V

NOTE BOOK

Manufactured with best ruled foolscap
Brings easier & cleaner writing

研究会記録
など
覚え書

VOL. V

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湯川秀樹

日章 BA4

Nissho Note

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Tanikawa,	Watanabe, Chirality	
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March 31, '58

Chirality of Tensors and Parity-Violating Interactions

Y. Tanikawa and S. Watanabe

G : group of congruent transf. in Minkowski space

$$G = I + P$$

P: space inversion class including

factor group G/I consists of I and P

$$I \cdot I = P \cdot P = I \quad , \quad P \cdot I = I \cdot P = P$$

If L is a faithful representation of I, PL will be a faithful repres. of G, provided $P \neq I$.

Definition of chirality operator X:

$$X \cdot X = I \qquad X \cdot I = I \cdot X = X$$

$$X \cdot P = - P \cdot X$$

Lowest rank of repr. of this group consisting of I, P and X is <u>two</u>. In this case, there are four linearly independent operators I, P, X and Y satisfying ~~$X \cdot Y = Y \cdot X$ and~~ $Y \cdot P = - P \cdot Y$ and $X \cdot Y = - Y \cdot X$ etc.

setting $X \cdot Y \cdot P = 1$, X, Y and P can be expressed by Pauli matrices.

eigen-vectors of P : S_+, S_-

X : T_+, T_-

Y : R_+, R_-

$$T_\pm = (S_+ \pm S_-)/\sqrt{2} \qquad R_\pm = (S_+ \mp i S_-)/\sqrt{2}$$

$$S_+ = \bar{\psi} \varphi \qquad S_- = i \bar{\psi} \gamma_5 \varphi$$

$$S_{\mu +} = i \bar{\psi} \gamma_\mu \varphi \qquad S_{\mu -} = i \bar{\psi} \gamma_\mu \gamma_5 \varphi$$

$$T_\pm = \bar{\psi} (1 \pm i \gamma_5) \varphi \qquad T_{\mu \pm} = i \bar{\psi} \gamma_\mu (1 \pm \gamma_5) \varphi$$

$$R_\pm = \bar{\psi} (1 \pm \gamma_5) \varphi \qquad R_{\mu \pm} = i \bar{\psi} \gamma_\mu (1 \mp i \gamma_5) \varphi$$

charge conjugate of $S_\pm \rightarrow S_\pm^*$
$T_\pm \rightarrow T_\pm^*$

$R_\pm^*$ is the result of c.c. and reversal of γ-chirality.

Any one of $S_\pm$, $T_\pm$, $R_\pm$ is invariant under $P \cdot X \cdot Y$

In order that a quantity is invariant also under one of X, Y, P, it should have the form

(i) $S_+ S_+'$, $S_- S_-'$, (ii) $T_+ T_+'$, $T_- T_-'$

(iii) $R_+ R_+'$, $R_- R_-'$

such as

(i) $i \bar{\psi} \gamma_\mu \varphi \cdot A_\mu$, $i \bar{\psi} \gamma_5 \varphi \pi$

(ii) $i \bar{\psi} \gamma_\mu (1 \pm \gamma_5) \varphi T_\pm$, $\bar{\psi} \gamma_\mu (1 \pm \gamma_5) \varphi \times \bar{\xi} \gamma_\mu (1 \pm \gamma_5) \eta$

(iii) $\bar{\psi} (1 \pm \gamma_5) \varphi R_\pm$

$\bar{\psi} (1 \pm \gamma_5) \varphi \, \bar{\xi} (1 \pm \gamma_5) \eta$

Y. T. : boson interacting with nucleon : R_-
" " muon : R_+

G. Källén and W. Pauli
Lee model
~~Danske V. S.~~ (30 (1955), Nr. 7,
Dan. Mat. Fys. Medd.)

$$\frac{g^2}{g_0^2} = 1 - A \cdot g^2 \quad \rightarrow -\infty \;!!!$$

A: divergent integral
g_0: unrenormalized coupling const.

(which should lie between 0 and 1 from very general principle.)
S-matrix is not unitary!!!

$$g = g_0 \cdot N \qquad \psi_V'(\bar{p}) = \psi_V(\bar{p}) \frac{1}{N}$$

$$\langle 0 | \psi_V'(\bar{p}) | V \rangle = 1$$

$$N^2 = 1 - \frac{g^2}{g_{crit}^2}$$

$$g^2 = \frac{g_0^2 g_{crit}^2}{g_0^2 + g_{crit}^2}$$

$N^2 < 0$ for $g^2 > g_{crit}^2$
→ indefinite metric
metric operator η.

$$\langle n_V, n_N, n_K | \eta | n_V', n_N, n_K \rangle = \delta_{n_V n_V'} \delta_{n_N n_N'} \times \delta_{n_K n_K'} (-1)^{n_V}$$

$$\langle V | \eta | V \rangle = \langle N, \theta | \eta | N, \theta \rangle = 1$$

$$\langle V_{-\lambda} | \eta | V_{-\lambda} \rangle = -1$$

W. Zimmermann

Bound State Problem

$$\lim_{\varepsilon \to \pm 0} \left\{ [A(\vec{x}, x_0) \dot{A}(\vec{y}, x_0+\varepsilon)] - (\Omega, [A(\vec{x}, x_0), \dot{A}(\vec{y}, x_0+\varepsilon)]\Omega) \right\}$$

$$= 0$$

etc.

Lehmann:

$$[A(x), A(y)] = 0 \quad \text{for } (x-y)^2 \geq 0$$

$$(\Omega, A(x)\Phi) \neq 0 \quad \text{if } -P_\mu^2 \Phi = m^2 \Phi$$

$$\left\{ \begin{array}{l} (\Omega, A(x)\Phi) = 0 \\ (\Omega, A(x)A(y)\Phi) \neq 0 \end{array} \right\} \text{if } -P_\mu^2 \Phi = M^2 \Phi$$

$$B(x, \xi) = T A(x+\xi) A(x-\xi)$$

CERN Conference on High Energy Physics 1958

第1回 開会： 東京, 阪大工, 物性研究.

武田暁： Meson physics, strange particles etc.,
主として field theory.

岡林氏： Minkowski 空間と charge space

× parity 非保存
particle mixture) → strong int で parity 保存

× $(\gamma\partial + m\zeta_a\gamma_5)\psi = 0$

$\zeta_a = \zeta_1$: $\psi = \begin{pmatrix}\psi \\ \pm\psi\end{pmatrix}$

岡林氏 $\left\{\begin{array}{l} \zeta_2 \quad \begin{pmatrix} \frac{1\pm\gamma_5}{2}\psi \\ \pm i\gamma_5\frac{1\pm\gamma_5}{2}\psi \end{pmatrix} \\ \zeta_3 \quad \begin{pmatrix}\psi \\ 0\end{pmatrix} \quad \begin{pmatrix}0 \\ \psi\end{pmatrix} \end{array}\right.$

○ $\{(\alpha+\beta\gamma_5)\gamma\partial + m\}\psi = 0$

$\psi_r = \sqrt{a}\, e^{\delta\gamma_5}\psi'$

$a(\gamma\partial(1+\delta\gamma_5) + m')\psi' \to$ Dirac

$\{(\alpha+\beta\gamma_5)\gamma\partial - m)\gamma_5\psi = 0$

$(i(\alpha+\beta\zeta_1)\zeta_2\gamma_5\gamma_\mu\partial_\mu - m)\varphi = 0 \quad \zeta_1\gamma_5$

$(i(\alpha+\beta\zeta_2)\zeta_1\gamma_5\gamma_\mu\partial_\mu - m))\varphi = 0 \quad \zeta_2\gamma_5$

$\begin{pmatrix}\psi \\ \gamma_5\psi\end{pmatrix} \qquad \begin{pmatrix}\psi \\ i\gamma_5\psi\end{pmatrix}$

hyperonic charge:

$$\eta = \zeta_1\gamma_5 \qquad N \ \Xi$$
$$\text{or } \zeta_2\gamma_5 \qquad \Lambda \ \Sigma$$

charge conjugation:

$$\left.\begin{array}{l}\vec{T} = \zeta_1\vec{\tau} \\ T_4 = \zeta_2\end{array}\right\} \text{charge space}$$

hyperonic charge は保存 → parity 保存
しない → しない

$$K : \frac{1+\zeta_{1,3}}{2}$$

$$\bar{X} N K \bar{T} : \quad \bar{\Psi}_X \frac{1+\zeta_3}{2}\psi_N = \bar{\Psi}_X\gamma_5\psi_N$$

$$\pi : \frac{1+\zeta_2}{2} \qquad \bar{\Psi}_\Lambda \frac{1+\zeta_2}{2}\psi_N = \bar{\Psi}_X(1-\gamma_5)\psi_N$$

6次元:

$$\left\{\begin{array}{l}\theta^0 \\ \theta^0_2\end{array}\right. \qquad \text{life} \quad \text{hyp. charge}$$

parity non-conserv.
→ renormalization

$K \to e+\nu$ X
$\pi \to \mu+\nu$ O
lepton の hyperonic charge

原論的: Minkowski と charge 空間.

Gauge 変換

$\psi \to e^{i\alpha}\psi$

$A_\mu \to A_\mu + \partial_\mu \alpha$

$A_\mu = \mathfrak{a}_\mu + \partial_\mu \Lambda$

$\mathfrak{a}_\mu \to \mathfrak{a}_\mu$

$\Lambda \to \Lambda + \alpha$

α は回転角.

Kaluza　$x_1 \cdots x_4,\ x_5$

$$\left(\begin{array}{c|c} g_{mn} & A_i \\ \hline A_i & ? \end{array}\right)$$

$\frac{\partial}{\partial x_5} \propto e$

$x_5 \to x_5 + \alpha$

$x_5 \equiv \alpha$　　$\frac{\partial}{\partial x_5} \propto I_3$

$I_3 \to \mathrm{I}$

Klein　1956. 相対論 50 年記念.

$x_5 \to x_5 + \alpha(x_1 \cdots x_4)$

$\downarrow$

$x_5 \to x_5 + \alpha(x_1 \cdots x_4, x_5)$

$\langle \varepsilon \rangle$ ← Fourier expansion

$\psi = \varphi_n U_n(x_5)$

$$\psi_n \to (\delta_{nm} + \varepsilon Q_{nm})\psi_m$$

$$Q_{nm} = \xi_0(I_3) + a(I_1) + b(I_2)$$

$$[I_i, I_j] = iI_k$$

charge space の回転 と equivalent
5次元 invariance から出てくる.

$$s_{max} = \begin{cases} 0 & \text{gauge inv.} \\ 1 & \text{C.I.} \\ 2 & \cdots \end{cases}$$

電磁場は5番目の成分. 故に電磁場 [illegible] 失われると5次元 inv. が破れる.

lepton
inversion は CP

metric への影響

$$L = -\hbar^2 c^2 (\partial_\mu \psi^* g^{\mu\nu} \partial_\nu \psi + \kappa^2 \psi^* \psi)$$

町田君：核子の構造.

magnetic moment μ_p, μ_n

$\langle r^2\rangle^p_{ch}$ $\quad \langle r^2\rangle^n_{ch}$

$\langle r^2\rangle^p_{mag}$ $\quad \langle r^2\rangle^n_{mag}$

討論

$\mu_p + \mu_n : 0$

$\langle r^2\rangle^p_{ch} \sim (0.8\times 10^{-13}\,\mathrm{cm})^2$

$\langle r^2\rangle^n_{ch} \sim 0 \quad \rightarrow \quad \dfrac{\langle {}^2\rangle^p_{ch}}{\langle\ \rangle^n_{ch}} > 70$

$\cong \langle r^2\rangle^p_{mag} \cong \langle r^2\rangle^n_{mag}$

static meson theory

$\mu_p + \mu_n = 0$

$|\langle r^2\rangle^n_{ch}| = |\langle r^2\rangle^p_{ch}|$

K-particle effect

$n \rightarrow \Sigma^- + K^+$
$\quad \rightarrow p + \pi^-$ } へる

$p \rightarrow \Sigma^0 + K^+$
$\quad \rightarrow n + \pi^+$ } ふえる.

$\dfrac{\langle {}^2\rangle^p_{ch}}{\langle {}^2\rangle^n_{ch}}$: $4 \Rightarrow 6$

nucleon source と π cloud のひろがり の関係.

virtual pair

$$\Gamma_\mu(x,y,z) = \gamma_\mu \frac{1+i\gamma_5}{2}\delta(x-z)\delta(z-y)$$
$$+ie\Box_x\Box_y'\langle T\psi(x)\bar\psi(y)A_\mu(z)\rangle_0 \Box_y$$

nucleon current　　　　meson current

$$\frac{1}{2}\Gamma_\mu^{ns} + \frac{1}{6}\Gamma_{3\mu}^{nv} + \Gamma_\mu$$

dynamical correlation が問題

Za講演: Comment
Jamm

小講演: Weak Interaction
Conservation Laws for Lepton Processes (P. R. Feb. 25 109(1958), 1404)

Γ の符号
C 不定でない。

① β-ray, μ-decay, μ-capture に関する実験:
Co^{60}　　Eu^{152} の K-electron capture の後の γ の circular polarization

polarized $n \to e, \nu$: 角分布の asym.

$e^\pm$ の polarization $\sim \pm\frac{v}{c}$ (80°)

結論: $(1+\gamma_5)\psi_\nu$　$\begin{cases}\nu: \text{左巻き} & \nu_L \\ \bar\nu: \text{右巻き} & \bar\nu_R\end{cases}$

が正しい。

$\pi \to \mu \to e^+$ 右巻き (Liverpool)

1つの結果： (i) νは2成分
(ii) lepton number 保存
(iii) neutrino charge に右まきとcouple するか、左まきとcouple するか）．
(iv) V-A-interaction (Fermi)
~~[illegible]~~

◎ Λ, Σ inv.

$\Lambda^0 \Sigma^+$; $\bar{P}\alpha$ 大 $(1+\bar{P}\alpha\cos\theta)$

仮定； $\Lambda(1+\gamma_5)N\pi$

S, P ×

V, A ○

$g\bar{\psi}_\Lambda \gamma_\mu (1+\gamma_5)\psi_N \partial_\mu \phi_\pi$

くりこみできない

$\Sigma^+ \to p+\pi^0$ $\bar{P}\alpha_0 = -0.37 \pm 0.19$

$\to n+\pi^+$ $\bar{P}\alpha_+ = -0.36 \pm 0.21$

$$\left.\begin{array}{l} \dfrac{\alpha_0}{\alpha_+} \sim 1 : \\ \dfrac{\tau(\Sigma^+)}{\tau(\Sigma^-)} \sim \dfrac{1}{2} \\ \dfrac{w(\Sigma^+ \to p+\pi^0)}{w(\Sigma^+ \to n+\pi^+)} \sim 1 \end{array}\right\}$$

仮定 T に対して inv.
one-to-one law $(1+\gamma_5)$
$|\Delta I| = 1/2$
の3つから出ない．

Weak Interaction $g \sim 10^{-17}, 10^{-20}$ cm
Fermion F.-B.

2nd kind
parity 1:1

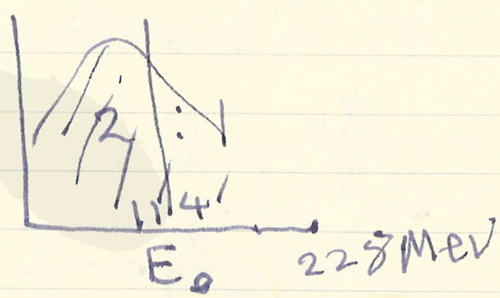
$\dfrac{\pi \to \mu + \nu}{\pi \to e + \nu,\ e + \nu + \gamma}$

N, Ξ

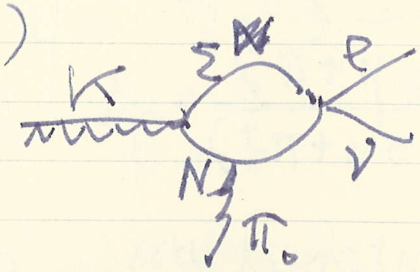
{ global symmetry
{ cut-off

neutrino capture
Davis　$\nu + N \to p + e^-$　(Cl^{37})
King　$\nu + p \to N + e^+$

阪G: K-decay
$K^{\pm}_{e3} \to e^{\pm} + \pi^0 + \nu$　energy spectrum
$K^{\pm}_{\mu 3} \to \mu^{\pm} + \pi^0 + \nu$
$K^0_{e3} \to$
$K^0_{\mu 3} \to$



43例.
S, T が多い
(半減期は同じ)



C. W. Misner and J. A. Wheeler
Geometrodynamics
(Annals of Physics dec. 1957
2 (1957), 525; 604)

中西氏の紹介 (May 24, 1958)

Physics is geometry

1. grav. without grav → general relat.
2. el. mag. .. → Rainich
3. charge .. → wormhole
4. mass .. → geon

unquantized

Einstein 方程式

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = f_{\mu\alpha} f_\nu{}^\alpha + {}^*f_{\mu\alpha} {}^*f_\nu{}^\alpha$$

$${}^*f_{\mu\nu} \equiv \frac{1}{2}\sqrt{-g}\,[\mu\nu\alpha\beta] f^{\alpha\beta}$$

$${}^{**}f = -f$$

dual rot. $e^{*\alpha} f \equiv f \cos\alpha + {}^*f \sin\alpha$

$$\left.\begin{aligned} R_\mu{}^\alpha R_\alpha{}^\nu &= a\,\delta_\mu{}^\nu \\ R_\alpha{}^\alpha &= R = 0 \end{aligned}\right\}$$ Rainich condition

5 成分

$$R_{00} = 0$$

から逆に dual rot. を除いて $f_{\mu\nu}$ が unique にきまる..

Maxwell 方程式

$$f^{\mu\nu}{}_{;\nu} = 0 \qquad {}^*f^{\mu\nu}{}_{;\nu} = 0$$

$$\frac{\partial\alpha}{\partial x^\beta} = \alpha_\beta \qquad \alpha_\beta \equiv \sqrt{-g}\,[\beta\lambda\mu\nu]\frac{R^{\lambda\gamma;\mu} R_\gamma{}^\nu}{R_{\sigma\tau}R^{\sigma\tau}}$$

$$\alpha_{\beta;\gamma} - \alpha_{\gamma;\beta} = 0$$

$$\alpha = \int_0^x \alpha_\beta\, dx^\beta + \alpha_0 \qquad \oint \alpha_\beta\, dx^\beta = 2\pi n$$

Charge as Flux in Multiply-Connected Space

Topology: 連続一対一連続で不変な性質をしらべる。
（可微分にたいして）

n-Manifold: $\mathfrak{M}$

1) locally Euclidian (n次元)

2) Hausdorff 空間 (T-2)

2点で互いに交わらない近傍がつくれる.

3) countable basis をもつ

可算無限の open set でおおえる.

R^n, S^n 等. T^n (torus)

W (長田の wormhole)

differentiable manifold

$\forall x \in \mathfrak{M}$ 任意 属する　　$U = N(x)$

$x_i(x)$　$(i = 1, 2, \cdots n)$

(a) $x \in U \xrightarrow[\text{into}]{} (x_1(x) \cdots x_n(x)) \in R^n$

$f(x)$　　$f(x_1 \cdots x_n)$

(b) $f(x)$ differentiable ← diff.

local coord.　coord. patch

Cartan: intrinsic notation

$\mathbf{v} = v^m \mathbf{e}_m$

$\mathbf{v} = v_\alpha \,\mathrm{grad}\, x^\alpha \equiv v_\alpha \, d x^\alpha$

(1-form)

$df = \dfrac{\partial f}{\partial x^\alpha} dx^\alpha$

exterior product: $\alpha = \mathbf{u} \wedge \mathbf{v}$

2-form　hook

$$u \wedge v = - v \wedge u$$

$$\alpha = \frac{1}{2} \alpha_{\mu\nu} dx^\mu \wedge dx^\nu$$

$$\alpha_{\mu\nu} = u_\mu v_\nu - v_\mu u_\nu$$

$$a = \frac{1}{p!} a_{\alpha_1 \alpha_2 \cdots \alpha_p} dx^{\alpha_1} \wedge \cdots \wedge dx^{\alpha_p}$$

$$da = \frac{1}{p!} da_{\alpha_1 \cdots \alpha_p} \wedge dx^{\alpha_1} \cdots \wedge dx^{\alpha_p}$$

$p+1$-form

$$d(a+b) = da + db$$

$$d(a \wedge b) = (da) \wedge b + (-1)^p a \times db$$

a p-form

$$d(da) = 0 \qquad \text{curl grad} = 0$$

Maxwell

$$\begin{cases} df = 0 \rightarrow \text{2-form} \Rightarrow f = da \\ d{*f} = 0 \end{cases}$$

Integration

p-form を p次元の surface 上で積分すると scalar になる。(座標に依存しない)

R^n 中の p次元立方体 I^p $\qquad 0 \leq \lambda^a \leq 1 \quad a = 1, \cdots p$

$I^p \xrightarrow{\text{into}} M$ map した像を C.

$C \xrightarrow{\text{onto}} I^p$ ~~(φという mapping)~~.

$I \leftleftarrows ? \; C \rightarrow p$ a mapping を φ:

$$x^a = \varphi^a(\lambda^1, \ldots, \lambda^a)$$

$$a \rightarrow a^\varphi = \frac{1}{p!} a_{a_1 \cdots \alpha_p} dx^{a_1}(\lambda) \wedge \cdots dx^{\alpha_p}(\lambda)$$

$$= \sum_{\alpha_1 < \cdots < \alpha_p} a_{\alpha_1 \cdots \alpha_p} \frac{\partial(x^{\alpha_1} \cdots x^{\alpha_p})}{\partial(\lambda^1 \cdots \lambda^p)} d\lambda^1 \wedge \cdots \wedge d\lambda^p$$

$$\int_c a = \int_{I^p} a^\varphi = \int \sum_{\alpha_1 < \cdots < \alpha_p} a_{\alpha_1 \cdots \alpha_p} \frac{\partial(x^{\alpha_1} \cdots)}{\partial(\lambda^1 \cdots)} d\lambda^1 \cdots d\lambda^p$$

p-chain: → Poincaré: Combinatorial topology

$C_1 \pm C_2 \pm \cdots$

$$C^p = \sum_{i=1}^{N} s^i C_i^p$$

free Abelian group

s^i: 実数.

$$\int_C a \equiv \sum_{i=1}^{N} s^i \int_{C_i} a$$

boundary: C の j 番目の上端面 C^{j+}

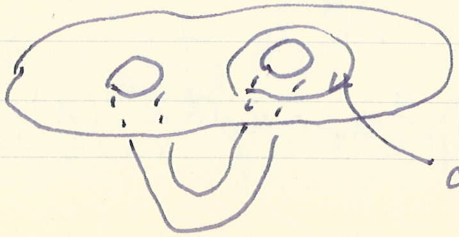
$$C^{j+} \equiv \{x^a = x^a(\lambda^1 \cdots \lambda^{j-1}, 1, \lambda^{j+1} \cdots \lambda^p)\}$$

下端面 $C^{j-} \equiv \{x^a = x^a(\cdots 0 \cdots)\}$

$$\partial C = \sum_{j=1}^{p} (-1)^{j-1} (C^{j+} - C^{j-})$$

$$\left.\begin{aligned} c &= \sum s^i C_i \\ \partial c &= \sum s^i \partial C_i \end{aligned}\right) \partial(\partial c) = 0$$

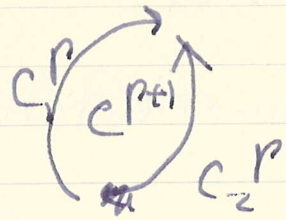
$\partial c = 0 \to c$: cycle



cycle だが boundary でない

homology:

$C_1^p - C_2^p = \partial C^{p+1}$



$C_1^p \sim C_2^p$: homolog

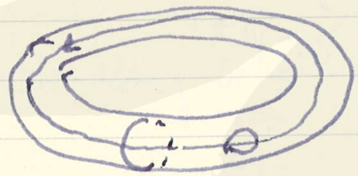
$C^p = \partial C^{p+1}$: $C_1^p \sim 0$

homology class: $\{C_i^p\}$

𝔐: $\{C_1^p\} \dots \{C_{R_p}^p\}$

R_p : p-th Betti number

Euclid R^n: $R_0=1$, $R_p=0$ $n \geq p \geq 1$



$R_0 = 1$
$R_1 = 2$
$R_2 = 1$

torus

Z^p: p-cycle の集合
B^p: p-boundary の集合

$H^p = Z^p/B^p$: homology group
その次元が R_p.

{ free な部分が R_p $p=0, \dots, n$
{ 捩り率. {0, 1}
$2C = 0$

2次元閉曲面の分類 : ~~捩率~~

1) Orientable $R_0=1$ $R_1=2\gamma$ $R_2=1$
把手の数

2) Non-orientable $R_0=1$ $R_1 \geqq 1$ $R_2=0$

$\gamma=1$:

2次元では 把手と wormhole が一致

Stokes の定理:

$$\int_{C^{p+1}} d\alpha^p = \int_{\partial C^{p+1}} \alpha^p$$

$p=1$: Stokes
$p=2$: Gauss

(1) $\partial C^{p+1}=0$ なら $\int_{C^{p+1}} d\alpha^p = 0$

(2) $d\alpha^p=0$ なら $\int_{\partial C^{p+1}} \alpha^p = 0$
conservation law

$C_1^p \sim C_2^p$: $\int_{C_1^p} \alpha^p = \int_{C_2^p} \alpha^p$

Maxwell: $df=0$ $d{}^*f=0$

$M = M^3 \times R$ time は直積
space-like wormhole

$\int_{C^2} f = 4\pi p$ $\int_{C^2} {}^*f = 4\pi q$

↑closed mag. charge ↑closed electric charge

$f = d\alpha$

$$\int_{C^2} f = \int_{C^2} d\alpha = \int_{\underset{0}{\overset{\shortparallel}{\partial C^2}}} \alpha = 0$$

$f=0$ なら α が存在する.
しかし $g \neq 0$ では…,

p-chain c^p
$\partial c^p = 0 \Rightarrow c^p$: cycle
$c^p = \partial c^{p+1}$: boundary
$\partial^2 = 0$
boundary なら cycle $\underset{\not\Leftarrow}{\Rightarrow}$

p-form f
$df = 0 \Rightarrow f$: closed form
$f = d\alpha$: exact
$d^2 = 0$
exact $\underset{\not\Leftarrow}{\Rightarrow}$ closed

$\int_{c^p} f = 0$ for $df = 0$, $c^p = \partial c^{p+1}$ } for $\partial c^p = 0$, $f = d\alpha$ }

divergence $*d* = \delta$

Universal Fermi Interaction

by R. E. Behrends

(P. R. 109 (1958), 2217

1) Interaction Hamiltonian has the same form and coupling strength for any four fermions and any order of writing four particles. (subject to conservation of charge, baryons and leptons

2) lepton fields anticommute.
Baryon fields anticommute.

3) neutrino mass is zero. Hamiltonian is invariant under $\psi_\nu \to \pm\gamma_5\psi_\nu$.

From these assumptions, we obtain the conclusion:

$$H = g[(V-A) \pm (V'-A')]$$

where g is independent of the processes and V', A' are parity non-conserving terms.

CERN準備会

基研　May 29, 1958

午前: Nucleon-pion

Introductory Talk: 宮沢氏

4. ~~anti-prot~~ nucleon Annihilation

100 ~ 300 MeV

(4) ~~absorb~~ annihilation と scattering が comparable

$$\sigma_{sc} \approx \sigma_{ab}$$

core ρ_c の [illegible] ††

~~(5)~~ π-multiplicity > statistical theory*

6. (6) π-p 2nd max.

(7) π-p high energy limit

$$\lim_{\omega\to\infty} \sigma_{\pi p} \approx 30 \text{ mb}$$

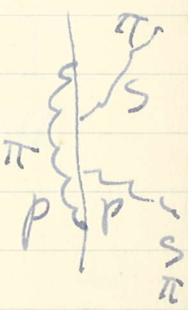
$$\sigma_{pp}$$

* (K の multip < S.T.)

1. nucleon の anomalous mag. mom.†
2. S wave π-N scatt.*
4. spin-orbit がでるか; **

$$* \quad \frac{a_1 - a_3}{2} = 2\frac{f^2}{4\pi} + \frac{1}{4\pi^2}\int_0^\infty \frac{\sigma^- - \sigma^+}{\omega}\, dk$$

0.175　　0.16　　-0.03



$$\pi^+ + p \to \pi^+ + p$$

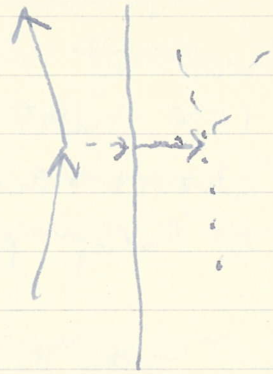
(p)

**  遠いπからはでない

† $\mu_{pc} \approx 3\frac{e}{2M}$)　$\boxed{M_{core} = \frac{M_p}{3}}$ †††

†† $\overline{er^2} \sim \left(\frac{3}{M}\right)^2$

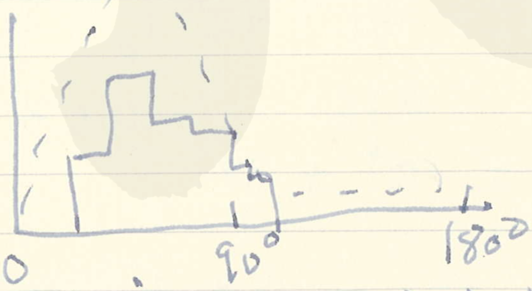
7. High energy limit
Hamiltonian
1. $\lambda\varphi^4$　λ?
2. ps. pv. ?



附　$\sigma(\infty) = \frac{\pi}{M_c^2} = 20$ mb
for $M_c = \frac{M}{3}$

$\simeq \frac{\pi}{\mu c^2}$　for π-π int.

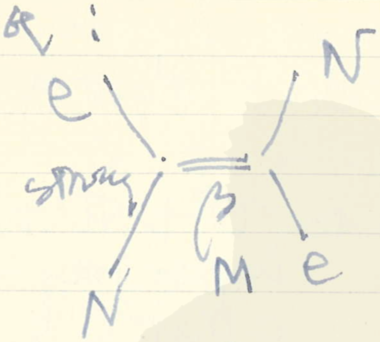
木楽氏: I, $\bar{p}$-p Elastic and exch. scattering (Segré et al)
Propane bubble chamber
30 ~ 215 MeV　(av. ~120 MeV)
37 events　(theory 68 mb)
$41 \pm^{10}_{7}$ mb　15°~165° mb
[$\bar{p} + p \to n + \bar{n}$　1 例　$\bar{n} + p \to 3\pi^+ + 2\pi^-$]
角分布



Chew et al.

II, $\bar{p}$-p emulsion　G. Goldhaber et al
$\sigma^{scatt}_{\bar{p}-p} = 71 \pm^{30}_{21}$ mb　20 events ~ 350 MeV
$\sigma^{ann.}_{\bar{p}-p} \leq 86 \pm^{58}_{37}$ mb　5 events (no heavy prong even charge)

中間子論 :



$$\frac{f^2}{M_\beta} \approx \frac{e^2}{M_\pi}$$

$\beta^+ \to p^+ + \nu$
weak

$N + \bar{N} \to e + \bar{e}$?

T III. $\bar{p} - p$ liq. H_2 counter Coombes et al.

T	σ_{tot}	σ_{elas}	$\sigma_{ch.ex}$
133±13	170±12	78±12	10^{+8}_{-4}
197±16	156±9	69±9	11^{+9}_{-5}
265±17	127±12	58±9	8^{+6}_{-5}
333±17	117±6	53±5	8^{+6}_{-3}

Strange Particle

Introductory talk 討論会.

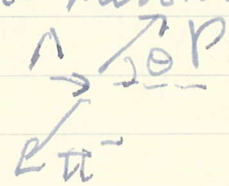
① spin & parity

spin	K	0
	Λ	1/2
	Σ	1/2(?)
parity		± ?

$\tau^+ \to \pi^+ + \gamma$

hyperfragment

a mesonic v. non-mesonic decay



$\langle \cos\theta \rangle$

② I-spin が strange particle で保存されるか?

$m_{\Sigma^-} - m_{\Sigma^+} \sim 7$ MeV

m_{Σ^0}

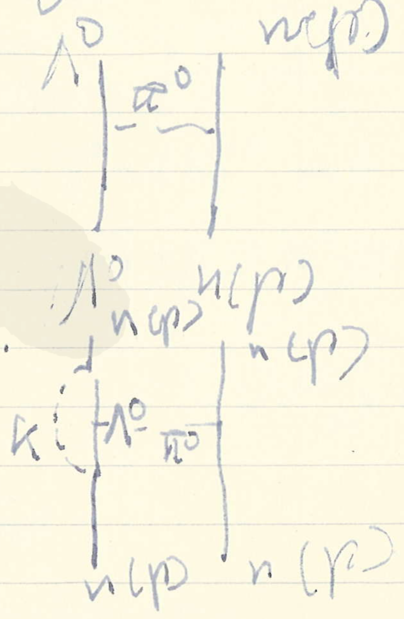
$\Lambda^0 \to \Lambda^0 + \pi^0$

decay で

$\Delta I = 1/2$ が主である.

しかし $\Delta I = 3/2,\ 5/2$ がいる.

electro-mag で破れないか.



③ global symmetry
実験的には確かでない.
a. G.S. と ($g_K \ll g_\pi$) X
b. K, π-coupling Σ と Λ で違う, X

$$\left(\begin{pmatrix} \Sigma\text{-}\Sigma\pi \\ \Sigma\text{-}\Lambda\pi \end{pmatrix} \quad \begin{pmatrix} \Sigma K N \\ \Lambda K N \end{pmatrix} \quad \begin{pmatrix} \Sigma K \Xi \\ \Lambda K \Xi \end{pmatrix} \right.$$

が判れば g.S. の答えを示す筈

④ g_K の大きさ
⑤ Weak Int. $\begin{cases} \pi \to e + \gamma + \nu \\ \pi \to e + \nu \end{cases}$
CP invariance
Type V−A
{ a Feynman-Gell-Mann → a: p.v.
b. mass-reversal
c. one-to-one } law / rule

$((A/V)_{exp.} \sim 1.14)$

⑥ New particle ?
g.S. との関連
composite model

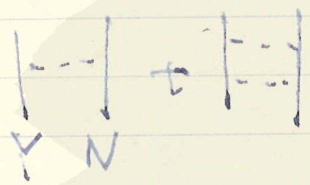
討論：

~~[illegible]~~ :

山口 ; global symmetry (preprint)

bound state of hyperon-nucleon system

π, S, π $\left\{ \begin{array}{l} n, p \\ \Sigma \ \Lambda \\ \Xi^- \ \Xi^0 \end{array} \right.$



$(\Sigma^-, n) = (n, n)$ etc

K の力) を無視.
mass の差)

binding energy が出ない.

$N + N \to \underbrace{Y + N}_{} + K$

野上 ; g_K の大きさ.

中野 ; [illegible]

Λ Σ K
$\frac{1}{2}$ $\frac{1}{2}$ 0

$g_\Lambda \ \Lambda N K$) scalar
$g_\Sigma \ \Sigma N K$)

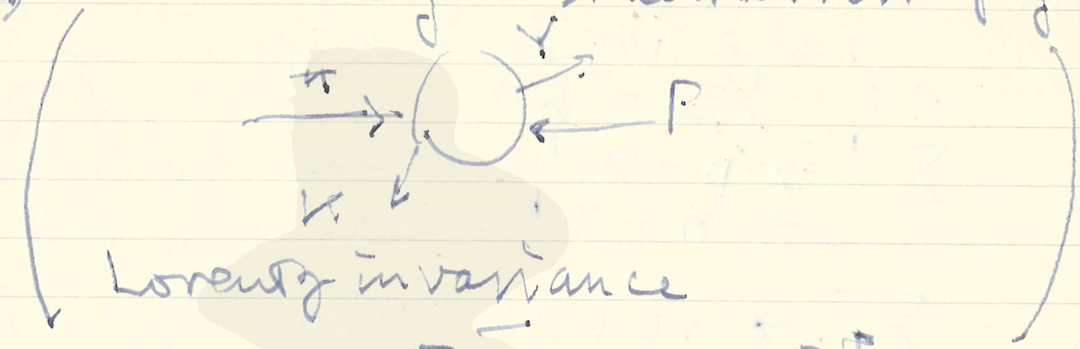
$1 > g_\Lambda^2 + g_\Sigma^2$

KN a bound state はない.

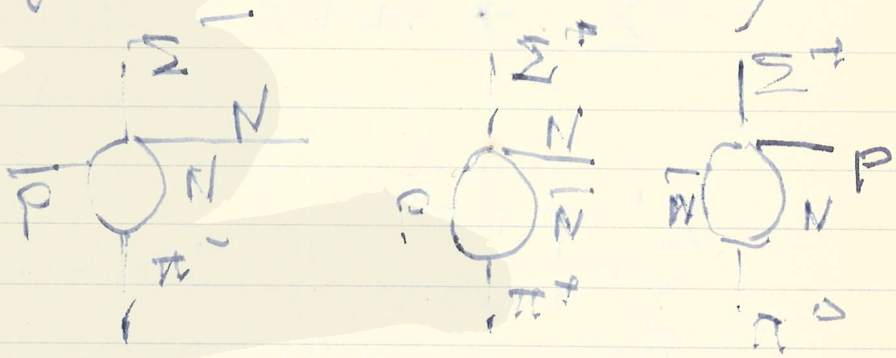
$0.2 \sim g_\Lambda^2 + g_\Sigma^2$

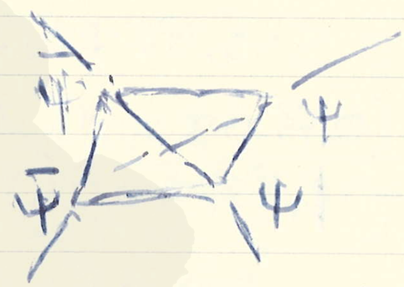
photo-K-production $g_\Lambda^2 \approx 3 \sim 1$

(補)問題：Σ-decay の smaller anisotropy



Lorentz invariance





$$\sum_{1,2,3,4} \bar{\Psi}(1)\,\Psi(2)\,\Psi(3)\,\Psi(4) \qquad F(1\,2\,3\,4)$$

回転

identical なら parity 保存

永田: Isotopic spin selection rule in Σ decay

$\Delta I = 1/2$ or $3/2$,

$\Sigma^+ \to p + \pi^0$ $\quad a_0 + b_0 (\vec{k}\vec{\sigma})$

$\quad\to n + \pi^+$ $\quad a_+ + b_+ (\vec{k}\vec{\sigma})$

$\Sigma^- \to n + \pi^0$ $\quad \sqrt{2}(a_0 + b_0 (\vec{k}\vec{\sigma}) + a_+ + b_+ (\vec{k}\vec{\sigma}))$

$$\frac{\Sigma^+ \to p + \pi^0}{\Sigma^+ \to n + \pi^+} = 0.88 \pm 0.12$$

$\bar{P}\alpha_0 \quad -0.37 \pm 0.19$

$\bar{P}\alpha_+ \quad -0.36 \pm 0.21$

$\to \frac{\alpha_0}{\alpha_+} \lesssim 1$

$$\frac{\tau_{\Sigma^+}}{\tau_{\Sigma^-}} = \frac{(0.86 \pm 0.17) \times 10^{-10}}{(1.87 \pm 0.26) \times 10^{-10}}$$

$\alpha_+ \quad \pm 0.68$

$\alpha_0 \quad \pm 0.70$

$\bar{P}^+ = \mp 0.53$

$\alpha_- \quad \mp 0.84$ $\quad \bar{P}^- = \pm 0.15$

$\bar{\Sigma}^- \gamma_\lambda (g_0 + g_0' \gamma_5) n (\partial_\lambda \pi^+)$

$g_+/g \quad \pm 0.46 \quad \pm 3.0$

$g_0'/g_0 \quad \pm 2.9 \quad \pm 0.48$

$g_-'/g_- \quad \mp 2.2 \quad \mp 0.64$

H. Yukawa :

H. Fukutome :

1) $\mathfrak{H}$: Hilbert space

Ω : vacuum　　$\psi(x)$: operator

$$\Psi = c_0\Omega + \int \psi(x) f(x)\,dx\,\Omega$$

$$+ \cdots\cdots$$

$$X = c_0 + \Psi + \Psi\Psi + \cdots\cdots$$

$$\Psi_X = X\Omega$$

$$\|\Psi_X\|^2 = (\Omega, X^* X\Omega) \qquad X \in \mathfrak{A}$$

$$= \omega(X^* X) \geq 0$$

$$(\Omega, X\Omega) = \omega(X)$$

$$\omega(X+Y) = \omega(X) + \omega(Y)$$

$\mathfrak{A}$: complete set of X

2) $\mathfrak{H}$: irreducible

$$\omega(X) \neq \omega_1(X) + \omega_2(X)$$

3) $\mathfrak{A}$: inhom. Lorentz transf. によって $\mathfrak{A}$ は不変である.

この変換は unitary operator

$$U^*_{\Lambda a}\mathfrak{A}\, U_{\Lambda a} = \mathfrak{A}$$　　Λ : h. p. L. t.,
　　a : transl.

$$U_{\Lambda a} \in \mathfrak{A}$$　　$U_{\Lambda a}\Omega = \Omega$

Field theory では $\mathfrak{A}$ から出発すると,
然るに いくつもの Hilbert space (separable)

$$\omega(U_{\Lambda a}^{*} X U_{\Lambda a}) = \omega(X)$$

Heisenberg 表示から出発すると、
Hilbert space は一義的にきまらない。
(Haag, Wightman)
相対論の表示をとると、Free
particle の Hilbert space と一致する。

$$L_A X \Omega = A X \Omega \qquad \{L_A\} = \mathcal{L}$$ (有限系)

$$R_A X \Omega = X A \Omega \qquad \{R_A\} = \mathcal{R}$$

weak topology:
任意の f, g $\quad (f, X_n g) \to (f, X g)$
X が $\mathcal{L}$ (or $\mathcal{R}$) に属する。

$$R_{A(x)} = A(x) - i \int \Delta(x-y) \frac{\delta}{\delta A(y)} dy$$

$\mathcal{L} \cdot \mathcal{R} = (a I)$

Hilbert space の dimension type

I, II, III

I_n, I_∞ $\quad$ II_1 $\quad$ II_∞

n-Euclid $\quad$ Hilbert $\quad$ II_1 $(0 \sim 1)$

II_∞ $(0 \sim \infty)$

III: 0 or ∞

free field: I_∞

I type なら $U^*\Omega U \to \Omega_0$ free

これは interaction がきかない場合.
それは II か III である必要がある

$$U_{\lambda a} = \sum_{J=0}^{\infty} \int_0^{\infty} U_{\lambda a}(\varkappa J)\, d\mu(\varkappa J)$$

$$J^2 \varkappa = P_\mu^2 M_{\mu\nu}^2 - (M_{\mu\nu})^2$$

$$\varkappa = P_\mu^2$$

$$S = \sum \int S(\varkappa J)\, d\varkappa(\varkappa J)$$

II ∞, III ∞

Ω は P_μ の 0 [illegible] で
zero measure の space

第2日　May 30

午前　I. Pion-Nucleon

本論：Collision time について.

1. extreme high energy coll. data

いままでの model は全部 × ← F. L. / H. / T.

$\overline{p_\perp} \sim$ const. はいいとしても

n(E) が困る

→ n(E, α)

2. 1) new model

{ Bohm
{ 丹生-Cocconi
{ Ezawa

2) つぎはぎ

master theory

LOW　F.L.　H

早い近似　おそい近似

3. 2段階

1) excited state　　F

2) { decay
   { secondary interaction　} L. H.

1) kin. energy → excitation energy

effective time τ　　disturbance の大きさ

excitation mode の energy ω

effective matrix element V

$V \ll \omega$　　　　$V \gg \omega$

$\tau \gg \frac{1}{\omega}$ adiabatic　　(statistical theory)

$\tau \sim \frac{1}{\omega}$ perturbation

$\tau \ll \frac{1}{\omega}$ sudden approx. —— sudden approx.

LOW, Bloch-Nordsieck

$\longrightarrow \oplus \longrightarrow$

$E_0 \xrightarrow[\tau]{} E_1 \qquad \frac{dE}{dx} = \text{const}$

Lorentz factor
$\omega \sim \mu$
$\sim M$

nucleon-anti-nucleon annihilation

$N + \bar{N} \rightarrow \text{(compound)} \rightarrow 5\pi$

$\tau_1 \qquad \tau_2$

π k.e. 180 MeV
E 300 MeV

$\tau_1 = \frac{V}{\sigma v}$

V : effective volume $\sim \left(\frac{1}{\mu}\right)^3$

$\tau_2 \simeq \frac{1}{300\,\text{MeV}}$

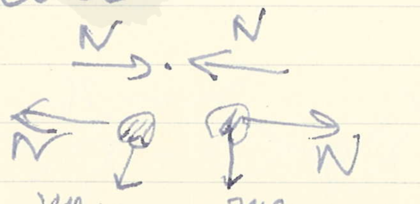
$\tau_1 \sim \tau_2$

2 [illegible] かもしれない。
$\frac{1}{v}$ law は s-wave だけ。
$\frac{a}{v} + bv + \cdots$

$\tau_1 \sim 5\tau_2$

statistical model に近づく。

Ag. Lorecini

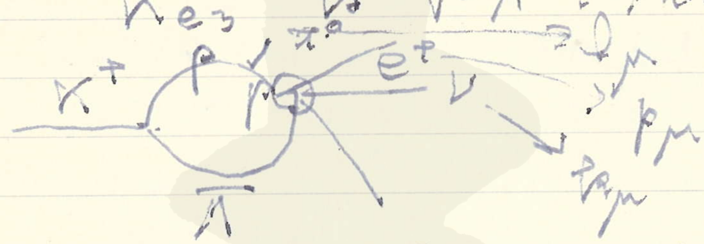


meson のあたり　meson のあたり
どちらも π-μ decay.

II. Strange Particle

米誌: Fermi int. で V-A が悪いかも.

K^+_{e3} は V-A で説明が困難.



$$\mathfrak{M} = \bar{\Psi}_e O_j \Psi_j \Phi_\pi \Phi_K K_j \cdot F(\nu)$$

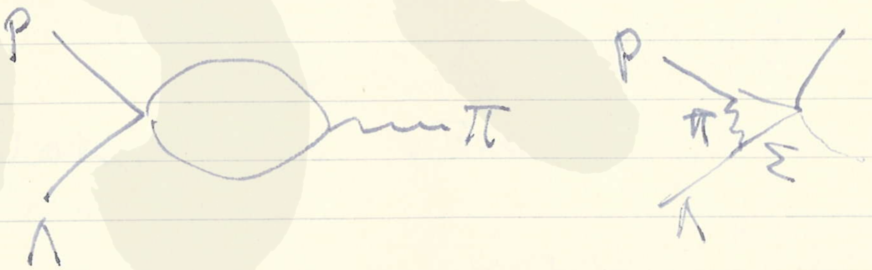
χ^2 probability

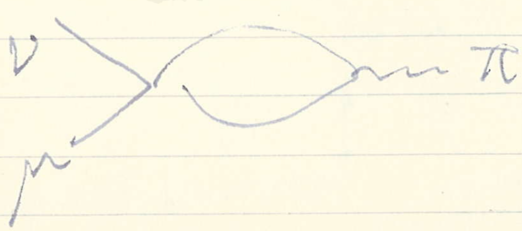
type	V±(A)	S+T±(P)	
F=const	0.004	pure < 0.0001 mixed 0.03 < 0.86	S T
F~δ(W-...)	0.07	> 0.86 ?	

pion max. energy

$$F^S / F^T m_\pi^2 \doteq 0.6$$

中間: (i) V±A の Correction





~~$K^{\pm}$ と π^0 の mass diff~~

one-to-one rule :

2成分 neutrino

neutrino charge

$$\psi_\nu \to e^{i\lambda\gamma_5}\psi_\nu$$

$$\psi_f \to e^{i\lambda}\psi_f$$

に対して invariant.

従って ν-process のみなら one-to-one rule は OK.

Strong int.

spin 0 . derivative coupling

$$\bar{\Psi}(a\gamma_\mu + b\gamma_5\gamma_\mu)\vec{\tau}\psi\,\partial_\mu\vec{\varphi}$$

$$\partial_\mu(\bar{\Psi}\gamma_\mu\vec{\tau}\psi) + \text{pion current} = 0$$

$$\nabla\vec{\varphi}\times\vec{\varphi}$$

$$\bar{\Psi}\,b\,\gamma_5\gamma_\mu\vec{\tau}\psi\,\partial_\mu\vec{\varphi}$$

+ pion term

おかしい

(ii) Feynman-Gell-Mann theory

$$\begin{cases}\psi = (\gamma\partial - m)\chi \\ \gamma_5\chi = \chi\end{cases}$$

$$\{(\Box^2 - m^2) + (\ )\sigma_{\mu\nu}F_{\mu\nu}\}\chi = 0$$

V-A しかでてこない.

$$(\Box^2 - m^2)\hat{\chi} = (\ )\,\varphi\chi \quad \hat{\chi}$$

~~χ^*~~, χ 困る.

(iii) K^+ と K^0 の mass-difference

$\left.\begin{array}{l}\pi^+ - \pi^0 \\ p - n \\ K^+ - K^0\end{array}\right)$ dynamical anomalous mag. moment.

$$H_{int} = ie A_\mu (\partial_\mu \varphi^* \varphi - \varphi^* \partial_\mu \varphi)$$
$$- e^2 A_\mu^2 \varphi^* \varphi$$
$$- \frac{ie}{\mu^2} K F_{\mu\nu} \partial_\mu \varphi^* \partial_\nu \varphi$$

$K^+ - K^0$:　$\delta\mu = (3.79 + 1.21 K - 1.15 K^2) m_e$ (cut-off M_n)

$\delta\mu = (15.1 + 11.8 K - 17.2 K^3) m_e$ (cut-off $2M_n$)

(K^0 の [illegible] の2乗はなし.)

$g_2 \Lambda K N$ と γ_5

$g_1 \Sigma K N$

P_1	P_2		$K^\pm$
1	1		$0.027 g_1^2 + 0.011 g_2^2$
1	$i\gamma_5$		⋮
$i\gamma_5$	1		⋮
$i\gamma_5$	$i\gamma_5$		

K^0

$0.004 g_1^2$

⋮

磯：

C·P 保存の条件で str. e.m.
Weak をしらべる
Soloviev.

$$\left.\begin{array}{l}(\bar{N}\tau_4 N)(\bar{N}\tau_4 N)\\(\bar{N}\tau_k N)(\bar{N}\tau_k N)\\(\bar{N}\tau_4 N)(\bar{\Lambda}\tau_4 \Lambda)\end{array}\right\}$$

$$\text{CP}: \left.\begin{array}{l}\psi' = S\bar{\psi}^{T}\\ \bar{\psi}' = \psi S\end{array}\right\} \quad S\gamma_\mu S^{-1} = -\gamma_\mu^T$$

S, T, P から CP invariant なのはできない.
V, A から CP invariant なのができる.
(∵ P: ⊖
C: V, T で ⊖)
charge independence

{ CP inv.
$O_i = S, T, P$ } P inv.
C.I

{ CP inv
$-S+T+P$ } → P inv

e.m. gauge

問: global symmetry の test.

$I \sim 3/2$ $\quad$ $I = 1/2$

A $\quad$ B

$\pi - N$

$\pi - \Sigma^+$, Y^0

$$\pi - \Sigma \begin{cases} T_2 = A \\ T_1 = (A + 2B)/3 \\ T_0 = B \end{cases}$$

$\pi - \Sigma^0$, Σ^-

$\pi - \Lambda_0 \quad T = (2A + B)/3$

$\sigma(\pi^- + \Sigma^+ \rightleftarrows \pi^+ + \Sigma^-) = 0$

T_1 only

$$K^- + p \to \begin{cases} \Sigma^0 + \pi^0 : \to \text{[illegible]} \\ \Sigma^- + \pi^+ : \\ \Sigma^+ + \pi^- : \end{cases}$$

final 3体.

i) $N \quad \Sigma(\text{or } \Lambda) \quad \pi$

ii) Σ, π, O

$K^- + D \to \Sigma^+ + N + \pi^- \to 3/2$ 3体..

$\to \Lambda^0 + p + \pi^-$

3体 ナリ global sym.

$\pi^+ + p \to \Sigma^+ + \pi^+ + K^0$

3体 ナリ global sym.

III Field Theory

N. Nakanishi, A theory of clothed unstable particles

unstable particle の state

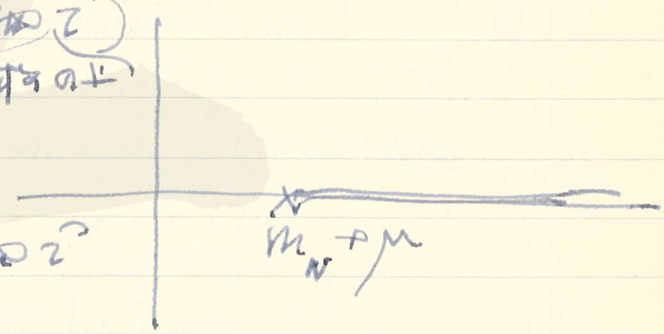
Lee model　　$V \rightleftarrows N + \theta$

V: 安定なら　$[S_V^c(E)]^{-1} = 0$ が V

real solution をもつ　$E = m$

不安定なら ~~real~~ solution はない…

(Riemann 面で)
(第1葉の上)



次の Riemann 面で

$E = m_V - i\gamma/2$

にある筈。

$H|V\rangle = \cancel{m|V\rangle} (m_V - i\gamma/2)|V\rangle$

を満す state $|V\rangle$ をもつ。

complex distribution　　ω: complex

$\int F(\omega)\varphi(\omega)\, d\omega = F[\varphi]$

a rational

linear function を define するのに、
~~pat~~ ~~pro~~ $F(\omega)$ の pole のまわりで
積分路を変えればよい。

$$|V\rangle = |V\rangle - g \int \frac{G(\omega_k)/\sqrt{2\omega_k}}{\omega_k^{(-)} + m_N - m_V + i\gamma/2} a_k^* \, dk\, |N\rangle$$

$\langle \tilde{V} | \tilde{V} \rangle = 0$

$\int = \cdots$ wave packet

(ψ) 〃　1) H の eigenstate

2) $0 \le Z \le 1$　　$g \to 0$　$Z \to 1$

3) くりこみ可能.

4) decay spectrum

5) $e^{-\gamma t}$ からの近似.

表面: Non-linear spinor field theory

重力を spinor で出せるか?

general invariance

$$\boxed{\gamma^\mu(\partial_\mu - \Gamma_\mu)\psi = 0} \qquad \Gamma_\mu = \Gamma_{\mu\rho}^{\ \ \rho} + \frac{1}{4}\,\mathrm{tr}\,\Gamma_\mu$$

$$\boxed{\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}\cdot I}$$

$$\boxed{\frac{\partial\gamma_\mu}{\partial x_\nu} = \Gamma^\rho_{\mu\nu}\gamma_\rho + \Gamma_\nu\gamma_\mu - \gamma_\mu\Gamma_\nu}$$

Spur Γ_ν は I の倍数で γ_μ と commute するから不定,

$g_{\mu\nu}(\psi, \bar\psi)$

Spur Γ_μ は 電磁場と看做す.

$$\boxed{\frac{\hbar}{4}\,\mathrm{tr}\left(\frac{\partial\Gamma_\mu}{\partial x_\nu} - \frac{\partial\Gamma_\nu}{\partial x_\mu}\right) = l^{-2}(\bar\psi\gamma_{[\mu\nu]}\psi)}$$ rot. part

$$\begin{aligned} g_{\mu\nu} = {} & (\bar\psi\gamma_{[\mu\rho]}\psi)(\bar\psi\gamma_{[\nu\sigma]}\psi)g^{\rho\sigma} \\ & - (\bar\psi\gamma_5\gamma_\mu\psi)(\bar\psi\gamma_5\gamma_\nu\psi) \\ & + (\bar\psi\gamma_\mu\psi)(\bar\psi\gamma_\nu\psi) \end{aligned}$$

Γ_μ は vector でないが, tr Γ_μ は vector

rot. part とは ; divergence のないもので
あらわれる場合を除くことを意味する.

Energy-momentum of Matter

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \equiv -4\pi k\, T_{\mu\nu}$$

(symmetric tensor)

Angular-momentum
Thomson-Bergmann

Charge-current

$$\frac{i}{4}\,\mathrm{trace}\left(\frac{\partial \Gamma_\mu}{\partial x_\nu} - \frac{\partial \Gamma_\nu}{\partial x_\mu}\right) = \cdots \equiv F_{\mu\nu}$$

$$\sqrt{g}\, g^{\rho\mu} g^{\sigma\nu} F_{\mu\nu} = f^{\rho\sigma}$$

$$\frac{\partial f^{\rho\sigma}}{\partial x_\sigma} \equiv j^\rho$$

→ charge の保存則.

Flat space solution は存在.

$$\psi = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad g_{\mu\nu} = \delta_{\mu\nu} \qquad \gamma_\mu^0$$

$$\gamma_4 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$\Gamma_\mu = 0$$

$$\psi = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{i\Lambda(x)} \qquad \Gamma_\mu = i\,\partial_\mu \Lambda$$

$$\psi = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

縮退は残る

Schwarzschild

$$\left.\begin{aligned} g_{4i} &= 0 \\ g_{ij} &= \delta_{ij} + \lambda(r)\frac{x_i x_j}{r^2} \\ g_{44} &= \mu^2(r) \end{aligned}\right\}$$

$\psi = \begin{pmatrix} F \\ 0 \\ iG \\ 0 \end{pmatrix}$ と取ると, μ は含まるが, λ は含まらぬ.

中野: Euclid parameter による
場の方程式.
Wightman function の
analytic domain について.

$$\Delta^{\pm} = \int \delta(p_\mu^2 + m^2)\, e^{ip_\mu x_\mu}$$

analytic continuation
$t \to it$

$$\Rightarrow \int \frac{e^{ipx}}{p_\mu^2 + m^2} \qquad x_4 \gtrless 0$$

$$\int \psi(-x)\, x_4\, \psi(x) = it$$

張園: Gürsey formalism
(N.C. 1958)
Dirac γ, ψ の代りに σ, ρ

$\gamma_j = \sigma_j \times \rho_k$

$\gamma_4 = 1 \times \rho_e$

$\gamma\psi = \sigma_{ip}\, \rho_{kq}\, \Psi_{pq}$

$= \sigma_{ip}(-\Psi_{pq}')(-\rho_{qk}'^{T})$

$= \sigma_{ip}\Psi_{pq}'\rho_{qk}'$

$\Psi \to \psi_+ + \underbrace{\sigma_{123}(\psi_-)^{\rho}}_{\psi}$ → charge conj.

$\phi \to \Psi$

$\gamma_t\psi = \psi_t$

$\psi \to \Psi$

$\psi\rho \to -\Psi\tau_l$

$i\psi \to i\Psi\check{\tau}_m$

$\gamma_j\psi \to -i\sigma_j\Psi^{\rho}\check{\tau}_m$

$\gamma_4\psi \to \Psi^{\rho}$

$\tau_k \quad \tau_l \quad \check{\tau}_m$

$1 \quad 2 \quad \check{\tau}_3 = \varepsilon_{klm}\tau_m$

$(\gamma_\mu\partial_\mu + \varkappa)\psi = 0$

$\to (\partial_t + \sigma_j\partial_j)\Psi^{\rho} + i\varkappa\Psi\check{\tau}_m = 0$

	ψ	Ψ
Lorentz 変換	$e^{\gamma_j\gamma_k a_{jk}}\psi$; a_{jk} : real	$e^{i\sigma_j l_j}\Psi$; l_j : real
4j	$e^{i\gamma_j\gamma_4 l_j}$; l_j : real	$e^{\sigma_j k_j}\Psi$; k_j : real
full proper		$e^{h_j\sigma_j}$; h_j : complex (6-parameter)

general
Gauge Transf.
Pauli Transf.

$\psi \to a\psi + b\gamma_5\psi^P$ $\quad \Psi e^{i\vec{p}_j \cdot \vec{l}_j}$
$(|a|^2 + |b|^2 = 1)$ $\quad l_j$: real

T. K. T. transf.
$\psi \to a\psi + b\psi^P$ $\quad \Psi e^{\vec{p}_j \vec{k}_j}$
$(|a|^2 - |b|^2 = 1)$

① general gage. Transf. は
Lorentz 群と
isomorph

γ_5 変換 $\quad e^{i\beta\gamma_5}\psi$ $\quad \Psi e^{i} = e^{i}\Psi$

解析

① ψ
Ψ

$$\begin{pmatrix}\psi_p' \\ \psi_n'\end{pmatrix} = e^{i\vec{\tau}\vec{\epsilon}}\begin{pmatrix}\psi_p \\ \psi_n\end{pmatrix}$$

$\to \psi = \chi' \to a\chi + b\gamma_5\chi^P$
$\hat{\psi} = \zeta' \to a\zeta - b\gamma_5\zeta^P$

①′ n-conj. $\quad \hat{\psi} = \gamma_1\gamma_2\gamma_3\psi$
$\chi' = e^{i\zeta\gamma_5}\chi$
$\zeta' = e^{-i\zeta\gamma_5}\zeta$

② $(\gamma_\mu\partial_\mu + \kappa)\psi = 0$
T. K. T. 変換に対して invariant か

Lagrangian は如何か?

$i\bar{\psi}(\gamma_\mu \partial_\mu + \kappa)\psi \quad \times A \quad \text{real}$

$+ i\bar{\psi}^P(\gamma_\mu \partial_\mu + \kappa)\psi^P \quad \times B \quad \text{real}$

$+ i\bar{\psi}(\qquad)\psi^P \quad \times D$

$+ i\bar{\psi}^P(\qquad)\psi \quad \times D^*$

Hermite

T.K.T. 変換に対する invariance

i) $A = B$ $\quad \nearrow \; k = \frac{D}{A}$

$\alpha \neq 0$ の場合には、

$b = ik\alpha' \qquad a = \alpha + i\alpha'$

$\alpha^2 + (1-K)\alpha'^2 = 1 \qquad K = k \cdot k^*$

1-parameter group

a) $K = 0$ $\quad$ T.K.T. 変換

$e^{i\alpha}\psi \qquad \alpha = \cos\theta, \; \alpha' = \sin\theta$

b) $K = 1$ $\quad |k| = 1 \qquad k = e^{i\rho}$

$\alpha = 1 \qquad \alpha'$: arbitrary

$\psi \to (1 + i\theta)\psi + i\theta e^{i\rho}\psi^P$

c) $K \neq 0$, $K \neq 1$,

$0 < K < 1$

$\psi \to \left(\cos\theta + i\frac{\sin\theta}{\sqrt{1-|k|^2}}\right)\psi + i\frac{k\sin\theta}{\sqrt{1-|k|^2}}\psi^P$

Jauch field

$$\kappa > 1: \quad \psi \to \left(\cosh\theta - i\frac{\sinh\theta}{\sqrt{|\kappa|^2-1}}\right)\psi$$

$$+ \frac{i\kappa}{\sqrt{|\kappa|^2-1}} \sinh\theta \cdot \psi^P$$

iii) $A = 0$: Tokuoka neutrino

$a = a^*$ $\qquad b = -b^*$

$\alpha^2 - \beta'^2 = 1$ $\qquad \left.\begin{array}{l}\alpha = \cosh\theta \\ \beta' = \sinh\theta\end{array}\right]$

$$\psi \to \cosh\theta\,\psi + i\sinh\theta\,\psi^P$$

[illegible]: wave front

$$\Box\varphi + f(\varphi) = 0$$

宏武G
May 31, 1958 D. F. Kurdgelaidze
On the Nonlinear Generalization of the Meson and Spinor Field Equations
J. E. T. P. 5 ('57) 941

1. Scalar

$$L = -\frac{1}{2}\left((\nabla\varphi)^2 - \varphi_t^2 + \Phi(\varphi)\right)$$

$$\varphi_{tt} - \varphi_{nn} + F(\varphi) = 0$$

$$F(\varphi) = \frac{1}{2}\frac{d}{d\varphi}\Phi(\varphi)$$

$\varphi = \varphi(\sigma)$ $\sigma = k_\mu x_\mu$ $k_4 = i\omega$

$$\bar{H} = \frac{1}{2T}\int_0^T \{(\nabla\varphi)^2 + \varphi_t^2 + \Phi(\varphi)\}\,dt = a(k^2 + k_0^2)/\omega$$

$$G_{\bar{H}} = -\frac{1}{T}\int_0^T (\nabla\varphi)\varphi_t\,dt = ak$$

$$(\omega^2 - k^2)\varphi_\sigma^2 + \Phi(\varphi) = h \equiv \text{const}$$

$$k_0^2 = h\omega/2a$$

$$a = \frac{\omega}{T}\int_0^T \varphi_\sigma^2\,dt$$

$F(\varphi) = k_0^2\varphi$ (i)

$= k_0^2\varphi + \alpha\varphi^2$

$= k_0^2\varphi + \beta\varphi^3$

(i) $\varphi = \varphi_0 \begin{pmatrix} \cos(\sigma - \sigma_0) \\ \sin(\sigma - \sigma_0) \end{pmatrix}$

$\omega^2 - k^2 = k_0^2$

$a = \frac{\varphi_0^2\omega}{2}$ $h = k_0^2\varphi_0^2$

$k_0 = k_0 \qquad \overline{H} = a\omega \qquad G_n = a k$

(ii) $\varphi = \varphi_0 \begin{pmatrix} cn^2(\sigma - \sigma_0) \\ sn^2(\sigma - \sigma_0) \end{pmatrix} + \varphi_1$

$$\omega^2 - k^2 = \left(\frac{k_0}{2}\right)^2 \left[\frac{2}{3}\left(\frac{\alpha\varphi_0}{k_0^2}\right) \pm \sqrt{1 - \frac{1}{3}\left(\frac{\alpha\varphi_0}{k_0^2}\right)^2}\right]$$

$$\varphi_1 = -\frac{k_0^2}{2\alpha}\left[\left(1 + \frac{\alpha\varphi_0}{k_0^2}\right) - \sqrt{\quad ''\quad}\right]$$

$$\int^{\tau} \frac{d\tau}{\sqrt{(1-\tau^2)(1-k_1^2\tau_1^2)}} \qquad k_1^2 = 2\,\frac{\alpha\varphi_0}{6(\omega^2 - k^2)}$$

$$\tau \simeq 1 \;\Big|\; < \begin{matrix} cn^2 \\ sn^2 \end{matrix}$$

$$h = k_0^2\left[1 + \frac{2}{3}\left(\frac{\alpha\varphi_1}{k_0^2}\right)\right]\varphi_1^2$$

(iii) $\varphi = \varphi_0 \begin{pmatrix} cn(\sigma - \sigma_0) \\ sn(\sigma - \sigma_0) \end{pmatrix}$

(iv) $k_0^2\varphi + \alpha\varphi^2 + \beta\varphi^3$

Heisenberg anharmonic oscillator
Landau-Lifschitz
quantized energy:

$$H_q = k_0 h\left(n + \frac{1}{2}\right) + \frac{3\beta}{8k_0^3}\left[n^2 + n + \frac{1}{2}\right]$$
$$- \frac{5}{12}\left(\frac{\alpha}{k_0^2}\right)^2 \frac{1}{k_0}\left[\qquad\right]$$

[illegible].

$$a = n + \frac{1}{2}$$

classical にちかく考えるには $n = \frac{1}{2}$ の場合,

spinor:

$$\left\{ \gamma_\mu \frac{\partial}{\partial x_\mu} + a(\bar{\psi}, \psi) \right\} \psi = 0$$

$a(\bar{\psi}, \psi) = a(\rho)$ $\qquad \rho = \bar{\psi}\psi$

$\psi = \chi(s)\,\Phi(x)$ $\qquad \bar{\chi}(s)\chi(s) = 1$

$$\left\{ \gamma_\mu \frac{\partial \Phi}{\partial x_\mu} + A(\Phi) \right\} \chi(s) = 0 \qquad A(\Phi) = a(\rho)\Phi$$

$$\frac{\partial^2 \Phi}{\partial x_\mu^2} - B(\Phi) = 0$$

$$B(\Phi) = A(\Phi) \frac{dA(\Phi)}{d\Phi}$$

Φ: real function

$$A(\Phi) = \pm \sqrt{k + 2\int_0^\Phi B(\Phi)\, d\Phi}$$

ゆえに $k = 0 \leftarrow A(0) = 0$.

$\Phi(\sigma)$ $\qquad \sigma = k_\mu x_\mu$

$$A(\Phi) = \lambda \frac{d\Phi}{d\sigma} \qquad \lambda = \sqrt{k_\mu^2}$$

$$(\gamma_\mu k_\mu + \sqrt{k_\mu^2})\chi(s) = 0$$

Heisenberg:

$$\frac{\partial^2 \Phi}{\partial x_\mu^2} - g\Phi^8 = 0$$

$$b = 3a^2 \qquad \Phi = (\sigma + c)^{-\frac{1}{2}}$$

$$(2a)^2 = k_\mu^2$$

素粒子論 と Heisenberg 理論の関係,

$$\gamma^\mu(\partial_\mu - \Gamma_\mu)\psi = 0$$

$$g^{\alpha\beta} = \overset{\circ}{g}{}^{\alpha\beta} - \beta\chi^{\alpha\beta}, \qquad \beta \sim \sqrt{k}$$

$$\overset{\circ}{\gamma}{}^\mu\partial_\mu\psi + \beta\,\eta - \frac{1}{2}(\delta_\gamma^\sigma \overset{\circ}{g}_{\delta\mu} - \frac{1}{2}\delta_\mu^\sigma \overset{\circ}{g}_{\gamma\delta})$$

$$\times \chi^{\gamma\delta}\overset{\circ}{\gamma}{}^{\sigma\mu}\partial_\sigma\psi$$

$$+ \frac{1}{8}(\qquad\qquad)\;(\quad)\;\psi_\gamma = 0$$

$$\shortparallel$$

$$l^2\psi(\bar{\psi}\psi)$$

PC Conservation in Strong Interactions
S. D. Drell and S. C. Frautschi
(Standford)
A. M. Lockett, III
(Los Alamos)

S. N. Gupta, PC-invariance for neutral ps. meson theory
(Can. J. of Phys. 35 (1957), 1309)

V. G. Solovieu, Possible Test of Conservation of Parity in Production of K-mesons and Hyperons.

PC invariance plus charge independence imply separate P and C invariance for (ps. + sc.)-meson theory without derivative coupling:

$$\mathcal{L}'_\pi = g_1 (\bar\psi \cdot \gamma_5 \tau_3 \psi \phi_0 + \sqrt{2}\,\bar\psi \cdot \gamma_5 \tau_+ \psi \phi + \sqrt{2}\,\bar\psi_0 \gamma_5 \tau_- \psi \phi^*)$$

$$+ i g_2 \sqrt{2} (\bar\psi \cdot \tau_+ \psi \phi - \bar\psi \cdot \tau_- \psi \phi^*) \quad (1)$$

g_1, g_2: real $\qquad \bar\psi \cdot O \psi = \frac{1}{2}(\bar\psi O \psi - \psi O^T \psi)$

ϕ : ps.

$g_2 = 0$ because of charge independence

(sc- and ps.-coupling have opposite PC symmetry resulting in (1). derivative V- and A. coupling have same symmetry leading to P violation in meson physics as in β-decay)

Neutral K meson is distinct from its anti-particle with opposite strangeness. PC invariance plus charge indep. do not lead to P conservation in hyperon, nucleon, K-meson interactions:

$$\mathcal{L}'_{KN\Lambda} = f\left(\bar{\psi}_N(\gamma_5 + i\rho)\psi_\Lambda \phi_K + \bar{\psi}_\Lambda(\gamma_5 + i\rho)\psi_N \phi_K^*\right) \quad (2)$$

$$\phi_K = \begin{pmatrix} \phi_{K^+} \\ \phi_{K^0} \end{pmatrix}$$

High energy 会議，
6月12日　1958

磯：Elastic Scattering and Intrinsic Structure of Elementary Particles
Blokhintsev et al.

$\pi^- + P \rightarrow$ 　1.3 BeV

$\sqrt{\langle r^2 \rangle} = 0.82 \pm 0.06 \times 10^{-13}$ cm

上田：Pion multiplicity in anti-nucleon annihilation
E. Eberle　　multiplicity
Fermi 理論.　　3.4
exp.　　4.7 ± 0.4　　K ≤ 6%
Fermi + Dyson model
Belenkij　　resonance
伊藤.　?　　結論　?

K ~~6%~~　4.4　　0.7 %

上田：Proper Meson Field of a Physical Nucleon
Ning Hu

outer region ———— core
$\pi \longrightarrow$　　K, $\bar{N}$
virtual π　1.3 → 3.3
Fock space　　Tomonaga approx.
$g^2 = R g_i^2$　　R = 7 or 2.2

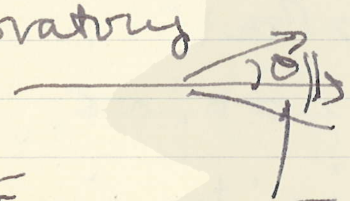
c033-507-2挟込

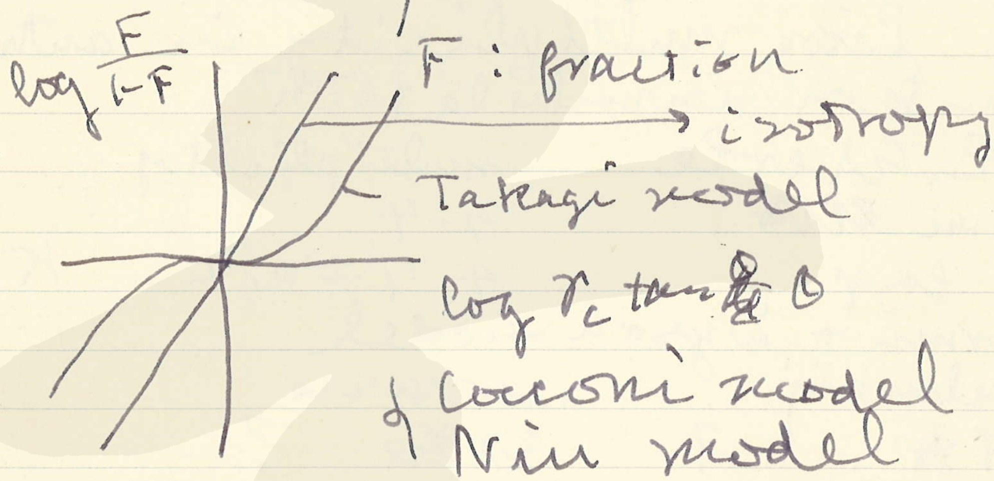
Cocconi : Jet

CMS isotropic

$$\cancel{\log}\ \frac{F}{1-F} = \gamma_c^{\,2} \tan^2 \theta$$

laboratory





結局：核子有効エネルギー[illegible]

June 14 木曜日

N. N. Bogoliubov, B. V. Medvedev and M. K. Polikarov

On the problem of indefinite metric in q. field theory

1. Gupta-Bleuler
2. Heisenberg-
3. Bogoliubov

$$\mathcal{L}_{int}(x) = g\,\bar{f}(x) f(x) \bar{f}(x) f(x)$$

$$S = T\{e^{i\int \mathcal{L}_{int}\,dx}\}$$

$$f(x) = \underbrace{\psi(x)}_{\mathrm{I}} + \underbrace{\sum_u c_u \psi_u(x) + \sum_\nu c_\nu \psi_\nu(x)}_{\mathrm{II}}$$

$$i\{\psi(x), \bar{\psi}(y)\} = S(x-y)$$

$$i\{\psi_u(x), \bar{\psi}_u(y)\} = -S_{M_u}(x-y)$$

$$i\{\psi_\nu(x), \bar{\psi}_\nu(y)\} = S_{M_\nu}(x-y)$$

$$i\{f(x), \bar{f}(x)\} = S(x-y) - \sum c_u^2 S_{M_u}(x-y) + \sum_\nu c_\nu^2 S_{M_\nu}(x-y)$$

$$H = H_1 + H_2$$

$$\Phi = P\Phi + (1-P)\Phi = \varphi + F$$

$$\varphi \in H_1, \quad F \in H_2$$

$P^\dagger = P$, $P = P^2$

$\|\Phi\|^2 = \|\varphi\|^2 + \|F\|^2$ $\quad \|\varphi\|^2 > 0$

Forbidding rule:

(1) Gupta-Bleuler

$H_1 + H_2$

$\left.\begin{array}{l}\varphi + F_1 \\ \qquad F_2 \\ \quad \vdots\end{array}\right\}$ physically equivalent

Lorentz condition によって
F の方の norm が 0 になり
観測上の effect が出ない。

(2) Heisenberg, R. M. P.

$t \to -\infty$ では H_1 の状態) となるように
$t \to +\infty$ でも 同様
相互作用と無限遠時間を選ぶ。

(3) Bogoliubov

$\left\{\begin{array}{l}\text{physical part} \\ \text{non-physical part}\end{array}\right.$ $\begin{array}{l}\varphi \\ F\end{array}$

F は φ からきめられる。

$t = -\infty$; $\Phi_{-\infty} = \varphi_{-\infty} + F_{-\infty}$

$t = +\infty$; $\Phi_{+\infty} = \varphi_{+\infty} + F_{+\infty}$

$\|\Phi_{-\infty}\| = \|\Phi_{+\infty}\|$

故に $\|\varphi_{-\infty}\| = \|\varphi_{+\infty}\|$ でなければ

$\varphi_\infty = \tilde{S}\,\varphi_{-\infty}$ $\quad \tilde{S}$: unitary

$\varphi^*_\infty \underbrace{P_\mu}_{\text{energy-momentum}} \varphi_\infty = \varphi^*_{-\infty} P_\mu \varphi_{-\infty}$

$$F_{\infty} + \underbrace{F_{-\infty}}_{e^{i\alpha}} = 0 \qquad \| F_{\infty} \| = \| F_{-\infty} \|$$

$$F_{-\infty} + (1-P)S(F_{-\infty} + \varphi_{-\infty}) = 0$$

$$F_{-\infty} = -\frac{1}{1+(1-P)S}(1-P)S\varphi_{-\infty}$$

$$\phi_{-\infty} = \varphi_{-\infty} - \frac{1}{1+(1-P)S}(1-P)S\varphi_{-\infty}$$

$$\phi^*_{as} P_\mu \phi_{as} = \varphi^*_{as} P_\mu \varphi_{as} + F^*_{as} P_\mu F_{as}$$

P_μ : asymptotic ie additive

H_1 2 H_2

$$\varphi_{\infty} = \tilde{S}\varphi_{-\infty}$$

$$\varphi_{\infty} = PS\{\varphi_{-\infty} + F_{-\infty}\}$$

$$= PS\left\{1 - \frac{1}{1+(1-P)S}(1-P)S\right\}\varphi_{-\infty}$$

$$\tilde{S} = PS\frac{1}{1+(1-P)S}$$

Reactance matrix;

$$K = i\frac{1-S}{1+S} \rightarrow S = \frac{1+iK}{1-iK}$$

K: pseudo-Hermite

$$\varphi_{\infty} - \varphi_{-\infty} = iK(\varphi_{\infty} + \varphi_{-\infty})$$

$$\varphi_{\infty} - \varphi_{-\infty} = iPK(\varphi_{\infty} + \varphi_{-\infty})$$

$$= iPKP(\varphi_{\infty} + \varphi_{-\infty})$$

$$\equiv i\tilde{K}(\varphi_{\infty} + \varphi_{-\infty})$$

$$\tilde{K} = PKP$$

~~Example~~:

$$K = \sum f(x_1)\cdots f(x_n)\, K(x_1 \cdots x_n)$$

Example: q physical oscillator

q_f non-physical "

$$\left.\begin{aligned} \frac{d^2q}{dt^2} + \omega^2 q &= \sum A_f q_f \\ \frac{d^2q_f}{dt^2} + \omega_f^2 q_f &= A_f q \end{aligned}\right\}$$

1) $q_f(-\infty) = \dot{q}_f(-\infty) = 0$

$$\frac{d^2q}{dt^2} + \omega^2 q = \int_{-\infty}^{\infty} dt'\, K^{ret}(t-t')\, q(t')$$

$$K^{ret} = \theta(t-t') \sum_f \frac{A_f^2}{\omega_f} \sin\omega_f(t-t')$$

damped oscillation であるから一般に
physical oscillator の energy が non-physical
oscillator へ流れ込む.

2) $q_f(-\infty) + q_f(+\infty) = 0$

$$\bar{K} = \frac{K^{ret} + K^{adv}}{2}$$

$$= \frac{\varepsilon(t-t')}{2} \sum \frac{A_f^2}{\omega_f} \sin \omega_f (t-t')$$

$$(\dot{q}_f(-\infty) + \dot{q}_f(+\infty) = 0)$$

non-locality

fictitious local fields
+ non-local boundary condition
⟹ non-local field theory
$\tilde{S}$: unitary

Heisenberg: $\partial F/\partial t = 0$, $F^* P_\mu F = 0$ }

maximum program
Heisenberg's idea
minimum program
Schwinger

J. Podolanski
Unified field theory in six dimensions
(Proc. Roy. Soc. A ~~2~~ 201 (1950), 234)

Dirac matrices $15 = \binom{6}{2}$
four space-like and two time-like dimensions, because of hermiticity.

R. Ascoli and E. Minardi

1. On the Unitarity of the S-Matrix in Q.F.T. with Indefinite Metric (N.C. 8 (1958) 951)

2. On Quantum Theories with Indefinite Metric (Nuclear Physics (1958),)

If the state vector φ satisfies a Schrödinger equation

$$i\frac{\partial\varphi}{\partial t} = H\varphi$$

with self-conjugate Hamiltonian

$$H^\dagger = H$$

and if the eigenstates of H, which belong to real positive eigenvalues a...s are permitted by eventual supplementary conditions, have never negative norm, then we ~~obtain~~ can construct a definite unitary S matrix, such that the probabilistic interpretation of ~~S~~ scattering processes is always possible.

$$\varphi(+\infty) = S\varphi(-\infty)$$

$$S = \exp[-i\int H dt]$$

$$S^\dagger S = 1$$

On the contrary if there is an energy ~~state~~ eigenstate of negative norm as in the case of a state for a V-particle in Lee-model (G. Källen and W. Pauli, Dan. Mat. Fys. Medd. 30 (1955), No. 7), negative probabilities appear in scattering problems, although $S^\dagger S = 1$ holds.

$$\begin{cases} \psi(-\infty) = \psi_+(-\infty) + \psi_0(-\infty) \\ \psi(+\infty) = S\,\psi(-\infty) = \psi_+(+\infty) + \psi_0(+\infty) \\ \psi^\dagger(-\infty) = \psi_+^\dagger(-\infty) + \psi_0^\dagger(-\infty) \\ \psi^\dagger(+\infty) = \psi^\dagger(-\infty) S^\dagger = \psi_+^\dagger(+\infty) + \psi_0^\dagger(+\infty) \end{cases}$$

$$S^\dagger S = 1$$

$$\psi^\dagger(+\infty)\psi(+\infty) = \psi_+^\dagger(+\infty)\,\psi_+(+\infty).$$

If we define S' such that

$$\left.\begin{array}{l} S'\,\psi_+(-\infty) = \psi_+(+\infty) \\ S'\,\psi_0(-\infty) = 0 \end{array}\right\}$$

then $\psi_+^\dagger(-\infty)\, S'^\dagger S'\, \psi_+(-\infty) = \psi_+^\dagger(-\infty)\, S^\dagger S\, \psi_+(-\infty)$.

Since $S^\dagger S = 1$, also $\underline{S'^\dagger S' = 1}$