(φ) 4次元量子化 → 3次元量子化

$$[\varphi(D_1), \bar{\varphi}(D_2)] = V_{12}$$

$$\varphi_n = \sum_D f_n(D)\,\varphi(D)$$

$$\sum_D = \int\cdots\int d^4X\, d^3\theta\, d^3\vartheta \times d^4\Delta\, \delta\Big(\prod_\mu \Delta_\mu - l_0^4\Big)$$

$$\varphi_n = \int\int f_n(X_\mu, \theta_i, \vartheta_i, \Delta_j)\,\varphi(\ \ )$$

$$[\varphi_n, \bar{\varphi}_m] = \int\cdots\int f_n(X_\mu \ldots)\, f_m^*(X'_\mu \ldots)\, V(X_\mu, \theta_i, \vartheta_i, \Delta_j;\ X'_\mu, \theta'_i, \vartheta'_i, \Delta'_j) \times d^4X \ldots \times d^4X' \ldots$$

(4次元的)

$\varphi(D), \bar{\varphi}(D)$: Dにおける4次元
局所的 annihilation (creation) operator

通常の a a^* は一定の状態をもつ
particle の (annihil.) (creation) operator

(5) energy-momentum space

三次元における

$$[a(p_j), a^*(p_j')] = \delta^3(p_j, p_j')$$

$$[a(p_\mu), a^*(p_\mu')] = \delta^4(p_\mu, p_\mu')$$

$$a(p_j) = \int f(p_4)\, \delta(p_4 - \ldots)\, a(p_\mu)\, dp_4$$

$$a^*(p_j') = \int \frac{\delta(p_4')}{\tilde f(p_4)}\, a^*(p_\mu')\, dp_4'$$

$$[a(p_j), a^*(p_j')] = \int f(p_\mu) \tilde f(p_\mu') \, dp_\mu\, \delta^4(p_\mu, p_\mu') \ldots$$

$$\int f(p_4) \tilde f(p_4)\, dp_4$$

(6)

$$\left(\frac{\partial^2}{\partial x_\mu \partial x_\mu} - \kappa^2\right)\varphi(x_\mu, \dots)\Psi = 0$$

$$\cancel{\Psi = \Psi_0}$$

$$\varphi(x_\mu, \dots) = \int f(P_\mu)\exp i(P_\mu x_\mu) \times a(P_\mu\dots)\, d^4P_\mu$$

$$\int (P_\mu P_\mu - \kappa^2) f(P_\mu\dots)\, a(P_\mu\dots)\, d^4P_\mu\, \Psi = 0$$

$$\Psi = \int \delta(P'_\mu P'_\mu - \kappa^2)\, a^*(P'_\mu\dots)\, g(P'_\mu\dots)\, d^4P'_\mu\, \Psi_0$$

$$a(P_\mu\dots)\Psi_0 = 0$$

$$\int (P_\mu P_\mu - \kappa^2) f(P_\mu\dots)\, \delta(P_\mu P_\mu - \kappa^2)\, g(P_\mu\dots) \times d^4P_\mu$$

$$\Psi = 0$$

$$a(P_j, \kappa)\, g\cancel{(P_j, \kappa)}$$

$$= \int \frac{a(P_j, \sqrt{\ })}{\sqrt{P_j^2 + \kappa^2}}\, g(P_j, \sqrt{\ })\, d^3P_j$$