VIII

NOTE BOOK

Manufactured with best ruled foolscap
Brings easier & cleaner writing

研究会メモ
Oct. 1958 ～ Dec. 1958
(北海道行 Oct. ～ Nov. 1958)
(小関館研究会 I, II)

VOL. VIII

湯川秀樹

Nissho Note

日章 BA4

CERN における[illegible]の話 報告の会
Oct. 4, 1958 基研サロン

[illegible]氏:

木庭氏,

中野氏.

宗像氏.

次の談話会 (Oct. 25.) で
macrocausality について討論することに
きめる.

非線型場の理論　Oct. 16, 1958

京都研究会．打合せ会

世話人：高林　内山　中野　[illegible]

池田，　11月～12月

高多．

片山．

鈴田．

宮武．

山中．

I. Heisenberg 理論の検討．

II、古典 non-linear 量子化

III、重力場

IV、高エネルギー多重発生を統計流体力学．

12月に中間報告．

素粒子論シンポジウム
Oct. 19, 1958

[illegible]:

$V_\beta \sim$ ~~[illegible]~~ A_β $\boxed{A_\beta > V_\beta}$ $(A_r = A_0[1 - g_\pi^2 f(\Lambda)]$

$V_\mu - A_\mu$ $A_\beta < V_\beta$?)

ghost があれば $g_\pi^2 < 0$: $A_\beta > V_\beta$.

$\mu \to e$ $\rho = 0.68$

Σ の decay $\Delta I = 1/2$

V-A でない,

K_1^0 の decay $\Delta I = 1/2$,

dispersion relation がアメリカで流行

Pologoy

[illegible]: Columbia π-e-exp.

[illegible] bubble chamber

momentum error 3%

3 meV — (10 meV = 69.4 meV)

6 events average 72.9 ± 1.5 meV

$\frac{6}{65,000} \sim \frac{1}{10,400} \pm 40\%$

武田: decay of particle

荘から decay する

雲による相関性の破壊 or screening

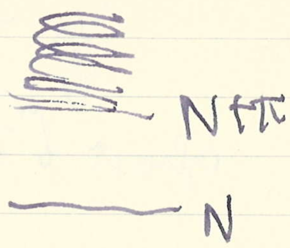
Λ, $\tilde{\Lambda}$ $\tilde{\Xi}$ + 中性 V-A なら [illegible]

Σ, $\tilde{\Xi}$ $\tilde{\Lambda} + \pi$ V-A が全ては成立する.

$$\tilde{\Sigma}^+ \to p + \pi^0$$
$$\quad\searrow n + \pi^+$$

$\tilde{\Sigma}^+$ or $\tilde{\Lambda} + \pi^+ \to n + \pi^+$

$g\,\bar{\psi}_\Sigma \gamma_\mu (1+\gamma_5) \underbrace{\psi_N \partial_\mu \phi_\pi}_{N,\ N+\pi \cdots}$



global symmetry は decay の範囲.

$$g\left[\begin{pmatrix}\tilde{\Sigma}^+ \\ \tilde{Y}^0\end{pmatrix}\vec{\tau}\begin{pmatrix}p \\ n\end{pmatrix}\vec{\phi}_\pi + \begin{pmatrix}\tilde{Z}^0 \\ \tilde{\Sigma}^-\end{pmatrix}\vec{\tau}\begin{pmatrix}\Xi^0 \\ \Xi^-\end{pmatrix}\vec{\phi}_\pi\right]$$

<u>isospin</u>　すべての hyperon の isospin は 1/2. (decay では破れるとする) strong interaction で保つてゐる.
○ I = 1/2 は K-interaction でやぶれる.

宮沢:
mass reversal

沢田:
分離状: strong と weak とは同根.
武田:
gage 場理論.
強いはこの新結合性が保存か.

内容：非線形方程式の一般的性質
第一回 10, 22, 1958

§1. 弦の例

$$L = \int \left(\sqrt{1+\psi_x^2} - \frac{1}{2}\psi_t^2 \right) dx$$

$$\psi_{xx} - (1+\psi_x^2)^{\frac{3}{2}} \psi_{tt} = 0$$

(i) 線形の場合：

$$\psi_{xx} - \psi_{tt} = 0$$

$$\psi_x = u, \quad \psi_t = -v$$

$$u_x + v_t = 0$$

$$u_t + v_x = 0$$

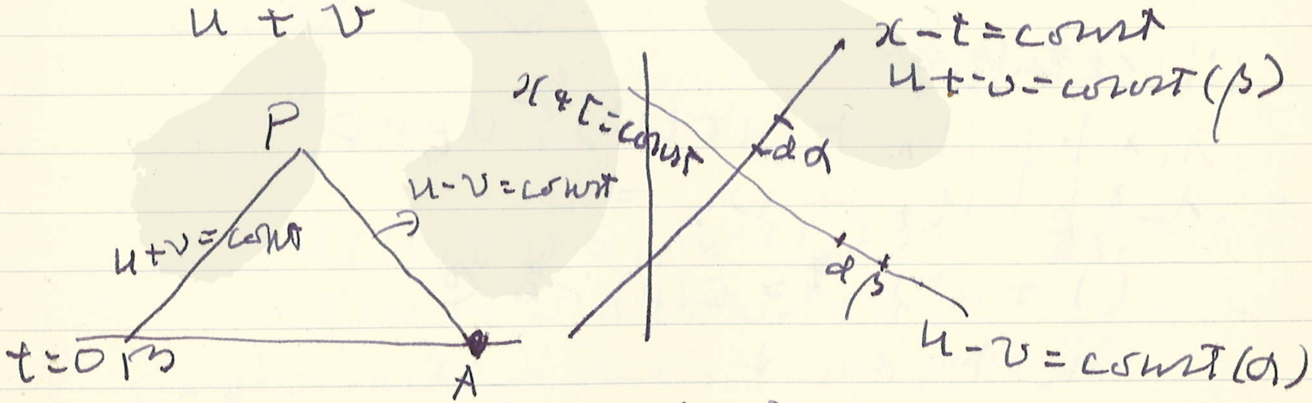
$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right)(u+v) = 0$$

$$\left(\frac{\partial}{\partial x} - \frac{\partial}{\partial t}\right)(u-v) = 0$$

$$\frac{\partial}{\partial x} + \frac{\partial}{\partial t} = \frac{\partial}{\partial \alpha} \qquad \left(\frac{x_\alpha}{t_\alpha} = 1\right)$$

$$\frac{\partial}{\partial x} - \frac{\partial}{\partial t} = \frac{\partial}{\partial \beta} \qquad \left(\frac{x_\beta}{t_\beta} = -1\right)$$

$u + v$



$\psi(x)$ $\qquad u(x) = f(x)$

$\psi_t(x)$ $\qquad v(x) = g(x)$

$$u - v = f(A) - g(A)$$

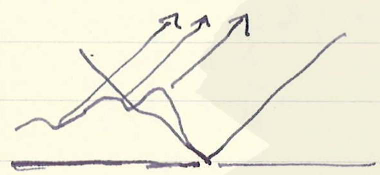
$$u + v = f(B) - g(B)$$

$$u = \frac{1}{2}\{(f_A + f_B) - (g_A - g_B)\}$$

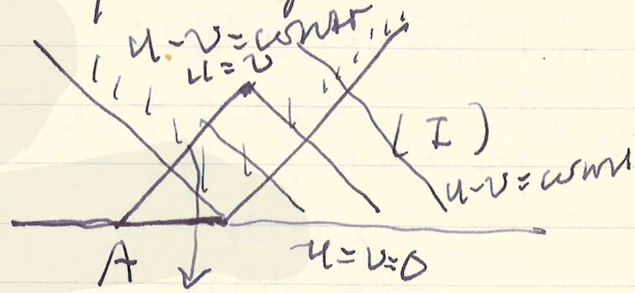
$$v = \frac{1}{2}\{ \qquad , \quad \}$$

const (β) = const

const (α)



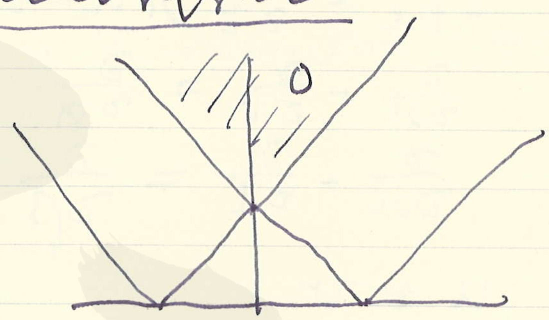
{ simple wave



$u = \frac{1}{2}(f_A + g_A) = v$

にとの解法すv

method of characteristics



(ii) 任意振動

$$\begin{array}{l|l} \lambda_1 \times & \\ \lambda_2 \times & \end{array} \left\{ \begin{array}{l} u_x + (1+u^2)^{3/2} v_t = 0 \\ u_t + v_x = 0 \end{array} \right.$$

$(1+u^2)^{-3/4} = c(u)$: 音速

$u \uparrow \qquad c \downarrow$

$$\left(\lambda_1 \frac{\partial}{\partial x} + \lambda_2 \frac{\partial}{\partial t}\right) u + \left(\lambda_2 \frac{\partial}{\partial x} + \lambda_1 (1+u^2)^{3/2} \frac{\partial}{\partial t}\right) v$$

$= 0$

$$\frac{\lambda_1}{\lambda_2} = \frac{\lambda_2}{\lambda_1 c^{-2}} = \frac{x_\sigma}{t_\sigma}$$

$$\lambda_1 \left(x_\sigma \frac{\partial}{\partial x} + t_\sigma \frac{\partial}{\partial t} \right) u + \lambda_2 (\quad) v = 0$$

$$\lambda_1 u_\sigma + \lambda_2 v_\sigma = 0$$

$$\lambda_2 u_\sigma + \lambda_1 c^{-2} v_\sigma = 0$$

$$t_\sigma^2 - x_\sigma^2 c^{-2} = 0$$

$$\frac{x_\sigma}{t_\sigma} = \pm c(u)$$

$$u_\sigma x_\sigma + v_\sigma t_\sigma = 0$$

$$\frac{x_\alpha}{t_\alpha} = c(u) \qquad \frac{x_\beta}{t_\alpha} = -c(u)$$

$$\left.\begin{aligned} c(u) u_\alpha + v_\alpha = 0 \\ -c(u) u_\beta + v_\beta = 0 \end{aligned}\right\}$$

$$\int c(u)\,du + v = \text{const}(\beta)$$

$$-\int c(u)\,du + v = \text{const}(\alpha)$$

$$\int \frac{du}{(1-u^2)^{3/4}} = \gamma(u)$$

$$\gamma(u) + v = \text{const}(\beta) \quad \left(\frac{dx}{dt} = c \right)$$

$$-\gamma(u) + v = \text{const}(\alpha) \quad \left(\quad = -c \right)$$

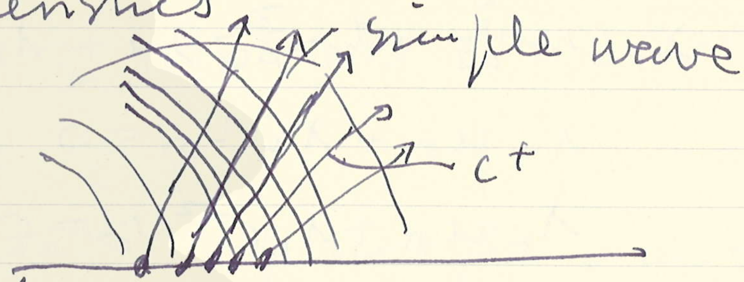
C-characteristics　[illegible]

Γ- characteristics

simple wave

t=0: $-\gamma(u) + v = \alpha_0$

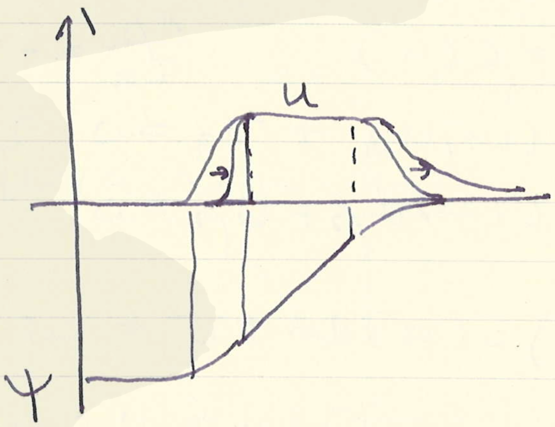
c^- characteristics

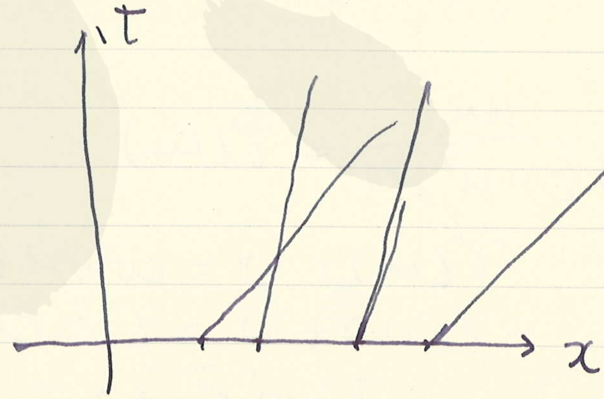


c^+は直線群
波形は縮まったり，伸びたりする．

$$c(u) = \frac{1}{(1+u^2)^{3/4}} \qquad u\uparrow \quad c\downarrow$$

初めに滑らかでも 結局は不連続性があらわれる．
shock





解の接続の問題

ψ, ψ_x, ψ_t 連続　滑らかな解.

δ-函数 → C^∞ という無限回微分可能な函数の limit として定義.

weak な解
微分係数で[illegible]

§2. 2変数の2階偏微分方程式

$F(\psi_{xx}, \psi_{xt}, \psi_{tt}, \psi_x, \psi_t, x, t) = 0$

$a\psi_{xx} + b\psi_{xt} + c\psi_{tt} + g(\psi_x, \psi_t, \psi, x, t) = 0$

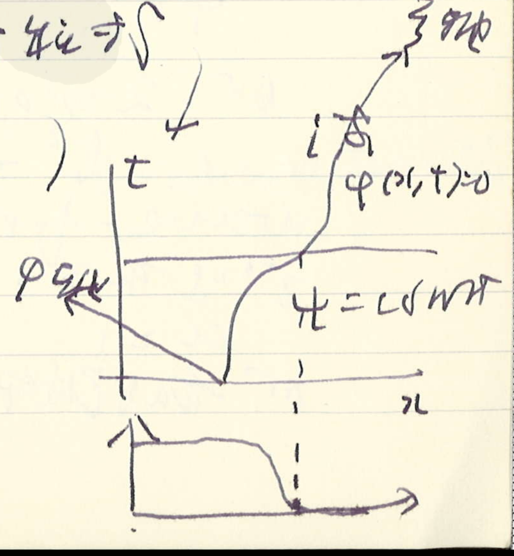
quasi-linear 準線型.

$a(\psi, \psi_x, \psi_t, x, t)$ etc.

特に a, b, c が x, t の函数のみ
semi-linear

波とは何か?
伝播していく. 有限の速さで伝わる signal. → 波動方程式

(harmonic solution)



境界面 $\varphi(x,t)=0$
両側からの limit は
ψ_t, ψ_x … は皆 0.

$\psi_{xx}, \cdots$ は $\varphi(x,t)$ の所で不連続となる.
(解析的ではない解)

$$2Q(\varphi,\xi)\frac{\partial^2\psi}{\partial\varphi\partial\xi} + Q(\varphi\varphi)\frac{\partial^2\psi}{\partial\varphi^2}$$
$$+ Q(\xi,\xi)\frac{\partial^2\psi}{\partial\xi^2} + L[\varphi]\frac{\partial\psi}{\partial\varphi}$$
$$+ L(\xi)\frac{\partial\psi}{\partial\xi} + g(\quad) = 0$$

$$Q(\varphi,\xi) = a\varphi_x\xi_x + \frac{b}{2}(\varphi_x\xi_x + \varphi_t\xi_x)$$
$$+ c\varphi_t\xi_t$$

$$L = a\frac{\partial^2}{\partial x^2} + b\frac{\partial^2}{\partial x\partial t} + c\frac{\partial^2}{\partial t^2}$$

$$Q(\varphi,\varphi)\,\Delta\,\psi_{\varphi\varphi} = 0$$

$Q(\varphi,\varphi) = 0$ という面が存在しなければ
特異点は生じない,

$$a\varphi_x^2 + b\varphi_x\varphi_t + c\varphi_t^2 = 0$$

or $\dfrac{\varphi_x}{\varphi_t} = -\dfrac{dt}{dx}$: $a\left(\dfrac{dt}{dx}\right)^2 - b\left(\dfrac{dt}{dx}\right) + c = 0$

が 2つの実根を持つ.

$$b^2 - 4ac > 0$$

wave-front の速さは a, b, c で定まる.

$a = 1 \quad c = -1 \quad b = 0$
が普通の波動方程式の一つ.

いつでも $\psi_{\varphi\xi} + F(\psi_\varphi, \psi_\xi, \psi, \varphi, \xi) = 0$

になる 標準形 に帰着できる

但し、之れは condition φ, ξ に depend する.

semi-linear の時には a priori に与えられる.

quasi-linear の時には 結合方程式ではない….

$$\psi_{xx} - \psi_{tt} = 0 \qquad \varphi = t - x,\quad \xi = t + x$$

$$\psi_{\varphi\xi} = 0 \qquad \varphi = \text{const.} \to \psi_\varphi = \text{const.}$$

$$\xi = \text{const.} \to \psi_\xi = \text{const.}$$

$$\psi_x = u, \quad \psi_t = -v$$

$$\psi_\varphi = -\frac{1}{2}(u+v) \qquad u + v = \text{const.}$$

$$\psi_\xi = \frac{1}{2}(u-v) \qquad u - v = \text{const.}$$

$$\begin{matrix} \lambda_1 \times \\ \lambda_2 \times \end{matrix} \left\{ \begin{aligned} & a u_x + b u_t - c v_t + g(u, v, \psi, x, t) = 0 \\ & u_t + v_x = 0 \end{aligned} \right.$$

$$\{a\lambda_1 \partial_x + (b\lambda_1 + \lambda_2)\partial_t\} u + \{\quad\} v + \lambda_1 g = 0$$

$$\frac{a\lambda_1}{b\lambda_1 + \lambda_2} = \frac{\lambda_2}{-c\lambda_1} = \frac{x_\sigma}{t_\sigma}$$

$$a\left(\frac{t_\sigma}{x_\sigma}\right)^2 - b\left(\frac{t_\sigma}{x_\sigma}\right) + c = 0 \qquad \text{C-characteristics}$$

$$\frac{t_\sigma}{x_\sigma} = \rho_\pm$$

$$t_\alpha - \rho_+ x_\alpha = 0, \quad t_\beta - \rho_- x_\beta = 0$$

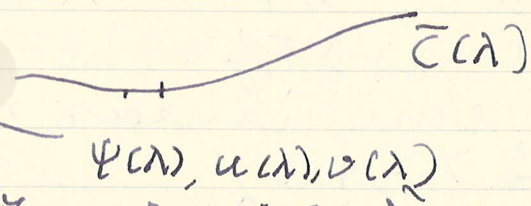
初期値問題

$\bar{C}(\lambda)$ はどこでも $\frac{dt}{dx} = \rho$ とはならない

初期条件

$$\frac{\partial \psi}{\partial \lambda} = x_\lambda \frac{\partial \psi}{\partial x} + t_\lambda \frac{\partial \psi}{\partial t}$$

$$= x_\lambda u - t_\lambda v$$



t, x, ψ, u, v に関する方程式 semi-linear は (t, x) と (ψ, u, v) に分かれる.

$$u_\alpha - \rho v_\alpha + \frac{q}{c} x_\alpha = 0$$

⋮

$$(x, t) \leftarrow (\alpha, \beta)$$

$$D = x_\alpha t_\beta - x_\beta t_\alpha \neq 0$$

quasi-linear の場合は D が solution に depend する.

P. D. Lax, Contribution to the theory of differential eq.

内山Q: A. Arnowitt and S. Deser:
Q. Theory of Gravitation - I.
General Formulation and
Linearized Theory.

$$\delta \langle b\,\sigma_2 | a\,\sigma_1 \rangle = i \langle b | \delta W | a \rangle$$

$$L = -\sqrt{-g}\, g^{\mu\nu} R_{\mu\nu}$$

$$R_{\mu\nu} = \frac{\partial \Gamma^{\alpha}_{\mu\nu}}{\partial x^{\alpha}} - \frac{\partial \Gamma^{\alpha}_{\mu\alpha}}{\partial x^{\nu}} + \Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha} - \Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta}$$

$$\Gamma^{\alpha}_{\rho\sigma} = \frac{g^{\alpha\beta}}{2}\left(g_{\rho\beta,\sigma} + g_{\sigma\rho,\beta} - g_{\rho\sigma,\beta}\right)$$

$$L = -\varphi^{\mu\nu}\left(\frac{\partial \Gamma}{\partial x} - \frac{\partial \Gamma}{\partial x} + \Gamma\Gamma - \Gamma\Gamma\right)$$

φ と Γ は独立として variation.

$$\Gamma \to \Pi = -\varphi$$

$$\varphi \to \pi = 0$$

$$H = \Pi\dot{\Gamma} + \pi\dot{\varphi} - L$$

$$(\Pi + \varphi)\Psi = 0$$

$$\pi\Psi = 0$$

湯理論 [illegible]

Causality Oct. 25, 1958

紹介者: Macrocausality & S-matrix

Schützer-Tiomno, P.R. ~~73~~ (1951) 249. (non-relativistic, R-matrix)

van Kampen, P.R. 91 (1953), 1267

S.T.:

R(E) E-plane

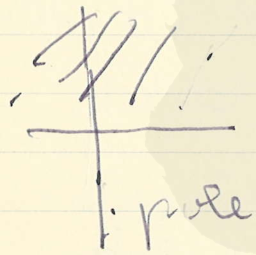
1. 上半面で analytic
2. Im R(E) ≥ 0
3. 実軸上 boundary fn. is real

入射波が ~~到達~~ scatterer に到達する前に out-going wave は出ない。

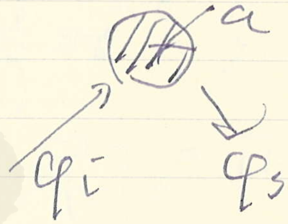
If $\varphi_i(a, t) = 0$ for $t < t_1$

then $\varphi_s(a, t) = 0$

S(k)



pole



v.K.:

$$\int_a^\infty 4\pi r^2 |\psi_{in}(r) + \psi_{out}(r)|^2 dr \leq 1$$

$$\int_a^\infty 4\pi r^2 |\psi_{in}(r, -\infty)|^2 dr = 1$$

or

$$\int_a^\infty 4\pi r^2 |\psi|^2 dr \leq 1 \quad \text{for } \int_a^\infty 4\pi r^2 |\psi|^2 dr = 1$$

for t for $t' < t$.

1. S の analyticity
2. 虚軸上で singular
3. S の微分も有、

$$S_a(\omega) - i = H + \frac{1}{\pi}\int_0^\infty \frac{1+E\omega}{E-\omega}\,\frac{\mathrm{Im}\,S_a(E)+1}{1+E^2}\,dE$$

$$+ \sum \rho_n \frac{1+E_n\omega_n}{\omega-E_n} + \int_0^\infty \frac{E\omega-1}{E+\omega}\,d\tilde{\rho}(E)$$

新しい Ph :

G. Wataghin　N. C. V ('57), 689
N. C. 9 ('58), 519

non-local ; finite , macrocausal

$\pi \cdot N$: ps. pv

cut-off operator

$I_t = k_\mu u^\mu$　　$I_s = \sqrt{I_t^2 - k_\mu k^\mu}$

$u_\mu = \frac{P_\mu}{m}$　　$P_\mu = \sum p_\mu$

$m^2 = P_\mu P^\mu > 0$

$G^2(I_s)$

②′ としそへてなほに見る 唐国兵への 姿のなしい
　　いや静かなり

① 朝霧の はれゆく 釧路 家々に
　　干す 大根の 数限りなく　　(ハ泡)
② なゝかまど 赤き実 あかし 海つるゆ ししゃもの
味のことによろしき　　(ハ泡)
③ 日の本にて 湖沼を 恵み 風はげし 摩周湖
　に 御霧して
⑤ 黒岳の 頂き 白し 月白し 層雲峡の
　秋晴れの朝.

北海道行

Oct.29, Wednesday, 晴天

13.00 羽田 日航(507) 小林氏
16.00 千歳　田中、古市、宮原.
~~18~~17.00 札幌 グランドホテル 歓迎会.

Oct.30, Thursday 札幌

11.30～13.30 京大同窓会
14.30～16.30 北大・物理へ
　素粒子物理学の現状
　中谷氏. 低温研究所.

Oct.31, Friday 札幌
11.30～13.00 産業クラブ午餐会
13.30～15.00 講演
　科学と国際協力
~~20~~16.30～20.00 エルム山荘.

~~Oct. Nov. 1~~
20.34 札幌発

Nov.1　かに、たらばがに.　「釧路」の間
7.15 釧路着.　八浪にて朝食
12.00 東洋(ソバ屋)で昼食
(ライオンズ)
14.30～15.00 公開講演

科学と人間 探究精神
15.15～15.30 高校生に対する言葉
16.00 釧路空港
18.00 摩周グランドホテル（弟子屈）着
Nov. 2, Sunday
弟子屈 8.30 摩周湖.
クッシャロ湖 — ミルクを飲む — 美幌峠

北見 13.30
北見 14.24 発
上川 17.46 着
層雲峡 層雲閣着 18.30

Nov. 3, Monday 中谷. 大沢. 田中. 小泉朗.
層雲峡 9.00
旭川 11.00 記者会見
ニュー北海ホテル
小泉氏邸で昼食.
講演 婦人会 : 13.30～14.15
高校生 (2,200人) : 14.30～15.15
歓迎パーティー : 17.30～
夕食 : 花月 市長招待

Nov. 4 Tuesday ニュー北海ホテル
午前中ホテル. こけしかり, 花束贈呈
3時～4時 : 旭川市公会堂, 公開講演
"科学文明のゆくえ"
北教大 滝田博士 訪問, シャボン玉. ぶらんこ

⑧ 人境の ~~[illegible]~~ とかかわる ~~詠評~~ ~~この北の国~~
人間らしく生くる ~~[illegible]~~ 職さぞ 余地ある 詠評にさえ
性を生かす 職[illegible]

④ 落葉松の黄をあはれがりし ~~[illegible]~~の
師と弟と ~~[illegible]~~ の記憶 ゆたかなり
酒北の秋、 ~~[illegible]~~ (北見市郊)

⑥ 旭川へまばゆく降りし秋の日の
大雪山の雲岩かな

⑦ うれしくのアイヌの酋長来てまつれば
伴侶となく 織さしぐさ
サルルンドリリンヤ

Nov. 5, Wednesday
8.45: 旭川
10.50: 札幌
12.05: 港石 (小樽): 学勝園, 昼ジンギスカン
堀武男 (~~[illegible]~~ [illegible]会館[illegible])
16.50 千歳 (日航508)
19.50 羽田
20.30 東京. 細川家.

⑩ 静かなる 波を 湯の宿 ~~窓閉に~~
0.3 ~~[illegible]~~ 紅き
紅き実 目にしむ うめもどき
朝のひととき

⑨ あたたかき 黄 ~~すも~~ ストーブあり
0.4 庭試に 書き散らしたる 文字を
たどるも

⑪ 小樽港 打いかの かたみ 落ちくるしし
0.1 せいりい[illegible]周[illegible] 続く[illegible]して

[illegible]

基研。　Nov. 8, Saturday

德岡氏

V. Votruba and M. Lokajiček
An Algebraic System of Fundamental Particles

M. Lokajiček, Algebra of Elementary Particles.

strong interaction について coupling constant に [illegible].
spurion

四次元 γ_i, β_i
Dirac and Duffin-Kemmer Algebra

$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$.　$\mu, \nu = 1, 2, 3, 4$

$\sigma_{(\alpha)\bar{\lambda}} = -\frac{i}{4} \epsilon_{\bar{\lambda}\bar{\mu}\bar{\nu}} \gamma_{\bar{\mu}} \gamma_{\bar{\nu}}$　$\bar{\lambda}, \ldots = 1, 2, 3$

$\beta_\lambda\beta_\mu\beta_\nu + \beta_\nu\beta_\mu\beta_\lambda = \delta_{\lambda\mu}\beta_\nu + \delta_{\nu\mu}\beta_\lambda$

$\sigma_{(\beta)\bar{\lambda}} = -i\,\epsilon_{\bar{\lambda}\bar{\mu}\bar{\nu}}\beta_{\bar{\mu}}\beta_{\bar{\nu}}$

1　5　10

$\beta^{(0)}$　β' pion　β'' photon

三次元 γ_i, β_i

$\{\rho_j, \rho_k\} = 2\delta_{jk}$

(1)　2×2　$\rho \to \sigma$　$\to \sigma$　spin (t_j)

(2)　2×2　$\rho \to -\sigma$

$U(\rho) = \frac{2}{3} t_j \rho_j = \frac{2}{3}\rho_j t_j \begin{cases} 1 & (1) \\ -1 & (2) \end{cases}$

c033-577～579挟込

$U(\rho)$ で表示が区別できる。

(1) $\begin{pmatrix} N_+ \\ N_0 \end{pmatrix}$ $\begin{pmatrix} K_+ \\ K_0 \end{pmatrix}$

(2) $\begin{pmatrix} \Xi_0 \\ \Xi_- \end{pmatrix}$ $\begin{pmatrix} \kappa_0 \\ \kappa_- \end{pmatrix} \equiv \begin{pmatrix} \bar{K}_0 \\ \bar{K}_+ \end{pmatrix}$

$$q(\rho) = \tau_3 + \frac{1}{2} U(\rho)$$

$$\xi_i \xi_j \xi_k + \xi_k \xi_j \xi_i = \delta_{ij} \xi_k + \delta_{kj} \xi_i$$

spin $(0;\ )$
0

1×1 $\xi^{(0)}_j = 0$

3×3 $\xi^{(1)}_j \to \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \cdots > \xi^{(1)}_j$ pl... $(e_+$ $(\mu_+$

3×3 $\xi^{(2)}_j \to -\xi^{(1)}_j$

4×4 ξ'_j $\xi'_1 = \frac{1}{\sqrt{2}} \left(\begin{array}{ccc|c} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \hline 1 & 0 & 1 & 0 \end{array}\right)$ $\left(\begin{array}{c|c} \xi^{(1)}_j & 0 \\ \hline 0 & 0 \end{array}\right)$

$$U(\xi) = \frac{2}{3} \theta_j \xi_j = \frac{2}{3} \xi_j \theta_j$$

0
4/3
−4/3
0

$$q(\xi) = \theta'_3 + \frac{1}{2} U'(\xi')$$

$\begin{pmatrix} \Sigma_+ \\ \Sigma_0 \\ \Sigma_- \\ \Lambda \end{pmatrix}$ $\begin{pmatrix} \pi_+ \\ \pi_0 \\ \pi_- \\ \pi' \end{pmatrix}$

Algebra of ω & λ in Isospace

$$\omega_j = \begin{pmatrix} \rho_j^{(1)} & & \\ & \rho_j^{(2)} & \\ & & \xi_j' \end{pmatrix}$$

$$\left\{\begin{array}{l} \sum_P (\omega_j \omega_k \omega_l - \delta_{jk}\omega_l) = 0 \\ [\omega_j, \lambda_k] = i\epsilon_{jkl}\omega_l \\ [\lambda_j, \lambda_k] = i\epsilon_{jkl}\lambda_l \\ \lambda_j\omega_k + \lambda_k\omega_j = \omega_j\lambda_k + \omega_k\lambda_j = \delta_{jk}U \end{array}\right.$$

$$U = \frac{2}{3}\lambda_j\omega_j$$

...tion

$(\nu = \bar{\nu}, e_-)$ (μ_+, ν, e_-)

$(\mu_0 = \bar{\mu}_0, \mu_-)$ $(e_+, \bar{\nu}, \mu_-)$

$$[\omega_j, U] = 0$$

$$U^3 = U \rightarrow (\pm 1, 0)$$

$$(\{\omega_j, \omega_k\} - 2\delta_{jk})U = 0$$

$$(\mathcal{D} - \kappa)(1 - U^2) = 0$$

$$\omega_j^T = \Omega^{-1}\omega_j\Omega$$

Ω: unitary symmetric

R: unitary hermitian

$$\{R, U^2\} = R$$

$$\{R, q\} = 0$$

$$\Omega R^T = -R\,\Omega$$

R の存在を仮定すれば $\xi^{(0)}$ が出る.

R : spurion

$$R = \begin{pmatrix} 0 & T^\dagger \\ T & 0 \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

Reflection

$$Y_j\,\omega_k = (2\delta_{jk} - 1)\,\omega_k Y_j$$

$$Z_j\,\omega_k = (1 - 2\delta_{jk})\,\omega_k Z_j$$

$$\Omega Y^T = Y\,\Omega$$

etc.

$$[Y_j, U] = 0$$

$$[R, q] = 0$$

$$\{Z_j, U\} = 0$$

$$\lambda = \begin{pmatrix} t_i & & \\ & t_j & \\ & & \theta_j \end{pmatrix}$$

(U, λ, λ_3) の 同W. 固有値　<u>8</u>

Ψ | Ψ_{N_+} Ψ_{N_0} Ψ_{Ξ_0} Ψ_{Ξ_-} . . . .
↓ ψ 成分

φ | (K_+) (K_0) . . . .
↓ 5 成分

$$\{\Psi_{a\alpha}, \overline{\Psi}_{b\beta}\} = \frac{\delta_{ab}}{i} S_{\alpha\beta}(\ ,\ )$$

$$\{\varphi \quad \varphi\} = \frac{\delta_{ab}}{i} T\ (\ ,\ )$$

$$\Psi^{(a)} = \Omega C \overline{\Psi}^T$$

$$\Psi^{(c)} = Z_2 \cancel{\Omega} \Psi^{(a)}$$: Charge Conj.

$$\underline{Z_2 \Omega = 1}$$

$\varphi^{(a)}$

$\varphi^{(c)}$

$\overline{\Psi} \gamma_\mu \varphi \Psi \rightarrow \Psi \to \Psi^{(a+)}$ で 符号が かわる.

$\varphi^{(a)} = \varphi$　　$K_0 = \overline{K_0}$　　$\pi_0 = \overline{\pi_0}$

$\pi_\pm = \overline{\pi_\mp}$

$$\Phi = \frac{1}{4\sqrt{m}} \Gamma^{+}_{\mu} \beta_\mu \varphi$$

5次元から1次元への projection (マーガ)

interaction は Φ として入る

(derivative coupling はとらない)

hermite $\left(\begin{array}{l} a_1 = :\bar{\Psi} i\gamma_5 \omega_j \psi L_j^+ \Phi: \\ a_1' = :\bar{\Psi} i\gamma_5 \psi \frac{1}{3} L_j^+ \omega_j \Phi: \end{array} \right.$ herzmit

$a_2 = :\bar{\Psi} i\gamma_5 \omega_j \Phi L_j^+ \psi:$

$a_2' = :\bar{\Psi} i\gamma_5 \Phi \frac{1}{3} L_j^+ \omega_j \psi:$

hermite $\left\{ \begin{array}{ll} b_1 = & \omega_j \to \lambda_j \\ b_1' & \frac{\cup}{2} をはさむ \end{array} \right.$

b_2 $\quad \omega_j \to \lambda_j$

b_2' $\quad \frac{\cup}{2}$ をはさむ

Lorentz invariant,
space reflection に對して不變
c.c. で hermite conj にうつる
Ino-reflection

$$\mathcal{L}^{(5)} = G\{(a_1 - b_1 + a_1' - b_1')$$
$$-(a_2 - b_2 + a_2' - b_2')\} + h.c.$$
$$= iG\{\tfrac{1}{2}(\bar{N}\gamma_5\vec{\tau}N)\vec{\pi}$$
$$-\tfrac{3}{2}(\bar{\Xi}\gamma_5\vec{\tau}\,\Xi)\vec{\pi}$$
$$+2i(\bar{\vec{\Sigma}}\gamma_5\times\vec{\Sigma})\vec{\pi} + \tfrac{1}{2}(\bar{N}\gamma_5 N)\pi'$$
$$+\tfrac{3}{2}(\bar{\Xi}\gamma_5\Xi)\pi' - \tfrac{1}{2}(\bar{N}\gamma_5 K)\Lambda$$
$$-\tfrac{3}{2}(\bar{\Xi}\gamma_5 i\tau_2 K^*)\Lambda$$
$$-\tfrac{1}{2}(\bar{N}\gamma_5\vec{\tau}K)\vec{\Sigma}$$
$$+\tfrac{3}{2}(\bar{\Xi}\gamma_5\vec{\tau}\,i\tau_2 K^*)\vec{\Sigma}\}$$
$$+ h.c.$$

---

weak

$$a_1^W = i\bar{\psi}_i\gamma_5\omega_j\psi L_j^i R\psi_i$$

$$\Delta I = 1/2$$

松田氏：オーストラリヤの記

Nov. 11, 1958

Sydney 大学　　　　1925 物理教室
名物一つあり

Camberra : 国立大学

単色大学.

Sydney

1952 H. Messel : 物理の head

基金 £ 500,000 の寄付を受けた基金の fund.

Dr. Adolph Basser £ 50,000

Dr. G. B. S. Falkiner £ 50,000

building

Silliac : Bennett

州政府 £ 50,000

連邦政府 £ 50,000

Nuclear Research Foundation

Governor	基金 2,000 以上
Member	400 〜 1,999
Associate	100 〜 399

"Nucleus" 第21回

理論 J. M. Blatt　　実験 H. Rathgeber

S. T. Butler　　宇宙線 (A. J. Herz

R. Schafroth　　D. D. Millar

S. T. Ma　　Ogilvie

Makinson

Progress meeting

N. Austern
Prof. McCusker
北[illegible]氏 ([illegible])
国際会議に人を送る.
給与はよくない.

	£1 = 8250
teaching fellow	£950 ~ 1,100
lecturer	1400 ~ 2250
senior lecturer	2120 ~ 2500
reader / associate prof.	2750
professor	3,450

General Covariance and
Elementary Particles
R. Finkelstein
(P.R. 110 (1958), 1200)

Sakata's composite model
8-component spinor (Watanabe)

$$H = i\Gamma_4 \boldsymbol{\Gamma}\cdot\boldsymbol{P} + m\Gamma_4$$

$$\Gamma_\mu = \begin{pmatrix} \gamma_\mu & \\ & -\gamma_\mu \end{pmatrix} \qquad \Gamma_5 = \begin{pmatrix} \gamma_5 & \\ & \gamma_5 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & \gamma_5 \\ \gamma_5 & 0 \end{pmatrix} \qquad P = \begin{pmatrix} \gamma_4 & \\ & \gamma_4 \end{pmatrix}$$

$$Q = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

$$(\Gamma_i, P)_+ = (X, P)_+ = (X, Q)_+ = 0$$

$$(X, \Gamma_\mu)_- = (Q, P)_- = (Q, \Gamma_\mu)_- = 0$$

$$i = 1, 2, 3.$$

eigenstates of chirality

$$X\Psi = \Psi \quad \text{or} \quad X\Psi = -\Psi$$

eigenstates of achirality

$$Q\Psi = \Psi \quad \text{or} \quad Q\Psi = -\Psi$$

X and Q commute with H.

Elementary baryon is identified as
A when it is in eigenstates of Q
or B X

assign parity to A but not to B
strangeness to B A

Strangeness of a composite structure is the number of B (chiral) particles
~~baa~~ baryon number is the number of elementary particles.

8-row spinors provide a representation of 6-dimensional rotation group.

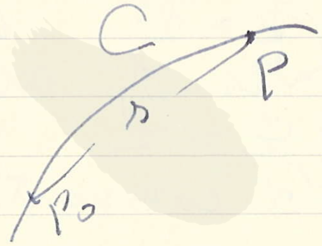
(a) general covariance of classical equations of motion

(b) covariance of ~~the~~ q. ~~m~~ eq. of motion under linear orthogonal coordinate transformations only.

normal coordinates

$$\frac{d^2x^i}{ds^2} + \Gamma^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0$$

s is arbitrary up to a linear transformation



$$\xi^i = \left(\frac{dx^i}{ds}\right)_0$$

$$y^i = \xi^i s$$

arbitrary transf. $x^i \to x^{i'}$:

$$y^{i'} = \xi^{i'} s' = \left(\frac{dx^i}{ds}\right)'_0 s'$$

$$s' = s, \quad \left(\frac{dx^i}{ds}\right)'_0 = \left(\frac{\partial x^{i'}}{\partial x^p}\right)_0 \left(\frac{dx^p}{ds}\right)_0$$

Hence $\quad y^{i'} = a^i_p y^p$

$$a_p^{i} = \left(\frac{\partial x^{i'}}{\partial x^p}\right)_0$$

the matrices $\| a_p^i \|$ provide a representation of the group of arbitrary analytic transf. It is not orthogonal, but is isomorphic to the orthogonal group in six dimensions, L^{ij} be a tensor in y^i-space and be antisymmetric

$$\varepsilon_{ijkl} L^{'ij} L^{'kl} = |a_p^i|\, \varepsilon_{ijkl} L^{ij} L^{kl}$$

$|a_p^i| = 1$: $\varepsilon_{ijkl} L^{ij} L^{kl}$: absolute invariant

or $L^{12} L^{34} + L^{23} L^{14} + L^{31} L^{24} = \text{invariant}$

$L^{12} = Y^1 + iY^2$, $L^{23} = Y^3 + iY^4$, $L^{31} = Y^5 + iY^6$

$L^{34} = Y^1 - iY^2$, . . .

$$\sum_{\mu=1}^{6} (Y^\mu)^2 = \text{invariant}$$

Thus fifteen-parameter unimodular group on y^i $(i = 1, .. 4)$ is isomorphic to ~~the~~ fifteen-parameter orthogonal group on Y^μ $(\mu = 1, \dots 6)$

The group of generally covariant coordinate transformations may be mapped on the orthogonal group in (O_6) six mensional Euclidian space (E_6).

Construction of a Complete Set of Independent Observables in the general Theory of Relativity
A. Komar, P.R. 111 (1958), 1182

Coordinates in general relativity are unobservables because a given world point can be given ~~at~~ arbitrary coordinates relative to any other set of world points. Metric Tensor is also not observable because one can not readily tell whether ~~they rep~~ two metric Tensor fields represent two distinct physical situations or the ~~phs~~ same physical situations in two ~~distinct~~ different systems.

Choose four independent scalar which can be expressed in terms of metric Tensor and its derivatives in order to identify a point in space, provided that the space is sufficiently asymmetric

Symmetry Laws and Strong Interactions

J. J. Sakurai　　Sept. 1958

UCRL - 8440

Research for the relations between space-time symmetry and intrinsic symmetry

第I分科会 — 討論会

Franchi

neutron charge distribution

idea of Nathan

old experiment

$N + Z \to N + Z$

$M.E. = a_{nucleus} + a_{spin}$

$+ a_{Foldy} + a_{charge\ dis.\ of\ N}$

$a_{spin} = 0$ (Noble gas)

$\sigma \cong a_{Nucl}^2 + 2 a_N a_{Foldy}$

Foldy Term → V_0

Experiment → $V_0(1 \pm 0.1)$

Nathan (Osaka);

$a_{Nucleus} = 0$ Two isotopes

$a_N > 0$

$a_N < 0$

(check: wide angle scattering)

$a_{Nucleus}$ isotropic

a_{spin} is known

a_{Foldy} is known

obtain 3% accuracy

checks (1) polarize N

(2) diffraction of a_{spin}

" of a_{Foldy}

$a_{charge} \sim \langle r^2 \rangle_N$

Franchi

Indefinite metric (Bogolubov)

Pauli-Villars cut-off

(1) $m_e \sim \alpha M_x$

(2) M_{x_i}: affects phys. propag.

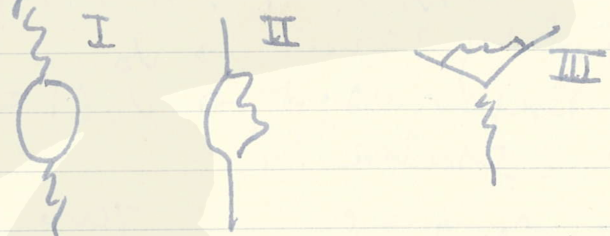
anomalous moment of e, μ

(3) e - p scattering

$\gamma + P \to e^+ + e^- + P$

Q.E.D.

divergences are removed



minimal requirement

1. X Fermion
2. " a.s X fermion

(1) $H_I = \bar{\psi}_e \gamma_\mu \psi_e A^\mu + a \bar{\psi}_x \gamma_\mu \psi_e A^\mu$

$+ (H.C.) + b \bar{\psi}_x \gamma_\mu \psi_x A^\mu$

I or III: $1 + 2a + b = 0$

II: determines a

($a = -1$, $b = 1$: identically 0.)

(2) $m_x \gtrsim 500\, m_e$ ($e + P \to e + P$)

m

(1) If $a=0$, gage invariant

T. H. R. Skyrme (Harwell)

A Non-linear theory of
strong interactions
(P. R. S. 247 (1958), 260)

$$\mathcal{L} = -\frac{1}{2}\sum\left(\frac{d\phi_\rho}{dx_\mu}\right)^2 + \frac{1}{4}\gamma^2\Sigma\phi_\rho^4$$

$$-\psi^\dagger\beta\left(\gamma_\mu\frac{\partial}{\partial x_\mu} + g(\phi_4 + i\gamma_5\tau_i\phi_i)\right)\psi$$

$\rho = 1, 2, 3, 4$

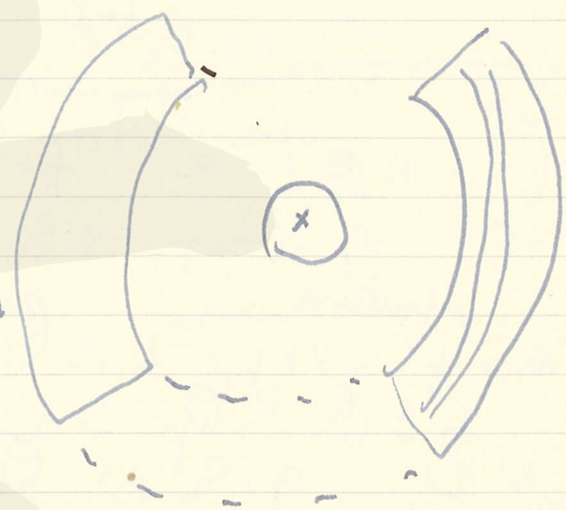
meson 一元論, v. spinor 一元論

i) non-singular meson field

ii) singular meson field

Nov. 1958

Andy McReynolds
General Atomic   John J. Hopkins
Fred de Hoffmann
Edward Creutz
June 56   seminar
safe research reactor
Dyson, Taylor, McReynolds
→ Triga
accelerator
30 MeV   linac
0.5 Amp
neutron
thermonuclear research
TAERE
Kerst
Rosenbluth



Triga
Be-U
o   ZrH-U
30 KW 10^{12} n/cm²/sec
max. 250 MW (40 ms)
10^{16} n/cm²/sec
(1.5% excess suddenly)
$ 140,000   < 200,000

S行列の理論　加藤敏夫

Nov. 18, 1958

Jauch, Helv. Phys. Acta. 1958, 127

scattering operator

simple scattering system

H

H_0　　　$\mathfrak{h} \ni f$

$$e^{-itH} f \xrightarrow[t \to +\infty]{\text{asymptotic}} \sim e^{-itH_0} g$$

$$t \to -\infty \quad \sim e^{-itH_0} h$$

$$\left(\| e^{itH_0} e^{-itH} f - g \| \to 0 \quad t \to +\infty \right)$$

$$\underset{\text{future}}{g} \leftarrow f \rightarrow \underset{\text{past}}{h} \qquad Sh = g$$

あるfに対して上記のことが成立すればよい

$f \in \mathfrak{R} \in \mathfrak{h}$

しかし f, g, h is $\mathfrak{h}$の全空間をおおわなければならない。

[I]
$$\begin{cases} e^{itH} e^{-itH_0} g \to f & t \to +\infty \\ e^{itH} e^{-itH_0} h \to f & t \to -\infty \\ \text{or } e^{itH} e^{-itH_0} \to \Omega_+ & t \to +\infty \\ e^{itH} e^{-itH_0} \to \Omega_- & t \to -\infty \end{cases}$$

強収束 (strong convergence)

(II) Ω_+, Ω_- の range は共通 $\mathfrak{R}$.

(III) $H/\mathfrak{R}^{\perp}$ は点スペクトルだけをもつ.

($H/\mathfrak{R}$ と H_0 連続スペクトルだけ)

wave operators Ω_+, Ω_- are isometric

$$\| \Omega_\pm g \| = \| g \|$$

they are not necessarily unitary.

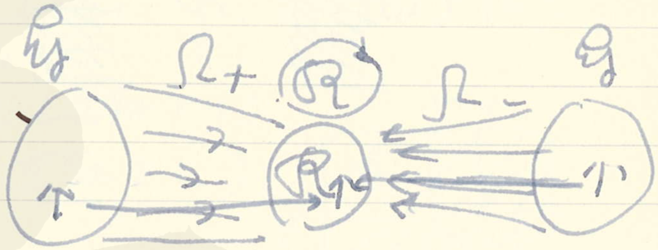
$$\Omega_\pm^* \Omega_\pm = 1$$

but $\Omega_\pm \Omega_\pm^* = P$ projection on $\mathfrak{R}$

$\mathfrak{R}$: closed

$$g = \Omega_+^{-1} \Omega_- h$$

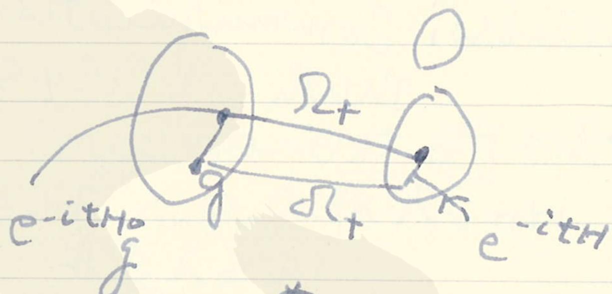
$$= \Omega_+^* \Omega_- h$$



Ω_+^{-1} は $\mathfrak{R}$ の中だけでしか定義されていない。$\mathfrak{R}$ の中では Ω_+^* と一致。

$$S = \Omega_+^* \Omega_- \quad \text{: unitary}$$

$$e^{-itH} \Omega_\pm = \Omega_\pm e^{-itH_0}$$

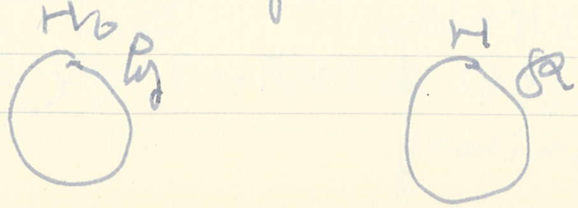


$$\Omega_\pm^* e^{itH} = e^{itH_0} \Omega_\pm^*$$

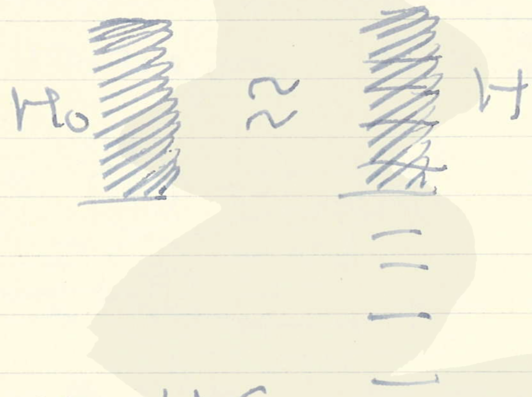
$$\therefore \quad P e^{itH} = e^{itH} P$$

$\therefore$ $\mathfrak{R}$ は e^{-itH} による unitary 変換に対して不変：

infinitesimal generator



H_0 : contin. spectra
H : point spectra
の下にある場合にも上の論をいれたいもの.



$S H_0 = H_0 S$
H_0 : simple ; $S = \Phi(H_0)$
Jauch が論文で証明したことによる.

実際そういう system があるか?
Friedrichs: Communication Pure Appl Math. 1 (1948), 361

$H = H_0 + \varepsilon V$

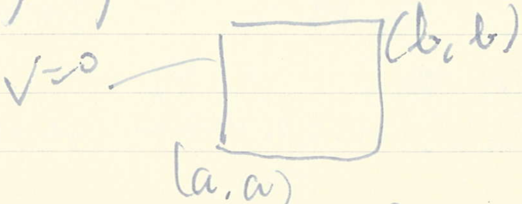
$H_0 = \lambda \times$ $\qquad H_0 f(\lambda) = \lambda f(\lambda)$

$a \leqq \lambda \leqq b$

$V(\lambda, \mu)$ Hölder 函数.

$$\frac{|V(\lambda, \mu) - V(\lambda', \mu')|}{|\lambda - \lambda'|^\alpha + |\mu - \mu'|^\alpha} \leq C$$

$0 < \alpha < 1$



$|\varepsilon|$ が十分小さければ,
I, II の条件が満される. $\mathfrak{R} = \mathfrak{H}$.
Ω_{I} : unitary

Rosenblum, Pac. J. Math. 1957

$H_0 + V$

degenerate

V の range が有限

degenerate operator

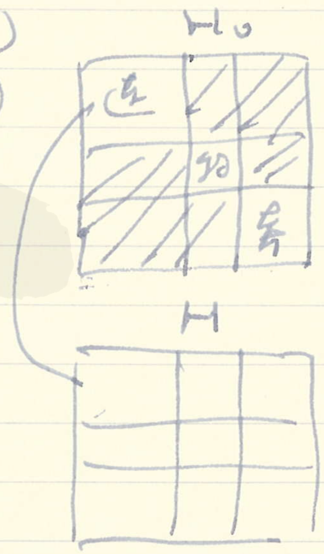
$$V(\lambda,\mu) = \sum_{n=1}^{N} \lambda_n \varphi_n(\lambda) \overline{\varphi_n(\mu)}$$

Aronszajn, Amer. J. Math. 1957

連続スペクトル

{ 絶対連続（密度あり）
{ 特異（密度なし）

離散スペクトル



Rosenblum

$V \in$ trace class

$$V(\lambda,\mu) = \sum_{n=1}^{\infty} \lambda_n \varphi_n(\lambda) \overline{\varphi_n(\mu)} \quad \begin{cases} \sum_n |\lambda_n| < \infty \\ \|\varphi_n\| = 1 \end{cases}$$

Cook, J. Math. Phys. 36 (1957), 81

$$\left.\begin{aligned} H_0 &= -\Delta \\ V &= V(x,y,z) \end{aligned}\right\} \quad \int |V|^2 dx\,dy\,dz < \infty$$

(I) を仮定.

Kuroda, $A = |V|^{1/2} (H_0 - i)^{-1} \in$ Schmidt class

$$\sum_{m,n} |(A\varphi_n, \varphi_m)|^2 < \infty$$

$$\int |V| \, dx\,dy\,dz < \infty$$

(I), (II) が云える

Kato :

$$V(x, y, z) \sim O\left(\frac{1}{r}\right)$$

c033-581挟込

日本数学会 函数論

Nov. 19, 1958

石田氏: Introduction

Lax: Two variables

弱い解 → singular になる

解の uniqueness

P. D. Lax, Initial Value Problem for Nonlinear Hyperbolic Eq. in two indep. variables.

$$U_t + AU_x + B = 0$$

$$U = \begin{pmatrix} \vdots \\ \vdots \end{pmatrix} \quad A = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \quad B = \begin{pmatrix} \vdots \end{pmatrix}$$

A, B: 函(x, t, U)

A: 函(x, t): semi-linear

Aの固有値はreal

x, t, U

$U(x, 0) = \phi(x)$ $\quad -\infty < x < \infty$

解は $0 \leq t \leq \delta$ しか存在しない

norm:

$$U_t + DU_x + BU + C = 0$$ $\quad t = 0$

$$U(x, 0) = \phi(x)$$

$|\phi(x)|$, $|B(x,t)|$, $|C(x,t)|$

maximum value $\quad |f| = \max |f(x)|$

$$|B| \equiv \max_U \frac{|BU|}{|U|} \qquad |U| = \max_n |u_n|$$

$$|U(x,t)| \leq \phi_0 e^{\beta t} + \int_0^t c(\tau) e^{\beta(t-\tau)} d\tau$$

$U = TV \qquad T^{-1}AT = D$

$$V_t + DV_x + \tilde{B}V + \tilde{C} = 0$$

$\tilde{\beta} = 0$ ならすべてのx, tについて解が存在する

$\tilde{\beta} \neq 0$:

$V = \tau(W) \qquad V_t + DV_x + \tilde{B}W + \tilde{C} = 0$

$V(x,0) = \phi(x)$

$$V_t + DV_x + \tilde{B} + \tilde{C} = 0$$

後者から解いてゆける. Vをみつけ

semi-linear では solution は t について どこまでもある. (shock は生じない.)

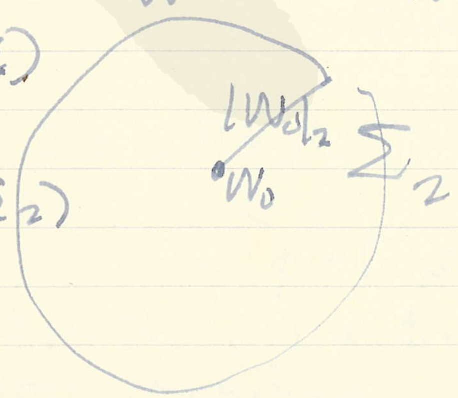
quasi-linear

$$U_t + A^{(W)}U_x + B^{(W)} = 0$$

$U(x,0) = \phi(x)$

$U = \tau(W) \qquad W \xrightarrow{\tau} U = \tau(W)$

$W_0(x,t) = \phi(x)$



$\tau(W)$

a. $W \in (C_k\ k > 1 \cap \Sigma_2)$

$W \xrightarrow{\tau} \tau(W) \in \Sigma_2$

$t < \delta_0$

b. $|\tau(W) - \tau(W')|_0 \leq \lambda |W - W'|_0$

$\| \ \|_{mod} \leq \lambda \| \ \|_{mod}$

$\underline{\lambda < 1}$

Lipschitz contin.

shock は処理できない.

ex. $u_t + u u_x = 0$

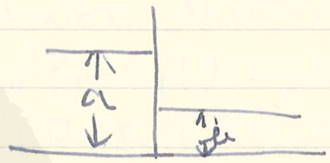
$u = \phi(x - ut)$

$F(u, x, t) = u - \phi(x - ut)$

$u \in C_1$ $\quad \frac{\partial F}{\partial u} \neq 0$ $\quad$ i.e. $1 + t\phi' \neq 0$

$t\phi' > -1$

$\phi(x) = \begin{cases} a & \\ b & \text{for} \end{cases} \quad \begin{matrix} x<0 \\ x<0 \end{matrix}$



$a > b$: $\phi_i \to \phi$ $\quad \phi_i' \to -\infty$

$t \to 0$ or $\delta_0 \to 0$

$a < b$: $\phi_i' \to +\infty$ $\quad \forall t$ (すべての t) に対して一様に収束する. $\quad \phi_i \to u_i$ $\quad \{u_i\} \to u \in C_1$

~~so~~ $u_i \to u \in C_1$ $(t > 0)$

$u \notin C_1$ $(t = 0)$

$U_t + AU_x + B = 0$

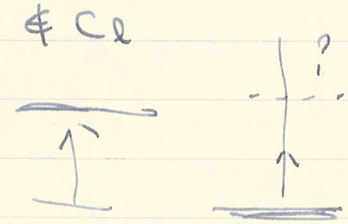
$U_0(x,t) \in C_1$

$t \to t_0$ の limit で滑かでない.

$U_0(x, t_0 - t)$: $U_t - AU_x - B = 0$

$t > 0$　$U_0(x, t_0 - t) \in C_1$

$t = 0$　$U_0(x, t_0) \notin C_1$

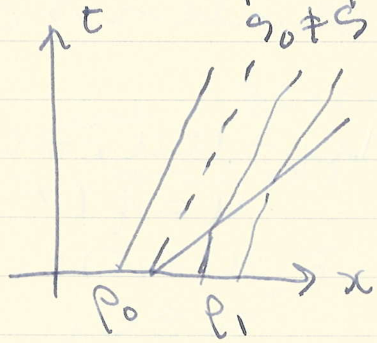


例外.

Shock

contact surface　p 同じ.　$\rho_0 \neq \rho_1$, $s_0 \neq s_1$

(exceptional case)

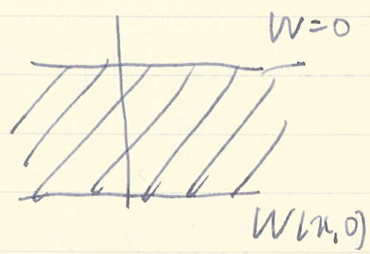


weak solutions

$U_t + F_x(x,t,U) + B = 0$

$U(x,0) = \phi$

$-\int W(x,0)\Phi\, dx + \int\{-W_t U - W_x F + WB\}\,dx\,dt = 0$



$V = H(x,t,U)$

$V_t + G_x(x,t,V) + C = 0$

$V_0 \neq H(x,t,V_0)$

conservation laws
mass
momentum
energy
(entropy)

weak solutions is <u>not</u> unique

$$u_t + \left(\frac{u^2}{2}\right)_x = 0$$

$$\phi(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0 \end{cases}$$

has infinitely many solutions

1. $u(x,t) = \begin{cases} 0 & x \le 0 \\ x/t & 0 \le t \\ 1 & 0 < t \le x \end{cases}$

2. $u = \begin{cases} 0 & 2x < t \\ 1 & 2x > t \end{cases}$

additional principle!!!
initial data を少しかえれば、あとの solution も少しかわる.
すると weak solution is unique (conjecture)

$$u_x + (u^2/2)_x = \lambda u_{xx} \qquad u(x,0) = \phi(x)$$

$\forall t > 0 \quad \exists u_\lambda$

$\lambda \to 0 \qquad u_\lambda(x,t) \to u(x,t) \quad (\cancel{\forall x, t})$
$(x, t \quad$ a.e.w. $)$

$$\left.\begin{aligned} U_t + F_x(x,t,U) + B &= \lambda U_{xx} \\ U(x,0) &= \phi(x) \end{aligned}\right\} ?$$

steady solution

$U(x,t) = \phi(x - cT)$

irreversibility in time

逆変化.

41 続き

加藤氏：微分方程式の weak solution

(Nov. 20, 1958)

I. 変分法.

$$\delta\int_a^b F(x, u, u')\,dx = 0 \; ; \; u' = \frac{du}{dx}$$

$$F_u - \frac{d}{dx}F_{u'} = 0$$

? $$\int (F_u\,\delta u + F_{u'}\,\delta u')\,dx = 0$$

$$\frac{d}{dx}F_{u'} = F_{u'x} + F_{u'u}u' + F_{u'u'}\boxed{u''}$$? 存在?

(du Bois-Reymond)(2階)

$$\int\left(-\int^x F_u\,dx + F_{u'}\right)\delta u'\,dx = 0$$

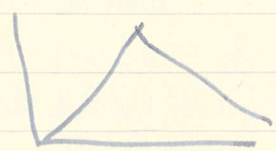
$$-\int^x F_u\,dx + F_{u'} = \text{const.}$$

for $\delta u = 0$ $\quad x = a, b$

∴ $$\frac{d}{dx}\left(F_u - \frac{d}{dx}F_{u'}\right) = 0$$

不連続解. u'が不連続

u'は両方から連続.



$F_{u'u'} \neq 0$ ならば (区間)

Euler式が導かれる.

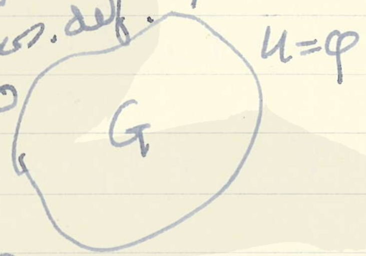
$$\int (F_u w + F_{u'} w')dx = 0$$

weak solution (Euler 方程式 ~~[illegible]~~ を弱い形にかいたもの)

Dirichlet 問題:

$$\delta \int_G (u_x^2 + u_y^2)\,dx\,dy = 0$$

$$\begin{pmatrix} F_{u_x u_x} & F_{u_x u_y} \\ F_{u_y u_x} & F_{u_y u_y} \end{pmatrix} > 0 \quad \text{pos. def.}$$



正則な場合と同じく weak solution が genuine solution と一致する

$$\Delta u = 0 \qquad u = \varphi$$

$$\delta \int (u_x^2 - u_y^2)\,dx\,dy = 0$$

の場合は正則でない。weak solution が genuine solution と限らない。

II. a. 正則なら weak と genuine が一致するということが (又要証明)
楕円型偏微分方程式の問題.

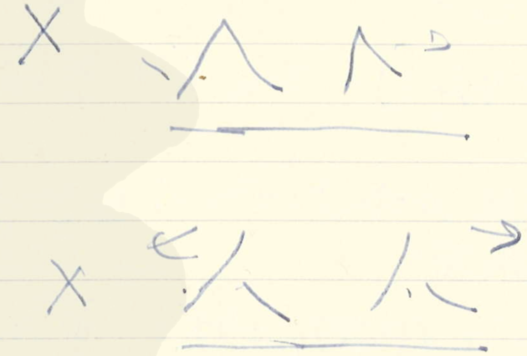
b. 正則でない
双曲型微分方程式

$$u_{tt} - u_{xx} = 0$$
$$u = f(x-t) + g(x+t)$$

$f, g \in C^2_{xy}$ (for genuine solution)

とがつた波

不連続な場合.



$$\int (u_x w_x - u_y w_y)\, dx\, dy = 0$$

$$\times \quad \int u(w_{xx} - w_{yy})\, dx\, dy = 0$$

<u>distribution</u>

III. $I = \int (u_x^2 + u_y^2)\, dx\, dy \qquad \delta I = 0$

$I(u_n) \to d \qquad n \to \infty$

minimizing sequence

$u_n \to u$ を証明.

Courant 直接法.

IV linear 方程式では weak は genuine の limit, 両者の間は差はない.. non-linear 方程式では genuine な解は なく, weak しかない場合が一般的である.

V. 非線形~~[illegible]~~方程式.

$$U_t + AU_x + B = 0$$

$$U_t + F(t, x, U)_x + B = 0$$

$$\iint (-W_t U - W_x F + WB)\, dt\, dx = 0$$

weak solution の uniqueness?

一次元流体: $\xrightarrow{\rho,\ p}$ $\vec{u}$

$$\begin{cases} \rho_t + (\rho u)_x = 0 \\ u_t + u u_x = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} \\ p = p(\rho) \end{cases}$$

weak solution
shock の所で entropy が変わる.
$p = p(\rho, s)$

積分形の法則から導けるか?

Maxwell 方程式

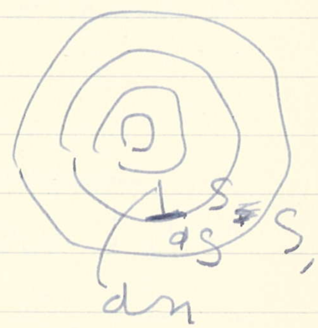
$$\left.\begin{aligned} \frac{\partial \vec{E}}{\partial t} &= \operatorname{rot} \vec{H} \\ \frac{\partial \vec{H}}{\partial t} &= -\operatorname{rot} \vec{E} \\ \operatorname{div} \vec{E} &= 0 \\ \operatorname{div} \vec{H} &= 0 \end{aligned}\right\}$$

$$\oint E_s\, ds = -\frac{d}{dt}\int H_n\, dS$$

$$\oint H_s\, ds = \frac{d}{dt}\int E_n\, dS$$

$$\int_{t_1}^{t_2} dt \oint E_s\, dS = -\int H_n\, dS \Big|_{t_1}^{t_2}$$

$$\left.\begin{aligned} &\oint E_n\, dS = 0 \\ &\oint H_n\, dS = 0 \end{aligned}\right\}$$



$$\int \vec{E}\, \mathrm{grad}\, \varphi\, dx\,dy\,dz$$

$$= \int E_n \frac{\partial \varphi}{\partial n}\, dS\, dn$$

$$= \int E_n\, dS\, d\varphi = 0$$

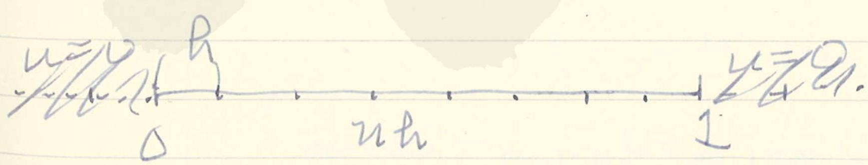
(変分原理から ~~[illegible]~~
~~[illegible]~~) も出そう)

微分方程式の近似解法
(差分法)

$$\frac{d^2u}{dx^2} - q(x)u = 0 \qquad 0 < x < 1$$

$Nh = 1$

$u(0) = u(1) = 0$



$u(nh)$

$$u((n+1)h) + u((n-1)h) - 2u(nh)$$
$$- q(nh)\,u(nh) = 0$$

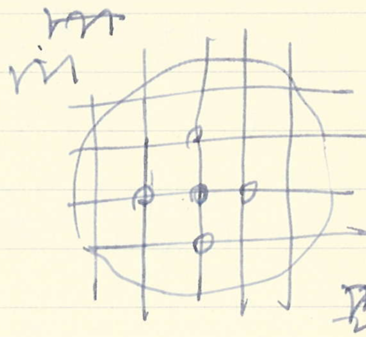
~~$n = 0, \ldots N$~~ $n = 1, \cdots N-1$

$u(0) = u(Nh) = 0$

Dirichletの問題

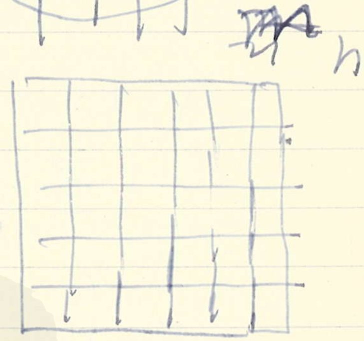
$$u_{xx} + u_{yy} = 0$$

$$u = \varphi$$



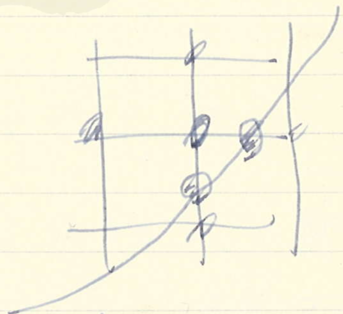
内部

$$h^2 \Delta u = u_{n+1,m} + u_{n-1,m} + u_{n,m+1} + u_{n,m-1} - 4u_{nm} = 0$$



へり: weightをつける.

Iteration



双曲型

$$u_{tt} - u_{xx} = 0$$

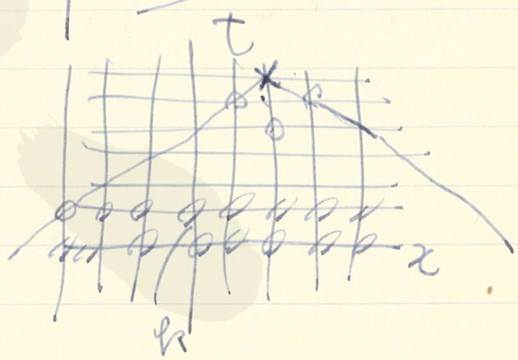
$$t = 0, \quad u = \varphi, \quad u_t = \psi$$

$$\frac{u_{n1} - \varphi_n}{k} = \psi_n$$

$$u_{n1} = \varphi_0 + k\psi_n$$

$(\cancel{dt}\ u(t) + u_t\, dt)$

$u(t+dt) =)$



$$\frac{1}{k^2}(u_{n,m+1} + u_{n,m-1} - 2u_{n,m}) = \frac{1}{h^2}(u_{n+1,m} + u_{n-1,m} - 2u_{nm})$$

$$u_{n,m+1} = 2u_{n,m} - u_{n,m-1} + \frac{k^2}{h^2}[\ \cdots\ ]$$

$m=1$ では $u_{n,2}$ が定められる

$h \to 0$

$k \to 0$

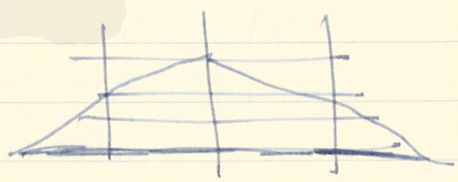
$k=h$; $u_{n\,m+1} = -u_{n,m-1} + u_{n+1,m} + u_{n-1,m}$

$k>h$ はおかしい

$k<h$ ならばなりたつのか?

　よろしい

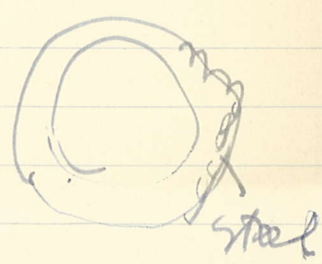
差分のとり方,



Nov. 21, Friday
1300 Kyoto ~~10.4?~~ ~ Odawara
with K. Mori to Fujiya Hotel
in Miyanoshita, Hakone.
10 p.m. Sir John Cockcroft
arrives with Kikuchi, Sagane
and others.

Nov. 22, Saturday
Komagatake — Hakone Villa
— Daikanyama — Hakone
Hotel (Lunch)
— Odawara — Kyoto
(Seikai)

Accelerator Program in England
7 BeV Proton Synchrotron
at Harwell (three years)
100 time more intense
than Bevatron (injection)
weak focussing
Fusion, 500 scientists
and engineers
Zeta 1000 MF.



Budget of Universities
independence,
five year budget.

非局所場理論に関する若干の考察
(素研 研究会)　Nov. 25, 1958

1. $g_{\mu\nu}$ は ψ から作れない
2. 電磁場は ψ から (non-local に) 微分型で与えられる
3. strong と weak を共に $(\bar\psi O\psi)(\bar\psi O\psi)$ 型にしようとすると
   ψ_s, ψ_w, χ
   による3種の場が必要
4. proton, electron, neutrino
   graviton?. (photon) という
   stable particles に対応する場が必要
   という意味での多元論

2'. non-local と indefinite
   metric の関係.

[illegible]（つづき）

池田峰夫：大域微分幾何学とその応用

第1回　　Nov. 26, Wed. 1958

$(\xi^1, \cdots \xi^n)$　一般座標.

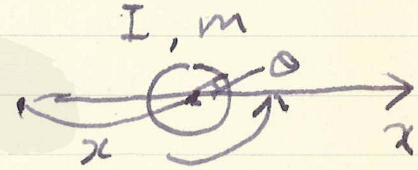
$T = \frac{1}{2} a_{\mu\nu}(\xi)\, \dot\xi^\mu \dot\xi^\nu \geq 0$　　正定値

scalar　tensor covariant　$\dot\xi^\mu$: contravariant vector

Riemann metric

$ds^2 = 2T\,dt^2$

例：



1) $T = \frac{m}{2}\dot x^2 + \frac{I}{2}\dot\theta^2$

$-\infty < x < +\infty$

$0 \leq \theta < 2\pi$

2) $T = \frac{I_1}{2}\dot\theta_1^2 + \frac{I_2}{2}\dot\theta_2^2$

$0 \leq \theta_1, \theta_2 < 2\pi$

どちらも

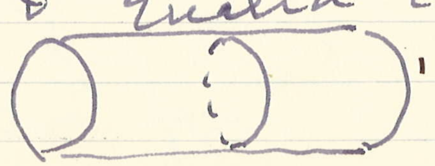
$ds^2 = (d\xi^1)^2 + (d\xi^2)^2$

1) $\begin{cases} \xi^1 = \sqrt{m}\, x \\ \xi^2 = \sqrt{I}\,\theta \end{cases}$　　　$\begin{cases} \xi^1 = \sqrt{I_1}\,\theta_1 \\ \xi^2 = \sqrt{I_2}\,\theta_2 \end{cases}$

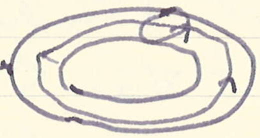
Riemann metricは同じ

Topologicalな違いはある.

どちらも Euclid である….

1) 　cylinder

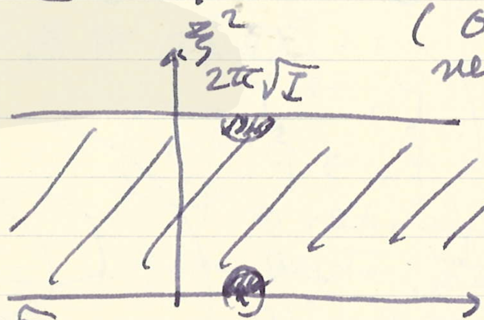
2) torus
但し計量は Euclidian
ではない。



3) plane
違いの一つは 閉曲線を一点に縮めない
ものが 1) 1組 2) 2組 3) 0

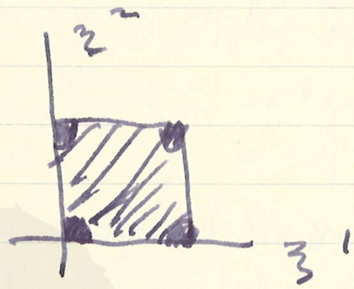
Riemann 計量だけからは、空間の性質はきまらぬ

1) の場合、 $\theta=0$ に近い所と $\theta=2\pi$ に近い所は座標では
離れている。 (open neighborhood)



2) の場合

1) の場合
$0 < \xi^2 \leq \xi^2 \leq \xi^2 < 2\pi\sqrt{I}$



2) の場合、3) と計量を同じにする
とすると topology がこ
われ、違った topology
を持たせて、計量が違ってくる
但し、[illegible]

$$ds^2 = \sum_{\alpha=1} (dz^\alpha)^2$$

$$\left.\begin{aligned} z^1 &= \cos \xi^1 \\ z^2 &= \sin \xi^1 \\ z^3 &= \cos \xi^2 \\ z^4 &= \sin \xi^2 \end{aligned}\right\}$$

closed
finite
flat space

小域的には 一般の 2次元面は ~~Euclid~~ flat 3次元空間に入る。 $V_2 < S_3$

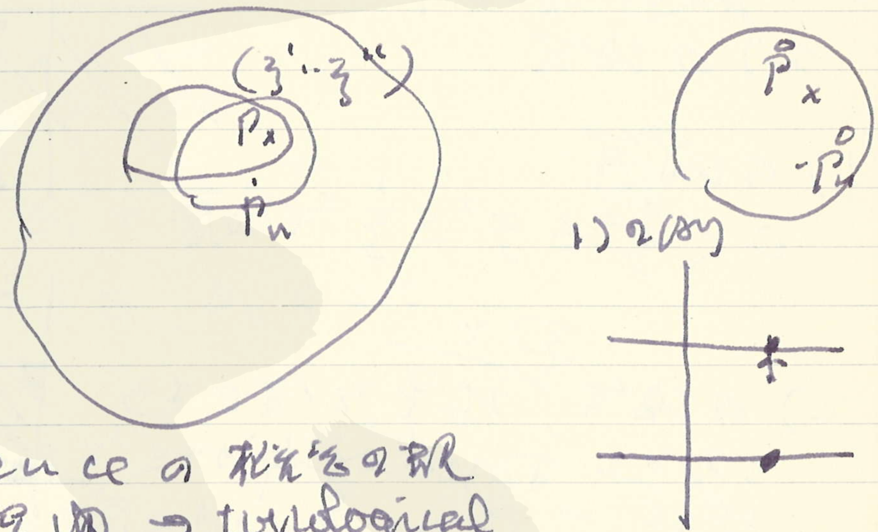
小域的構造では 座標系ごとに空間がかわる。

大域 ~~的~~ 的構造

一般的には 総合的構造がつかめない。

$(\xi^1 \cdots \xi^n)$: 座標系, n次元の Euclid 空間の open set と 一対一対応を考えている. プラス 連続性

$\left(\begin{array}{l} P_n \to P \\ \mathring{P}_n \to \mathring{P} \end{array} \right.$



1) cylinder の場合, 一つの座標系ではおおえない。

1) convergence の概念の拡張 それによる空間 → topological space

2) 両方で convergence の定義が一致 homeomorph 同位相

Definitions of Topological space:

M: set

1) open set O

2) neighborhood

3) closure (閉包) / の導入

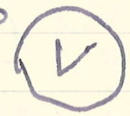
Mの中の部分集合 $\mathcal{O} = \{U, V \ldots\}$

1. $U \cap V \in \mathcal{O}$ 　$\mathcal{O}$の元

2. 任意個数の ~~open set~~
   U_λ ~~集合~~
   $\bigcup U_\lambda \in \mathcal{O}$

3. $M \in \mathcal{O}$ 　　$\phi \in \mathcal{O}$

$U \cap V = \phi$

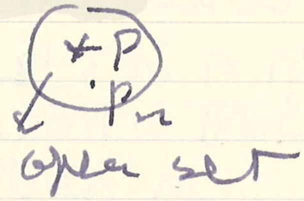


$\mathcal{O}$ は M に位相づける

U, V …: Mの open set

$\mathcal{O}$: topological ~~set~~ space

$\{P_n\} \to P$
集積点.



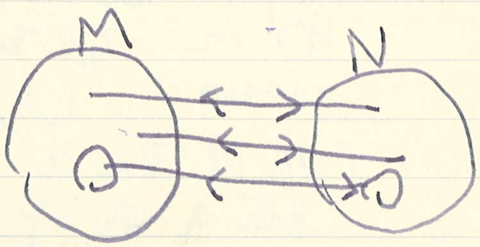
$P' = f(P)$ 　　$P \in M$

写像

連続性

P'を含む任意の open set U' に対し,
f(P)∈U' なる P の全体が M で open set.

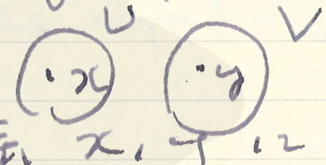
同位相:
1対1対応.
開集合が1対1対応.
しかも f, f^{-1} が連続 ~~連続で~~,



大域微分幾何の定義

1) topological space　M

2) Hausdorff space



任意の2点 x, y に対して共通点のない open set U, V がいつでも存在
(任意の2点は任意に近くない)
(lattice space　x　y　・ ・ ・ ・)
$\overset{x}{\underset{x}{\cap}}$　$\overset{y}{\underset{y}{\cap}}$

3) connected

Mの部分集合 M', M'' で
$M = M' \cup M''$
$M' \cap M'' = \phi$
M', M'' : open (かつ closed)
(closed　$M \supset N$ closed とは
$N^c = M - N$: open　)
となるようなものがない.

4) 可附番無限個の U_λ からなる base がある*.
M の open set の集り U_λ の組.
$\cup U_\lambda = M$ であれば, U_λ を M の open covering.
特に任意の U が $U = \vee U_{\lambda'}$ であれば U_λ を基 (base)
(抽象空間を中心とする物理学者の頭の内容) (separable Hilbert space)

* 2nd axiom of countability

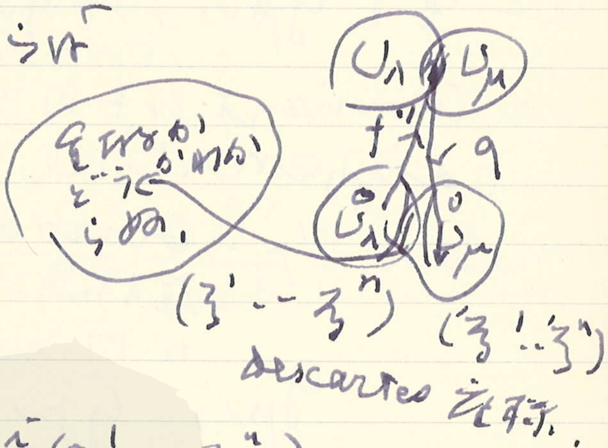
(differential manifold)

5) M は n 次元の C^r 級の微分可能な多様体

i) M が $\{U_\lambda\}$ という開集合の系列をもつ。 $\bigcup U_\lambda = M$

ii) $U_\lambda \xrightarrow[f]{\text{homeo.}} \mathring{U}_\lambda \in E^n$

iii) $U_\lambda \cap U_\mu \neq \phi$ ならば



$\mathring{U}_\lambda \xrightarrow[f^{-1}]{} U_\lambda$

$U_\mu \xrightarrow[g]{} \mathring{U}_\mu$

$g \circ f^{-1}$:

$$\xi^i \to \xi'^i = F^i(\xi^1 \cdots \xi^n)$$

F^i: r 回まで連続微分可能.

F^i は C^r 級. $r \geq 1$

$$\left[\det\left(\frac{\partial \xi'^i}{\partial \xi^j}\right) \neq 0,\ \det\left(\frac{\partial \xi^i}{\partial \xi'^j}\right) \neq 0 \right]$$

iv) ii), iii) を満足する U_λ は i) の中にふくまれる.

M のその開集合 U_λ で上記の条件を満すものを (座標)局所座標近傍 U_λ

$\xi^1 \cdots \xi^n$: 局所座標.

[illegible]: Seminar

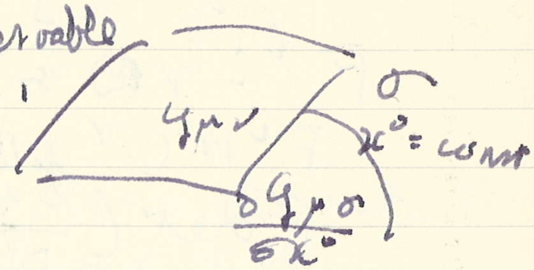
[illegible]: A. Komar, Independent Observables in General Relativity (P. R. 111, No. 4,)

$I = \Phi[g_{\mu\nu}, \partial_\rho g_{\mu\nu}, \partial_\rho\partial_\sigma g_{\mu\nu} \cdots]$

で invariant なものを observable.
(differential invariant)

x_μ: observable でない.
座標系のとり方によって、値がかわるから、

$g_{\mu\nu}$; $\bar{g}_{\mu\nu}(x)$: [illegible]
[illegible]
$g_{\mu\nu}$ 自身を observable とは考えられない



(Special relativity
Lorentz 群
Gage 群)

invariant なものは 4つしかない,
4つあれば、group of motion が存在しない,
3つしかないなら、$g_{\mu\nu}(x^2, x^3, x^4)$ で
3つの invariant と x^4 で一定座標系をつくればよい。

4つある場合

$R_{\mu\nu\rho\sigma}$　　$g_{\mu\nu\rho\sigma} = g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho}$

$$[R_{\mu\nu\rho\sigma} - \rho\, g_{\mu\nu\rho\sigma}] V^{\rho\sigma} = 0$$

(6次元)

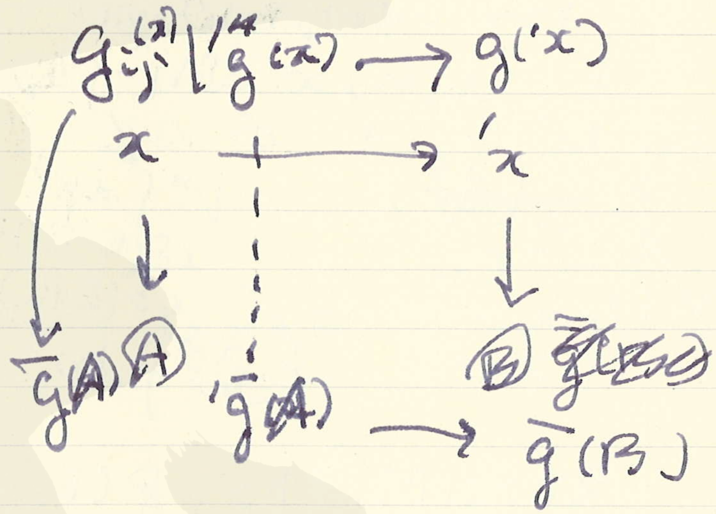
$\rho \to A^i(x)$　$i = 1, 2, \cdots 4$

scalar functions

$$\bar{x}^i = A^i(x)$$

$$\bar{g}^{ij} = g^{ab} \frac{\partial A^i}{\partial x^a} \frac{\partial A^j}{\partial x^b} = \text{scalar}$$

observable



$$R_{ij} = 0$$

$A^i(g_{ab}) = x^i$; 座標条件

$10 - \underline{\underline{4}} = 6$

Cauchy problem

$20 - 16 = \underline{\underline{4}}$　　linear theory

true observables → transverse wave

High energy Colloquium
Nov. 27, 1958

Koba, Fejnberg's Comment
on Koba-Takagi's Article

1. transverse momentum for heavier particles ~~is~~ are higher than those for lighter particles on the average

$p_\perp(\pi) \sim 0.5$ BeV

$p_\perp(K, N, \bar{N}) \sim 1.5$ BeV

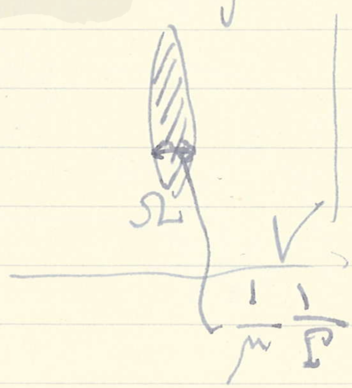
Thermal equilibrium (Landau)
(Iso, Franklin)

2. Fermi's statistical theory
Two interpretations

1. $W(n) \propto |M|^2 \cdot \rho(E_0)$

$\left(\frac{\Omega}{V}\right)^n$ $\qquad$ $V^n \int dp^3 \times \delta(E-E_0) \times \delta(\Sigma p_n)$

$\Omega^n \int dp^3$



2. $|M|^2 = 1, \quad \rho(E_0) = \Omega^n \int dp^3$

Ishida, Belenky's contradication

Fejnberg is strongly against 2nd interpretation.

Koba, Classical and quantum quantities in high energy phenomena

Tsiao $\bar{p}$-N-annihilation Process II
Chamberlain et al.

A.C.E. I: 36 $\bar{p}$
II: 185 $\bar{p}$) 221 $\bar{p}$ stars
700 MeV/c
(95 in flight)

$\sigma = (1.9 \pm 0.2)\sigma_0$

$\sigma_0 = \pi(1.2 \times 10^{-13} A^{\frac{1}{3}})^2$
except H
(emulsion nuclei)

$\langle n_\pi \rangle = 5.36 \pm 0.3 \rightarrow 1.3$ at rest $\bar{p}$
~~heavy~~ $\bar{p}$
1.9 in flight

energy branching	charged π	neutral (π^0)
at rest	$48 \pm 6\%$	28 ± 7
in flight	45 ± 7	22 ± 7

	K	cascade nucleon excitation
at rest	3 ± 1.5	21 ± 7
in flight	3 ± 1.5	$30 \pm 2.$

($T_p < 10$ MeV at rest)

Fermi model $r = 2.5 r_0$

$\frac{M}{V} \approx 4$

$\langle E_\pi \rangle = 350 \pm 18$ MeV

$\langle N_\pi \rangle = 4.1 \pm 0.3$

$\pi^+ / \pi^- = 0.45 \pm 0.12$
(0.58 理論値)
(0.56 Coulomb の影響なし)

electron:
β-decay?
5 ~ 10 MeV
↓
5 ~ 30 KeV

Dalitz pair 1つ.

Bubble chamber　$\bar{p}$ - H
$\langle N_\pi \rangle = 4.7 \pm 0.5$
$\langle E_\pi^1 \rangle = 374 \pm 25$ MeV

Observation in Q. M.

H. S. Green (Dublin and Adelaide)

(N. C. 9 (1958), 880)

Observation of an individual micro-system is possible only by means of its interaction with a macro-system in metastable state.

It is found, in the case of a simple model, that states in which the eigenvalue of the micro-observable is indeterminate cannot affect the detectors and that they are rapidly suppressed when the interaction between micro-system and the detector is « switched on ». After the interaction, the state of the micro-system is no longer pure and cannot be represented by a single wave-function. There is no discontinuity in the transition. ~~However~~ Without inspecting the measuring device, one cannot tell what the result of the measurement was, but the uncertainty is due to incomplete information rather than q. m. indeterminism.

(IX 終り)

Connection between spin and statistics

G. Lüders and B. Zumino

(P.R. 110 (1958) 1450)

Postulates

I. Invariance under proper inhomogeneous Lorentz group. (continuous)

$$\langle \varphi(x)\, \varphi(y) \rangle_0 = f(z)$$

vacuum : non-degenerate, invariant

$$\langle [\varphi(x), \varphi(y)] \rangle_0 = \cancel{f}\, 0$$

z : space-like

II. Two operators of the same field at points ~~sp~~ separated by a space-like interval either commute or anti-commute (Locality)

If $\quad \langle \{\varphi(x), \varphi(y)\} \rangle_0 = 0 \qquad z$: space-like

$$\langle \varphi(x) \varphi(y) \rangle_0 = 0 \qquad \text{''}$$

III. Vacuum is the state of lowest energy.

$$\langle \varphi(x) \varphi(y) \rangle_0 = 0 \quad \text{for all } z$$

(by analytic continuation)

IV. The metric of the Hilbert space is positive definite:

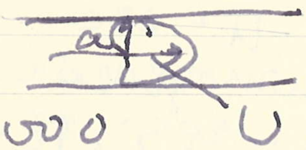
$$\varphi(x)\, \Omega = 0 \qquad \Omega : \text{vacuum}$$

V. The vacuum is not identically annihilation by ~~vacuum~~ field

非線型現象としての乱流
巽友正
(Dec. 2. 1958)

Reynolds number

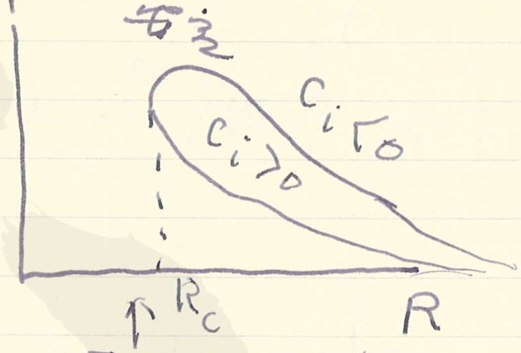
$$R = \frac{Ua}{\nu}$$



水 : $R_{cr} = 1000 \sim 20000$
層流 ⇅ 乱流
安定 - 不安定　disturbance
disturbance を小さくしても 1000 より下からぬ.

$$\left.\begin{aligned} \frac{\partial u}{\partial t} + (u \cdot \nabla) u &= -\nabla p + \nu \nabla^2 u \\ \nabla \cdot u &= 0 \end{aligned}\right\}$$

$u = U + u'$　$\overline{u'} = 0$
($\overline{u}$: ensemble 平均)
linear theory ($u' \cdot u'$ を省略する)
$u' \propto f(r) \exp[i\alpha(z - ct)]$
$c_i \gtreqless 0$

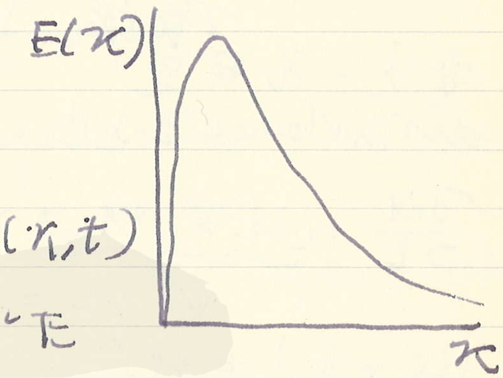


円軸対称流は
Poiseuille flow においては c_i 常に<0
すべての α, R
において安定.

平行板の場合
$R_c \fallingdotseq 6000$

non-linear theory
Taylor の不安定性
secondary flow　$R \gtrsim R_c$
遠心力

統計量だけで表わす.
完全に発達した turbulence
$\overline{u^2}$ etc
inhomogeneous, anisotropic
小さい部分で、[illegible] 一様.
渦のスペクトル
κの大きいところで
νが効く.
Kolmogorov
Heisenberg: $E(\kappa, t)$

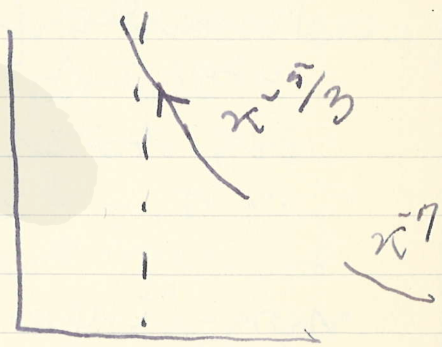


$$\frac{\partial E}{\partial t} = T(\kappa, t) - 2\nu\kappa^2 E$$

$E \propto u^2$
$T \propto u^3$

$$\int_0^u E(\kappa, t)\, du$$

$$T \to \int_\kappa^\infty \cdots d\kappa \int_0^\kappa \kappa^2 E\, d\kappa$$



$$\frac{\partial T}{\partial t} = \{u^4\} + \cdots$$