Research Institute for Fundamental Physics
Kyoto University

Appendix I.

Conservation of Volume in Hilbert space

eigenvalues of H_0: $E_1, E_2, \cdots$

eigenfunctions $\psi_1, \psi_2, \cdots$

$$H_0 \psi_j = E_j \psi_j \qquad j = 1, 2, \cdots$$

$$\psi(t) = \sum c_j(t) \psi_j$$

$$i\hbar \frac{\partial \psi}{\partial t} = H \psi$$

$$i\hbar \, \dot{c}_j = \sum_k H_{jk} c_k$$

$$H_{jk} = \int \tilde{\psi}_k H \psi_j$$

$$\tilde{H}_{jk} = H_{kj}$$

$$i\hbar \, \dot{\tilde{c}}_j = \sum_k \tilde{H}_{jk} \tilde{c}_k = \sum_k \tilde{c}_k H_{kj}$$

$$H_{jk} = 0 \qquad j \leq N \text{ and } k > N$$
$$\text{or } j > N \text{ and } k \leq N$$

$$i\hbar \, \dot{c}_j = \sum_{k=1}^{N} H_{jk} c_k \qquad j = 1, 2, \cdots N$$

$$c_j(t) = \sum_k \left(e^{-iHt/\hbar}\right)_{jk} c_k$$

$$= \sum_{k=1}^{N} U_{jk} c_k(0)$$

$$\tilde{c}_j(t) = \sum_{k=1}^{N} \tilde{U}_{jk} \tilde{c}_k(0)$$

$$= \sum_{k=1}^{N} U^{-1}_{kj} \tilde{c}_k(0)$$

$$C(t) = \overline{U} \, C(0)$$

$$C(t) = \begin{pmatrix} c_1 \\ \vdots \\ c_N \\ \tilde{c}_1 \\ \vdots \\ \tilde{c}_N \end{pmatrix} \qquad \overline{U} = \left(\begin{array}{c|c} U & 0 \\ \hline 0 & \tilde{U} \end{array}\right)$$

No. 5. 1957. 10. 20,000

Research Institute for Fundamental Physics
Kyoto University

$$c_j = x_j + i y_j \equiv X_j + i X_{N+j}$$
$$\tilde{c}_j = x_j - i y_j \equiv X_j - i X_{N+j}$$

$$\begin{pmatrix} c_j \\ \tilde{c}_j \end{pmatrix} = \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}_j \begin{pmatrix} X_j \\ X_{N+j} \end{pmatrix}$$

$$C = SX$$

$$X(t) = S^{-1} C(t) = S^{-1} \bar{U} S X(0)$$

$$\| S^{-1} \bar{U} S \| = \| S^{-1} \| \, \| \bar{U} \| \, \| S \| = \| \bar{U} \|$$
$$= \| U \| \cdot \| \tilde{U} \| = 1$$

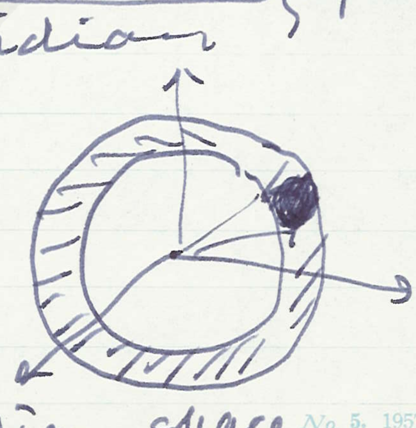
In ~~two~~ 2N-dimensional (real) Euclidian space, the <u>volume</u> does not change with time.
There is also ~~the~~ conservation of probability:

$$\sum_{j=1}^{N} |C_j(t)|^2 = \sum_{j=1}^{N} |C_j(0)|^2$$

or $$\sum_{j=1}^{2N} |X_j(t)|^2 = \sum_{j=1}^{2N} |X_j(0)|^2$$

Thus, the change of states with time corresponds to a ~~tran~~ rotation (of points X_j) in 2N-<u>dimensional</u> Euclidian space.

~~The~~ Classes of states of a q.m.s. can be defined as those states in a small portions of ~~the~~ ~~definite~~ a different common thin spherical shell, of in ~~2N~~ 2N-dimensional Euclidian space.



Research Institute for Fundamental Physics
Kyoto University

A(3)

Around ~~each~~ any one of the $2N$ axes, take a small portion with between $(R, R+dR)$ $(0, d\alpha_1)$

$$x_1 = R\cos\alpha_1$$
$$x_2 = R\sin\alpha_1\cos\alpha_2$$
$$x_3 = R\sin\alpha_1 \cancel{\sin}\,\alpha_2\ \cancel{\cos}\,\alpha_3$$
$$\vdots$$
$$x_{2N-1} = R\sin\alpha_1\sin\alpha_2 \cdots \cancel{\sin}\,\alpha_{2N-1}\ \cancel{\cos\alpha_{2N}}$$
$$x_{2N} = R\sin\alpha_1\sin\alpha_2 \cdots \sin\alpha_{2N-1}\ \cancel{\sin\alpha_{2N}}$$

$$\mathbb{X} = \mathbb{X}_1 + \mathbb{X}_2$$
$$\left.\begin{array}{l} |\mathbb{X}_1| = R\cos\alpha_1 \\ |\mathbb{X}_2| = R\sin\alpha_1 \end{array}\right\}$$

$$dX_1\,dX_2 \ldots dX_{2N} \propto \cancel{R^{2N-1}\,dR\,\sin}$$
$$J \cdot dR\,d\alpha_1 \cdots d\alpha_{2N-1}$$

$$J = \left|\begin{array}{llll} \cos\alpha_1 & -R\sin\alpha_1 & 0 & \cdots \\ \sin\alpha_1\cos\alpha_2 & R\cos\alpha_1\cos\alpha_2 & -R\sin\alpha_1\sin\alpha_2 & 0 \\ \sin\alpha_1\sin\alpha_2\cos\alpha_3 & R c_1 s_2 c_3 & R s_1 c_2 c_3 & -R s_1 s_2 s_3 \cdots \\ \cdots & \cdots & \cdots & \cdots \end{array}\right|$$

$$\propto (\sin\alpha_1)^{2N-2} R^{2N-1}\,dR\,d\alpha_1$$

$$\left.\begin{array}{l} x_1 = R\cos\alpha_1 \\ x_2 = R\sin\alpha_1\cos\alpha_2 \\ x_3 = R\sin\alpha_1\sin\alpha_2 \end{array}\right\} \quad J = \left|\begin{array}{lll} c_1 & -Rs_1 & 0 \\ s_1c_2 & Rc_1c_2 & -Rs_1s_2 \\ s_1s_2 & Rc_1s_2 & Rs_1c_2 \end{array}\right|$$

$$= R^2 \left|\begin{array}{lll} c_1 & -s_1 & 0 \\ s_1c_2 & c_1c_2 & -s_1s_2 \\ s_1s_2 & c_1s_2 & s_1c_2 \end{array}\right| = c_1^2 s_1 + s_1^2 \cdot s_1 = s_1$$

No. 5. 1957. 10. 20,000