A simple arguments for the Irreversibility in Q.M. systems

§1. Introduction

Dec. 2, 1958 (1)

~~Suppose that~~ a quantum mechanical system ~~satisfying~~ ruled by a Schrödinger ~~wa~~ equation

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \qquad (1)$$

~~with~~ where, ~~ψ is a fn of $q_1 \cdots q_n$ and t~~ ~~and~~ H is a real ~~fn of $q_1 \cdots q_n$, and the $\frac{\partial}{\partial q_1}, \cdots \frac{\partial}{\partial q_n}$ and $\frac{\partial^2}{\partial q_j \partial q_k}$~~ operator.

The reversibility ~~of~~ in this system can be stated in the following way:

If $\psi_0(t)$ is a solution of eq. (1), then $\varphi_0(t) = \tilde{\psi}_0(t_0 - t)$ is also a solution and ~~they are re ψ_0 and φ_0 are related~~ the relations hold:

$$\left.\begin{aligned} \varphi_0(t_0) &= \psi_0(0) \\ \varphi_0(0) &= \tilde{\psi}_0(t_0) \end{aligned}\right\} \qquad (2)$$

for $t=0$ and $t=t_0$. ~~This shows that~~ Thus, if there is a state of the system which starts with $\psi_0(0)$ at $t=0$ and ends up with $\psi_0(t_0)$ at $t=t_0$, there is also another state which starts with $\tilde{\psi}_0(t_0)$ and ends up with $\tilde{\psi}_0(0)$. Things happen in these cases in order of time opposite to each other. ~~in the sense that~~

~~Now if we consider all possible states of the system, there are as many all possible states The ensemble of all possible states can be divi~~

Let us call ~~these two states~~ ~~them~~ dual states.
one of these states
each two
the ~~dual~~
of the other.

No. 5. 1957. 10. 20,000

Research Institute for Fundamental Physics
Kyoto University

~~There can be no~~ ~~As long as we~~ This situation seems to ~~exclude~~ ~~completely~~ contradict the thermodynamical behavior of a macroscopic system which, after all, could be regarded as a quantum mechanical system with a great many degrees of freedom. ~~Namely~~, ~~supp~~ ~~if I'm~~ In order to ~~make clear the apparent point~~, ~~contradiction~~ show it as clearly as possible, let us adopt the standard method of statistical mechanics. ~~and~~ ~~suppose that we have~~ Imagine an ensemble Σ of identical q. m. systems, each of which is at a definite state at $t=0$. Now, ~~if the ensemble has the property~~ ~~that~~. ~~any~~ ensemble Σ which we think as appropriate for deducing the thermodyn behavior of the system has always the following property:

If Ψ_0 belongs to ~~the~~ Σ, then φ_0 which is dual to Ψ_0 also belongs to Σ.

As long as we accept ~~Σ has~~ this, ~~property~~ it seems to be impossible to find out any kind of uni-directional trend in the ensemble, simply because if some of the states in the ensemble show a ~~certain~~ trend in ~~one dir~~ a certain direction, there are always ~~as many~~ equally many states in the ensemble which show a trend in the opposite direction.

This is a well-known dilemma which puzzled us for many years since the time when we knew nothing about quantum theory.

No. 5. 1957. 10. 20,000

The are two ways of thinking in this connection.

I. The dilemma should and could ultimately be resolved in the light of the theory of measurement in quantum mechanics. This point of view was ~~most or~~ developed by von Neumann to a great extent.[1)]

II. The dilemma should and could be ~~reduced to~~ resolved by taking ~~into~~ full account of the dynamical system, whether classical and quantum, with great many degrees of freedom.

§2. ~~Co~~ Time concept and quantum mechanical observation,
time ordering, memory, quantum jumps

§3. a simple model, of ~~dynamical ensemble~~ ~~q. m. ens.~~ which shows ~~one~~-directional trend.

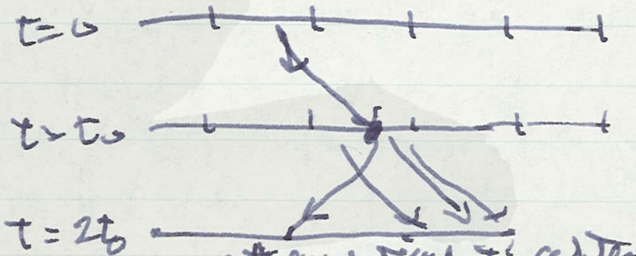
~~Ensemble~~						
class	1	2	-	-	-	L
number of states	N_1	N_2	-	-		N_L

$$\sum_{j=1}^{l} N_j \ll N_{l+1} \quad l = 1, \ldots L-1 \qquad (3)$$

Suppose that the q. m. system is in ~~a certain~~ one definite state which belongs to the class j at t = 0. at t = t₀, it ~~can~~ will be in a state belong to one definite state belonging to ~~the~~ definite class, l say. Now one can easily see that ~~among~~ N_j states of class j to start with end great majority of

Research Institute for Fundamental Physics
Kyoto University

up with states which belong to ~~class~~ either of $j, j+1, \dots L$. Only very small fraction of states of class j end up with states of belonging to ~~clas~~ either of the classes smaller than j. This ~~conclus~~ is the immediate conclusion from the reversibility as stated in §1 ~~and~~ together with the assumption (3).†



Thus, if we ~~looked~~ look at the states in any class j ~~$j=2, \dots L-1$~~, $(1<j<L)$, ~~almost~~ at any time 0, almost all the states will be ~~bound to have~~ ~~have the common tendency to~~ at any time t' after 0 go over to the higher class ~~or~~ $j'(>j)$ or at least ~~to~~ have remain in the same class j.* ~~The same trend~~ It is obviously true for states in the class 1, because they can not be otherwise. For states in the class L, this is also true in the sense that ~~they have the~~ most of them have the tendency to remain in the same class L.

§4. The system with a great many degrees of freedom.

The above argument ~~hold good~~ is equally true ~~how~~ whatever ~~be the degree~~ however large or however small be the number of degrees of freedom of a dynamical system.

† In the case of q.m.s. we have to ~~replace~~ define the number N_j by the volume in Hilbert space. (See Appendix I.)

* In ~~the~~ any time between t and t', only a small fraction of them might have sojourned in classes j' smaller than j.

Research Institute for Fundamental Physics
Kyoto University

Then, we may ask the following question:
Why is it usually regarded as essential for the irreversibility that the system is a macro scopic so that ~~the~~ number of degrees of freedom is practically infinite.