うが、相互
中性粒子としては
作用が違う　その違いは ~~[illegible]~~ 中性子と反中性子
を区別することによって　[illegible]
[illegible]。

(II) Non-local theories

References

1) H. Yukawa,

2) G. Wataghin, Z.S. f. Phys. 88 (1934), 92

3) M.A. Markov, J. of Phys. USSR, 2 (1940), 452

4) V.L. Ginzburg and I.E. Tamm

5) Rayski,

6) R.E. Peierls

7) Møller and Kristensen

II. Non-local field 関する文献.

M. Markow

On Dynamically Deformable Form Factors in the Theory of Elementary Particles, Nuovo Cimento 3 (1956), 760.

(I) Parity Non-Conservation 等

A. Salam, On Parity Conservation and Neutrino Mass
N. C. ~~4~~ 3^{rd} (1957), 299

Invariance with respect to

$\psi_\nu \to \gamma_5 \psi_\nu$ $\quad$ $\bar{\psi}_\nu \to -\bar{\psi}_\nu \gamma_5$

$L \to L$ $\quad$ $\bar{\psi}_\nu \psi_\nu \to -\bar{\psi}_\nu \psi_\nu$

Consequences:

(a) To β-decay coupling

$$\{\bar{p}(x)\,\Omega\, n(x)\}[\bar{e}(x)\,\Omega\, \nu(x)]$$

must be added pseudo-coupling

$$\{\bar{p}[x]\,\Omega\, n[x]\}[\bar{e}(x)\,\Omega\, \gamma_5 \nu(x)]$$

(b) π_μ-decay

$$f\,\bar{\mu}(x)\, i\gamma\partial\pi(x)\,(\gamma_5 \nu(x) + \nu(x)) + h.c.)$$

similarly for $K_{\mu\nu}$-decay.

(c) magnetic moment of neutrino must vanish because

$$\bar{\nu}(x)\,\sigma_{\alpha\beta}\,\nu(x) \to -\bar{\nu}(x)\,\sigma_{\alpha\beta}\,\nu(x)$$

Since

$$j_\mu = \bar{\nu}(x)\gamma_\mu \nu(x) \to +\bar{\nu}(x)\gamma_\mu \nu(x),$$

the neutrino and anti-neutrino are not identical

I

May '57

A. Salam, On Fermi Interactions

I Invariance with respect to

$\psi_e \to -\gamma_5 \psi_e$, $m_e \to -m_e$

and similarly for μ-meson.

$g_V = a + \frac{m_e m_\mu}{M^2} b$

$g_A = a - \frac{m_e m_\mu}{M^2} b$

for μ-e-decay

I

Lüders-Pauli Theorem について　April, 1957

W. Pauli, Niels Bohr and the development of Physics, London 1955

Bohr's favourite verse of Schiller:
"Nur die Fülle führt zur Klarheit
Und im Abgrund wohnt die Wahrheit."

Assumptions:
(A) Connection between spin and statistics
(B) Invariance of the Lagrangian with respect to continuous Lorentz group L_+.
(C) Local character of the field equation
(D) Kinematically independent spinor fields anticommute.

Ordering of products: ~~(inversion)~~
Each product of M Boson fields and N Fermion fields is to be replaced by the sum, divided by (M+N)! of all permutations of the factors, each of the terms being multiplied by +1 or −1 for an even or odd permutation of the Fermion fields, respectively.

Conclusion: Every ordered local vector changes its sign and every ordered local tensor or the second rank or scalar stays unchanged under SR.

Proof: (T): For SR fields have to be multiplied by $(-i)(-1)^m = i(-1)^n$ for $m+n$ odd
$(-1)^m = (-1)^n$ for $m+n$ even

If (T) holds for the original field quantities and for the space-time coordinates, it also holds for all ordered products of them or their derivatives of finite order after application of the inversion.

generalization of the invariance of charge with respect to WR

$$\left.\begin{aligned} j'_\mu(x) &= j_\mu(-x) \\ f'_{\mu\nu}(x) &= -f_{\mu\nu}(-x) \\ \phi'_\mu(x) &= \phi_\mu(-x) \end{aligned}\right\}$$

generalized gauge group:

$[Q, f] = -f \qquad [Q, f^*] = +f^*$ T [?]

---

f = 4(?) prototype is

$N = \sum_k a_k^+ a_k, \quad \left.\begin{aligned} [N, a_k] &= -a_k \\ [N, a_k^*] &= a_k^* \end{aligned}\right\}$

Q has the general meaning of the difference between the numbers of particles and ~~p~~ anti-particles which is the integral of the field equation owing to a generalized gauge group.*

Energy-momentum operator

$$i[P_\mu, f] = -\frac{\partial f}{\partial x_\mu}$$

(translation group)

postulate of the invariance of P_μ

$$P_\mu' = P_\mu$$

with respect to the space-time reflection

$$x_\mu' = -x_\mu$$

generalization to the invariance of the density:

$$T_{\mu\nu}'(x) = T_{\mu\nu}(-x)$$

and

$$L'(x) = L(-x)$$

<u>The inversion has to be an essential</u>

---

* $[Q, \bar{H}] = 0$

~~Q~~ $\bar{H} \propto \int f^* \cdots f$

$\to \int e^{-i\alpha Q} f^*$ $\qquad f e^{i\alpha Q} \propto e^{-i\alpha Q} \bar{H} e^{i\alpha Q}$

part of the transformation of the fields, if time is reversed, (that means both for WR and for SR), in order that ~~p_μ~~ p_μ ~~iE_μ~~, the left-hand side of (8) change sign. (According to Schwinger, P.R. 82 (1951), 914, inversion is defined by the rule that all operator relations have to read from right to left instead of from left to right.)

WR: (i) Q retains its sign.
(ii) For the inversion the commutators in (7) produce a change of sign.
Thus for WR any f has to be transformed into a f^* and vice versa

$$\phi'(x) = \phi^*(-x) \qquad \phi^{*\prime}(x) = \phi(-x) \qquad (10)$$

$$\bar\psi(x) = \psi^*(x)\gamma_4$$

$$\psi'(x) = \Omega^{-1}\bar\psi(-x), \qquad \bar\psi'(x) = \psi(-x)\Omega$$

Ω transforms the Hermitian Dirac matrices γ_μ ($\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}$) into

$$(\gamma_\mu)^T = \Omega\gamma_\mu\Omega^{-1} \qquad (11)$$

$$\psi'(x') = \cancel{\Omega}\,\psi'(-x) = \Omega^{-1}\bar\psi(x)$$

#

$$\Omega\Omega^* = 1, \quad \Omega^T = -\Omega$$

$$j_\mu(x) = ie\left(\frac{\partial\phi^*}{\partial x_\mu}\phi - \phi^*\frac{\partial\phi}{\partial x_\mu}\right)$$

and

$$j_\mu(x) = ie\bar{\psi}(x)\gamma_\mu\psi(x)$$

are both invariant. ##

(10) or (12) can be interpreted as replacing every emission operator by an absorption of operator of the same eigen-vibration without interchange of particle and anti-particle.

(:

$$\phi(x) = a_k \exp(i k_\mu x_\mu) + b_k^* \exp(-i\cdot\cdot)$$

$$\phi'(x') = \phi'(-x) = \phi^*(x)$$

$$= a_k^* \exp(-i \quad) + b_k \exp(+i\cdot\cdot)$$

$$= a_k^* \exp(+i k_\mu x_\mu)$$

$$+ b_k \exp(-i k_\mu x_\mu)$$)

$$\Omega\gamma_\mu\partial_\mu' \psi'(x') = -\Omega\gamma_\mu\partial_\mu\Omega^{-1}\bar{\psi}(x)$$

$$= -(\gamma_\mu)^T\partial_\mu\bar{\psi}(x)$$

$$\bar{\psi}'(x')\gamma_\mu\psi'(x') = \psi(x)(\gamma_\mu)^T\bar{\psi}(x)$$

For W.R. ~~de~~ commutation rules is indifferent.

(ps) $(\gamma_5)^T = \Omega\gamma_5\Omega^{-1}$ † $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$

(t) $\gamma_{[\mu\nu]}^T = -\Omega\gamma_{[\mu\nu]}\Omega^{-1}$

(a) $[\gamma_5\gamma_\mu]^T = -\Omega\gamma_5\gamma_\mu\Omega^{-1}$, (t) $[\gamma_5\gamma_{[\mu\nu]}]^T = -\Omega\gamma_5\gamma_{[\mu\nu]}\Omega^{-1}$

Thus for WR:

+ sign for $\bar\psi\psi$, $i\bar\psi\gamma_\mu\psi$, $i\bar\psi\gamma_5\psi$

− sign for $i\bar\psi\gamma_5\gamma_\mu\psi$, $i\bar\psi\gamma_{[\mu\nu]}\psi$, $\bar\psi\gamma_5\gamma_{[\mu\nu]}\psi$

Lagrangian which is invariant with respect to proper L.T.:

$\Phi\bar\psi\psi$; $\Phi(i\bar\psi\gamma_5\psi)$

$\frac{\partial\Phi}{\partial x_\mu}(i\bar\psi\gamma_\mu\psi) + H.c.$; $\Phi_\mu(i\bar\psi\gamma_\mu\psi)$

$\frac{\partial\Phi}{\partial x_\mu}(i\bar\psi\gamma_5\gamma_\mu\psi) + H.c.$; $\Phi_\mu(i\bar\psi\gamma_5\gamma_\mu\psi)$

$\left(\frac{\partial\Phi_\nu}{\partial x_\mu} - \frac{\partial\Phi_\mu}{\partial x_\nu}\right) i\bar\psi\gamma_{[\mu\nu]}\psi + H.c.$

$\left(\frac{\partial\Phi_\nu}{\partial x_\mu} - \frac{\partial\Phi_\mu}{\partial x_\nu}\right) \bar\psi\gamma_5\gamma_{[\mu\nu]}\psi$

---

† $(\gamma_\mu\gamma_\nu)^T = \gamma_\nu^T\gamma_\mu^T$

$[\gamma_\mu,\gamma_\nu]^T = -[\gamma_\mu^T,\gamma_\nu^T]$

$= -\Omega[\gamma_\mu,\gamma_\nu]\Omega^{-1}$

$$\phi'(x) = -\phi(-x) \qquad \phi_\mu' = \phi_\mu(-x)$$
$$\Phi'(x) = +\Phi(-x) \qquad \Phi_\mu' = -\Phi_\mu(-x)$$

complex field:

$$\phi'(x) = \phi^*(-x), \quad \phi^{*\prime}(x) = \phi(-x)$$
$$\phi_\mu'(x) = \phi_\mu^*(-x), \quad \phi_\mu^{*\prime}(x) = \phi_\mu(-x)$$

Summary:

(1) The transformation law of a quantity with respect to proper Lorentz-transf. does not ~~determine~~ determine uniquely its ~~b~~ behavior for WR. The latter depends on the assumed interaction energy.

(2) The invariance with respect to WR imposes further restrictions upon the Lagrangian density of the interaction besides its invariance for proper Lorentz transformations.

Strong Reflection

$$\mathcal{Q}' = -\mathcal{Q} \qquad j_\mu'(x) = -j_\mu(-x)$$

$$(27a)\begin{cases} \psi'(x) = i\gamma_5\psi(-x) \\ \bar\psi'(x) = -(-i)\bar\psi(-x)\gamma_5 \end{cases}$$

$$(27)^{or}\ \{\psi'(x) = \gamma_5\psi(-x), \quad \bar\psi'(x) = -\bar\psi(-x)\gamma_5$$

$$j_\mu(x) = ie\frac{1}{2}\{\bar\psi(x)\gamma_\mu\psi(x) - \psi(x)\gamma_\mu\bar\psi(x)\}$$

Particle-Anti-particle Conjugation:

$$\psi'(x) = C^{-1}\bar\psi(x) \qquad \bar\psi'(x) = \psi(x)C$$

$$C = \Omega\gamma_5 \qquad CC^* = 1$$

$$C^T = -C, \qquad \gamma_\mu^T = -C\gamma_\mu C^{-1}$$

1

S. Watanabe

Chirality of K-Particle

(IBM Research Laboratory
Poughkeepsie, N.Y., March 13 '57)

§1. Chirality (pronounced as kirality) means "handed-ness". ~~H~~ The chirality operator anticommutes with the parity operator and can be applied to both fermions and bosons. In the case of a spinor particle, the eigenvalues of chirality are ±1, which are good quantum numbers only when the mass is zero. In the case of a boson, the eigenvalues are ±2 and 0 and are good quantum numbers even if the mass is finite. The scalar particle can have only eigenvalues ±2. The eigenstates of chirality imply an indefinite parity.

§2. Mirage (space-inversion)*:

$$\Psi(r,t) \to \gamma_4 \Omega \Psi(r,t) = \gamma_4 \Psi(-r,t)$$

$$\Omega = \Omega^{-1} = \Omega^T = \Omega^*$$

$$(r'|\Omega|r'') = \delta(r'+r'')$$

* S. Watanabe, R.M.P. 27 (1955), 40.

parity operator:

$$P = \gamma_4 \Omega \qquad P^2 = 1$$

parity conjugation operator X:

$$[P, X]_+ = 0$$

$X \equiv \gamma_5$: $[P, \gamma_5]_+ = 0$

chirality conjugation operator Y:

$$[X, Y]_+ = 0$$

change conjugation C:

$$\Psi \to \bar{\Psi}\gamma_4 C, \quad \bar{\Psi} \to C^{-1}\gamma_4 \Psi$$

$\bar{\Psi}$: hermitian conjugate of Ψ

Covariantly $\Psi \to \bar{\Psi} C$

$$\bar{\Psi} \to \gamma_4^T C^{-1} \gamma_4 \Psi),$$

$$C^{-1}\gamma_\mu C = -\gamma_\mu^T \qquad C^T = -C, \quad \bar{C} = C^{-1}$$

C leaves the parity and chirality in q-number theory.

Helicity: twice the spin in the direction of propagation vector

$$\eta(p) = i\gamma_4\gamma_5\gamma_a p_a/|p|$$

$$a = 1, 2, 3.$$

Para-helicity ($\propto$ magnetic moment in a direction $\perp$ar to the propagation vector)

$$\lambda(p) = i\gamma_5 \gamma_a l_a$$

$$l_a l_a = 1, \quad l_a p_a = 0$$

$$[P, \eta(p)]_+ = 0 \qquad [X, \eta(p)]_- = 0$$

$$[P, \lambda(p)]_- = 0 \qquad [X, \lambda(p)]_+ = 0$$

$$H = i\gamma_4 \gamma_a p_a + \gamma_4 m$$

$m = 0$: X commutes with H, p and $\eta(p)$

$$\eta = -XH/|p|, \quad m = 0$$

		H	$\eta(k)$	X $(m=0)$
a_1	$u_1(k)$	$+$	$+$	$-$
a_2	$u_2(k)$	$+$	$-$	$+$
b_1	$u_3(k)$	$-$	$+$	$+$
b_2	$u_4(k)$	$-$	$-$	$-$

$$\int (\psi, \eta\psi)\,dv = \sum_k \{N_1(k) - N_2(k) + M_1(k) - M_2(k)\}$$

$$\int (\psi, X\psi)\,dv = \sum_k [-N_1 + N_2 - M_1 + M_2]$$

$$N_{1,2} = \bar{a}_{1,2} a_{1,2} \;; \quad M_{1,2} = \bar{b}_{1,2} b_{1,2}$$

C : $\quad N_1 \rightleftarrows M_1, \quad N_2 \rightleftarrows M_2$

M : $\quad N_1(k) \rightleftarrows N_2(-k), \quad M_1(k) \rightleftarrows M_2(-k)$

$$\lambda: \quad N_1(R) \rightleftarrows N_2(R)$$
$$M_1(R) \rightleftarrows M_2(R).$$

Two-component neutrino theory:

$$\psi_{(1)} \equiv (1+X)/2 \cdot \psi \quad ; \quad X\psi_{(1)} = +\psi_{(1)}$$
$$\psi_{(2)} \equiv (1-X)/2 \cdot \psi \quad ; \quad X\psi_{(2)} = -\psi_{(2)}$$

$$\psi_{(1)} \rightarrow N_2, M_1 \quad ; \quad \psi_{(2)} \rightarrow N_1, M_2$$

Only $\psi_{(1)}$ exists in the right-handed coordinate system.

Time reversal (q-number theory)

$$R: \quad \psi(x,t) \rightarrow \psi(x,-t)\,\gamma_5 C$$
$$\bar\psi(x,t) \rightarrow C^{-1}\gamma_5\,\psi(x,-t)$$

after the operation

$$\bar\psi(x,t)\,O\,\psi(x,t) \rightarrow \psi(x,t)\,O^T\,\bar\psi(x,t)$$

§3. Boson Field with Indefinite Parity

$$\gamma_\mu a_{\mu\nu} = S\gamma_\nu S^{-1}$$

$$C^{-1}SC = \sigma_t (S^T)^{-1}$$

$$\gamma_4 S \gamma_4 = \sigma_t (\bar S)^{-1}$$

where σ_t

$+1$ for S even reflection

-1 for S odd 〃

G : four-four Dirac matrix

$S: \; G \to G' = SG\bar{S}$

$F = G\gamma_4 \qquad G = F\gamma_4$

$S: \; F \to F' = \sigma_t S F S^{-1}$

G transforms like $\varphi\bar{\psi}$

F 〃 $\varphi\psi^{+} \qquad \psi^{+} = \bar{\psi}\gamma_4$

$G = (S + iP\gamma_5 + iV_\mu\gamma_\mu + iA_\mu\gamma_5\gamma_\mu + iT_{\mu\nu}\gamma_{[\mu}\gamma_{\nu]})$
$\times\gamma_4$ (μ, ν)

$T_{\mu\nu}\gamma_{[\mu}\gamma_{\nu]} = T'_{\kappa\lambda}\gamma_5\gamma_{[\kappa}\gamma_{\lambda]}$

$T'_{\kappa\lambda} = -T_{\mu\nu}\varepsilon_{\mu\nu\kappa\lambda}$

$\bar{G} = (\bar{S} + i\bar{P}\gamma_5 + i\bar{V}_\mu\gamma_\mu + i\bar{A}_\mu\gamma_5\gamma_\mu$
$+ i\bar{T}_{\mu\nu}\gamma_{[\mu}\gamma_{\nu]})\gamma_4$

(G is hermitian if S, P, V, A and T are all real)

$S: \; \bar{G}\gamma_4 = \sigma_t S(\bar{G}\gamma_4)S^{-1}$

$sp\, A = (1/4)\sum_i A_{ii}$

$S: \; sp(F_1F_2\cdots F_n) \to (\sigma_t)^n sp(F_1F_2\cdots F_n)$

S: $sp(\bar{G}\gamma_4 G\gamma_4) \to sp(\bar{G}\gamma_4 G\gamma_4)$
(regular scalar)

$$sp(\bar{G}\gamma_4 G\gamma_4) = \bar{S}S \pm \bar{P}P - \bar{V}_\mu V_\mu + \bar{A}_\mu A_\mu \pm \bar{T}_{\mu\nu} T_{\mu\nu}$$

$sp(\bar{G}G) = \bar{S}S + \bar{P}P$: regular scalar

$$G = G^{(+)} + G^{(-)}$$

$$G^{(+)} = [(G + \gamma_4 G\gamma_4)/2] \quad G^{(-)} = [(G - \gamma_4 G\gamma_4)/2]$$

$G^{(+)} = \gamma_4 G^{(+)}\gamma_4$, $\quad G^{(-)} = -\gamma_4 G^{(-)}\gamma_4$
positive parity $\qquad$ negative parity

If, $\quad \gamma_5 G = c_1 G \qquad c_1 = \pm 1 \qquad (3.18)$
we say that G has first chirality equal to c_1, which $+1$ or -1.

$$G = {}_{(+)}G + {}_{(-)}G \quad \begin{cases} {}_{(+)}G = \{[1+\gamma_5]/2\}G \\ {}_{(-)}G = \{[1-\gamma_5]/2\}G \end{cases}$$

$${}_{(+)}G = \gamma_5\, {}_{(+)}G \qquad {}_{(-)}G = -\gamma_5\, {}_{(-)}G.$$

$${}_{(\pm)}G = [(S \pm iP) + i(P \mp iS)\gamma_5 + i(V_\mu \pm A_\mu)\gamma_\mu + i(T_{\mu\nu} \pm T'_{\mu\nu})\gamma_{[\mu}\gamma_{\nu]}]\gamma_4/2$$

When G does not satisfy (3.18), we say that $c_1 = 0$.

$G\gamma_5 = c_2 G$ (second chirality)

total chirality:

$c = c_1 - c_2$ $(\pm 2, \pm 1, 0)$

$G_{(\pm)}\gamma_5 = \pm G_{(\pm)}$

scalar, tensor: $c_1 = -c_2$ $\quad c = \pm 2$

vector: $c_1 = c_2$ $\quad c = 0$

parity transformation changes the sign of c_1 as well as that of c_2.

Any transformation that anti-commutes with γ_5, changes the sign of both c_1, c_2 and is ~~called~~ a chiral conjugation for boson fields.

$C: \quad G \to -(C^{-1}\gamma_4 \bar{G} \gamma_4 C)^T$

A boson field with definite parity can be expressed by any one of the five terms in Eq. (3.2) and has a vanishing chirality

pion field: $G_\pi = i b \gamma_5 \gamma_4$

with chirality zero.

anti-pion: $G_{\bar{\pi}} = i \bar{b} \gamma_5 \gamma_4$

$(C: \; i b \gamma_5 \gamma_4 \to i \bar{b} \gamma_5 \gamma_4)$

K-particle:

$$G_K = a(1+\gamma_5)\gamma_4/2$$

$$c_1 = -c_2 = 1, \quad c = 2$$

(in a right-handed coordinate system)

C: $$G_{\bar{K}} = \bar{a}(1-\gamma_5)\gamma_4/2$$

$$c_1 = -c_2 = -1 \quad c = -2$$

Space-inversion:

$$M: \; a(1+\gamma_5)\gamma_4/2 \to a(1-\gamma_5)\gamma_4/2$$

Postulate: ~~Their~~ positive K-particles with chirality +2 and negative K-particles with chirality −2 alone exist in nature.

A combination of C and chiral conjugation result in a forbidden particles with opposite chiralities.

Neutral K- and anti-K- fields can ~~be~~ never be the same.

q-number theoretical transformation:

$$M: \; G(r) \to \gamma_4 G(-r)\gamma_4$$

$$R: \; G(t) \to \gamma_5 (C^{-1}\bar{G}(-t)C)^T \gamma_5$$

(In the time reversal, we have to

reverse the order of the factors, if a product is involved)

$$C \times R \times M : \begin{cases} S, P, T \to S, P, T \\ V, A \to -V, -A \\ G_K \to G_K \end{cases}$$

4. Free and Interaction Lagrangian

$$\mathcal{L}_F = -\int sp(\bar{G} W G) dx$$

with $W = (-\partial_\mu \partial_\mu + m^2) I = (\overleftarrow{\partial}_\mu \partial_\mu + m^2) I$

$$H_F \sim \int sp[\bar{G}(-2\overleftarrow{\partial}_4 \partial_4 + W) G] dx$$

γ_5 commute with $2\partial_4\partial_4 + W$ so that the chirality is a good q. n. even if $m \neq 0$.

Current vector

$$i_\mu \sim i\,sp[\bar{G}(\overleftarrow{\partial}_\mu - \partial_\mu) G]$$

or

$$i_\mu \sim i[(\partial_\mu \bar{a}) a - \bar{a}(\partial_\mu a)]$$

$$\mathcal{L}_I = \int L_I dx$$

$$L_I = f\, sp(\underbrace{F_K F_K \cdots}_{n} \underbrace{F_{\bar{K}} F_{\bar{K}} \cdots}_{l} \underbrace{F_\pi \cdots}_{m})$$

$+ h.c.$

which ~~was~~ does not vanish only if $n=0$ or $l=0$. When $l=0$ we have

$$L_I = f a^n (ib)^m/2 + h.c. \quad \text{if } n \neq 0$$

$$= f(ib)^m (1+(-1)^m)/2 + h.c. \quad \text{if } n=0$$

5. Possible Relation between Chirality and Strangeness:

(i) fusion theoretical interpretation
 cohesive field

(ii) composite particle model
(Goldhaber, (P.R. 101, 433, 1956)

Relationship between the symmetry property in the outer space and that in the inner (isotopic) space.

(Yukawa, Seattle Conference)

(I)

May, '57

S. Watanabe

Chirality of spin ½ particle

8-component theory of ~~the~~ spin ½ particle !!!

III Non-linearity

References

1) W. Heisenberg, Nachr. Göttinger Akad. Wiss. (1953), 111.
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III non-linearity に関する諸問題

(Feb. 1957)

W. Heisenberg, On Recent attempts concerning the quantum theory of fields and the elementary particles

1. There is no criterion by which we can distinguish between an elementary particle and a compound system.
2. The S-matrix is an important but very complicated mathematical quantity that should be derived from the fundamental field equations, but it can scarcely serve for formulating these equations.

A. General principles for a fundamental theory:

1. The field operators shall not refer to any specified particle like proton, meson, etc., but simply to <u>matter in general</u>.
2. The particles (elementary or compound) should be derived as ~~eigenfunction~~ eigensolutions of the field equations.
3. The fundamental field equations must be non-linear in order to represent interaction. The masses of

the particles should be a consequence of this interaction. Therefore the concept of a "bare particle" has no meaning.

4. The selection rules for the creation and decay of the particles should ~~be~~ follow from the symmetry properties of the fundamental equations. Therefore the empirical selection rules should provide the most detailed information on the structure of the equations.

5. Besides the selection rules and the invariance properties, the only other guiding principles available seems to be the simplicity of the equations.

B. A model for the theory of matter:

i) $\gamma_\nu \frac{\partial \psi}{\partial x_\nu} - l^2 \psi (\psi^\dagger \psi) = 0$

ii) Introduction of Hilbert space II:
The local behavior of $\psi(x)$ cannot

† R. Ascoli u. W. Heisenberg, to be published in Zeits. f. Naturf.

be interpreted in the usual manner, since $\psi(x)$ contains contributions from H II, and their conventional interpretation would lead to negative probabilities.

$$S_{\alpha\nu}(x,x') = \psi_\alpha(x)\psi^\dagger_\nu(x') + \psi^\dagger_\nu(x')\psi_\alpha(x)$$

$$\frac{1}{2}S(p) = \int \rho(\kappa)\,d\kappa \left[\frac{p_\mu\gamma_\mu + i\kappa}{p^2+\kappa^2} - \frac{p_\mu\gamma_\mu + i\kappa}{p^2} + \frac{p_\mu\gamma_\mu \kappa^2}{(p^2)^2} \right]$$

with $\int \rho(\kappa)\,d\kappa = 1$

new Tamm-Dancoff method:

$$\tau_{\alpha\beta\gamma}(x_1 x_2 | x_3) = \langle \Omega | T\psi(x_1)\psi(x_2)\psi^\dagger(x_3) | \Phi \rangle$$

C. Conclusions:

a. Fermion spectrum:

spin 1/2 : $\kappa = 7.426/l$

b. Boson spectrum

κl	spin	parity
0.33	1	-1
0.95	0	1
1.74	0	-1
3.32	0	1

c) electrodynamics
fine structure constant
$\alpha_F \approx 1/267$
d) Solution of an integral eq.
$\gamma_\mu \frac{\partial S_F}{\partial x_\mu} \sim G_F S_F$?

D. Extension of the model.
a. Isotopic spin.
b. Weak interactions.
c. leptons?

(IV) Structure of Particles

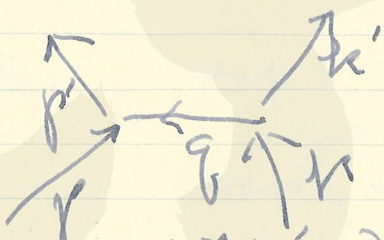
Electromagnetic Structure of Nucleons

D. R. Yennie, M. M. Lévy and D. G. Ravenhall
(R.M.P. 29 (1957), 144)

1. Electron-Neutron Interaction

$$-4\pi i\, j_\mu^{p,n}(p',p)(1/q^2)\, j_\mu^{e}(k',k)$$

(matrix element for the scattering of an electron ~~by~~ from a nucleon due to their electromagnetic interaction which includes both the Coulomb interaction and the exchange of transverse photons.)



$$q_\mu = p'_\mu - p_\mu = -(k'_\mu - k_\mu)$$

$$j_\mu^{p,n}(p',p) = ie\bar{u}(p')[\gamma_\mu F_1^{p,n}(q^2) + (\kappa^{p,n}/2M)\sigma_{\mu\nu}q_\nu F_2^{p,n}(q^2)]u(p)$$

$$\langle r^2\rangle_{1,n} = (0.000 \pm 0.006)\times 10^{-26}\ \mathrm{cm}^2$$

2. Electron-Proton Scattering

$$(\langle r^2 \rangle_{1p})^{1/2} = (\langle r^2 \rangle_{2,p})^{1/2} = 0.77 \times 10^{-13}\ cm$$
(± 0.10)

3. Electron-Deuteron Scattering

4. Hydrogen Spectra

$\langle r_{em} \rangle < 0.5 \times 10^{-13}\ cm$

$$\langle r^2_{em} \rangle = \langle r^2 \rangle_{ch} + \langle r^2 \rangle_{m}$$

(II) Non-local

G. Wataghin On a Non-Local Relativistic Quantum theory of Fields I
(Istituto di Fisica dell'Università - Torino)
Nuovo Cimento 5 (1957), March

2. Definition of the Cut-off Operators. First Approximation to the Construction of the S-Matrix
3. Internal Coordinates, Modified Propagators

$$S^n = \frac{(-i)^n}{n!}\int_{-\infty}^{+\infty} d^4x_1 \ldots d^4x_n \, P\left(H'(x_1)\cdots H'(x_n)\right)$$

$$H'(x) = i\frac{g}{m}\left(\bar\Psi(x)\gamma_5\gamma_\nu\tau_\sigma\Psi(x)\partial_\nu\varphi_\sigma(x) + \cdots\right)$$

Rules: $\exp[-ik'x] \to \exp[-ik'(x^+ + \eta')]$
for positive frequency
$\exp[ik''x] \to \exp[ik''(x^- + \eta'')]$
for negative frequency

form factor $F(\eta', \eta', \eta'')$
is a function of invariant operators

$$(I_s^\pm)^2 = \frac{(\eta^\nu i^* \partial_\nu^\pm)^2}{|i\partial_\nu^\pm i\partial^{\pm\nu}|} - \eta^\nu\eta_\nu$$
$$= (\eta^\nu U_\nu^\pm)^2 - \eta^\nu\eta_\nu$$

and

$$I_t^{\pm} = \frac{\eta^\nu i\partial_\nu^{\pm}}{|i\partial_\nu^{\pm}\cdot i\partial^{\pm\nu}|} = \eta_\nu U_\nu^{\pm}$$

where

$$U^{+} = \frac{k' + k'' + k^{\prime V}}{|k' + k'' + k^{\prime V}|}$$

$$U_\mu^{-} = \frac{k''' + k^{\prime V}}{|k''' + k^{\prime V}|}$$

(which are obtained after applying to

$$\exp[-i(k' + k'' + k^{\prime V})x^{+} - i(k'\eta' + k''\eta'' + k^{\prime V}\eta^{\prime V})]$$

$$\times \exp[i(k''' + k^{\prime V})x^{-} + i(k'''\eta''' + k^{\prime V}\eta^{\prime V})]$$

$$F(I_s, I_t) = \text{const} \neq 0 \quad \text{if } |\eta| \leq l$$

$$= 0 \quad \text{if } |\eta| > l$$

in C.M. system

Hamiltonian:

$$\mathrm{const}\int d^4\eta' d^4\eta'' d^4\eta''' \int \delta(x^+ - x)$$

$$\times\, \delta(x^- - x)\, dx^+ dx^- \left[F(I_s^+(\eta')) + F(I_s^-(\eta'))\right]$$

$$\times\, [\cdot\cdot]\cdots\left[F(I_s^+(\eta''')) + F(I_s^-(\eta'''))\right] \times$$

$$\times \left\{\left[\bar{\psi}^+(x^+ + \eta''') + \bar{\psi}^-(x^- + \eta''')\right]\cdots\right.$$

$$\left.\times \left[\varphi_0^+(x^+ + \eta') + \varphi_0^-(\varphi^- + \eta')\right]\right\}$$

The new hamiltonian remains hermitian and in the C.M. system contains only the derivatives ∂_ξ and ∂_0 with respect to the space co-ordinates. Form factors in the C.M. system do not depend on time coordinates.

Non-local propagator

$$(8)\quad \Delta'_c(x-y) = \frac{1}{(2\pi)^4}\int_c \frac{G(l I_s(k)) \exp(-ik(x-y))}{k^2 - m^2}\, d^4x$$

Stückelberg and Feynman's causality condition:
For $x^0 - y^0 \geq 0$, only the pole $k_0 = \pm\sqrt{k^2 + m^2}$ contributes to (8).

In C.M. system

$$\Delta_c'(x-y) = \frac{3}{4l}\int_{-l}^{+l}\left(1-\left[\left(\frac{\alpha}{l}\right)^2\right]\Delta_c(t, r+\alpha)\,d\alpha\right.$$

Δ_c' is regular everywhere and has non-vanishing values in the neighbourhood of the light cone $|t \mp r| < l$.

mass-zero-particle

4. Mean Life of Virtual States.
Convergency of the Perturbation Solution.

5. Causality, Concluding Remarks
Commutator vanishes for ~~a pair~~ two points mutual space-like if $|x' - x''| \gg l$

Ⅰ Non-local　　Ⅳ Structure

26,622

中村誠太郎　　49号 Dec. 1956

素粒子の本質（シアトル会議報告）

I. IV

E. C. G. Stueckelberg
Violation of Parity Conservation and General Relativity
(P. R. 106 (1957), 388, April 15)

Cosmological asymmetry (or local preponderance of the right-handed nucleons over the left-handed ones, which must amount to nearly 100% in order to avoid contradiction with the Pauli exclusion principle) is perfectly compatible with Riemannian space-time of ordinary general relativity.

$$D_\rho \varepsilon_{[\alpha\beta\gamma\delta]} = 0$$

covariant derivative — field

$\varepsilon_{[\alpha\beta\gamma\delta]}$ does not change sign in Riemannian space and can be regarded as a universal constant (in every local geodesic right-handed and orthochronous (or left-handed pseudochronous) Lorentz frame).
The surprising fact is that such constants appear only in the weak interactions.

(III) Non-linear

R. Ascoli und W. Heisenberg
Zur Quantentheorie nichtlinearer Wellengleichungen. IV
Elektrodynamik
Z. f. Naturf. 12a (1957) 177 (Heft 3, März)

b) kleinen Wechselwirkung

$$\mathrm{const.}[(\psi^+\chi)(\psi^+\psi) + \mathrm{conj.}]$$

Die Wechselwirkungsglieder ändern ihre Vorzeichen bei eine Drehung im Isotopenraum um 360°. Daher muß man die Ausdrücke mit einem unbestimmten Vorzeichen ± versehen, die beiden Wertes des Vorzeichens sind völlig äquivalent. Daraus folgt daß auch Ausdrücke der Form

$$\pm\,\mathrm{const.}[(\psi^+\chi)(\psi^+\gamma_5\psi) + \mathrm{conj.}]$$

zugelassen werden können. Bei Spiegelung im Raum wechseln diese Glieder zwar das Vorzeichen, gehen aber eben deswegen in sich über.

5

W. L. Bade and H. Jehle
Rev. Mod. Phys. 25 (1953), 714
An Introduction to Spinors.

XI Spinor Analysis

Covariant differentiation:

$$\psi_{\alpha;s} = \partial_s \psi_\alpha - \Gamma^{\rho}_{\alpha s} \psi_\rho$$

$$\psi^{\alpha}_{;s} = \partial_s \psi^\alpha + \Gamma^{\alpha}_{\rho s} \psi^\rho$$

○ $\partial_s \gamma - (\Gamma^{\alpha}_{\alpha s} + \Gamma^{\dot\alpha}_{\dot\alpha s})\gamma = 0$

4 real equations

○ $\Gamma^{k}_{rs} = \Gamma^{k}_{sr}$ 24 symmetry conditions

Remaining $4 = 32 - 28$ real parameters

ϕ_s: $\Gamma^{\alpha}_{\alpha s} - \Gamma^{\dot\alpha}_{\dot\alpha s} = 4 i \varepsilon \phi_s$

which could be identified with ~~poten~~ electromagnetic potential.

$$\gamma_{\dot 1 \dot 2} \gamma_{12} = \gamma$$

$$\chi^\mu \psi^\nu \gamma_{\mu\nu} = \chi'^\mu \psi'^\nu \gamma'_{\mu\nu}$$

$\gamma_{\mu\nu} = \begin{pmatrix} 0 & \gamma_{12} \\ -\gamma_{12} & 0 \end{pmatrix}$: fundamental spinor

$$g_{kl} \sigma^{k\dot\lambda\mu} \sigma^{l\rho\sigma} = \gamma^{\dot\lambda\dot\rho} \gamma^{\mu\sigma}$$

$$A_{\dot\lambda\mu} = \sigma^{k}_{\dot\lambda\mu} A_k$$

$$\sigma^{l\mu\dot\lambda} \sigma^{k}_{\dot\lambda\sigma} + \sigma^{k\mu\dot\lambda} \sigma^{l}_{\dot\lambda\sigma} = g^{kl} \delta^\mu_\sigma$$

Ⅱ Non-local

Ⅲ Non-linear

G. Heber, (Jena)

Zur Theorie der Elementarteilchen Ⅲ. (Z.S. f. Phys. 144 (1956), 39)

拡がつた粒子をあらわす particle-like state をもつ non-linear classical field を canonical formalism で量子化すると、拡がつているという性質は失はれ、divergence が出る (Ann. d. Phys. 16 (1955), 43; ibid.)

もつと違つた量子化法として、

1. ~~du~~ 交換関係に出てくる δ-fn を regular function で置きかえる。
2. 或る種の曲線は許される。

(5)

R. Utiyama
Invariant Theoretical Interpretation of Interaction
(Phys. Rev. 101 (1956), 1597)

NM: gauge invariance:
$L(Q, Q_\mu)$ なる Lagrangian が
$$Q \to e^{i\alpha} Q, \quad Q^* \to Q^* e^{-i\alpha}$$
$$\alpha = \text{const}$$
なる phase transformation に対して invariant なら、これを phase factor $\alpha(x)$ なる一般な gauge transf. に拡張できる。しかしそのためには A_μ なる新しい場を導入し、Lagrangian の中に Q, Q^* の derivative が $\frac{\partial Q}{\partial x^\mu} - ieA_\mu Q$, $\frac{\partial Q^*}{\partial x^\mu} + ieA_\mu Q^*$ なる組合せで入ってくる ~~ことが必要~~ になる。
まとめること
Yang-Mills (P.R. 96 (1954), 191) の B_μ-field や gravitational field もこのような立場から導入できる。

(5)

Displacement Operator について
素粒子論の新しい方向

H. Yukawa　　　May 31, 1959

$$i\frac{\partial A}{\partial x_\mu} = [A, P_\mu] \qquad (1)$$

operator P_μ は Lagrangian から

$$T_{\mu\nu} = \sum_\alpha \frac{\partial L}{\partial \frac{\partial \psi_\alpha}{\partial x_\mu}} \frac{\partial \psi_\alpha}{\partial x_\nu} - \delta_{\mu\nu} L$$

$$P_\mu = \int T_{\mu\nu}\, d^3x$$

として導き出される。

Lehmann, (N.C.)

は ~~[illegible]~~ P_4 が positive definite な場合につき、(1)を満す displacement operator が存在し、~~[illegible]~~ 交換関係や propagator が一般的性質を満たす 現[illegible]という意味で

所で displacement operator は一方的時間空間構造に連結すると同時に、他方では P_μ という operator という形で Hilbert 空間に関係している。前者は matter-energy content によって特徴づけられ、前者はそれを入れる形式、枠に関係している。結ぼう

いう見地から見ると、(1)なる関係は
従来、枠と中身の相互関係を取り
無している~~ものとする~~答のものである。
しかるに従来は両者を全く別々に考え、
space-time の方は Minkowski の枠、
system の state 及び operator は
無限の Hilbert space の vector と
transformation tensor と~~いう~~ rigid
に定められていた。
両者の関係のし方を明確にするためには、
~~a~~ space-time の方は lattice であると
考え、system を記述する physical
la quantity として、0, 1 だけの固有値を
もつ ~~op~~ occupation operators
だけを考えることにする (lattice の各点
について~~このような~~ついて、このような operator
を考える)

$n(l_1, l_0)$

抽象的にあらわれてくる要素間の互い
に依存する量で考える idea

5

L. H. Thomas
Relativistic Invariance
(R. M. P. 17 (1945), 182)

1. 観測者は 10 arbitrary parameters $\alpha, \beta, \ldots$ で specify される.
general relativity の意味で observer O に近い observer は
$O + (dx, dy, dt, d\tau;\ dl, dm, dn;\ du, dv, dw)$
で specify される. (du, dv, dw) は infinitesimal velocity

$$(2.11)\ \begin{cases} dx = \left(\dfrac{\partial x}{\partial \alpha}\right) d\alpha + \left(\dfrac{\partial x}{\partial \beta}\right) d\beta + \cdots \\ dy = \cdots \end{cases}$$

or

$$d\alpha = \left(\frac{\partial \alpha}{\partial x}\right) dx + \left(\frac{\partial \alpha}{\partial y}\right) dy + \cdots$$
$$d\beta = \cdots$$

但し,
$$\frac{\partial}{\partial \beta}\left(\frac{\partial x}{\partial \alpha}\right) = \frac{\partial}{\partial \alpha}\left(\frac{\partial x}{\partial \beta}\right)$$
は一般に成立せず,** (2.11) から true parameter $x, y, \ldots$ は求められるとは限らない.
しかし, $\dfrac{\partial \alpha}{\partial x}$ 等を使って
$$\frac{\partial f}{\partial x} = \left(\frac{\partial \alpha}{\partial x}\right)\frac{\partial f}{\partial \alpha} + \cdots$$
を define することにしよう.

2. ~~State~~ 物理系を記述する任意の座標系で任意点を x_μ とすると, *

A, B, …

2. 次に state は ~~を specify の~~
任意の observer が適当な装置を
用意して，適当な物理量の観測を行
つた場合の，種々な結果に対する
probability amplitude ~~として~~
~~speci~~ を要素とする函数によつて
specify される．それ等の物理量
の一組の固有値を a, b, … とおく
と probability amp. は a, b, …
の函数であるが ~~同~~ また同様な
観測を他の任意の observer が
行つたとすると prob. ampl. は違
つてくる．従つて state は一般に

$$f(a, b, \dots; \alpha, \beta, \dots)$$

なる形に書ける．

同じ状態に対し

この場合，観測される量 A, B, … 自体が

---

* その点を原点とする任意の local inertia
system (即ち その点で $g_{\mu\nu}dx_\mu dx_\nu$ が
standard form $dx^2+dy^2+dz^2-c^2dt^2$
となるように選んだ座標系 (x, y, z, t))
は x_μ 及び ~~適当~~ その中の一つの inertia system を基準として定めた
任意の inertia system の orientation
l, m, n, u, v, w の10 parameter で
表わされる；後の6つの parameter は x_μ の…

観測系に relative にきめられているという事を考慮すると、観測系の infinitesimal displacement による state function の変化は

$$\frac{df}{d\alpha} = D_\alpha f + \frac{\partial f}{\partial \alpha} \quad \text{etc.}$$

と書けるであろう。但し

$$D_\alpha f = \frac{\partial f}{\partial a}\frac{da}{d\alpha} + \frac{\partial f}{\partial b}\frac{db}{d\alpha} + \cdots$$

従って

$$\frac{df}{dx} = D_x f + \frac{\partial f}{\partial x}$$

$$\left(D_\alpha f = \frac{\partial x}{\partial \alpha} D_x f + \frac{\partial y}{\partial \alpha} D_y f + \cdots\right)$$

$$D_x f = \frac{\partial \alpha}{\partial x} D_\alpha f + \frac{\partial \beta}{\partial x} D_\beta f + \cdots$$

$$D(fg) = (Df)g + g(Df)$$

---

* ~~しかし~~ と考えても、やはり x_μ の他に local な orientation をきめる為にもの 6個の parameter が必要である。いずれにしても 10個の arbitrary parameter が必要である。

** x_μ が $\alpha, \beta \ldots$ 等の 10個の independent variable の函数というわけではない。寧ろ $dx, dy, \ldots$ 等の 10個が $d\alpha, d\beta, \ldots$ の linear combination. こういうことから $\frac{\partial x}{\partial \alpha}$ 等を形式的に[illegible]されたから。

3. Equivalence 概念の拡張。
もしも kinematics が observer に無関係で state function 独立の $g(a, b, \dots; \alpha, \beta, \dots)$
に対して

$$\frac{\partial g}{\partial \alpha} = 0 \quad \text{etc.}$$

が成立する場合 — これは（その意味で）三つの observer に対する A, B, C, … が互いに equivalent な場合 — には

$$\frac{dg}{d\alpha} = D_a g \qquad \text{etc.},$$

or

$$\frac{dg}{d\alpha} = D_\alpha g \qquad \text{etc.},$$

が成立する。

4. Dynamical law を表現するもう一つの方法は、Schrödinger picture に対応するものである。これは state function h が observer には totally independent で

$$\frac{dh}{d\alpha} = 0 \quad \text{etc}$$

従って

$$\frac{\partial h}{\partial \alpha} = -D_\alpha h \quad \text{etc.}$$

$$\frac{\partial h}{\partial x} = -D_x h \quad \text{etc.}$$

これが observer の変化による状態の specification の仕方の変化を表現すると考えられる。

5. Consistency は observable の値が循環的 circuit of observers を通過して、元へ戻ることで表現される。
4 節の表現からは

$$\frac{\partial}{\partial \beta}\frac{\partial h}{\partial \alpha} - \frac{\partial}{\partial \alpha}\frac{\partial h}{\partial \beta} = 0$$

or

$$-\frac{\partial}{\partial \beta}(D_\alpha h) = -\left(\frac{\partial}{\partial \beta}D_\alpha\right)h - D_\alpha \frac{\partial h}{\partial \beta}$$

$$-\frac{\partial}{\partial \alpha}(D_\beta h) = -\left(\frac{\partial}{\partial \alpha}D_\beta\right)h - D_\beta \frac{\partial h}{\partial \alpha}$$

or

$$D_\alpha D_\beta - D_\beta D_\alpha = \frac{\partial D_\alpha}{\partial \beta} - \frac{\partial D_\beta}{\partial \alpha}$$

従って

$$D_x D_y - D_y D_x = \frac{\partial D_x}{\partial y} - \frac{\partial D_y}{\partial x} + \sum_z C_{xy}^z D_z$$

が得られる。但し C_{xy}^{z} は structure constant

普通の general relativity では

此の45の関係式の中の15は

$$12\left\{ \begin{array}{l} D_m D_n - D_n D_m = D_l \\ \vdots \end{array}\right.$$

$$3\left\{ \begin{array}{l} D_\nu D_w - D_w D_\nu = -\frac{1}{2} D_l \\ \vdots \end{array}\right.$$

となり. observer は four parameter set of six parameter sub-families に分離できる。

6-parameter subfamily の中から one observer を continuous manner に選ぶと、残りの D_x, D_y, D_z, D_t によって、observer がある curve に沿って変化する時の state の specification の変化が与へられる。

更に特に minkowski space では残りの30の関係が

$$30\left\{ D_m D_z - D_z D_m = D_x \quad \text{etc} \right.$$

となり、

結論として、普通の general relativity よりも、もっと広い可能性が在る、ことがわかる。
それがどういう space-time structure に対応しうるかは論じられていない。

Reference (1) Doctor thesis B. Bakamjian.
Reference (2)

(2) L. H. Thomas, Relat. Dynamics of a System of Particles Interacting at a Distance. (P.R. 85 (1952), 868)

Infinitesimal displacement operators
(H, X, Y, Z) ; (L, M, N) (U, V, W)

alternative choice

$L = \sum L_r$ etc. $U = \sum U_r$ etc

which makes the invariance under homog. Lorentz group trivial. But, then the choice of (H, X, Y, Z) is more difficult.

In non-relativistic limit

$$H \to \text{mass term} + \sum_r \frac{1}{2m_r c}(X_r^2 + Y_r^2 + Z_r^2)$$

$$X \to \sum X_r \quad \text{etc},$$

spin? mass term $\to \sum_r m_r c^2$

$X = \frac{1}{2}(\mu \sum X_r + \sum X_r \mu)$ etc で μ は

$(H, X \ldots)$ 及び $L_r H_r + W_r Y_r - V_r Z_r$ etc 及び $L_r X_r + M_r Y_r + N_r Z_r$ の scalar 在る可!!!

(5) (4) R. I. Ingraham

Relation of Isotopic Spin Space to Space-Time

(P. R. 106 (1957), 595

$\tau X^0 = 1, \quad \tau X^\mu = x^\mu, \quad \tau X^5 = \frac{1}{2}(x^2 + \lambda^2)$

$x^2 = g_{mn} x^m x^n$

$G_{\mu\nu} X^\mu X^\nu \equiv X^2 - 2X^0 X^5$

$= 0 \quad (\cancel{4}\ x^2 =$

gives the locus of the points ($\lambda = 0$) of space-time.

signature: $(+ + + - ; + -)$

$\bar{X}^\mu = L^\mu{}_\nu X^\nu$

8 x 8 matrices

$\Gamma^\mu \Gamma^\nu + \Gamma^\nu \Gamma^\mu = 2G^{\mu\nu} \cdot 1$

$\Gamma^m = \gamma^m \times \tau^3, \quad \Gamma^0 = \ell^{-1} \cdot 1 \times \tau^+$

$\Gamma^5 = -\ell\, 1 \times \tau^-$

$\gamma^m \gamma^n + \gamma^n \gamma^m = 2g^{mn} 1$

$\tau^\pm = \frac{1}{\sqrt{2}} (\tau^1 \pm i\tau^2), \quad \tau^3$

$L^\mu{}_\nu \Gamma^\nu = S \Gamma^\mu S^{-1}$

$S_{\text{trans.}} = 1 - (a/2\ell)\gamma^1 \times \tau^+$

$(\bar{x}^m = x^m + a^m \quad a^m$; x-direction$)$

References
Murai,

R. Ingraham and J. Ford, Boson Family from Quantized Finite-Particle Maxwell Theory, P.R. 106 (1957), 1324
mass parameter λ (continuous mass spectrum).

(5)

Ryoyu Utiyama
Invariant Theoretical Interpretation of Interaction
(P. R. 101 (1956), 1597)

1. General Theory

$$L(Q^A, Q^A{}_{,\mu})$$

If

$$I = \int_\Omega L\, d^4x$$

is invariant under

$$Q^A \to Q^A + \delta Q^A \qquad (1)$$

$$\delta Q^A = T_{(a)}{}^A{}_B\, \varepsilon^a Q_B$$

ε^a : infinitesimal parameters
$a = 1, 2, \cdots n.$

T : constant coeff.

structure constants:

$$[T_{(a)}, T_{(b)}]^A{}_B = f_{ab}^c\, T_{(c)}{}^A{}_B$$

~~etc.~~

$$\frac{\delta L}{\delta Q^A} T_{(a)}{}^A{}_B Q^B + \frac{\partial L}{\partial Q^A{}_{,\mu}} T_{(a)}{}^A{}_B Q^B{}_{,\mu} \equiv 0$$

$(a = 1, 2, \ldots n)$

Together with the field eq., we have

$$\partial J^\mu{}_a / \partial x^\mu = 0$$

$$J^\mu{}_a = \frac{\partial L}{\partial Q^A{}_{,\mu}} T_{(a)}{}^A{}_B Q^B$$

If we extend the transformation (1) such that ε^a are arbitrary functions and we want to keep the invariance of the Lagrangian, we have to introduce a new field

in $A'^J(x)$ $J = 1, 2, \ldots M$.

$$L'(Q^A, Q^A{}_{,\mu}, A'^J)$$

2. Examples:

(i) gauge transformation

$$\delta Q^A = i\alpha Q^A$$

$\alpha \to A\lambda(x)$ A_μ = electromagnetic potential

(ii) Rotation group in isospin space

$\to B^c_\mu(x)$: Yang-Mills field.

(iii) Lorentz group

$\to$ gravitational field.

(II) Non-local

On Relativistic Wave Equations with Mass Spectrum
V. L. Ginzburg
Acta Physica Polonica
15 (1956), 163

Comparison of theories of mass spectrum. Mostly new continuous variables are ~~comp~~ components of some space-like four-vector in connection with non-local field theories.

T. Tati
An Attempt in the Theory of Elementary Particles
Nuovo Cimento, 4 (1956), 75

1. Probability amplitude are functionals of the numbers of particles specified by intrinsic quantities, (m, σ) and coordinates

Observations O_1 O_2

Probability of observables Ω_1 Ω_2

$$w = C(O_1; O_2) W[\mathcal{H}, \Omega_1, \Omega_2]$$

$C(O_1; O_2)$: undetermined quantity independent of $\mathcal{H}$, Ω_1, Ω_2

Ideal ~~μ~~-meson clock: $T(\mu)$

$$C(O_1; O_2) = (2\pi/\hbar) T(\mu)$$

基礎論研究会　箱根　June 29, 1959

1. 湯川：時空構造と素粒子構造

I. Einsteinの理論　Reduction to geometry of space-time world

II. 量子的時空理論
Tati　時空の統計的解釈

III. 時空の構造と素粒子の構造の相互規定.
予定調和.
spurion — 時空の[illegible]状態
— 相互作用[illegible]（弱い相互作用）
非局所的[illegible]
時空構造と素粒子系の依存的.
両者的[illegible]
gravitational field { metric / graviton

基礎[illegible]研究会

2. 宮武：一般相対論的粒子模型
と[illegible]の相対論的[illegible]

(June 29, 1957)

Bade & Jehle R. M. P. 25 ('53) 714)

外 $\beta^k \partial_k \psi = \mu \psi$

$$\beta^k \beta^l + \beta^l \beta^k = -2 g^{kl}\, 1$$

$$\beta^k = \sqrt{2} \begin{pmatrix} 0 & a^k \\ a^{k*} & 0 \end{pmatrix}$$

内 $\{\beta^k(x)\,\partial_k + \Xi(x) - \mu\}\,\psi = 0$

$$\Xi = \sqrt{2} \left(\begin{array}{c|c} 0 & \\ \hline \sigma^k(x)\,\Sigma\,\dot{\sigma}_k(x) & 0 \end{array} \right)$$

$$R_{kl} - \frac{1}{2} g_{kl} R = \frac{8\pi\kappa}{c^4} T_{kl}$$

H. L. Jordan

3. 困難：一つの相互作用の時間的距離

全体空理(?)

1. 素粒子の存在

2. 素粒子の自由度　$\Sigma^{(m)}, \Sigma^{(n)}, \ldots$

3. $n_k(p,s)$

4. $\Psi[n_k(p,s)]$

5. $a_k^*(p,s)$, $a_k(p,s)$

$[a_k, a_{k'}^*] = \delta_{kk'}$

6. Ω: observable

H_λ: total energy momentum

7. 観測の[illegible]

$O_1, O_2, \ldots$

8. $\Omega = \Omega_1 \oplus \Omega_2 \oplus \cdots$

$$\langle \Psi_\omega | e^{-i\Sigma H_\lambda C_\lambda(O_1 O_2)} \Omega_2 e^{i\Sigma H_\lambda C_\lambda} | \Psi_\omega \rangle$$

4. 中村. 弱い相互作用の一つの試み

森田氏: F. (G.T. PS あるらしい

e^- S (V,S)? T

e^+ V (V,S)? ? ?

$Pr^{144} \to O_- \to O_+$ (PS)

(K-capture)

e^- S T P

e^+ {VT, VA}

Morita

$$\left.\begin{aligned} C_S &= C_S' \\ C_V &= -C_V' \\ C_T &= -C_T' \\ C_A &= C_A' \end{aligned}\right\} \quad \left|\frac{C_V}{C_S}\right|^2 \sim \left|\frac{C_T}{C_A}\right|^2 \sim 3$$

real にとればtime reversal になっている

中村理論:

1. coupling constant

2. $[\bar{\psi}\gamma_\mu\psi(x),\ A_\mu(x')] = e f(x, x')$

3.

$$\{\gamma_\mu(\partial_\mu - ieA_\mu) + m\}\psi_e = 0$$

$$\left.\begin{aligned} x_4' &= x_4 \\ \psi' &= C\psi \qquad C = \gamma_1\gamma_2\gamma_3 \\ e' &= e \qquad A_k' = +A_k, \quad A_4' = A_4 \end{aligned}\right\}$$

5. 大林：風船の法(弱い相互作用)

1. strong P, T, C, I, η ... conserve

2. weak ≐ lepton process
μを仲介にする.

K : spin 0　Isospin ½　$\varkappa$
parity +

K′ : spin 0,　I = ½ (3/2)
parity −　$\varkappa' > \varkappa + m$

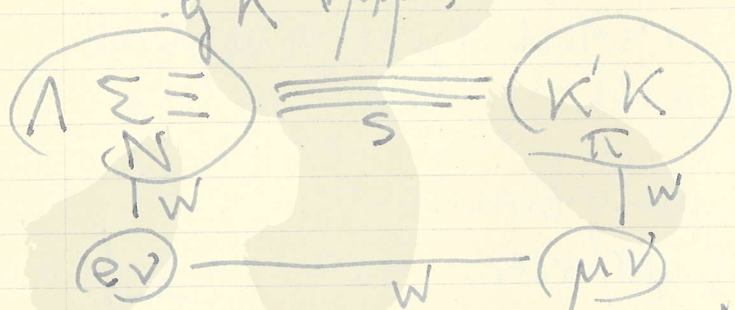
$G K'^{*} K \pi$: strong

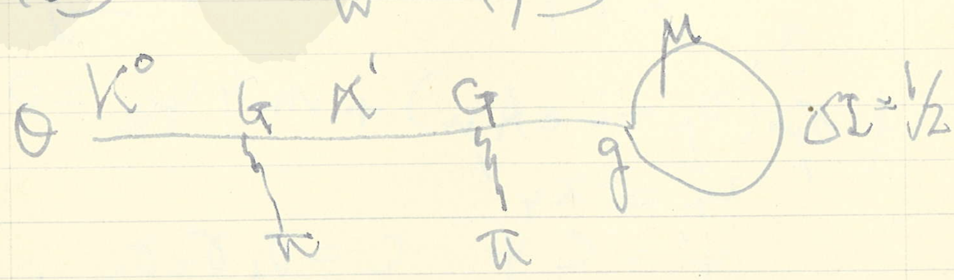
$K^{\pm} \to \mu^{\pm} + \nu$

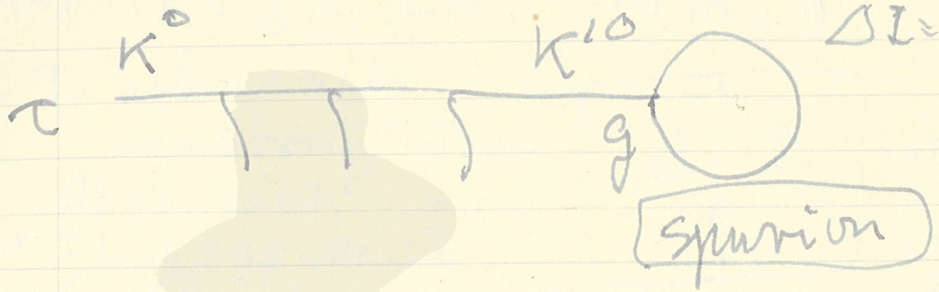
$K^{0} \to \bar{\mu} + \mu$

$g\ K^{0}(\bar{\mu}\mu)$

$g\ K'^{0}(\bar{\mu}\mu)$







$\Delta I = B/2$

$3/2$

6、空間うらがえしの種類

粒	正	反
電	反	正

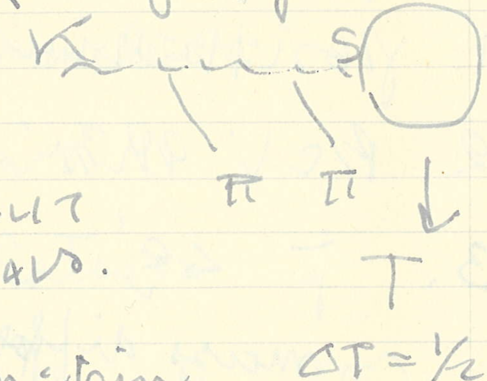
x　x_μ　ψ (粒子)

① proper L
② space refl.
粒子をひっくりかえす
③ charge conj.
粒子
④ Time rev (Wigner)

Deser, General Relativity and divergence (Preprint, July, 1957)

学術講演会. 素粒子と空間 July 2, 1957
崎山泰久

J. Schwinger
y-charge



素粒子を分離して
空間へ入れる.

Lorentz → Einstein

物 I) 規定量の役割
1) charge space
conformal space 6-dim.
2) PTA
3) mass reversal
charge
natural constantの 表[illegible]

II) 空間　general invariance

$T_{\mu\nu} + \frac{1}{2\kappa} t_{\mu\nu}$
non-(pseudo) tensor
幾何学的な場　Cond. Ʃμυρα, G.
Stückelberg
$R=0$　$\Gamma_{\mu\nu}^{\lambda} \neq \Gamma_{\nu\mu}^{\lambda}$ (捩れ)

de Sitterの空間.
$$d\tau^2 = \frac{dr^2}{1-K^2r^2} + \cdots + (1-K^2r^2)dt^2$$
$$= dx_1^2 + \cdots - dx_5^2$$

高エネ研究会　July 25 1957

武田氏：

1. positronium

2. BeV 現象

3. $\bar{p}$ の e　　p の e

mass difference : 1% 程度以下

武田氏：

$\bar{p} + p$　　σ_{int}

$\sim 450\,\mathrm{MeV}$　　$\sigma_{annih.}$　　$89 \pm 7\,\mathrm{mb}$

$(\sigma_{geo} \sim 60\,\mathrm{mb})$

$\bar{p} + A \to n\pi$　　n は大きい

$e + p \to e + p$

$\sigma_{annih} \simeq \pi (a + \lambda\!\!\!^{-})^2$

木庭氏：

July 26

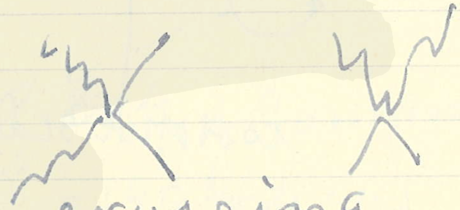
究極

1. ~~a~~ Free antinucleon

2. virtual pair

1. Dispersion Relation
local ?

$M(E) = M_{NN}(E) - M_{N\bar{N}}(E)$



crossing

$M(-E) = M^*(E)$　$\mathrm{Im}\, M(E)$ odd in E

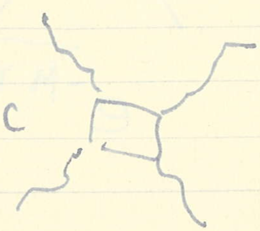
$\sim (\sigma_{NN}(E) - \sigma_{N\bar{N}}(E))$
$\overset{\shortparallel}{c}$　odd in E

$E,\ 1/E, \dots$

$\sigma_{pp} - \sigma_{p\bar{p}} \simeq 0$　for large E

E : labatory system

2.　π-π scattering

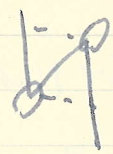


$\pi^0 \to 2\gamma$　10^{-17} sec

$\pi^\pm \to e^+ + \nu$

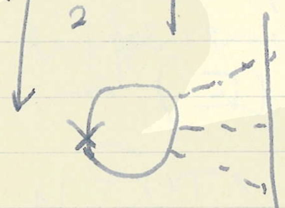
強[illegible]α : e-nucleon scattering
nucleon core の拡がり)

nucleon pair
Tamm
Chew



強い ?

$\bar{\Psi}\gamma_\mu\frac{1+\tau_3}{2}\psi$

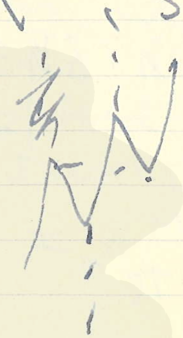


isotopic scalar

Suura, Hiida
$\langle r^2\rangle$

e^{-Mr} $\qquad \langle r^2\rangle^{1/2} = \sqrt{12}\,\frac{1}{M} \sim \frac{1}{2\mu}$

南氏, S-wave



$\frac{g^2}{4\pi} = 0.11$
(ps-pv)

Be pair cloud

伊藤氏 : Composite Model

$T^{res}_{\pi\text{-}N} = 190\,\mathrm{MeV}$

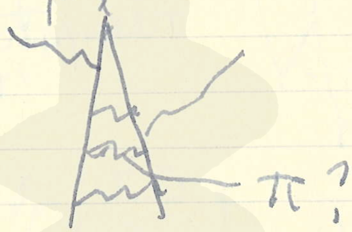
$T^{res}_{\pi\text{-}\pi} = 150\,\mathrm{MeV}$

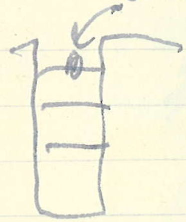
$\langle T_\pi \rangle = 180\,\mathrm{MeV}$

早川氏 : 宇宙論

検出器

antiproton が消える寸前.





原理.

charge conjugation の拡張.

(N. Bohr etc) p. 96

O. Klein Quantum Theory and Relativity

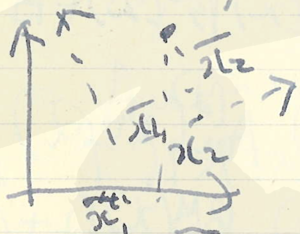
General space-time transformation and quantum conditions!

$$\bar{x}^\mu = x^\mu + \xi^\mu(x)$$

infinitesimal

$$\bar{A}_\mu(\bar{x}) = A_\mu(x) - \varepsilon_\mu^\nu(x) A_\nu(x)$$

$$\varepsilon_\mu^\nu(x) = \frac{\partial \xi^\nu(x)}{\partial x^\mu}$$



$A_\mu(x)\,dx^\mu$ = invariant

$$\bar{A} = A(1-Q)$$

where $Q = (\mu, x|Q|\mu' x')$

so that

$$(AQ)_{\mu,x} = \int A_{\mu'}(x')\,d^4x' \times (\mu', x'|Q|\mu x)$$

$$(\mu', x'|Q|\mu, x) = \int \varepsilon^\nu_\tau(y)\,d^4y \,(\mu', x'|Q^\tau_\nu(y)|\mu, x)$$

$$(\mu, x|Q^\tau_\nu(y)|\mu', x') = \delta_{\mu\nu}\,\delta_{\mu'\tau}\,\delta^4(x-y)\,\delta^4(x-x')$$

$$[Q_\nu^\tau(x), Q_{\nu'}^{\tau'}(x')] = \delta^4(x-x') \times \{\delta_{\tau\nu'} Q_\nu^{\tau'}(x) - \delta_{\tau'\nu} Q(x)\}$$

$$[A_\mu(x), A_{\mu'}(x')] = [B^\mu(x), B^{\mu'}(x')] = 0$$

$$[A_\mu(x), B^{\mu'}(x')] = i\delta_{\mu\mu'}\delta^4(x-x')$$

$$Q_\nu^\tau(x) = \frac{1}{2}\{A_\nu(x)B^\tau(x) + B^\tau(x)A_\nu(x)\}$$

$$\left.\begin{aligned} \bar{A}_\mu(x) &= (1-Q)\, A_\mu^{(x)}(1+Q) \\ \bar{B}^\mu(x) &= (1-Q)\, B^\mu(x)(1+Q) \end{aligned}\right\}$$

(coordinate transformation に対応する field operators の transformation)

この transf. に対して comm. relat. は invariant.

General displacement operators

$$\left.\begin{aligned} A_\mu(x) &\to A_\mu(x+\delta x) \\ B_\mu(x) &\to B_\mu(x+\delta x) \end{aligned}\right\} \quad \frac{\partial \delta x^\mu}{\partial x^\mu} = 0$$

(so that the elementary volume $d^4x = dx^1 dx^2 dx^3 dx^4$ is invariant)

$$T_\nu(x) = -\frac{1}{2}\left[\frac{\partial A_\tau(x)}{\partial x^\nu}\beta^\tau(x) + \beta^\tau(x)\frac{\partial A_\tau(x)}{\partial x^\nu}\right]$$

$$T = i\int \delta x^\nu(x)\, d^4x\, T_\nu(x) \quad *$$

$$[A_\mu(x), T] = \delta x^\nu(x)\frac{\partial A_\mu(x)}{\partial x^\nu}$$

$$(1-T)A_\mu(x)(1+T) = A_\mu(x) + \delta x^\nu(x)\frac{\partial A_\mu(x)}{\partial x^\nu}$$

$$= A_\mu(x+\delta x)$$

$$(1-T)\beta^\mu(x)(1+T) = \beta^\mu(x+\delta x)$$

$$P_\nu = \int T_\nu(x)\, d^4x$$

$$i\frac{\partial A_\mu(x)}{\partial x^\nu} = [P_\nu, A_\mu(x)]$$

etc.

Electromagnetic field without sources:

Dirac field:

* $T_\nu(x)$ は互いに commute する必要なし!!!

原子核研究会　July 29

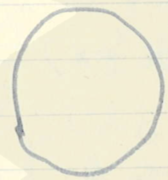
I、議題：反陽子と陽子との衝突断面積の解析。（武田・宮沢）

$\bar{p}\ p$ $\quad$ σ_{tot}

450 MeV $\quad$ $\sigma_{an} \sim 90$ mb

$\sigma_{el}/\sigma_{an} \ll 1$.

$\xrightarrow{l}$



$l_{min} \sim 3$

$\frac{\sigma_{el}}{\sigma_{an}} \le \frac{1}{5}$ $\Big\}$ σ_{tot} 280 mb

(black $= \frac{1}{2}$)

$l=4$ までいれないと説明できる.

$$\sigma_t = 4\pi \lambda^2 \sum (2l+1) a_l$$

$$a_l \le 1$$

$$\sigma_{an} = \pi \lambda^2 \sum (2l+1) b_l$$

$b_l \sim 0.8 \quad l = 0, 1, 2, 3$

$\downarrow$

$\sigma_{el}/\sigma_{tot} \sim 1/5$

II, π-Nucleon scattering（武田）

$\pi^- + p \rightarrow$, $\pi^+ + p \rightarrow$ $\Big\}$ $=$ high energy で 30 mb.

$\pi^\pm p \begin{cases} I = 1/2 & E_{max} \sim 0.9 \text{ BeV} \\ I = 3/2 & E_{max} \sim 1.4 \text{ BeV} \end{cases}$

$\pi^{\pm} p$	BeV	1	1.3~1.4	4.5
	σ_{el}/σ_{in}	1	1/2	1/4

1.5 BeV

$$\frac{\pi^- + p \to p + \pi^0 + \pi^-}{\pi^- + p \to n + \pi^+ + \pi^-} \sim (0.7 \sim 1)$$

$$\frac{1\pi}{2\pi} \sim \frac{5 \sim 10}{1}$$

1 BeV 大して違わない

解釈 i) $I = 1/2$ の max は ∧ 等の production の threshold に近いが、関係をつけることは難かしい

ii) Resonance

$J,\ l,\ T = 1/2$

$2\pi \lambda^2 \sim 9\,mb$

$$\sigma_T^{J,l} \leq 4\pi \lambda^2 \frac{(2J+1)}{2} \frac{\sigma_{el}}{\sigma_T^{J,l}} \equiv R$$

$R = 1/2 \qquad J \gtrsim (4 \sim 4.5)$

$= 1 \qquad J \gtrsim 2$

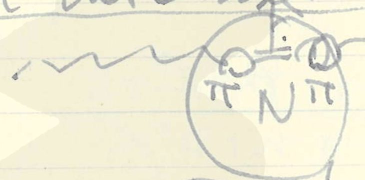
iii) $\frac{Q}{2} \sim 330\,\text{MeV} = 140 + 190$

それ等の max: $\frac{Q}{3} \sim 375\,\text{MeV}$

(Shutt)

大きさが足りない. (≤ 10 mb)
(論理：ひろがり)

iv) π-π interaction



止ってる状態にする

$$\sigma_{p,\pi^+} = P\left(\frac{1}{3}\bar{\sigma}_1 + \frac{5}{3}\bar{\sigma}_2\right)$$

$$\sigma_{p,\pi^-} = P\left(\frac{4}{9}\bar{\sigma}_0 + \frac{5}{9}\bar{\sigma}_2\right)\bar{\sigma}_1$$

$$\begin{cases} E_{res} \sim 150\ \mathrm{MeV} \\ \Gamma \lesssim 50\ \mathrm{MeV} \end{cases}$$

(charge ratio から $T=1$ がよい)

v) Dispersion relation
absorption potential
(optical)

III. π-π (実験)

$T = 0, 1, 2.$

one-N-system +nπ　　$T = 1/2,\ 3/2, \cdots$
　　　　　　　　gr. st., isobar

nπ-system　　$T = 0, 1, 2$
　　　　　　　　vacuum 2μ, 3μ

$N + N \to N + N + 2\pi$ (2 BeV)

π-π-correlation

$\pi^0\pi^0$: angular corr. あり

$\pi^+\pi^-$: なし

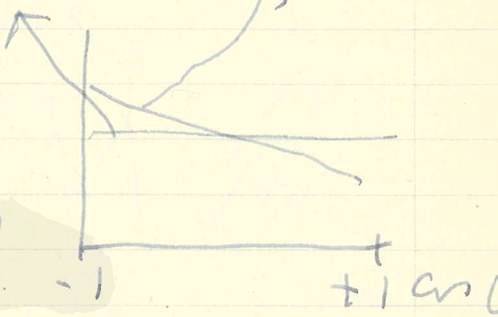
$T = 0$ なら 同じ

$T = 1$　$\pi^0\pi^0$ なし

$\pi^+\pi^-$ あり

$T = 2$　$\pi^0\pi^0$ 3

$\pi^+\pi^-$ 1.



$N + \bar{N} \to n\pi$

$\langle n \rangle$

$T = 0$ ×

$T = 1$ O.K. (by [illegible])

$T = 2$ OK

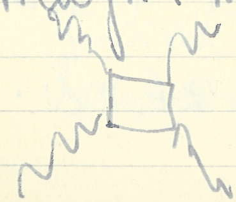
One Nucleon

$\langle n_{\pi^+} \rangle_p = 0.6$

$\langle n_{\pi^0} \rangle = 0.45$

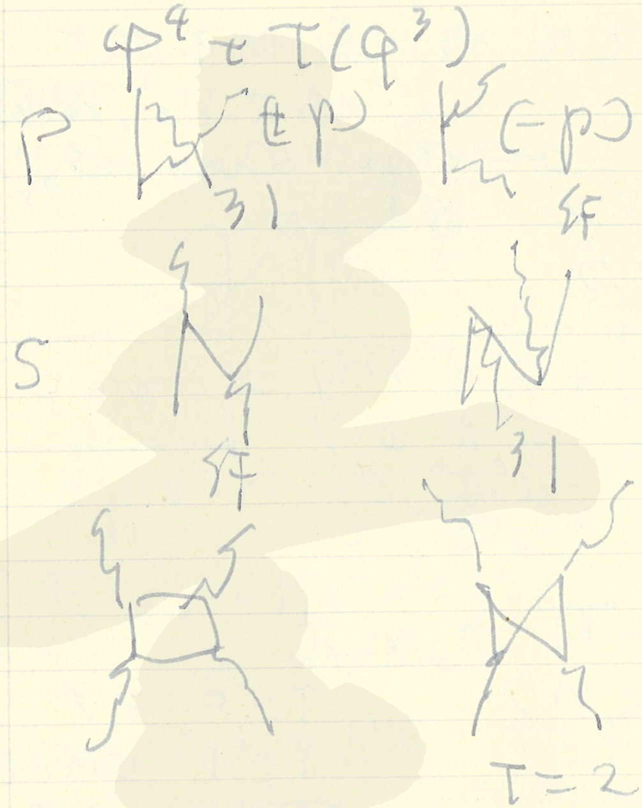
$\langle n_{\pi^-} \rangle = 0.3$

magn. mom.



π-N $\begin{cases} P_{\frac{3}{2},\frac{3}{2}} \text{ attr.} \\ \text{other rep.} \end{cases}$

S $\begin{cases} 3/2 \text{ rep.} \\ 1/2 \text{ attr} \end{cases}$



IV. N−N collision (BeV) (論)

Fowler et al. P.R. 103 ('56), 1489

Smith et al. 97 ('55), 1186

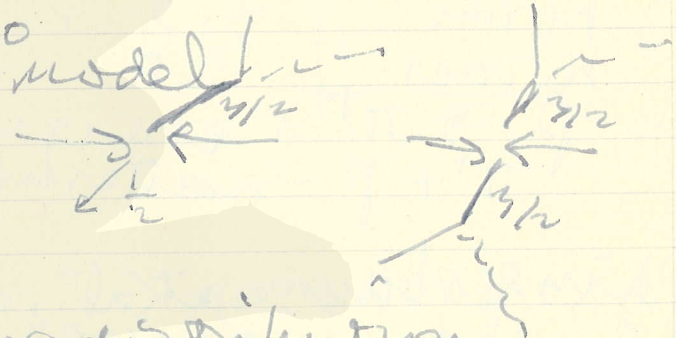
Wellenmeyer 105 ('57) 1058

6th Rochester Conf. ('56)

Total cross-section

$\sigma_{pp}(T=1)$ $\sigma_{inel} \sim 26$ mb. $\sigma_{T=1}$

$\sigma_{N\cdot p}$ ($D_2O - H_2O$)
Glauber の補正.
$T=0$ の contribution
$\sigma_{T=0}$
Isobar model



angular distribution
~~shadow~~ shadow scattering (前方に偏り)

multiplicity
Fermi より大きい
Kovacs (final interaction)
おおきすぎる

Branching ratio
$p+p \rightarrow \dfrac{(p\,n\,+)}{(p\,p\,0)}$ { 4.3 (0.65) BeV
4.1 (0.92
17±8 (0.8)?

Fermi 3
Isobar model 5

$n+p$: $pn\pi^-$: $pp\pi^0$ - : pp -

3.2 : 1 : 0.8

(1 ~ 2.2 BeV) (1.25 BeV)

Fermi 3.3 : 1 : 20.5

Kovacs 3.0 : 1 : 0.9

$p-p \to \pi^+$ の energy spectrum

$\pi^+ + p$-scattering に似ている,

Lindenbaum et al. 105 ('57), 1874

Barclay 106 ('57) 572.

Isobar model

$N+N \to N^* + N \to \pi + N + N$

$\searrow N^* + N^* \to \pi + N + \pi + N$

isobar mass m,

弦浦氏: Electron-Nucleon Interaction

$\rho(r)$ $\mu(r)$

$F_1(q^2)$ $\kappa F_2(q^2)$

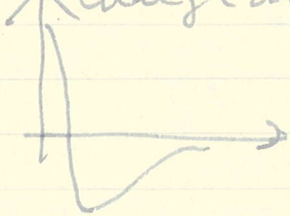
1. e-n interaction

3860 ± 350 eV

$\langle r_{1,n}^2 \rangle = (0.35 \times 10^{-13}\,\mathrm{cm})^2$

↓ magnetic moment

ɪ l ν < r neutron radius
$\langle r_{1,n}^2 \rangle = (0.00 \pm 0.08 \times 10^{-13} cm)^2$
図 ~ 分布.　↑charge distrib.



2. e – p
$\langle r_{1,p}^2 \rangle = (0.77 \pm 0.10 \times 10^{-13} cm)^2$
$\langle r_{2,p}^2 \rangle \simeq \langle r_{1,p}^2 \rangle$

3. e – D
$\langle r_{1,p}^2 \rangle + \langle r_{1,n}^2 \rangle = \langle r_c^2 \rangle$
$= (0.8 \sim 0.85 \ \ ,, \ )^2$
(Hofstadter
Yennie
electron; 500 MeV)
650 MeV – electron

$e^{-r^2}, e^{-r}, re^{-r}$

---

* p, n の違いが, π-meson 以外の相互作用ででてくる.
(shorter range)
anti-neutron と anti-proton の違い

550 MeV　　135°

σ_n/σ_p

D　　0.5

$He^4 - \frac{2}{3}C^{12}$　　0.85　　$\simeq \left(\frac{1.91}{2.79}\right)^2 = 0.46$

$Li^7 - Li^6$　　0.46　　mag. mom.

$$\left[\frac{F_{1n}(q^2) + \kappa_n F_{2n}(q^2)}{F_{1p}(q^2) + \kappa_p F_{2p}(q^2)}\right]^2 \sim \left(\frac{\kappa_n}{1+\kappa_p}\right)^2$$

$$\langle r_{2,n}^2 \rangle \cong \langle r_{2,p}^2 \rangle \simeq \langle r_{1,p}^2 \rangle$$

[illegible]、

pion [illegible]: 0.3×10^{-13} cm

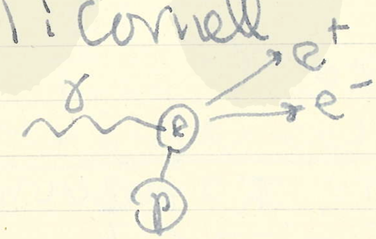
nucleon recoil　0.2

core の拡がり:

K では [illegible].

$\langle r_c^2 \rangle \simeq \langle r_v^2 \rangle$　些か大きい

実験: Cornell



(原因: neutral vector meson)

july 30

[illegible] : Positronium と $\bar{P}$

	P	C	
^3S	−	−	3γ
^1S	−	+	2γ
^3P	+	+	2γ
^1P	+	−	3γ

$\bar{p}$の減速過程

$E_{\bar{p}} = 300$ MeV　　Al　Pb

ionization range　20　9 cm

a. n. h. mean free path　25　13

10 MeV → trapped　$\sim 10^{-10}$ sec (in H_2)

free a.n.h. life　$\sim 10^{-8}$ sec (〃)

$\bar{P} + P$ - system

energy levels

Coulomb or period: 10^{-19} sec, 10 keV

nuclear forces

$\underbrace{\text{Selection Rule}}$ $\underbrace{\text{Exchange}}$

[illegible] : Pion Potential

$p + \bar{p} \rightleftarrows \bar{p} + p$

$p + \bar{n} \rightleftarrows \bar{p} + n \rightarrow p + \bar{n}$

J, P, C, CT, I, S

$N - \bar{N}$

$C = (-1)^{l+S} = (-1)^{l}$

$G = (-)^{S+l+I}$

$I = 0, 1$

π-system
ps-ps

$UHU^{-1} = \bar{H}$ (C.C.)

$U\psi U = C\bar{\psi}$

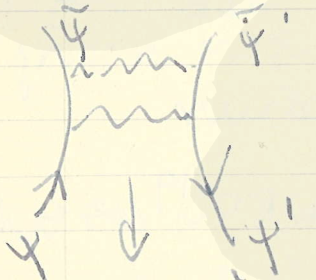
$U\varphi_2 U^{-1} = -\varphi_2$

charge exchange $i\tau_2$ $\begin{pmatrix} p \\ n \end{pmatrix} \to \begin{pmatrix} n \\ p \end{pmatrix}$

$U\varphi_3 U^{-1} = -\varphi_3$

$G\varphi_\alpha = -\varphi_\alpha$ $\qquad G = (-1)^N$

Lee-Yang N.C. 3 749 ('56)
Amati-Vitale N.C. 2, 719, ('55)



$(-1)^N$ だけ N-Nで
がある

$\psi' = G\bar{\psi}$

$\begin{pmatrix} p \\ n \end{pmatrix}$ $\begin{pmatrix} \bar{n} \\ -\bar{p} \end{pmatrix}$

N-πion

$\delta(x)$

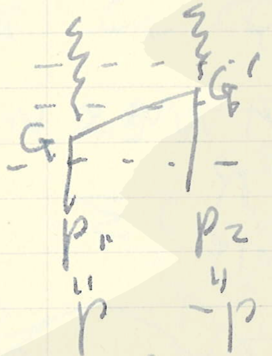
$\frac{1}{M}$・a ひろがる.

核力 : $p+\bar{p}$-annih.

$$-\frac{1}{2m}\nabla^2\psi + iW(r)\psi = E\rho$$

$$\mathrm{div}\,\vec{J} = 2W\rho$$

$W<0$: absorption



$a_3(k_1, k_2)$

$a_2(p_1 p_2 k)$

$a_1(p_1 p_2)$

$$(E-2E_p)a_1(p) = \int d\vec{q}\,[U_p^{(1)}(q) + U_p^{(2)}(q)] \times a_1(\vec{p}-\vec{q})$$

$$U_p^{(2)}(q) = 2G^2G'^2\int\frac{d\vec{k}}{\omega_k^2}\,\frac{1}{E-E_p-E_{p-k}-\omega_k}$$

$$\times\frac{1}{E-2\omega_k}\left[\overline{E-E_{p-}}\;\;\overline{E_{p-q-k}-\omega}\right]$$

$+$

$$\frac{1}{E-2\omega_k} = \frac{P}{E-2\omega_k} - i\pi\delta(E-2\omega_k)$$

↓ imag. potential

$p\to 0$: $W(q) \sim \frac{1}{q^2+\kappa^2}$ $\quad q \ll M$

$\kappa = 1.5M$ $\quad\sim\frac{1}{q}$ $\quad q \gg M$

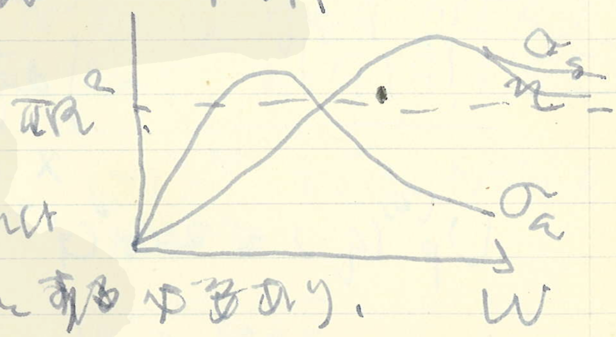
$W(r) \sim \dfrac{e^{-\kappa r}}{r}$ $\quad r \gg \dfrac{1}{\kappa}$

$\sim \dfrac{1}{r^2}$ $\quad r \ll \dfrac{1}{\kappa}$

cross-section は余り大きくないから吸

$V(r) = 0$

$W(r) = \begin{cases} 0 & r > R \\ -W & r < R \end{cases}$



実験にあわすれば

$R \simeq 1.2 \dfrac{1}{\mu}$ に取る必要あり。

武田の: multiplicity

$\bar{p} + (N) \to n_\pi \pi$ $\quad \langle n_\pi \rangle = \begin{cases} 5.1 \\ 4.9 \pm 0.4 \end{cases}$

$\dfrac{\hbar}{M\pi c}$

$2Mc^2$

$\langle n_\pi \rangle = 3.0$

$\to 3.5$ (I-spin)

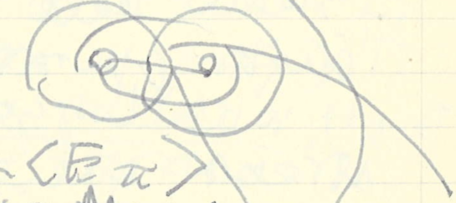
(selection rule) (Sternheimer)

$\to 5.1$ (Itoh)

(final interaction)

	p	$\bar{p}$
0π	C_0	C_0
1π	C_1	C_1
2π	C_2	C_2

$\longrightarrow n_\pi = 2 \sum \langle n_\pi^i \rangle$



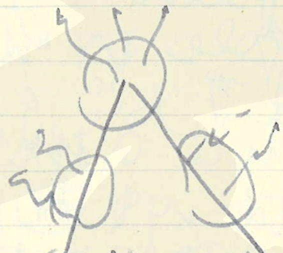
Fermi

$\frac{\hbar}{m_\pi c}$

$2M \to 2M - 2\langle E_\pi \rangle$

$\langle E_\pi \rangle = 340$ MeV

$\Sigma C_n n = 1.3 \to 2.6$



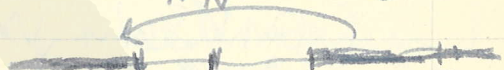
$\frac{0.75}{4}$

$\uparrow$

[illegible]：$\langle n_\pi \rangle \sim \Omega^{1/4} W_0^{3/4}$ · (Fermi

[illegible]：dispersion

$M_{N\bar{N}}(E) = M_{NN}(-E)$



$0 \quad M \quad 2M$

analytical continuation

$\sigma_{pn}(E) - \sigma_{p\bar{n}}(E)$

is energy a odd function

$$\frac{\langle n_\pi \rangle_{2M-2(2\mu)}}{\langle n_\pi \rangle_{2M}} \simeq \frac{2}{3}$$

$\Omega = 4\left(\frac{1}{\mu}\right)^3$

S. Deser (Copenhagen)
General Relativity and the Divergence Problem in Q.F.T.
(Preprint)

I. Introduction

Small scale world では metric tensor operator の意味は明瞭でない。しかし general relativity は stress-energy tensor の満足すべき definition を与えるのであるから、gravitational field はすべての場と同じ様に couple する唯一の場であ~~所~~ることは注目すべきである。又 general covariance の概念は small scale world にも適用されるべきものと考えられるから、gravitational field と他の場との相互作用も含めての上で micro の世界にまで拡張すべきであろう。(O. Klein はこの考えを長い間、主張してきた)

従来は special relativity の framework の中で divergence を除く可能性について悲観的であった。所が general relativity では energy density には上限が有りうるということでは事情が違ってくる。(以前 Klein)

(異常 singularity の避けられない)
Schwarzschild solution で ~~mass を大きくすると~~ 境界をさけるとき、
mass の上限が生ずる $(1-\frac{m}{r})^{-1}$)
それに伴って wave packet の大きさ
linear wave eq. から作れる
に制限があらわれる。 E を energy で
~~対して~~ dimension を d とすると

κ^2 = [illegible] Einstein const

$$E < c^4 d/2\kappa \qquad E = \hbar c/d$$

$$d > l_0 = (2\hbar c\kappa)^{1/2} \doteqdot 10^{-32}\,\mathrm{cm}$$

又 Q.F.T. では "clothed" particle
の ~~singula~~ propagator の singularity
が light cone の内外にあらわれ、
renormalization constant Z
あるものを ∞ にする。(perturbation
theory に関係ない、Lehmann
Källén) しかし その根拠にある
一般的仮定は P_μ が存在 して

$$\partial_\mu O(x) = i[O, P_\mu] \quad (1)$$

等である。従って (1) などは overall
properties についていえても、
inner property or region に
ついてはいえない。 (Pauli, Relativitäts

-merie, §21†) "clothed" electron
の position を正確にきめようとすれば
energy density の limit を超える
ことになる。
混れ又、Lagrangian は free
part を持っていない。kinetic energy
part に gravitational field
が入ってくるから non-interacting
particle を考えられない。
~~equal-time matter commutation
relation が metric tensor (operator)
を含む matter distribution によるから~~

T Viervektor s^k → Vektordichte f^k
$$\frac{\partial f^k}{\partial x_k}=0 \qquad \left(\int s_n\, dS=0\right)$$

f^k は Weltröhre の中でのみ 0 でないが
それ以外では充分早くへる。
$$J=\int f^4\, dx^1 dx^2 dx^3$$
ist unabhängig von x^4 である。J は
広い範囲の座標に対して不変 —
そして $U^k=U_i^k p^i$ は linear transf.
に対して vector. $J=J_{k}p^k$ は invariant,
従って $J_k=\int \mathfrak{U}^4_k\, dx^1dx^2dx^3$
は $\frac{\partial \mathfrak{U}^k_k}{\partial x_k}=0$ ならば ~~invariant~~ vector
J_k は free vector と呼ぶ。

に depend する。それ故 high energy domain で 強い相互作用が [illegible] 従って gravitational cut-off が convergent future theories への backstop として役立つであろう。

II. Quantization

Feynmann の方法を使う (Matthews and Salam, N.C. 2, 20 (1955)) (Misner, R.M.P. 1957,) (Laurents, N.C. 4 (1956), 1455)

I Propagators

$$\langle T(\varphi(1), \cdots \varphi(n)) \rangle_0 = \frac{\int \varphi(1) \cdots \varphi(n) \exp[-iI[\varphi(x)]]\, \delta\varphi}{\int \exp[-iI[\varphi(x)]]\, \delta\varphi}$$

$$I = \int \mathcal{L}\, dx^4 \qquad \beta = 2\varkappa$$

$$\mathcal{L} = \sqrt{g}\left\{\frac{1}{\beta^2} R - i\varphi^+ (\gamma^\alpha D_\alpha - \mu)\varphi\right\}$$

$$R = g^{\mu\nu} R_{\mu\nu}, \quad g = -|g_{\mu\nu}| \qquad \varphi = a\psi$$

$$[\gamma^\mu, \gamma^\nu]_+ = 2g^{\mu\nu}, \quad D_\mu = \partial_\mu - \Gamma_\mu$$

$$\frac{\partial \gamma_\mu}{\partial x^\nu} = \gamma^\alpha \Gamma_{\alpha\mu\nu} + \Gamma_\nu \gamma_\mu - \gamma_\mu \Gamma_\nu$$

The general framework is as in ordinary theory: the vacuum expectation values of the operators of interest is the functional integral of a corresponding c-number quantity over all classical configurations of the system. Thus, the contributions of the "scalar" and "transverse" parts of the gravitational fields will limit the density of matter in a given region and ~~to~~ "smear out the light cone" (summation over all possible Riemann spaces) and the singularities of the propagators on the cone.
(Pauli, Bern Conference 1955)

IV. Conclusions
In a flat space-time, and effectively in a non-flat but externally given geometry, equation (1) holds.

P_μ determines the translational properties of the system, and because of the homogeneity and isotropy of the space. P_μ does have, as a canonical conjugate the center of mass coordinate X_μ of the system with respect to ~~the~~ a flat background ~~space~~ frame. However, P_μ cannot be used to locate closely any component of the system.

The non-singularity of the S" and G' is an expression of the fact that one can never force such ~~consideration~~ concentration of energy of a field that its gravitational couplings cease to matter — the induced geometry would cause a repulsion before such a stage could be reached. Non singular matter distributions and non-singular metric go hand in hand.

Appendix; Conservation laws in General Theory of Relativity, P. G. Bergmann, Introduction to the theory of Relativity, 1942

$T^{\mu\nu}{}_{;\nu} = 0$

are covariant, but are not precisely what ~~are~~ called conservation laws. Instead, a proper conservation law should have the form, for instance,

$\mathfrak{T}_\mu{}^\nu{}_{,\nu} = 0$

so that the space integral $\int \mathfrak{T}_\mu{}^4 \, d\xi_1 d\xi_2 d\xi_3$ would ~~will~~ not change with time for an isolated system. ~~In~~ Now, we can construct $\mathfrak{T}_\mu{}^\nu$ from $T_\mu{}^\nu$ by adding the term

$\frac{1}{16\pi\kappa} t_\mu{}^\nu$

which is not a tensor. $t_\mu{}^\nu$ depends on $g_{\mu\nu}$ and their derivatives.

原核子研究会. 岩[illegible]
July 31, 1957

I、[illegible] 武谷G:

II、湯川: gravitational field
spurion

neutrino $\bar{\Psi}\gamma_\mu \partial_\mu \Psi + \eta\rho_3' \bar{\Psi}\gamma_5\gamma_\mu\partial_\mu\Psi$
$+ \ldots$
$+ g_0(\ \text{''}\ ) + g_3\rho_3(\ \text{''}\ )$

ρ_1 S.R., ρ_2 T.R., ρ_3 S.T.R.

Parity non-conservation
$\eta = \pm 1$: Two component theory

(I)

On Symmetries in Elementary Particle Interactions

B. D'Espagnat and J. Prentki
Cern, Geneva
and Abdus Salam
Imp. College, London

(Nuclear Physics 3 (1957), 446)

charge independence:

$$\mathcal{L} = g_1 \bar{N} i \gamma_5 \tau \cdot \pi N - g_2 (\bar{\Lambda} i \gamma_5 \pi \cdot \Sigma + h.c.)$$

Classical Physics as Geometry
Gravitation, Electromagnetism,
Unquantized Charge and Mass
as Properties of Curved Empty Space.
by C. W. Misner and J. A. Wheeler
(Preprint, Aug. 1957)

Yoichiro Nambu

Possible Existence of a Heavy Neutral Meson
(P. R. 106 (1957), 1366)

ρ^0: neutral vector, isospin 0
mass $\sim 2\sim3\, m_\pi$

Decay: (a) $\rho^0 \to \pi^0 + \gamma$, $2\pi^0 + \gamma$
$\pi^+ + \pi^- + \gamma$

(b) $\rho^0 \to e^+ e^-$, $\mu^+ \mu^-$

(c) $\rho^0 \to \pi^+ + \pi^-$

$P_a \sim (\mu c^2/\hbar)(G^2/\hbar c)(e^2/\hbar c)(\mu/M)^2$

$P_b \sim (\mu c^2/\hbar)(G^2/\hbar c)(e^2/\hbar c)^2(\mu/M)^2$

P_c: forbidden

Form factor due to exchange of ρ^0 between nucleon and electron $F(k^2) \sim Gg/(\mu^2 + k^2)$

g: ρ^0-electron coupling
$g^2/\hbar c \sim (G^2/\hbar c)(e^2/\hbar c)^2(\mu/M)^2$

(same sign for p and n)

pion exchange: $F(k^2) = \int_{2m_\pi}^{\infty} \frac{\rho(m)}{m^2 + k^2} dm$

$Gg/\mu^2 \sim e^2 \langle a^2 \rangle / 6$

$$(G^2/\hbar c)(g^2/\hbar c) \sim [(e^2/\hbar c)\cdot b]^2$$

$$\sim 10^{-6}$$

$\tau_a \sim 10^{-19} \sim 10^{-20}$ sec.

$\tau_b \sim 10^{-17} \sim 10^{-18}$ sec $(\sim \tau_c)$

1) neutral component increase in high energy reaction
2) second max (at 1 BeV) of $\pi^- + p \to n + \rho^0$
3) Repulsive nuclear force of Wigner type
4) Anomalous mag. mom.
5) K and hyperons may decay into $\rho^0 + \cdots$

J. A. McLennan, Jr.
Parity Nonconservation and the Theory of Neutrino
(P. R. 106 (1957), 821)

Majorana ~~eq~~ neutrino:

$$\gamma_\mu p_\mu \psi = 0$$

$$\gamma_j = \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix} \qquad \gamma_4 = \begin{pmatrix} 0 & -i \\ i & \end{pmatrix}$$

$$\psi^C = C\psi^T \qquad C = i\gamma_1\gamma_3$$

Majorana condition:

$$\underline{\psi = \psi^C}$$

or $\psi_3 = \psi_2^*$, $\psi_4 = -\psi_1^*$

Two component field

$$\varphi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$\psi = \begin{pmatrix} \varphi \\ i\sigma_2\varphi^* \end{pmatrix}$$

$$H = i\gamma_4\gamma_j p_j = i\gamma_4\gamma p$$

or $H = \begin{pmatrix} 1 p & 0 \\ 0 & -1 p \end{pmatrix}$

→ Lee-Yang neutrino
Majorana eq. is invariant under full Lorentz transf. group.
The interaction violates parity in either case.

Non-Local and Non-Linear Field Theories
by D. I. Blockhintsev
(Progress of Physics Sc, 1956)

I. Non-local theories

1. G. Watagin, Zs. f. Phys. 88 (1934), 92

2. M. A. Markow, GETP, 10 (1940), 1311; Jour. Phys. USSR, 2 (1940), 2;

3. H. Yukawa, P. R.

2.) $[X_\nu, A_\mu] = i\lambda_\nu A_\mu$

λ_ν: elementary (length) vector

$\Delta X_\nu \cong \lambda_\nu$, $\Delta A_\mu \cong A_\mu$

4. Blockhintsev, GETP 16 (1946), 480; 22 (1952), 254

5. H. McManus, P. R. S. (A) 195 (1948), 323; R. Peierls, P. R. S. (A) 214 (1952), 143

$$W = \int J_\mu(X) F(X-X') A_\mu(X') dX dX'$$

$F \sim e^{-s^4/s_0^4}$ $\quad s^2 = (t-t')^2 - (x-x')^2$

6. M. A. Markow, GETP 16 (1946), 790.

$[H(X_n, t_n), H(X_m, t_m)] \neq 0$
for space-like points

7. Incompatibility of non-local theory with Hamiltonian formalism:

$$i\hbar\,\delta\Psi = H\Psi\,\delta t$$

W. Pauli, N.C. 10 (1953), 658
V. S. Barashenkov, GETP 28 (1955), 579

8. S-matrix formalism in non-local theory
W. Heisenberg, ZS. f. Phys. 120 (1943), 513; 673.

9. Two point form factor $F(x-x')$ では vacuum polarization による divergence は除かれない、何故ならば [illegible], $A_\mu(x)$ による場による vacuum polarization は local theory になる
$$B_\mu(x) = \int F(x-x')\,A_\mu(x')\,dx'$$
による外場による vac. pol. と同じになるから。

10. Three point form factor:

J. Rzewuski, Acta. Phys. Polon. 11 (1951), 1, 9
J. Rayski, Phil. Mg. 42 (1951), 1289
P. Kristensen and C. Møller, Dansk. 27 (1952), 7.
11. C. Bloch, ibid. 27 (1952), 8.
R. Peierls and M. Chrètian P. R. S. (A) 223 (1954), 468.
Gage invariance
$e^{i\chi(x', x'')}$ $\quad \chi(x', x'') \sim \int_{x'}^{x''} A_\mu(x) d\xi_\mu$

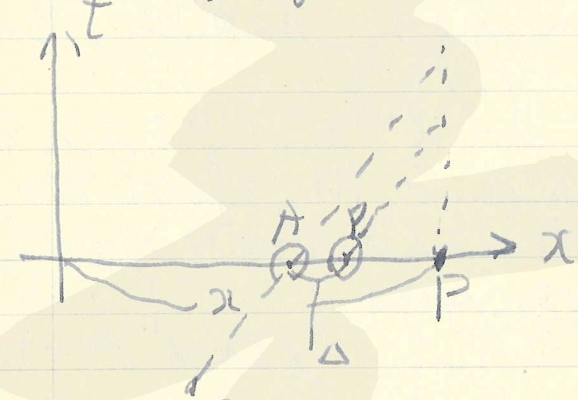
12. C. Hayashi, P. T. P. 10 (1953), 533
Non-unitarity of S-matrix
introduction of new fields involving initial field (influence of future).
13. B. V. Medvedev, Doklady of the Ac. Sc. 103 (1955), 37

$$S(g) = T \exp[i\int \Lambda(\xi, g) g(\xi) d\xi + \int M(\xi, g) g(\xi) d\xi]$$

Λ: Lagrangian, M: Hermitian operator

14. Dispersion relation

$$\Psi = \int f_A(\omega) a(\omega) e^{-i\omega(t-\frac{x}{c})} + \int f_B(\omega) a(\omega) e^{-i\omega(t-\frac{x}{c}+\frac{\Delta}{c})}$$

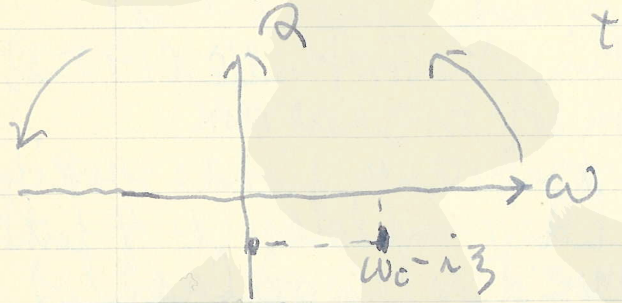


$$a(\omega) = \frac{a_0}{(\omega + i\xi - \omega_0)}$$

incident wave

$a_0 e^{-i(\omega_0 t - k_0 x)}$

for $t \geq 0$



$t<0$ $\quad e^{-i\omega t}$; $\omega \to i\infty$, $t<0$; $e^{-\infty}$

$$f_{AB}(\omega) = f_A(\omega) + f_B(\omega) e^{-i\omega\Delta/c}$$

$$f = g + ih \left\{ \begin{array}{l} g(a) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{h(x)\,dx}{x-a} \\ h(a) = -\frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{g(x)\,dx}{x-a} \end{array} \right.$$

if $f(\omega)$ is analytic in upper half-plane
f_{AB} is not so because of $e^{-i\omega\Delta/c}$, but

we may take instead $\Phi_{AB} = f_{AB} e^{\frac{i w \Delta}{c}}$

一般にしかし, non-local theoryでは dispersion relation が崩れることもある.

II. Non-linear theories:

1. M. Born, P. R. S. (A), 143 (1934), 410.

$$\delta \int \mathfrak{L}(\mathfrak{F}, \mathfrak{J}_{\mu} \ldots) dx\, dt$$

$$\mathfrak{L} = \left\{ 1 + \frac{H^2 - E^2}{E_0^2} - \frac{(EH)^2}{E_0^4} \right\}$$

$E_0 = e/s_0^2$: critical field

point charge:

$$E = \frac{e}{r} \left\{ 1 + \left(\frac{s_0}{r}\right)^4 \right\}^{-1/2}$$

total self-energy

$$U = 1.236\, e^2/s_0$$

signal velocity can be larger than c.*

In some case, we have to deal with a "bundle" of events which mutually cause one another but do not succeed one another.

* In such case, non-linear theory has

much in common with non-local theory.
W. Heisenberg, Z. f. Phys. 133 (1952), 65.

2. Theories with signal velocity smaller than c.
Zero-point energy divergence is serious in non linear theories.

$$E - E_0 = \sum N_k \hbar \omega_k' = \infty$$

$$\because \omega_k' = \sqrt{k^2 + \kappa^2} = \infty$$

$$\because \kappa^2(\varphi) = \infty \quad \because \overline{\varphi^2} = \infty$$

IV. Physics of Strong Interaction:
Concept of particle does not work for strong interaction. Interaction of particles is more important than individual particles so that the latter concept would be wholly meaningless except for ~~the~~ certain integrals of motion for the whole system.