3次元 oscillator

$$H = \sum_{i=1}^{3}(x_i^2 + p_i^2) = \sum_{i=1}^{3} u_i^* u_i + 3$$

$$u_i^* = x_i + ip_i \,;\quad u_i = x_i - ip_i$$

$$u_i^* u_i = x_i^2 + p_i^2 + i[x_i, p_i] = x_i^2 + p_i^2 + 1$$

$$u_i' = U_{ij} u_j$$

$$u_i^{*'} = U_{ji}^* u_j^*$$

$$u_i^{*'} u_i' = u_j^* U_{ji}^* U_{ik} u_k$$

$$U_{ji}^+ U_{ik} = \delta_{jk} \qquad\qquad \text{unitary} \therefore U(3)$$

$$U(3) \rightarrow SU(3)$$

位相変換 $\rightarrow U_{ij} = -\delta_{ij} \rightarrow$ 不変

$(|U| = -1)$