

N100

NOTE BOOK

Easy Write, Easy Read, Containing best White Paper.

R. III

R. III

July, 1972
~ Jan. 1974

100

c033-947

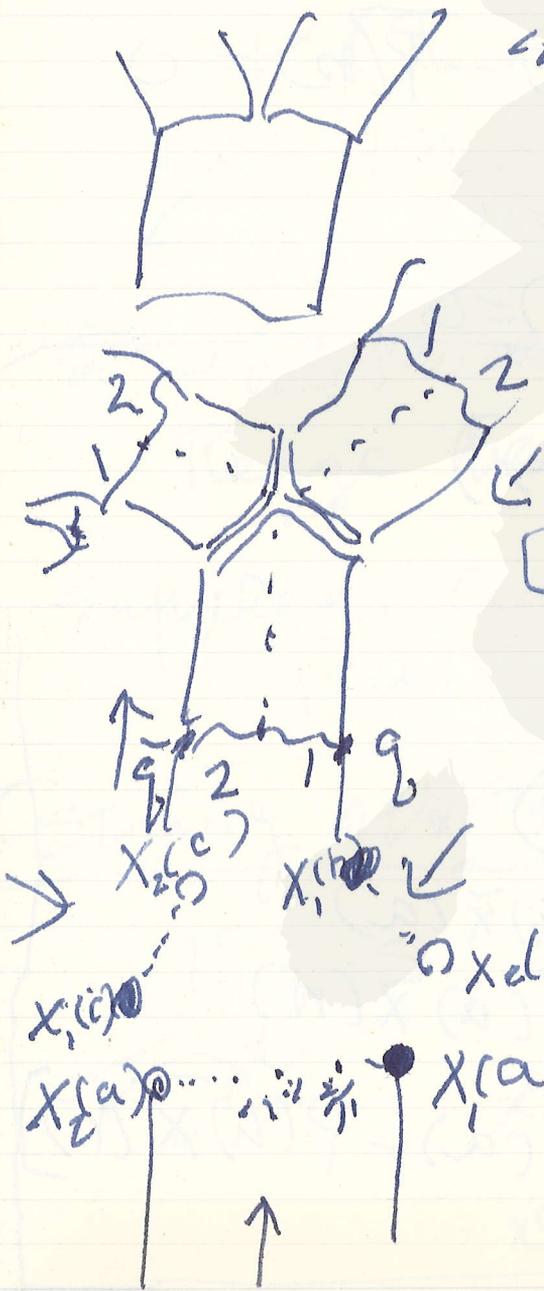
c033-948~992挟込

SPARTA
BOX 35

「大. 研究云
 弦理論の粒子の相互作用」
 1972年

7/18 (木) [湯川]
 原注:

後稿: bilocal 的 string model の特徴は vertex



string model:
 3本 弦を結ぶ in local,
 indefinite metric
 => bilocal 的 1'

(time component
 = 7117 indefinite metric)
Bilocal only 的 vertex

$$\delta(x_1(a) - x_2(b))$$

$$x \delta(\dots)$$

$$x \delta(\dots)$$

$\alpha x d(n)$

$$\frac{1}{2}(x_1 + x_2) = X \quad x_0$$

$$x_1 - x_2 = \bar{X} \quad i\bar{x}_0$$

$$x_1 = X + \frac{1}{2} \bar{X}$$

$$x_2 = X - \frac{1}{2} \bar{X} \quad \text{complex}$$

$$\begin{cases} X_1(a) - X_2(b) | \psi \rangle = 0 \\ [P_1(a) + P_2(b)] | \psi \rangle = 0 \end{cases}$$

$$\begin{aligned} X(a) - X(b) + \frac{1}{2} [\bar{X}(a) - \bar{X}(b)] &= 0 \\ \frac{1}{2} (P(a) + P(b)) + \bar{P}(a) - \bar{P}(b) &= 0 \end{aligned}$$

$$\begin{aligned} \bar{X}(a) + \bar{X}(b) + \bar{X}(c) &= 0 \\ P(a) + P(b) + P(c) &= 0 \end{aligned}$$



Vertex

$$\delta^4(\sum_{abc} \bar{X}(a)) \delta^4(\sum P(a))$$

$$X(a) \rightarrow i \frac{\partial}{\partial P(a)}$$

$$\bar{X} = (i \chi_0, \bar{\chi})$$

$$\bar{P} = \left(\frac{\partial}{\partial \chi_0}, -i \frac{\partial}{\partial \bar{\chi}} \right)$$

$$\begin{aligned} & e^{-\frac{1}{2} P_0(b) \bar{\chi}_0(a) - \frac{i}{2} \vec{P}(b) \vec{\chi}(a)} \\ & \times e^{+\frac{1}{2} P_0(a) \bar{\chi}_0(b) + \frac{i}{2} \vec{P}(a) \vec{\chi}(b)} \end{aligned}$$

$$\rightarrow \exp \left[\frac{i}{2} \{ P(b) \bar{X}(a) - P(a) \bar{X}(b) \} \right]$$

$$\bar{X} = \sqrt{\frac{1}{2\omega}} (a + a^\dagger)$$

$$\bar{P} = i\sqrt{\frac{\omega}{2}} (a - a^\dagger)$$

$$\bar{P}|X\rangle = 0$$

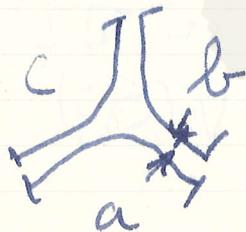
$$|X\rangle = e^{-\frac{1}{2}at^2}$$

~~Regge behavior~~

Regge behavior

supplem. condition
 $P a |\psi\rangle = 0$

String at $t=0$



$$x_a(\sigma) = x_a^{(1)}(\sigma) + x_b^{(2)}(\sigma)$$

$$x_b(\sigma)$$

$$x^{(1)} = x$$

$$= 0$$

$$x^{(2)} = 0$$

$$= x$$

$$\left. \begin{array}{l} 0 < \sigma < \frac{L}{2} \\ \frac{L}{2} < \sigma < L \\ 0 < \sigma < \frac{L}{2} \\ \frac{L}{2} < \sigma < L \end{array} \right\}$$

$$\left. \begin{aligned} \chi_a^{(1)}(\sigma) - \chi_b^{(2)}(L-\sigma) &= 0 \\ p_a^{(1)}(\sigma) + p_b^{(2)}(L-\sigma) &= 0 \end{aligned} \right\} 0 < \sigma < \frac{L}{2}$$

$$\left. \begin{aligned} \chi_a^{(1)}(n) - (-1)^n \chi_b^{(2)}(n) &= 0 \\ p_a^{(1)}(n) + (-1)^n p_b^{(2)}(n) &= 0 \end{aligned} \right\} \begin{aligned} n=0 \\ n \geq 1 \end{aligned}$$

spin

$$[\gamma_{\mu\nu}^{(1)}(a) + \gamma_{\mu\nu}^{(2)}(b)] | \psi \rangle = 0$$

$$\sigma_{\mu\nu}^{(1)}(a) + \sigma_{\mu\nu}^{(2)}(b) = 0$$

unitary spin

like physics

Koba: string model

$$H_0 = p^2 + \frac{1}{\alpha'} \sum_n n a_n^\dagger a_n$$

2nd side of the string is fixed to the other side of the string

$$H = \frac{\pi}{\alpha'} H_1 + \frac{\pi}{\alpha'} H_2 = H_0 + V(\sigma_i)$$

[大母]

著名: 場の量子論の相対論的場の理論の基礎となる
 の定式化

Wigner: inhomogeneous Lorentz
 groupの unitary representation

$$[P_\mu, P_\nu] = 0$$

$$[M_{\mu\nu}, P_\rho] = i\delta_{\mu\rho}P_\nu - i\delta_{\nu\rho}P_\mu$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = \dots$$

form factor $\rightarrow P_4, M_{4\mu}$ の helicity

$$U(\Lambda_{\mu\nu}, \tau) |p, \sigma\rangle = e^{i\tau \xi(p, p')} |p', \sigma\rangle$$

(stand) (stand)

[1] $|\Lambda p, \Lambda p'\rangle$

$$\langle p, \sigma | U | p', \sigma' \rangle$$

$$P_\mu \equiv e^{i\tau} P_\mu e^{-i\tau}$$

$$M_{\mu\nu} \equiv e^{i\tau} M_{\mu\nu} e^{-i\tau}$$

$$J = \dots$$

$$e^{iS_0} \underbrace{U(\Lambda, \tau)}_b = U(\Lambda, \tau)$$

reducible
 repres.

bound state

$\tau \rightarrow \dots$

$\tau \rightarrow \dots$

$$A^T A A$$

$$A^T A^T A^T A$$

finite self-energy
 mass renormalization

field theory is

$$A^T A^T A^T A^T$$

overlapping diagrams !!!

Dirac

crossing symmetry is form factor
 $\tau \rightarrow \dots$

field theory is

causality?

外場?

中場: Dual Field Theory
 与弦成3. 方程式

Vicarious

free $[L_r, L_s] = (r-s)L_{r+s}$

for V $[L_r - V, L_s - V]$

$= (r-s)(L_{r+s} - V)$

$V = e^{ikr\tau} e^{i\alpha' \sum \frac{p_{\alpha}^2}{\alpha}} e^{i\alpha' \sum \frac{p_{\alpha}^2}{\alpha}} e^{i\alpha' \sum \frac{p_{\alpha}^2}{\alpha}} e^{i\alpha' \sum \frac{p_{\alpha}^2}{\alpha}}$

$L_r = L_r + L_r$

$(L_0 - 1)|\psi\rangle = 0$

$L_n|\psi\rangle = 0$

$(L_0 + n - 1) L_n \frac{|\psi\rangle}{|\psi\rangle} = L_n (L_0 - 1)|\psi\rangle = 0$

quark model

【田畑】

名称: hadron a bilocal model
 (string model)

multilocal $N \rightarrow \infty$ string

primitive bilocal

$$\left[p^{(1)} + p^{(2)} - \frac{\pi_0}{2} (\alpha' - \alpha'')^2 + \frac{\pi_0}{2} \right] \psi = 0$$

$$\left[p^{(1)2} - p^{(2)2} - i\alpha' \pi (p^{(1)} + p^{(2)}) (\alpha' x^{(1)} - \alpha'' x^{(2)}) \right] \psi = 0$$

$$a_\mu = \frac{1}{2} \left(\frac{p_\mu^{(1)} - p_\mu^{(2)}}{\pi} \mp i \sqrt{\pi} (\alpha' x^{(1)} - \alpha'' x^{(2)}) \right)$$

$$a_\mu^\dagger = \dots \pm$$

$$[a_\mu, a_\nu^\dagger] = g_{\mu\nu}$$

$$p a \psi = 0 \quad \alpha = \pm 1$$

$$p a^\dagger \psi = 0 \quad \alpha = -1$$

$$\psi \rightarrow \psi^*$$

$$i(x) = i \int d^4 x \left(\frac{\partial \psi^*}{\partial x} \psi - \psi^* \frac{\partial \psi}{\partial x} \right)$$

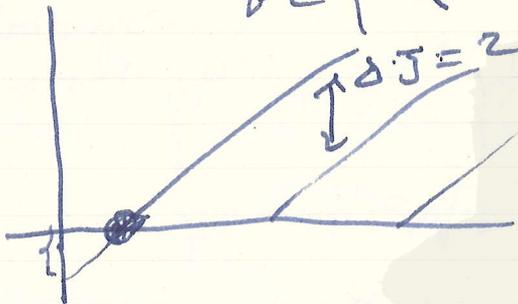
$$x' \leftrightarrow x''$$

$$M^2 = 4\pi (\tilde{n} + 2) + \pi_0$$

rest frame, mass part

$$(P + \mu a^2 + \kappa_0) \psi = 0$$

$$\begin{aligned} P a + \dots &= 0 \\ \alpha P a + \psi &= 0 \end{aligned}$$



form factor

$$(P - c A^2 + \dots)$$

$$(\kappa P) P_M - \dots$$

$$\frac{1}{\delta} = \frac{m_0}{\rho_0} \rightarrow \frac{1}{1 + t/2m_0^2}$$

$$\begin{cases} \alpha_0 < 0 \\ \text{parity} = (-1)^J \end{cases}$$

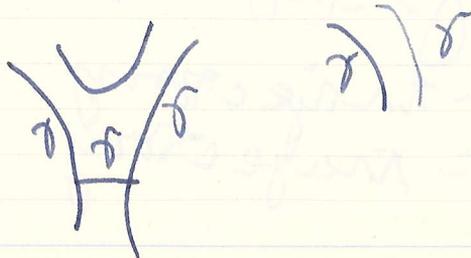
$$\text{check: } \frac{1}{4\kappa} = \rho_0^2 \rightarrow 0 \text{ local theory}$$

modified bilocal

1) baryon linear
 meson bilinear

2) subsidiary cond.: linear

meson $\gamma(x), \gamma(x')$
 baryon $\gamma, \gamma', \gamma''$



$$\alpha(\sigma) \leq \frac{1}{2}$$



$$\left[\gamma^{(1)} p^{(1)} + \gamma^{(2)} p^{(2)} - \frac{i\kappa}{2} \epsilon(\sigma^{(1)} - \sigma^{(2)}) (x^1 - x^2) \right] \Psi = 0$$

$$[\gamma^{(1)}_\mu, \gamma^{(2)}_\nu] = 0$$

$$\left[p^{(1)\mu} + p^{(2)\mu} + \frac{\kappa}{2} (x^1 - x^2)^\mu + \frac{\kappa}{2} \sigma^{(1)} \gamma^{(2)} \right] \Psi(x^{(1)}, x^{(2)}) = 0$$

(= meson)

$$M^2 = \mu^2 \left(a^\dagger a + \frac{1}{2} \tilde{d} d \right) + \kappa_0$$

↓ 0, 1, 2, 3, 4

$$0, \frac{1}{2}, 1, \dots$$

$$p = (-1)^k a^\dagger_k a_k + d^\dagger d$$

p - trajectory
 π - trajectory

(baryon)

$$\left(\Gamma^{(1)} p^{(1)} + \Gamma^{(2)} p^{(2)} + \frac{\kappa}{\sqrt{2}} \Gamma^{(3)} (x^{(1)} - x^{(2)}) \right)$$

$$+ i\mu_0 \sqrt{2} \psi = 0$$

$$\left\{ \Sigma^\alpha, \Sigma^\beta \right\} = 2 \delta_{\alpha\beta} g_{\mu\nu}$$

(4×64)
 $\gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)}$

$$M^2 = \mu^2 (a^\dagger a + b^\dagger b)$$

$SU(3), U(1)$

7月7日(金)
 [後編]

例: $\delta^2 \mu$ の Γ による粒子の相互作用。

Veneziano amplitude
 unitarity

1) satellite of unitarity

$$\Gamma_s(m-\alpha(s)) \Gamma_t(n-\alpha(t))$$

2) unitary spin \rightarrow $\alpha' \rightarrow \dots$

↓ realistic $q, m \rightarrow$ quark
 ↓ dual amplitude \rightarrow string

3) $\delta^2 \mu \rightarrow$ Γ の α の ϕ, ψ, χ

$$\alpha(x) = \sum_{j,m} \xi_{jm}^A \eta_{jm}^A$$

$$\xi(\alpha) \eta(\alpha)$$

$$H = \sum_{j,m} (\pi_{jm}^* \pi_{jm} + \mu \xi_{jm}^* \xi_{jm})$$

$$+ \frac{1}{2I} (S - J)$$

S : Total angular momentum

J : 振動数 μ の angular momentum

S : int. half int

J : int. int

non-rel. string energy world sheet
 S : int $S = \int$

S : half-int, $S = \int + \frac{1}{2}$

int: $h_1 = \frac{3}{2} \int_{\Sigma} (\rho \dot{x}^2 + \mu \dot{x} \cdot \partial x)$

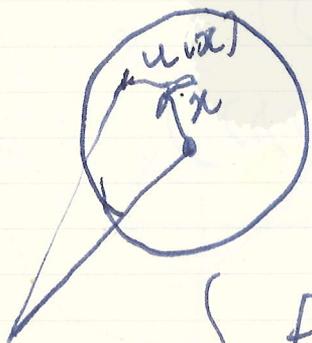
half-int: $\sum^r \rightarrow \sum^a$ (spinors)

$g_3 = \pm \frac{1}{2}$

$M_3 = \pm \frac{1}{2}$

$h_2 = \frac{1}{2} \int_{\Sigma} (\rho \sum_a^r \sum_a^r + \mu \sum_a^r \partial \sum_a^r)$
 x a v quark to

$h = h_1 + h_2$
 SU3 string quark



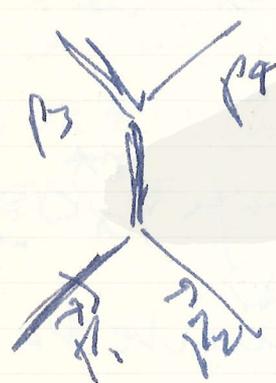
$A(x)$

$\int A(x + x + u) \alpha \alpha \leq \alpha A(x + \bar{x} + \bar{u})$

$u = (\alpha^{-1/2} / a) u$

$$\begin{aligned}
 & [P_\mu^2 + (m_0^2 - \pi_\mu^2)] \psi(x_\mu, \gamma_\mu(\sigma), \xi_0^r) = 0 \\
 & \downarrow \left[\frac{1}{2} \int P_\mu^2 + (Dx_\mu)^2 \right] d\sigma \psi = 0 \\
 & [P_\mu^2 + M_0^2 + \sum \gamma a_\mu^{r\dagger} a_\mu^r] \psi = 0
 \end{aligned}$$

scattering amplitude



$$S = \sum_N \frac{\langle f | H_{int} | N \rangle \langle N | H_{int} | i \rangle}{(p_1 + p_2)^2 + M^2}$$

$$M^2 = m_0^2 + N$$

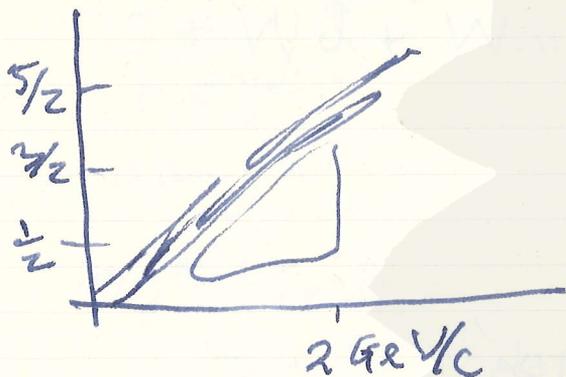
$$\chi \quad 2m^2 = -\alpha_p(0)$$

$$A(\eta, t) = B(1 - \alpha_p(\eta), 1 - \alpha(t))$$

$$\begin{aligned}
 \text{ans.} \quad & B(1 - \alpha_p(\eta), 1 - \alpha(t)) \\
 & + B(1 - \alpha_p(t), 1 - \alpha(\eta))
 \end{aligned}$$

Σ軌道)

課題: classification of baryons in a dual Regge SU(3) scheme

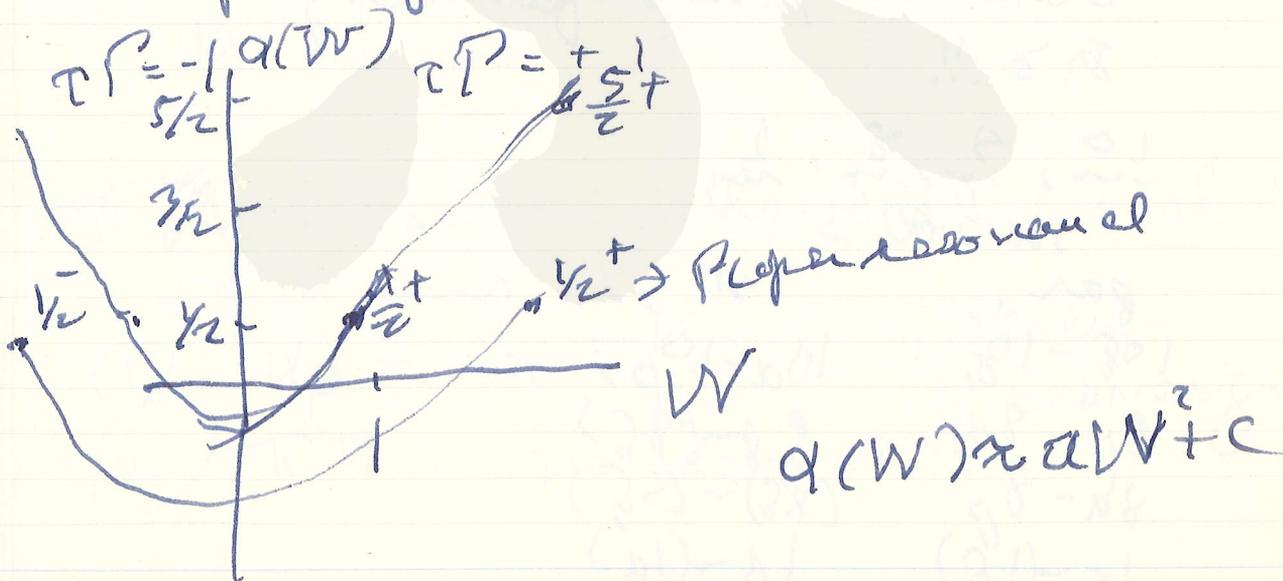


orbital excitation
 quark model
 $SU(6) \times O(3)$
 rad. exc.

1) Regge recurrence

2) approx. SU(3)

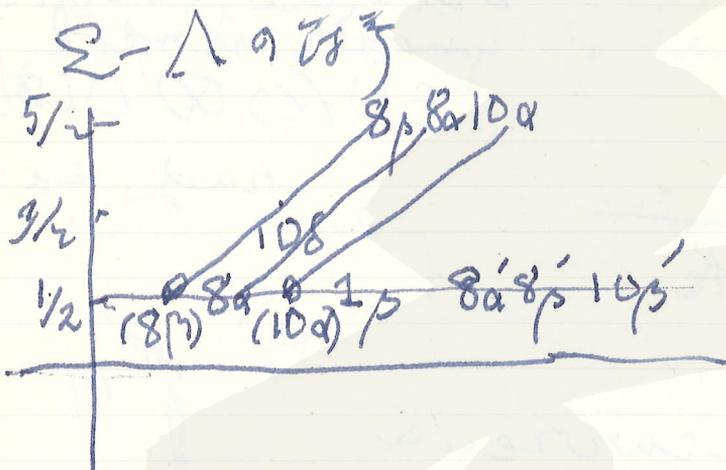
baryon trajectory a linearity?
 → parity doublet



i) $10_S, 1_P, 1_S \times$

ii) $8_S \alpha = 8_P$

Regge trajectories: $\alpha(W) \approx aW^2 + bW + c$



single Regge family

decuplet $\Delta \rightarrow 10$

singlet $\Lambda \rightarrow 1$

octet $\Sigma \rightarrow 2 \rightarrow$ family α'

$8_S \dots$

$10, 9, 8, 1$

$3 \otimes 3 \otimes 3$

par

$10_S - 10'_S$
 $8_P - 8'_P$
 $8_S - 8'_S$
 $1_S - (1'_S)$

1st

$10_P - 10'_P$
 $8_P - (8'_S)$
 $(8_S) - (8'_S)$
 $1_P - (1'_P)$

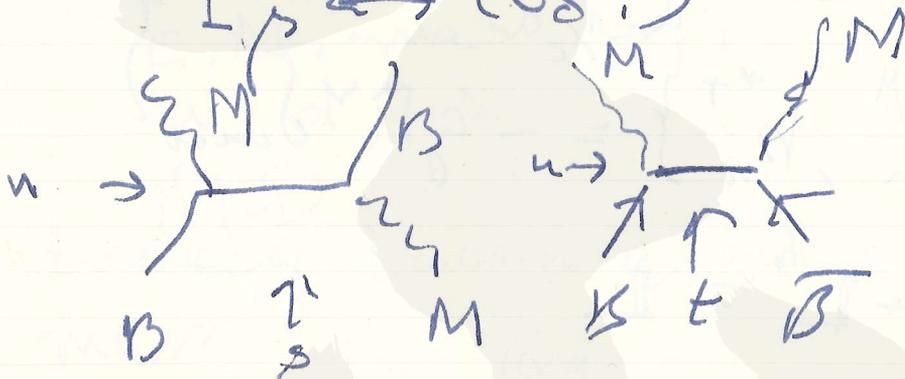
	$SU(6) \times O(3)$	bars
$N_0(4200)?$	yes	no
$1\alpha, 1\delta$	no	yes
Roper		
daughter		

2重対称性

$$8_p \leftrightarrow 10_S$$

$$(8 \oplus 10)_a \leftrightarrow (1 \oplus 8)_r$$

$$1_p \leftrightarrow (8_S?)$$



Factorizable Dual Model
 of π and K^0

(1) $\alpha_p - \alpha_\pi = 1/2$

(2) $\pi\pi$: no Tachyon

Neveu-Schwartz

$$\textcircled{3} \quad \alpha_{K^*} - \alpha_P = \alpha_P - \alpha_{K^*}$$

$$= \alpha_K - \alpha_{\pi}$$

$$\left. \begin{array}{l} L^a \\ L^- \\ L^+ \end{array} \right\} \begin{array}{l} h_0^a = R^a - P^a \\ \\ \\ \end{array} \quad R = - \sum_1^{\infty} n a_n^\dagger a_n$$

$$L^h = R^h = - \sum_{n=1/2}^{\infty} n b_n^\dagger b_n$$

$$\{ b_m^\dagger, b_n \} = - g^{\mu\nu} \delta_{mn}$$

$$L^a + L^h = L$$

↑ quantum : 1 (1個)
 N " " : 3 (3個) } 3 (3個)

↑ quantum : 2 (2個) 2 (2個) 2 (2個)

[BPA 10c]

漢語: quark 2 2 2 2 2 2 2 2 2 2

nuclear democracy

→ oligarchy

寡 3 4 5 6 7 8

1) quark current

2) $SU(6)$: $B(56)$, $M(35)$

3) quark counting
 $\mu_p: \mu_n = 3:2$
 $\sigma_B: \sigma_n = 3:2$

$\infty \infty$

E.M. mass sup

u, d, s

NHD

loosely bound quarks

mass

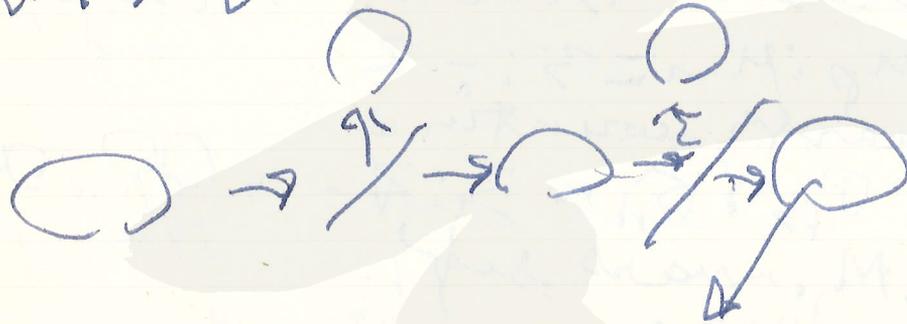
mag. mo $\frac{e q}{2M_q}$

$$\mu_p = \frac{e}{2M_q}, \quad \mu_n = -\frac{e}{3M_q}$$

$$M_q = 340 \text{ MeV} \rightarrow 450 \text{ MeV}$$

	u	d
$\langle N_B \rangle$	$\frac{1}{3}$	$\frac{1}{3}$
$\langle Q \rangle$	$\frac{2e}{3}$	$-\frac{e}{3}$

$\langle M \rangle$ 400 MeV 400 MeV



$$\langle Q \rangle = e \int \psi^\dagger \psi dx$$

$$= -\frac{e}{3}$$

state
 $\psi(x)$
 $Q = \text{continuous}$

observation
 $Q = ne$

quark
 a.F.T.
 $Q = \frac{e}{3} n$

$Q = ne$

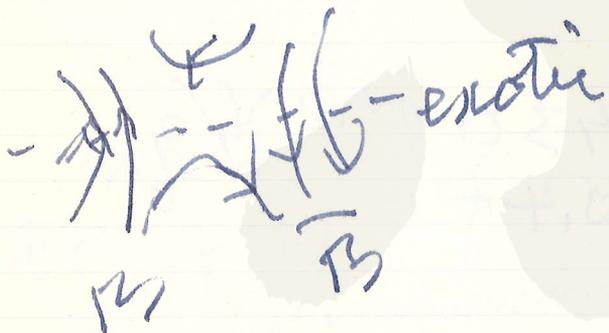
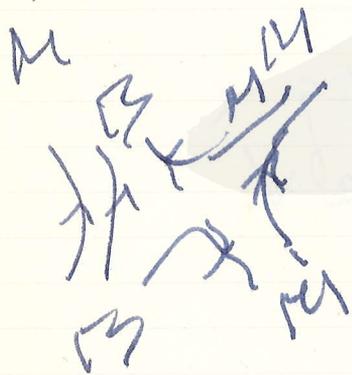
Two: Breiten scheme's
duality

Veneziano Amplitude

i) duality diagram



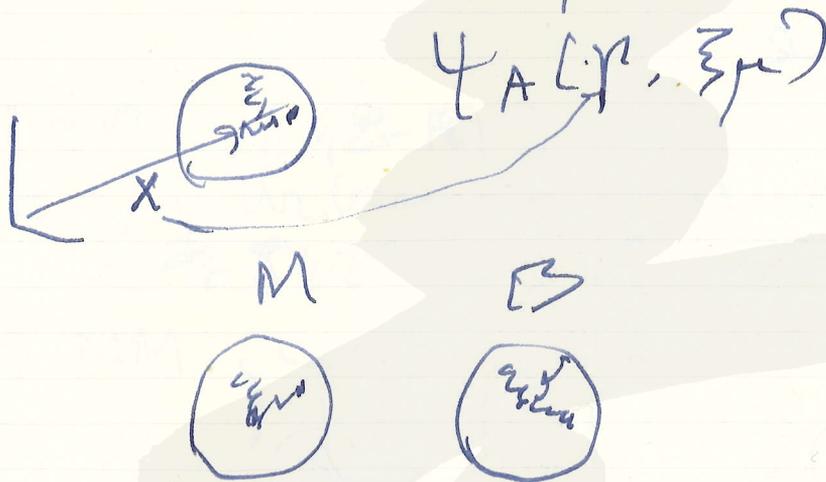
Okubo-Digheka rule



ii) spin, unitary spin

$$\xi_\mu \rightarrow \phi_\mu(\sigma)$$

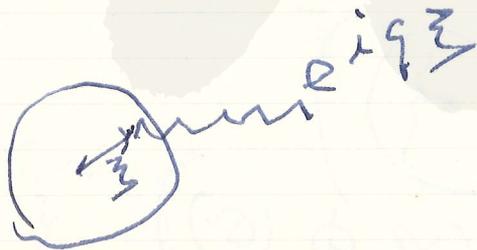
[II] dual amplitude
 (A) $\xi_\mu(\sigma) \rightarrow \xi_\mu$



$$|\Psi_M\rangle = \int \overline{\Psi}^A(p, \xi) \psi_B(p, \eta) \Psi_A(p, \xi) |0\rangle$$

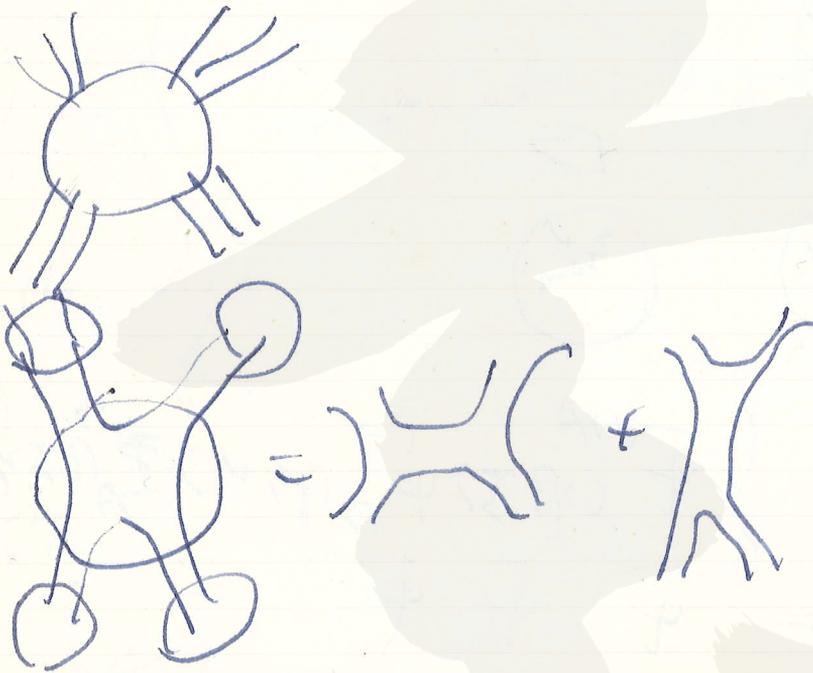
$$|\Psi_B\rangle = \overline{\Psi} \overline{\Psi} \overline{\Psi} \dots$$

connector





line picture



$$\vec{z}_\mu \rightarrow \epsilon \epsilon$$



$$\Psi_A(p, \vec{z}) \rightarrow \Psi_A(p, \phi_\mu(\sigma))$$

$$\phi_\mu(\sigma) = \sum_{\tau=1}^{\infty} \sum_{\alpha} \vec{z}_\mu \omega_\alpha \tau \sigma$$

$$e^{i\gamma_5} \rightarrow e^{i\gamma_5 \phi_\mu(0)}$$

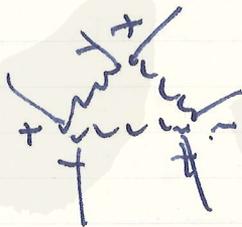
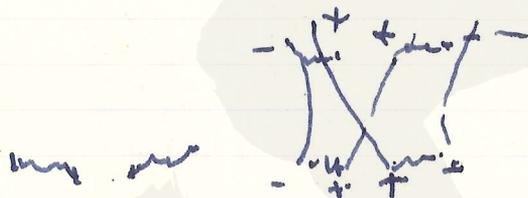
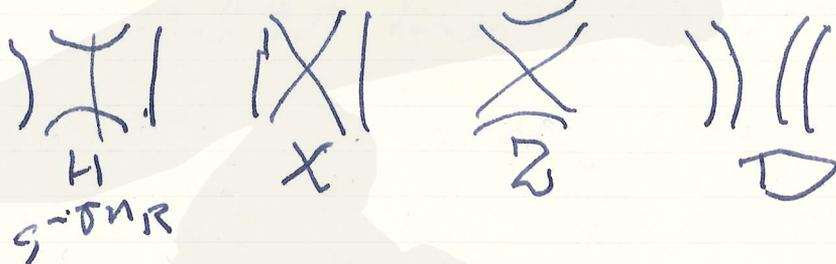
042° の γ_5 の 変換

7月8日

[徳用]

#105: Urbaryon Rearrangement
 と γ_5 の 2 行変換

1) MM



190 180 170 160 150 140 130 120 110 100 90 80 70 60 50 40 30 20 10 0

resonance の初稿 15 頁



0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
250

湯川会

Weak Interaction of Renormalizable
 Gauge Model と その歴史

花川 俊夫

7月13日, 1972

$$\frac{\delta_{\mu\nu} + \frac{k_\mu k_\nu}{M^2}}{k^2 + M^2}$$

Yang-Mills field \rightarrow Boulware

+ Hooft

$$\frac{\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{k^2 + M^2}$$

path integral $\int \mathcal{D}\phi$

$$= \frac{\delta_{\mu\nu} + \frac{k_\mu k_\nu}{M^2} \left(1 - \frac{k^2}{k^2 + M^2}\right)}{k^2 + M^2}$$

$$= \frac{\delta_{\mu\nu} + \frac{k_\mu k_\nu}{M^2}}{k^2 + M^2} - \frac{k_\mu k_\nu}{M^2 k^2}$$

Goldstone: $+$ $\frac{k_\mu k_\nu}{k^2}$
 Boson

on shell

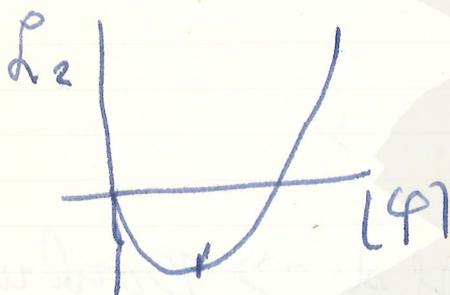
SU(2)

ϕ : doublet (scalar)

A: Gauge field

$$\mathcal{L}_1 = -\frac{1}{4}(\partial_\mu \varphi - ig A_\mu \varphi)^2$$

$$\mathcal{L}_2 = -\mu^2 \varphi^* \varphi - G(|\varphi^* \varphi|)^2 = \lambda$$



$$\langle |\varphi| \rangle = +0$$

$$\varphi = \begin{pmatrix} \psi_1 + i\psi_2 \\ \lambda + i\psi_3 \end{pmatrix}$$

$\underbrace{\hspace{10em}}_{\sigma^+}$

$$\langle \sigma \rangle = \langle \psi_i \rangle = 0$$

σ : massive

ψ_i : massless

Weinberg 1967

weak boson & gauge field

$$J^+ \rightarrow 0 \quad SU_W(2)$$

φ : doublet & $\frac{1}{2} \rightarrow 1$

$\varphi \rightarrow$ spontaneous breakdown

$A \rightarrow$ massive

hadron model は "うしろ" の
L₁ の系 と 対称性 2つ あり

1. weak boson \rightarrow gauge boson
gauge group G ($\rightarrow SU_N(2)$)
2. w. boson \rightarrow massive \rightarrow scalar
field ϕ として 何を 使うか?
3. lepton, hadron (or baryon) \in
 $SU_N(2)$ の "の" ϕ $\chi = \text{assign}$ する

Weinberg: $G = SU_N(2) \times U(1)$
 $W^\pm \quad Z \quad \gamma$

triplet
quartet
quintet

adder: unitarity $\phi^2 \in U(1) = \mathbb{R}$

self-consistent

L.S. 2, weak lin $\psi = \psi^{in}$

2) $\psi = \psi^{in}$ or com. relation is ~~is~~
 $\psi \rightarrow \psi^{in}$

canonical commutator ψ
 $\psi \psi^\dagger, \psi \psi \rightarrow 0$

generator

$$Q_f = \int f(x) T_4(x) d^3x$$

Number model

$$\bar{\psi}(\gamma_0)\psi + \frac{\lambda}{2}(\bar{\psi}\psi)^2 + \frac{g}{2}(\bar{\psi}\gamma_5\psi)^2$$

$$T_\mu = \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\square} \right) j_\nu(\psi^{in})$$

gauge
 transv.

$$+ a \partial_\mu B^{in}$$

$$\square B^{in} = 0$$

$$Q = \int T_4 d^3x = a \int d^3x B^{in}$$

$$\psi^{in} \rightarrow \psi^{in}$$

$$B^{in} \rightarrow B^{in} + a\theta$$

$$\psi(\psi^{\text{in}}) \rightarrow e^{i\theta T_5} \psi$$

scale invariance

$$\frac{1}{2} \partial_\mu \phi \partial_\mu \phi + \lambda \phi^4$$

$$\Theta_{\mu\nu} \quad (\Theta_{\mu\mu} = 0 \quad \partial_\mu \Theta_{\mu\nu} = 0)$$

$$D_\nu = x_\mu \Theta_{\mu\nu} \quad D = \int d^3x \mathcal{P}_4$$

$$[D, \phi] = (x \cdot \partial + 1) \phi$$

dimensional transformation

$$m^2 = 3\lambda \langle 0 | \phi^2 | 0 \rangle$$

mass difference $\propto \langle D \rangle$

山崎 浩
1972年8月17日

Aug. 17, 1972

spinless self-interacting field

$$L(x) = \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - m_0^2 \phi^2 - \frac{\lambda}{4!} \phi^4$$

Schwinger's chiral lagrangian
nonlinear ψ -part

$$L = \frac{1}{m_\pi^2}$$

$$\vec{\psi}_\mu(x) \rightarrow \partial_\mu \vec{\psi}^M = 0$$

$$\vec{Q}_\mu(x) \rightarrow \partial_\mu \vec{Q}^M = 0 \quad (M_0 = 0)$$

physical meson \rightarrow periodic
(stable) f_π

amplitudes or σ \dots limit?

世の世 世の世
 小カントール集公論の著者の手
 への手紙

1972年9月26日
 著者 湯川 秀樹

$X_0 \subset X \subset$

分離可能
 集合の性質

$X_0 \subset X_1 \subset X_2 \subset$

1908... : Cantor
 1930... : Gödel
 1962... : Cohen) 正統派の進化
 " 非正統派

van der Waerden: "Algebra"

$$x(t) = x_0 t^n + x_1 t^{n-1} + \dots + x_n$$

$t \in \mathbb{R}$ ならば $x(t)$ は t の多項式

$$\frac{x(t)}{y(t)} = \frac{x_0 t^n + \dots}{y_0 t^m + \dots}$$

$$\varepsilon > \frac{1}{2} > \frac{1}{t} > \dots > 0$$

$$x(\tau) = x_n \tau^n + x_{n-1} \tau^{n-1} + \dots + x_0 \\ + \frac{x_1}{\tau} + \frac{x_2}{\tau^2} + \dots$$

$$y(\tau) = f(x(\tau))$$

② $y \in \mathbb{R}^m$. $\vec{x} = \vec{x}_0 + \frac{\vec{x}_1}{\tau} + \frac{\vec{x}_2}{\tau^2} + \dots$

$SU(5) \uparrow$

湯川會 (湯川) 28. 1972
京都大学基礎物理学研究所 湯川記念館史料室

南政次 (叔精)

Duality or Topsy-Turvydom

October 26, 1972

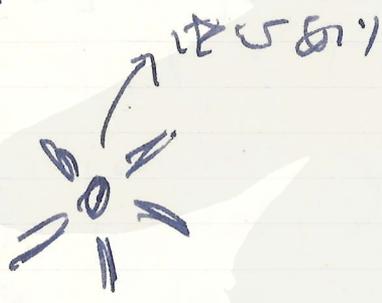
1532 ~ 1597

内政次郎

Topsy-Turvydom (Chamberlain)

西... 東...

建曆



西...型

東...型

R. Riemann (1826 ~ 1866): 西...型
K. Weierstrass (Göttingen, Berlin): 東...型

N. Bohr → Heisenberg
de Broglie → Schrödinger
西...型
東...型

④ τ - τ' duality

(domain, field) \rightarrow

(domain, field) \rightarrow
+
 $\tau \leftrightarrow \tau'$

(Nakanishi)

Veneziano

$x^\mu(\sigma, \tau)$

2D Minkowski space

Snyder 1946 Quantized space time

湯川記念館
水島真由と因果性

河村 11月30日, 1972
quark-antiquark \rightarrow bilocal field
3 quark \rightarrow trilocal field
 $SU(6) \times O(3) \rightarrow$

分類
級数

因果性
因果性

S-matrix

湯川-坂田系
場の相互作用の群論 \rightarrow 場の量子論
(Mandelstam
Yukawa)

$$p^2 \epsilon = 0$$

Intermediate Weak Boson and Weak Interaction at High Energy

1972年12月 (1972)

(1972年12月)

二重. 1972, 1972

Pure-leptonic $\nu + l \rightarrow \nu + l$

1972年

π -meson (1972), 1972年12月



$\nu + l \rightarrow \nu + l$ (10 GeV) (100 GeV)
 (100 GeV) (100 GeV)

$$\sigma \approx G^2 s / \pi$$

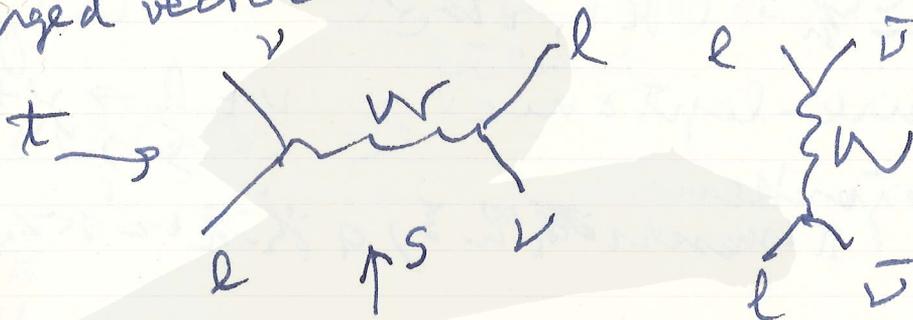
$$G \approx 10^{-5} m_p^{-2}$$

- $0 < s < s_A$: Perturbation region
- $s_A < s < s_B$: Resonance region
- $s_B < s < s_C$: Transition region
- $s_C < s$: asymptotic region

Pomranchnuk, I. Ya.
 Soviet Journal of Nucl. Phys.
 11 (1970), 477.

$$\bar{\Psi} \Gamma \Psi \leftarrow \infty$$

Weak boson
 charged vector m_W



$$f_{\pm}(s, t) = \frac{1}{2} [f_{e\nu} \pm f_{\bar{e}\bar{\nu}}]$$

$$= 2\sqrt{2} G m_W^2 \left(\frac{s}{m_W^2 - u - im_W \Gamma_W} \right)$$

$$s + t + u = 0$$

$$+ \frac{s}{\pi} \int_{s_0}^{\infty} ds' \operatorname{Im} \left[f_{\pm}(s', t) \left(\frac{1}{s' s' s} \right) \right]$$

$$m_W \gg \Sigma_W \quad \pm \frac{1}{(s'+t)(s'+0)}$$

$$S_A = m_W^2 \left[\exp\left(\frac{2\sqrt{2}\epsilon E}{G m_W^2}\right) - 1 \right]$$

m_W	$\sqrt{S_A}$	$\sqrt{S_c}$
50	2×10^8	2.5×10^9
150	4.6×10^3	10^5
350	3.5×10^2	2×10^4
\vdots	\vdots	
∞	$\frac{10^5}{3 \times 10^2}$	10^5

$$S_c > \frac{\sigma_{\infty} m_W^4}{2\epsilon^2} \left[\exp\left(\frac{2\sqrt{2}\epsilon E}{G m_W^2}\right) - 1 \right]$$

$(m_W \rightarrow \infty \quad S_c > \epsilon^2 \sigma_{\infty} / G^2)$
 $(3 \leq m_W \rightarrow \infty \rightarrow \text{Pomeranchuk } \epsilon - 3/2)$

$$\sigma_{\infty} = 1 \text{ GeV}^{-2}$$

大物録

湯川 昭三

Super-Quantization

(佐藤) 湯川 昭三 symmetry

(Dec. 21, 1972)

① Lagrangian-Hamiltonian

② Yang-Feldman

③ Super-Quantization

($\psi, \bar{\psi}, \psi, \bar{\psi}$)

unitarity?

④ Feynman-Path Integral

湯川 昭三

① $S \dagger S = 1$

② limit $i \rightarrow 0$ の場合 \mathbb{R}^4 と \mathbb{R}^3

$\phi(x)$ ← bilocal field $\phi(x, \xi)$

\downarrow
 $\psi = \psi(x) \text{ と } \psi$
 場の

\vdots
 $\phi(x_1, x_2, \dots)$

$\phi(0)$

Assumption ①

$$[\phi(x), \phi(y)] = C(x, y) = \text{c-number}$$

$$\phi^\dagger(x) = \phi(x)$$

$$G(x, y) = -G(y, x)$$

$$G^*(x, y) = G(y, x)$$

(2) partial ordering

$x \succ y$ (future-like) $x \sim y$ (space-like) $x \prec y$ (past-like)

causality $\Omega - \partial \Omega \subseteq \mathbb{C}$

$$G_D(x, y)$$

$$G_R(x, y)$$

$$G_A(x, y) \equiv \begin{cases} 0 & \text{if } x \succ y \text{ or } x \sim y \\ G(x, y) & \text{if } x \prec y \end{cases}$$

$$\left(\partial(y_0 - x_0) G(x, y) \right)$$

$x \sim y$:

(3) $G(x, y) = 0$ $x \sim y$ (causality)

$$G_A(x, y) \equiv \theta(y - x) G(x, y)$$

$$G_R(x, y) \equiv \theta(x - y) G(x, y) \\ \equiv -G(y, x)$$

$$G_A(x, y) - G_A(y, x) \equiv G(x, y)$$

$$\Lambda(x) \equiv \phi(x) - \int dy G_A(x, y) \frac{\delta}{\delta \phi(y)}$$

$$\hat{\Lambda}(x) \equiv \phi(x) + \int dy G_A(x, y) \frac{\delta}{\delta \phi(y)}$$

$$[\Lambda(x), \Lambda(y)] = [\Lambda(x), \hat{\Lambda}(y)]$$

$$= [\hat{\Lambda}(x), \hat{\Lambda}(y)] = 0$$

$$T(\phi(x_1) \dots \phi(x_n))$$

$$\equiv \Lambda(x_1) \Lambda(x_2) \dots \Lambda(x_n) \cdot \mathbb{1}$$

① x_1, x_2, \dots, x_n : symmetry

② $x_1 > x_2 > \dots > x_n$

$$\tilde{T}(\phi(x_1) \dots \phi(x_n)) = \mathbb{1} \hat{\Lambda}(x_1) \dots \hat{\Lambda}(x_n)$$

Interaction

$$L_I(x) \equiv h_I(x; \phi(x))$$

Quasi-local:

$$\frac{\delta}{\delta \phi(y)} L_1(x; \phi(x)) = 0$$

$$S \equiv \exp \{ iK \} \cdot \mathbb{1}$$

$$K = \int dx L_1(x; \Lambda(x))$$

S a unitarity:

$$S^\dagger \equiv \mathbb{1} \exp \{ -i\tilde{K} \}$$

$$\tilde{K} = \int dx L_1(x, \tilde{\Lambda}(x))$$

$$S^\dagger S = \mathbb{1} \cdot e^{-i\tilde{K}} e^{iK} \mathbb{1}$$

$$= \mathbb{1} e^{i(K - \tilde{K})} \mathbb{1}$$

$$K - \tilde{K} = \int dx D(x)$$

$$S S^\dagger = \mathbb{1}$$

$\mathbb{1}$

(is the
local
operator)

Example (2)

$$G(x, y) = 0 \quad x^2 - y^2 = \lambda^2$$

markov

Example (2)

bilinear
 $\phi(x, z)$

$$G(x, z; y, \eta) = 0$$

$$(x - y - \lambda z + \lambda \eta)^2 = 0$$

$$(x, z) \sim (y, \eta)$$

$$\iff x_0 - \lambda z_0 = y_0 - \lambda \eta_0$$

Example (3)

$$[\phi(\mathcal{D}), \phi(\mathcal{D}')] = G(\mathcal{D}, \mathcal{D}')$$

$$\mathcal{D} \succ \mathcal{D}'$$

Example (4)

non-flat space \mathbb{R}^n

$$(ds)^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

2nd level: 湯川電弱 symmetry

↓
symmetry

(1) ICP

(2) Crossing symmetry

$$\left. \begin{array}{l} \pi^+ N \rightarrow \pi^+ N \\ \pi^- N \rightarrow \pi^- N \end{array} \right\} \begin{array}{l} B \rightarrow -B \\ a \rightarrow -a \end{array}$$

discont.

(

cont.,

horizy

$$\begin{array}{l} SU(2) \rightarrow SU(3) \rightarrow SU(6) \\ \searrow \quad \nearrow \\ \quad \quad \quad \rightarrow SU(2) \rightarrow SU(3) \end{array}$$

時空の量子化 (湯川大) Space-time Quantization

↓ 湯川大の論文 (Can. J. Phys. 1973)
量子化 (湯川大の論文)

Riemann, 1854

リーマンの積分
リーマンの積分 → 曲率 (湯川大)
湯川大の論文 → 曲率 (湯川大)

Sobelman, 1891

Arnold, Planck との関係

1897:

原子の atom

Heisenberg の vector atom

Heisenberg, 1938
uncertainty length λ
湯川大の論文

Snyder, 1947 ←

Tokuda,
Tamm, 1965

1953 湯川大の論文

Landau, 1954

$$\epsilon^2 = \epsilon_0 \frac{1}{1 + \text{const} \epsilon_0^2 \ln \frac{\Lambda^2}{m^2}}$$

$$\frac{1}{\Lambda} = a \quad \text{diagram}$$

A diagram showing a circle with an arrow pointing to the right, labeled with 'g' below it.

Haag

$$U \phi U^{-1} = \psi$$

free $\psi \rightarrow$ free

Analytic continuation!!!

non-local interaction



convergence, Warden



"Minkowski space or Euclidean space"

量子場論 \rightarrow 素領域

spin : up, down

粒子のスピンの方向によって標識を
振る

このようにして

$P_n(x)$ の表現は次の通り

$$P_n(x) \rightarrow P_n(x')$$

macro-causality
↓
energy of macro の ϵ は
有限 ϵ である

この方向にエネルギーが
有限に増える

光速度の制限がある

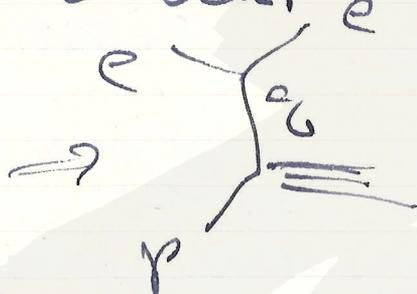
1955年 ("EPR" 論文)

湯川

UCC

A. Tarkenton
Auto model (Scaling)
Behavior in Field theory
(1973 2/18 to 3/10)

電子散乱 (e-p scattering)
Regge poles ... - Tarkenton
deep inela. e



$\zeta = q^2 / \nu$
Markov

$-q^2 \rightarrow \infty$ $\nu \rightarrow \infty$
Nakanishi

Causality
Sudakov

Limitations on Weight Functions

$\sum \lambda^k$
1) increasing \sum of λ . (polynomially)

190 180 170 160 150 140 130 120 110 100 90 80 70 60 50 40 30 20 10 0

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京都大学基礎物理学研究所 湯川記念館史料室

2) decaying



0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
250

田中

Bi-local Currents and Finite Q.E.D.

湯川会, Feb. 16, 1973

非局所電磁相互作用

→ くりこみ
 Q.E.D.

非局所場の理論

レプトン場の理論

(non-local interaction
 indefinite metric)



anomaly

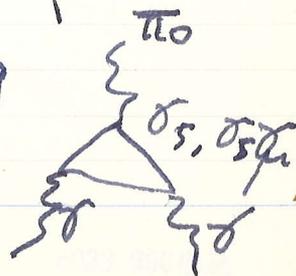
1) anomaly, gauge

$$e^{iR(\sigma)}$$

$$i \int \Lambda(\pi') \bar{\psi} \gamma_\mu \psi(\pi) d\sigma$$

$$[I, \bar{\psi}(\pi) \gamma_\mu \psi(\sigma)]_{\pi\sigma} = 0 \quad ?$$

Fukuda - Miyamao - 1949



Gauge Indep. AED
Valatin
Jauch - Rohrlich
Schwinger

高村茂典
 Theory of one-dim. Relativistic
 elastic Continuum and
 Hadronic Wave Eq.

基礎理論
 1973年3月13日(火)

$$H(\sigma) \neq [\pi(\sigma)] = 0$$

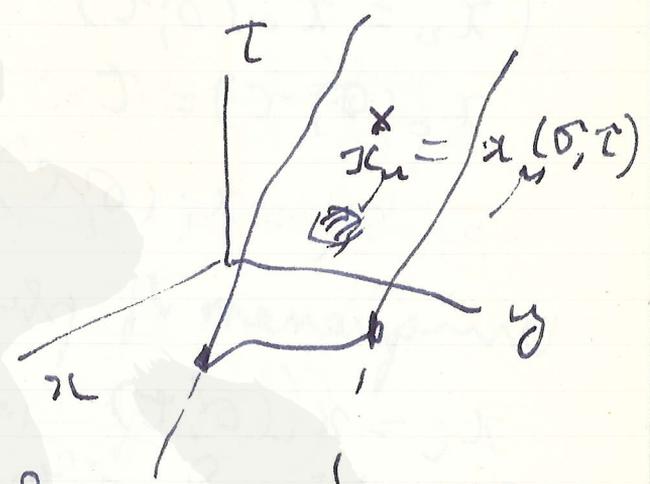
Hara
 Goto
 Tanihara
 Konishi

Nambu
 Classical theory
 world strip

σ : label

$[\sigma_1, \sigma_2]$

$\tau: [-\infty, +\infty]$



x_μ の変換 \rightarrow external transformation

$\sigma \rightarrow \sigma'(\sigma, \tau)$ の変換

$\tau \rightarrow \tau'(\sigma, \tau)$ の変換
 \rightarrow internal transformation

$x_\mu(\sigma, \tau) = x'_\mu(\sigma', \tau')$
 reparametrization

$$(dx^\mu)^2 = \left(\frac{\partial x^\mu}{\partial \sigma} d\sigma + \frac{\partial x^\mu}{\partial \tau} d\tau \right)^2$$

$$= G_{11} d\sigma^2 + G_{00} d\tau^2 + 2G_{01} d\sigma d\tau$$

$$G_{01} = -G_{00}G_{11} \neq 0$$

$$\sqrt{G_{01}^2 - G_{00}G_{11}} d\sigma d\tau$$

$$g_{\mu\nu} = (+ + + -)$$

$$\left\{ \begin{array}{l} x^i = x^i(\sigma, \tau) \\ x^0 = x^0(\sigma, \tau) \end{array} \right. \rightarrow \text{世界線} \text{の} \text{座標}$$

$$x^0(\sigma, \tau) = t \rightarrow \tau = \tau(\sigma, t)$$

$$x^i = x^i(\sigma, \tau) \rightarrow x^i(\sigma, t)$$

Uniqueness of phys. interpretation

$x^i = x^i(\sigma', t)$ is $x^i(\sigma, t)$ & identical τ' 's is τ '.

$$\sigma \rightarrow \sigma'(\sigma)$$

$$\tau \rightarrow \tau'(\sigma, \tau) \rightarrow \text{physical parameter transform.}$$

$$V_i = \frac{\partial x^i(\sigma, t)}{\partial \tau}$$

$$= \frac{\partial x^i(\sigma, \tau)}{\partial \sigma} / \frac{\partial x^0(\sigma, \tau)}{\partial \tau}$$

$$U_\mu = \frac{1}{\sqrt{-G_{00}}} \frac{\partial x^\mu}{\partial \tau}$$

$$G_{00} \leq 0$$

$$\frac{\partial \pi_\mu(\sigma, \tau)}{\partial \sigma} d\sigma = \underbrace{\left(\frac{\partial x_\mu(\sigma, \tau)}{\partial \sigma} \right)}_{W_\mu(\sigma, \tau)} - V_i \frac{\partial x^\mu(\sigma, \tau)}{\partial \sigma}$$

$$\int_0^1 \left(\frac{\partial x^\mu}{\partial \sigma} \right)^2 d\sigma^2 \rightarrow G_{11} \geq 0$$

(causality)
 $1 \neq \tau - \tau' \neq 0$

$G_{01} = 0$: orthogonal curvilinear
 coord. σ, τ !!!

$$L = \frac{\kappa}{2} \left(\left(\frac{\partial x^\mu}{\partial \sigma} \right)^2 - \left(\frac{\partial x^\mu}{\partial \tau} \right)^2 \right)$$

$$\delta W_{12} = \delta \int_{\tau_1, \sigma_1}^{\tau_2, \sigma_2} d\sigma d\tau L = 0$$

$$\frac{\delta^2 x^\mu}{\delta \tau^2} = \frac{\delta^2 x^\mu}{\delta \sigma^2}$$

$$\left. \frac{\partial x^\mu}{\partial \sigma} \right|_{\sigma_1, \sigma_2} = 0$$

$$\Lambda^0 = \frac{\kappa}{2} \int_{\sigma_1}^{\sigma_2} (G_{00} + G_{11}) d\sigma$$

$$= -\omega_0 \quad \omega_0 \geq 0$$

$\sigma' = d\sigma$
 $\tau' = d\tau$) internal
scale transf.

$$P_\mu(\sigma, \tau) = \kappa \frac{\partial \kappa}{\partial \tau}$$

$$P_\mu(\tau) = \int_{\sigma_1}^{\sigma_2} p(\sigma, \tau) d\sigma$$

$$M_{\mu\nu} =$$

spin

$$S = \frac{m^2}{2\pi \kappa} - \frac{\kappa \Omega}{2\pi}$$

$$(m^2 = -P_\mu^2)$$

佐藤・頭崎

New Series of Exact Solutions for Gravitational Fields of Spinning Masses

澤田 隆

1973年 3月 15日

Tomimatsu-Sato Solution black hole

1. $R_{\mu\nu} = 0$ 空時間的解
2. $\vec{v} = \vec{e}_i$ flat
3. 定常
4. 軸対称

$$g_{\mu\nu}(P, z)$$

Schwarzschild の解 (軸対称)

Weyl (回転対称内定常解)

回転対称の解

non-rotating (非回転)

Papapetrou

spinner 解

Kerr の solution (1963)

Schwarzschild type (m, δ)

Schwarzschild \rightarrow Kerr

Weyl \rightarrow T-S (1972)
 $(m, \delta) \rightarrow (m, \delta, \gamma)$

$$ds^2 = f^{-1} [e^{2\sigma} (\rho^2 + d\tau^2) + \rho^2 d\phi^2] - f [dt - \omega d\phi]^2$$

1968 Ernst

$$(\xi \xi^* - 1) \nabla^2 \xi = 2 \xi^* \nabla \xi \cdot \nabla \xi$$

$$f = \operatorname{Re} \frac{\xi - 1}{\xi + 1} \quad \varphi = \operatorname{Im} \frac{\xi - 1}{\xi + 1}$$

$$\rho^{-1} f^2 \nabla \omega = \hat{n} \times \nabla \varphi$$

ξ : real $\rightarrow g_{\mu\nu} \rightarrow \dots$

$$\downarrow \quad (\omega = 0)$$

$$\xi_0 = -\coth \Psi$$

$$\nabla^2 \Psi = 0$$

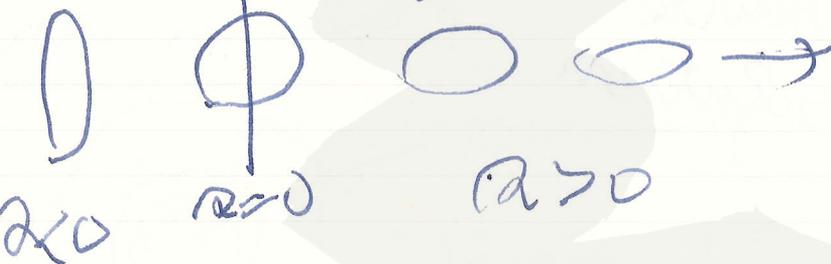
$$\Psi = \frac{\delta}{2} \ln \left(\frac{x-1}{x+1} \right)$$

$$\phi \sim -\frac{m}{\rho} + \frac{1}{3} \left(L - \frac{1}{\delta^2} \right) \frac{m^2}{\rho^3} P_2(\theta)$$

quadrupole moment

$\delta = 1$: Schwarzschild

$$f = 1 - \frac{2M}{r}$$



Schwarzs.

Weyl

Kerr

$$\bar{z} = x$$

$$\bar{z} = \frac{(x+1)^\delta + (x-1)^\delta}{(x+1)^\delta - (x-1)^\delta}$$

$$\bar{z} = p x - i q y$$

$$p^2 + q^2 = 1$$

T.S. $J = m^2 g$

$\delta = 2$:

$\delta = 3$:

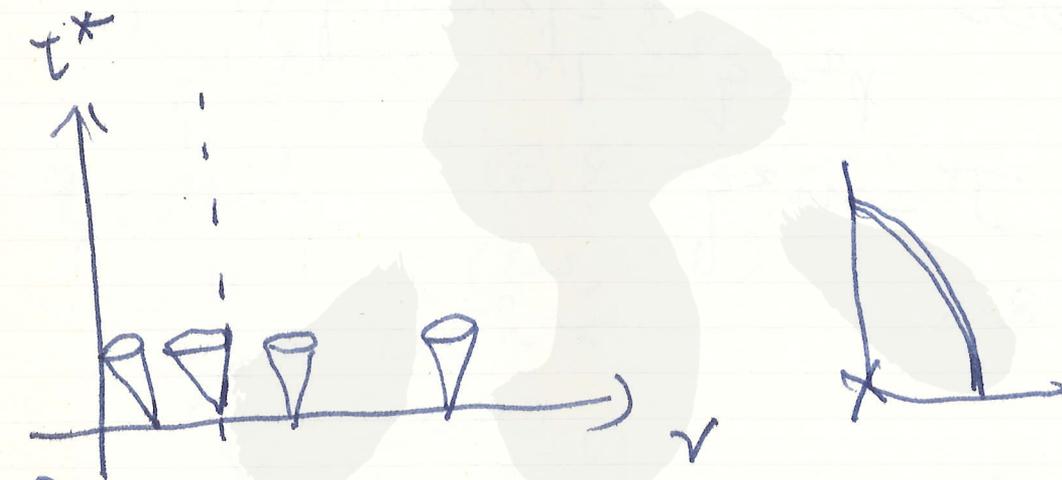
⋮

$$\xi = x \rightarrow \xi = ix \quad \text{dipole } i\gamma$$

$$\xi' = \xi e^{i\alpha}$$

charged source
 R-N solution

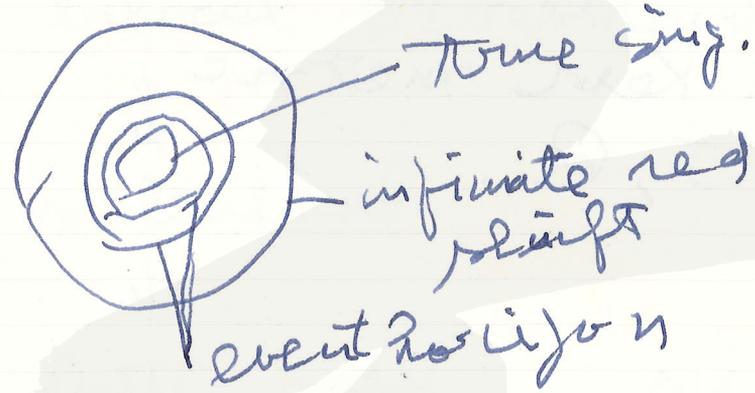
Black Hole の図



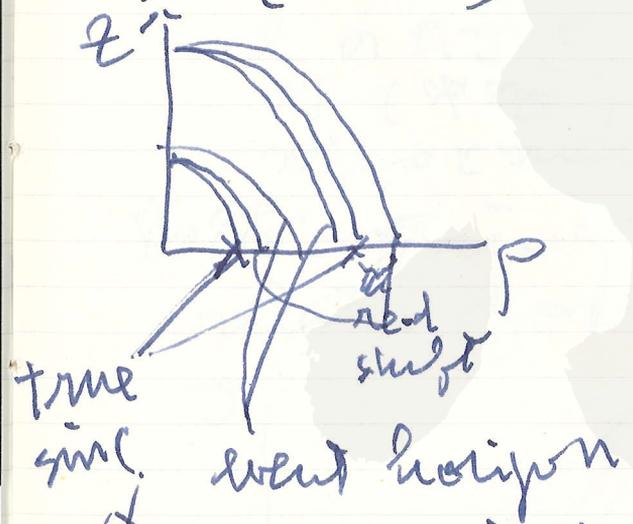
r_{in}
 null-surface $\rightarrow r = 2m$
 always surface \rightarrow even horizon
 \rightarrow red shift ∞

$\frac{dr}{dt} = \pm \sqrt{1 - \frac{2M}{r}}$
 $\lambda \rightarrow r < 2M$ \rightarrow $r > 2M$ \rightarrow $r < 2M$ \rightarrow $r > 2M$..

Kepler solution:



$T-S (\delta = 2)$



marked singularity
 \downarrow way not

Wheeler: 重力場の強さの結果として
"black hole has no hair"



changed Kerr metric
 M, J, Q .

(三次元の運動量)

ε va 基底: 運動量 → 粒子の相対速度

↑ infinitesimal deformation

$$H' = e^{i k_\mu (x_\mu + \epsilon_{\mu\nu})}$$

$$[\tilde{\Lambda}^\dagger, H'] = \epsilon H'$$

Covariance



variation
 of ϵ

$R_\mu = 1$: space-like

$$O_{\mu\nu} = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$$

$$\begin{pmatrix} 1 & & \\ & 1 & \\ & & 0 \end{pmatrix}$$

$$i k_\mu x_\mu = A$$

$$i p_\mu O_{\mu\nu} u_\nu = B$$

$$e^{A \pm i B} = e^A e^{B \pm \frac{1}{2i} [A, B] \pm \frac{1}{3!} [A, [A, B]]}$$

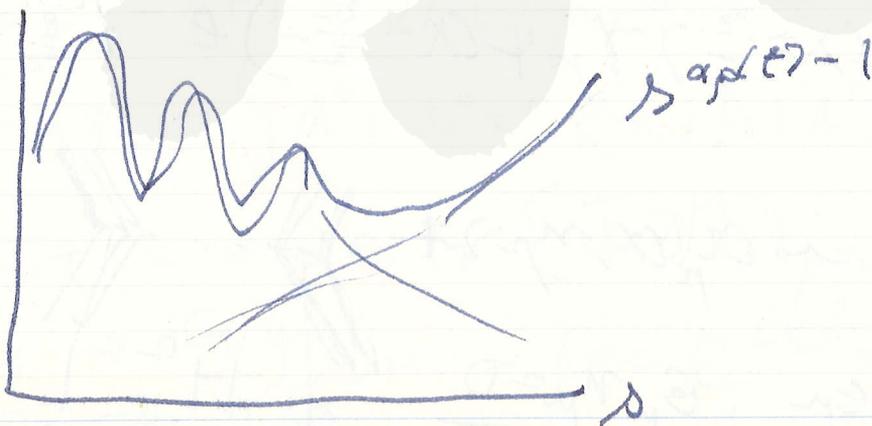
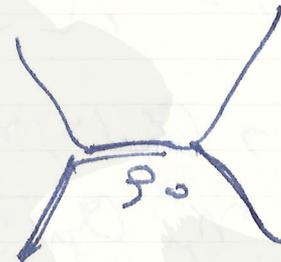
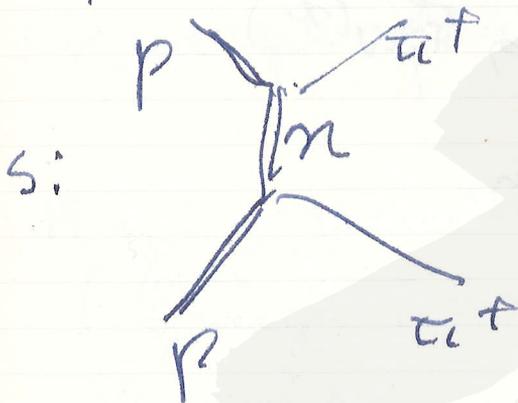
$$x_\mu = i \frac{\partial}{\partial p_\mu} \rightarrow \frac{(-1)^n}{(n+1)!} [A, \underbrace{[A, \dots [A, B] \dots]}_n]$$

$$e^A \cdot e^{\int_0^1 B(p + \lambda k) d\lambda}$$

$$\vec{P} = \vec{p} = \vec{R}_\mu \sqrt{\frac{2}{n}} (q_\mu^{(n)} e^{i q_\mu^{(n)} x}) \sqrt{\frac{2}{n}} \vec{p}_\mu$$

$$\vec{R}_\mu = \vec{\sigma}_{\mu\nu} (P, \lambda) \vec{R}_\nu$$

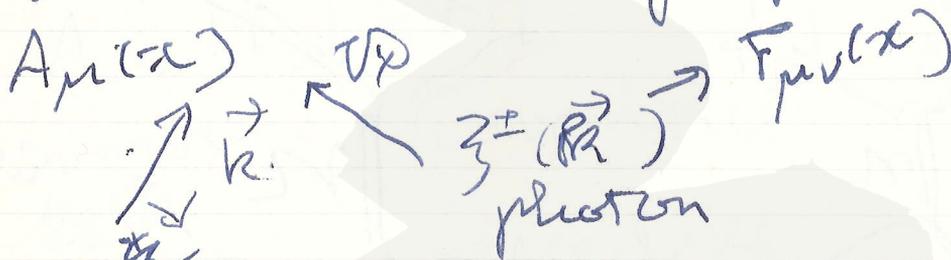
$$p + \pi^+ \rightarrow p + \pi^+$$



duality?

SU_3

QED: gauge invariance



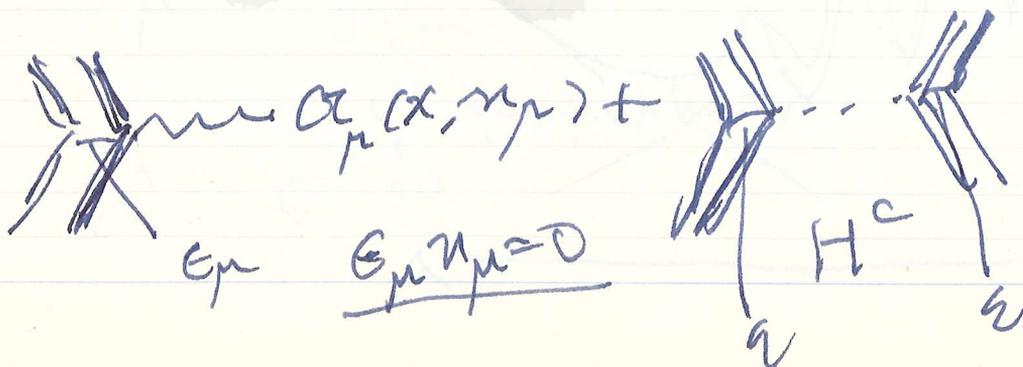
matter $\psi(x)$

$$\partial_\mu j^\mu(x) = 0, \quad [j^\mu(x), j^\nu(x')] \Big|_0 = 0$$

$$F_{\mu\nu}(x) \rightarrow \alpha_\mu(x, \eta_\mu) \hat{=} \eta_\nu \delta^{-1} F_{\mu\nu}(x)$$

$$\left. \begin{aligned} \eta_\mu \alpha_\mu(x) &= 0 \\ \partial_\mu \alpha_\mu(x) &= 0 \end{aligned} \right\}$$

$$F(x + \frac{\epsilon}{2}) \gamma_\mu \psi(x - \frac{\epsilon}{2}) e^{i\int_{x-\frac{\epsilon}{2}}^{x+\frac{\epsilon}{2}} \frac{A_\mu(x')}{c} dx'}$$



経路:

$$J_\mu(x, n_\mu) = \int d^4 \varepsilon \bar{\psi}(x + \frac{\varepsilon}{2}) \gamma_\mu \psi(x - \frac{\varepsilon}{2}) \\ \times \sqrt{\frac{\mu^3}{2\pi}} e^{-\varepsilon_\mu^2 \delta(\varepsilon_\mu - n_\mu)}$$

n_μ -dependence

How: non-local QED?

(test i.v.,
 gauge i.v.,

Peierls
 Bloch

local
 Marrow

Yukawa

$$A: \langle x | A | x' \rangle = A(\frac{x+x'}{2}, x-x')$$

$$\langle x | E | x' \rangle = \delta^4(x-x')$$

$$L_E = \text{tr} [\bar{\psi} \gamma^\mu (\partial_\mu \psi) + m_0 \bar{\psi} \psi]$$

$$\text{tr} AB = \int d^4 x d^4 x' \langle x | A | x' \rangle \langle x' | B | x \rangle$$

$$L_I = -ie \text{tr} [\bar{\psi} \gamma^\mu A_\mu \psi]$$

$$\psi \rightarrow G \psi \\ \bar{\psi} \rightarrow \bar{\psi} G^{-1}$$

$$G = e^{ie\psi A}$$

$$A_\mu \rightarrow G A_\mu G^{-1} - \frac{i}{e} [\partial_\mu, G] G^{-1}$$

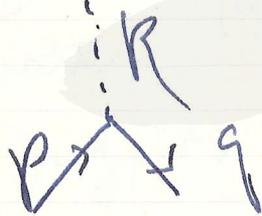
$\mathcal{L}_E \neq \mathcal{L}_I = \text{gauge inv.}$

$$F_{\mu\nu} = [\partial_\mu, A_\nu] - [\partial_\nu, A_\mu] - ie(A_\mu A_\nu - A_\nu A_\mu)$$

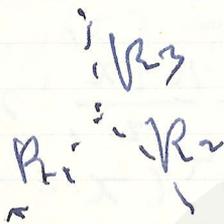
$$F_{\mu\nu} \rightarrow G F_{\mu\nu} G^{-1}$$

$$\mathcal{L}_P = \frac{1}{4} \text{tr} [F_{\mu\nu} F^{\mu\nu}]$$

Yang-Mills



$$\sum_g (P, g, R) = \sum_g (R_i \frac{P+g}{2})$$



$$\sum_{R2, R3} (R1, R2, R3) =$$

$$i_\mu = \bar{\Psi} \delta_\mu \Psi \quad [\partial_\mu, i^\mu] = 0$$

$$j_\mu = \delta_\mu \Psi \Psi - (A^\nu F_{\nu\mu} - F_{\nu\mu} A^\nu)$$

$$[\partial_\mu, j^\mu] = 0$$

$$[\partial_\mu, A^\mu] = 0$$

$$\text{tr} \rightarrow \text{tr}^* = \frac{1}{\sum c_i} \left\| c_i \langle x_1 | | x_2 - \frac{\epsilon}{2} \rangle \right.$$

$$\times \langle x_2 + \frac{\epsilon}{2} | | x_1 \rangle d^4 x$$

$$\langle x_1 | \exists | x_2 \rangle = \delta^4(x_2 - x_1 - \epsilon)$$

$$\tilde{\mathcal{L}}_F = \text{tr} \left[\exists \bar{\Psi} (\partial^\mu \Psi) + m_0 \bar{\Psi} \Psi \right]$$

$$\tilde{\mathcal{L}}_I = \text{tr} \left[\exists \bar{\Psi} \partial^\mu A_\mu \Psi \right]$$

$$\tilde{\mathcal{L}}_P = \frac{1}{4} \text{tr} \left[\exists F_{\mu\nu} F^{\mu\nu} \right]$$

$$\rightarrow \text{tr} (G^{-1} \exists G F_{\mu\nu} F^{\mu\nu})$$

$$\text{tr} (G \exists G^{-1} F_{\mu\nu} F^{\mu\nu})$$

帯取:



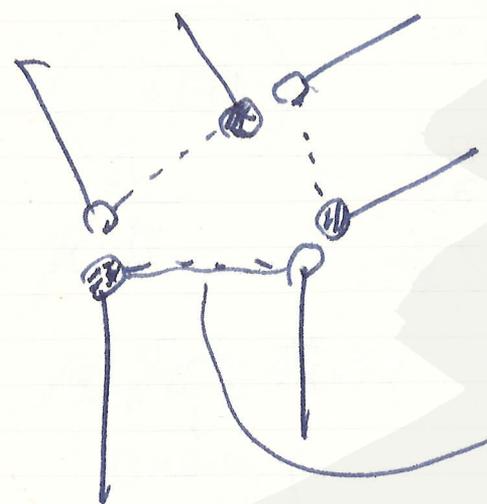
O.K.



X

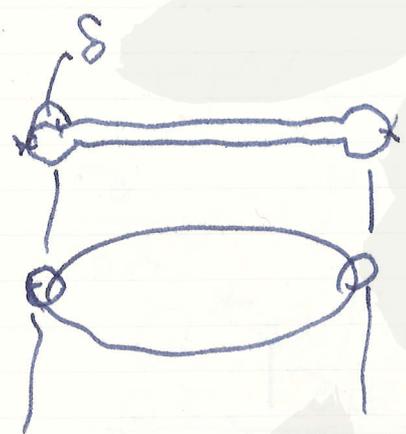
3月23日 H. Kawai
 模型

bilocal vs string



scattering angle,
 $\lambda \rightarrow \cos A \rightarrow E^{\cos}$
 causality?

bilocal \rightarrow string
 力の伝達



$$x_{\mu}(\sigma) \quad (0 \leq \sigma \leq 2L)$$

$$x_{\mu}(\sigma) - x_{\mu}(2L - \sigma) = 0$$

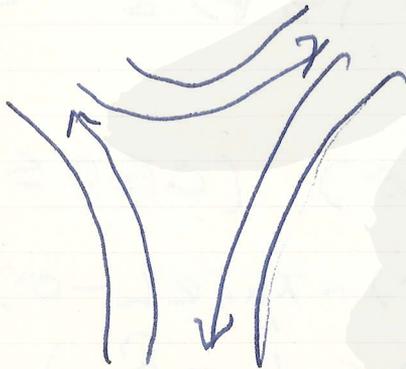
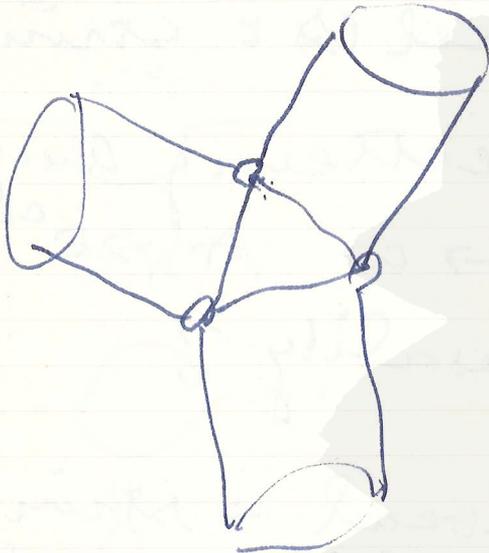
$$(\delta < \sigma < L - \delta)$$

$$p_{\mu}(\sigma) + p_{\mu}(2L - \sigma) = 0$$

$$\left. \begin{array}{ll} \bar{x}(0) & \bar{x}(L) \\ \bar{p}(0) & \bar{p}(L) \end{array} \right\}$$

$$\delta \rightarrow 0$$

$$\left. \begin{array}{l} \int_{\delta}^{L-\delta} \pi(\sigma) = \\ \int_{L-\delta}^L \end{array} \right\} (p_{\mu}(\sigma) + p_{\mu}(2L - \sigma)) = 0$$

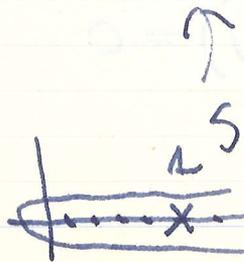


scattering amplitude

$$A \sim \frac{1}{\sin \pi \alpha(s)}$$

$$\times \int dt (-2)^{-\alpha(s)-1} (1-z)^{\cos}$$

$$c \times \left(\frac{1-z}{1+z} \right)^{c_1(t-u)} \times \square$$



$$\alpha(s) = c_2 s + c_3$$

今日の目録: 3次元理論 ~~の~~ 力の系
 質量密度 ρ の系



エネルギー密度
 energy

energy-momentum tensor

local rest system Σ^0
 local density ρ^* (scalar)
 質量密度 ρ

$$\rho = \frac{\rho^*}{\sqrt{1-v^2}}$$

$$= \rho^* u^0 u^0 = T^{00}$$

$$T^{\mu\nu} = \rho^* u^\mu u^\nu$$

$$\frac{\partial}{\partial x^\nu} T^{\mu\nu} = 0$$

$$Q^* u^\mu + \rho^* w^\mu = 0$$

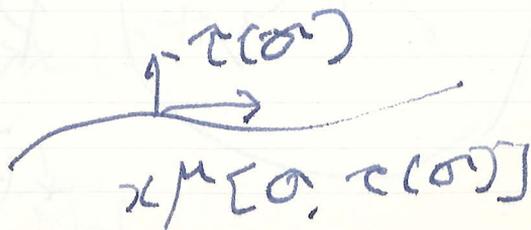
$$Q^* = \frac{\partial(\rho^* u^\mu)}{\partial x^\nu}, \quad w^\mu = \frac{\partial u^\mu}{\partial t}$$

$$u_\mu u^\mu = 1 \quad w_\mu u^\mu = 0$$

$$Q^* = 0 \rightarrow w^\mu = 0$$



string



Muler) 2594
 Four

pressure $P = \frac{1}{2} \left(\frac{\partial x^\mu}{\partial \sigma} \right)^2$

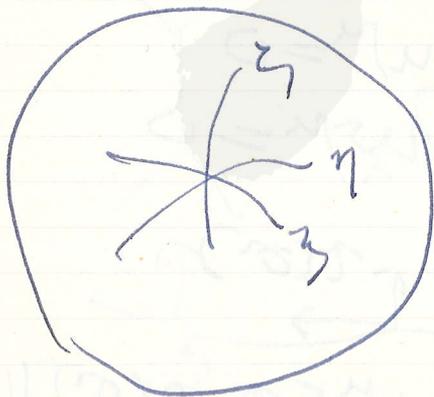
軌道
 $T^{\mu\nu} = \begin{pmatrix} -P & & \\ & -P & \\ & & -P_{\mu^*} \end{pmatrix}$

運動方程式
 $\rightarrow T^{\mu\nu} = (\mu^* + P) u^\mu u^\nu - P g^{\mu\nu}$

軌道方程式

$\frac{\partial T^{\mu\nu}}{\partial x^\nu} = 0$
 $\mu^* = f(P)$

$(\mu^* + P) u^\mu = g^{\mu\nu} \frac{\partial P}{\partial x^\nu} - \frac{\delta P}{\delta \tau} u^\nu$



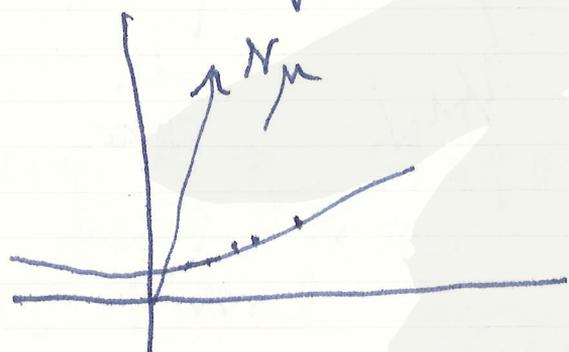
$x^\mu \{ \xi, \eta, \zeta, \tau(\xi, \eta, \zeta) \}$

$P = \frac{1}{2} \left\{ \left(\frac{\partial x^\mu}{\partial \xi} \right)^2 + \left(\frac{\partial x^\mu}{\partial \eta} \right)^2 + \left(\frac{\partial x^\mu}{\partial \zeta} \right)^2 \right\}$

23日 4/13
 図12.1: 湯川記号のU-121カウ



湯川記号の
 新L... dynamics



form factor

$$P_\mu = \int p_\mu a^\dagger a$$

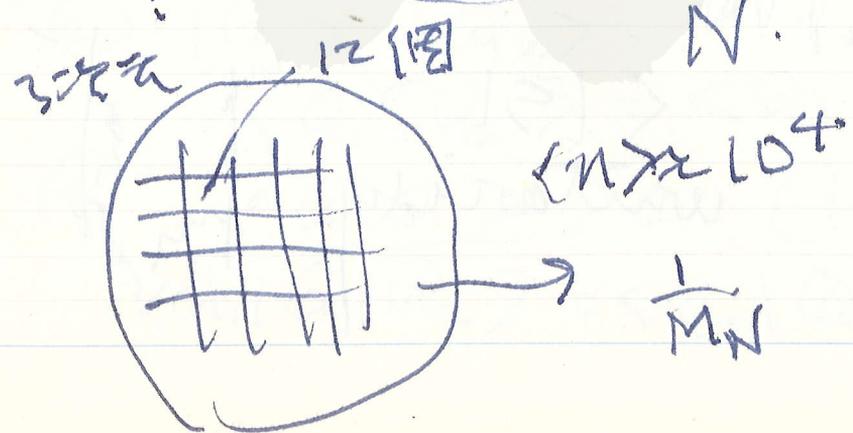
$$\int d^3x N_\mu \psi \psi \psi \psi$$

$\sim 1/M \sim \lambda \approx 10^{-15} \text{ cm}$

$$d\rho = \rho e^{-\rho} \frac{d^3p}{p_0} e^{-\lambda(p \cdot N)}$$

湯川記号のU-121カウ

hadronic matter

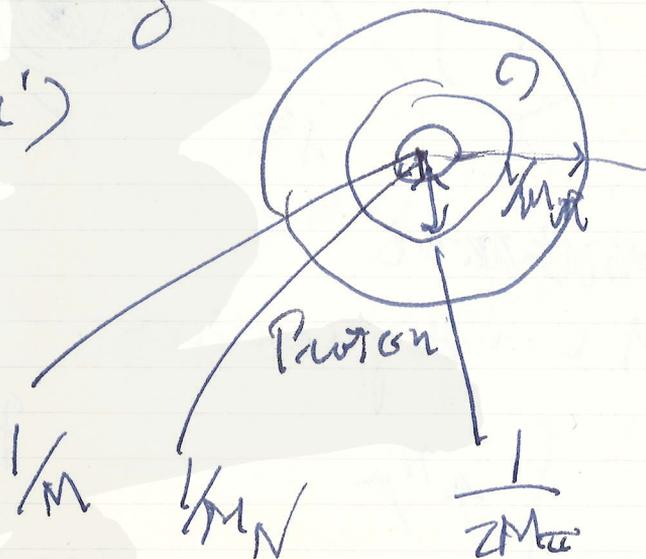


相互作用

$$g \Phi \Phi \Phi \quad g \pi \pi \pi$$

$$G(\alpha - \pi')$$

$$Y_{\text{had}} \sim 10^5$$



P-γ: F.F.
 E.M: F.F.

$$\sigma \sim \pi \left(\frac{1}{M\pi} \right)^2$$

$$\langle P_{\perp} \rangle \sim 2 \sim 3 M\pi$$

中間子: 2π or 3π のような状態
 の相互作用

経路積分
 表示)

$\langle |S| \rangle$
 unitarity



causality
macrocausality



山崎論文: 量子場の unitary 性とその物理的意義

(1) total norm of the field

(2) physical state condition

Hilbert space

(1) $|a\rangle \xleftrightarrow{|a\rangle} \langle a|$

(2) $|a_1\rangle, |a_2\rangle$

$$\langle a_1 | a_2 \rangle = \langle a_2 | a_1 \rangle$$

(3) $\langle a | a \rangle > 0 \quad (|a\rangle \neq 0)$

$$\langle a_1 | a | a_2 \rangle = \langle a_2 | a^\dagger | a_1 \rangle$$

$$\left. \begin{aligned} (Q^T)^T &= Q \\ (cQ)^T &= \bar{c} Q^T \\ (Q_1 Q_2)^T &= Q_2^T Q_1^T \end{aligned} \right\}$$

indefinite metric

③ $\langle \alpha | \alpha \rangle = \langle \alpha | \alpha \rangle$ \rightarrow $\langle \alpha | \alpha \rangle = \langle \alpha | \alpha \rangle$
 physical state $\langle \alpha | \alpha \rangle = 1$ ① ② ③
 $\langle \alpha | \alpha \rangle = \langle \alpha | \alpha \rangle = \langle \alpha | \alpha \rangle$

$$\left. \begin{aligned} (Q^*)^* &= Q \\ (cQ)^* &= \bar{c} Q^* \\ (Q_1 Q_2)^* &= Q_2^* Q_1^* \end{aligned} \right\}$$

$Q|0\rangle \leftrightarrow \langle 0|Q^*$
 1 \leftrightarrow 1 \leftrightarrow 1
 1 \leftrightarrow 1 \leftrightarrow 1

$\langle 0|Q^* Q|0\rangle$ is - a.c complex
 $S^* S = 1$ is total norm of $\{a, a^*\}$

Example

- $H = \int d^3k \left\{ \alpha^*(k) a(k) + \beta^*(k) a(k) + \beta(k) a^*(k) \right\}$
- $[a(k), a^*(k')] = \omega_0 \delta(k - k')$

$$[\alpha(k), \beta^*(k')] = \omega, \delta \quad \omega_0: \text{real}$$

$$[\beta(k), \alpha^*(k')] = \bar{\omega}, \delta$$

$$\text{Im } \omega, < 0$$

3. $a|0\rangle = \alpha|0\rangle = \alpha^*|0\rangle = 0$
 $\langle 0|a^* = \langle 0|\beta = \langle 0|\beta^* = 0$

4. $H_{int}^* = H_{int}$

$$S = T \exp(i \int H_{int} d^4x)$$

$$S^* S = 1$$

$$[S, H] = 0$$

$$|a\rangle = \underbrace{\alpha^{\dagger} \dots \beta \dots \beta^{\dagger} \dots}_{\mathcal{Q}} |0\rangle$$

$$E = \begin{cases} \text{real} & \text{for prep. state} \\ \text{complex} & \text{Im } E < 0 \end{cases}$$

$$|P\rangle = a^{\dagger} \cdot a^{\dagger} |0\rangle \quad \text{physical state}$$

$$|Q\rangle \neq 0 \quad \leftarrow \quad |0\rangle$$

$$|0\rangle \leftarrow \quad |P\rangle \neq 0$$

出 発 点 :

$$[\rho_\mu, [\rho_\mu, \phi]] + m^2 \phi = 0$$

$$[\rho_\mu, [\rho_\mu, \phi]] - \mathcal{L} \phi$$

$$-\hbar \frac{\partial}{\partial E} \phi' = \mathcal{L} \phi'$$

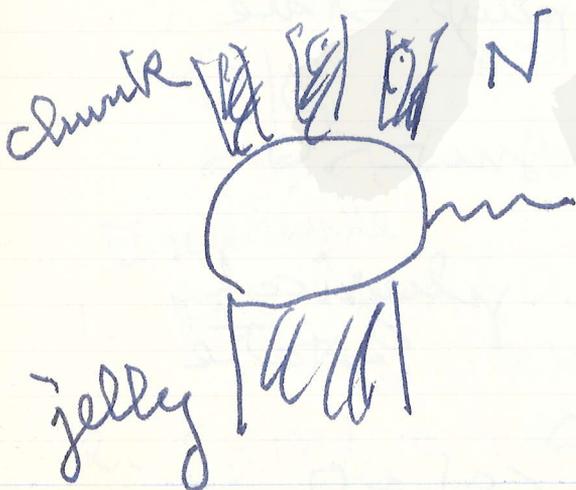
$$\mathcal{L} \phi' = t \phi'$$

$$\mathcal{L} = \mathcal{L}(\rho, \phi)$$

出 発 点 : 物 理 系 \rightarrow 1D 弦 振 動 系
 a-弦 光

H. B. Nielsen

d=1 string



transverse
momentum

\rightarrow zero point
oscillation

24日(土)

高橋 邦: 4次元電磁気

Casimir,
 K. T. Yang,
 J. Schwinger, *Rev. Mod. Phys.*
 Heitler

$$W(x) = \int d^4x' d^4x'' d^4x''' \bar{\psi}(x') \gamma_\mu \psi(x'') \\
 \times A_\mu(x''') F(x-x', x-x'', x-x''')$$

$$m(v) = \frac{m(0) - v^2 S(0)}{\sqrt{1-v^2}}$$

J. S. Bell, 1959~1960 $\frac{1}{\sqrt{1-v^2}}$
Nuovo Cimento 9.11

gauge invariance

convergence,
 (relativistic invariance
 1+共変性) τ^2 変換

Gauge

$$\delta F = [F, H] \\
 \frac{\delta \Psi}{\delta t} = 0$$

$$a v = E \\
 a v_{\perp} = E_{\perp} - E_{\parallel}$$

$$+$$

$$m(v) = \frac{m(0)}{\sqrt{1-v^2}}$$

para statistics

Pauli spin-statistics

$\mathbb{R} \rightarrow$ symmetry \rightarrow conservation

$\phi(x) \phi^\dagger(x')$

$-i \partial_\mu \phi = [\phi, P_\mu]$: identity

$[\phi(x) \phi^\dagger(x')]_{\pm} = \delta^4(x-x')$

$T_M = -\frac{i}{2} \int d^4x \{ \phi^\dagger(x) \partial_\mu \phi(x) - \partial_\mu \phi^\dagger(x) \cdot \phi(x) \}$

$(\square - m^2) \phi(x) \Omega = 0$

S-matrix? \rightarrow unitarity
 spin-statistics \rightarrow

(1) $T^* \in \mathcal{H} \otimes \mathcal{H}$
 relativistic theory
 unitarity $\forall \mathcal{H} \otimes \mathcal{H} \subset \mathcal{H}$

(2) higher spin
 spin $3/2$, e.m.f.
 Sudarshan - Johnson Paradox

(3) unitary representation
 Tadpole $\forall \mathcal{H}$?

[4] 重力波

参考文献: unitarity of S.M.

unitarity overall
 cons. ↑

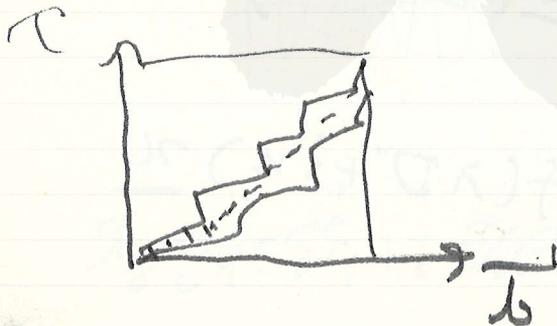
	$h(\nu) = \frac{m(\nu)}{\sqrt{1-\nu^2}}$	$h\nu = E$	u.e.c.	overall cons.
Møller (T)	yes	?	no	no
Heider (T)	no	yes	yes	yes
Tanaka	no	yes	yes	yes
Katayama	yes	?	no	no
4-2122222222	yes	yes	?	no
Q.E.V.	yes	yes	yes	no

参考文献: stochastic theory

known 490

impact parameter b

(\vec{p}_\perp is 8888 canonical conjugate)



$$|\dot{f}(\vec{b}, \tau)|^2 d\vec{b} \equiv \text{Prob}(\vec{b}, \tau) d\vec{b}$$

$$d\sigma = \int \text{emp} \{ -\vec{k}^2 d(d-1 + e^{-\alpha}) \} d\vec{k}$$

$$d = \langle \vec{u} \rangle \tau_c$$



M. Bander, Y. R. 6D July (1972)

条件

1P: 差分方程式

$$v(\tau) - v(\tau + \Delta\tau) = 0$$

$$L_\lambda = \frac{m}{2} D_\lambda \cdot x f(\lambda D) D_\lambda x$$

$$f(\tau) = \frac{\rho \sin \tau}{\pi \tau} \quad , \quad D = \frac{d}{d\tau}$$

$$f(\lambda D) v^z = \rho x (\tau + \pi \lambda)$$

$$- \rho x (\tau - \pi \lambda) = 0$$

$$x[0] = f(\lambda D) x$$

$$x[n] = \sqrt{2} \text{Re} f(\lambda D + i\eta) x$$

考察: 量子の「中」の波動性

(益川)

波動: 古典的連続波

空間: 量子状態

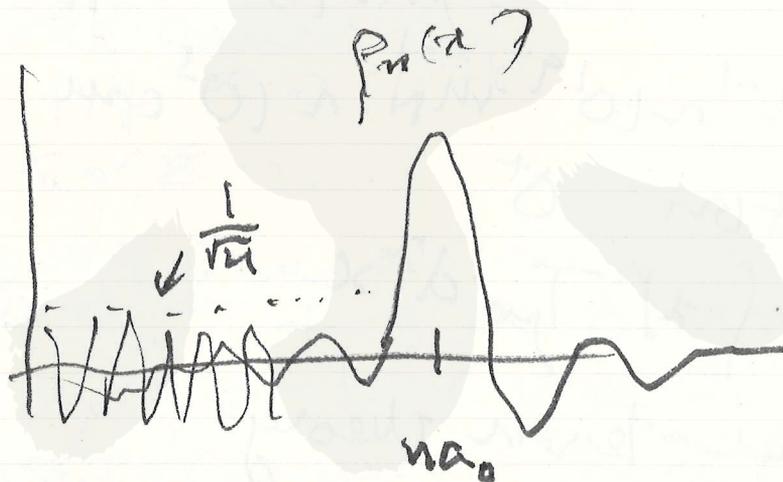
$$\varphi_n = \varphi_{n_0, n_1, n_2, n_3}$$

$$\varphi'_n = L_n(\Lambda, a) \varphi_n$$

$$a_0 \rightarrow 0; \quad \varphi'(x') = \varphi(x)$$

$$x' = \Lambda x + a$$

$$\Lambda^\mu L_n^\nu(\Lambda, a) n_\mu + a_\nu = n_\nu$$



量子状態は波動Lなる。

検証: scale invariance
重力

$$\varphi = v + \sigma$$

$$v \sim \frac{1}{\sqrt{G}}$$

$$m_{\nu} = g v$$

$$V \sim \frac{G}{r} (1 + A e^{-r})$$

$$c = \hbar = 1 \quad \sqrt{G m_{\nu}} \sim 10^{-19}$$

$$\mu \sim 10^{-19} \text{ MN}$$

$$M^{-1} \sim 10^{19} \text{ MN} \sim 10^5 \text{ cm}$$

dilatation δ^+

$$\mathcal{D} = \int \frac{1}{2} T_{\mu\nu} d^4x$$

Scalar-tensor theory

Dicke

$$\frac{GM}{R} \sim 1$$

Nonlocal QED

片山 隆久

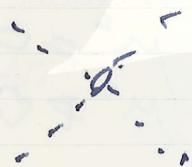
河津 公 1973, 4, 19

発行

Gauge Invariance

Peierls: 相互作用の表示.

Yang-Mills:



問題点

1. Σ_4 , Σ_{48} 既知因子 \leftarrow $\sim \epsilon^{\mu\nu\rho\sigma}$ の存在
 A) 4-成分 gauge fields
 $f, g \propto e^{-r/2\lambda}$
 長距離相互作用.

B) 3-成分 gauge fields
 方程式: $f, g \propto \delta(nr) e^{-\frac{r}{2\lambda}}$
 $n_\mu n^\mu = -1$ (方程式)

\sim n -dependent, self-stress ∇
 $\delta(nr) e^{-\frac{r}{2\lambda}}$ (2次元)
 $n_\mu = \frac{v_\mu}{\sqrt{-v^2}}$

photon self-energy $\omega \sim \omega^2$

2. $\omega = \tau \int \dots$
 有質量? $\omega \sim \omega^2$

$e^{-\lambda^2 (k^2)^2}$?

3. 有質量? $\omega \sim \omega^2$

$\lambda \neq 0 \rightarrow \text{mass} [M \bar{\psi} \psi]$

$\lambda = 0 \rightarrow \mu^2 = 0$ (質量)
 $\mu^2 \neq 0$ (有質量)



4. $\psi \bar{\psi} \psi$

$\psi (\bar{\psi} \psi)$

5. $\psi \bar{\psi} \psi$

湯川と宇佐美

May 17, 1973

仙崎文隆

土曜例会

Redshift of OH 472

Carswell, Strickmatter (P1171)

Name 242 April 6

Ohio State Univ 天竺望遠鏡

Radio-telescope Survey

Palomar Survey (光学)

$$m_B = 18 \sim 8.5 \text{ m}$$

$$3200 \sim 5800 \text{ \AA}$$

2# of emission line

$$\left\{ \begin{array}{l} 4548 \text{ \AA} \\ 5351 \text{ \AA} \end{array} \right. \quad 1:1.177 \quad \text{OVI} \quad 1033.8 \text{ \AA}$$

$$z = \frac{\Delta\lambda}{\lambda_0} = \frac{\lambda - \lambda_0}{\lambda_0} = 3.40 \quad 1:1.176$$

~~$z = \frac{v}{c}$~~

$$z = 2.827 \quad 4C05.34$$

QSO '63 0.158 }
 '65 2.2 }
 '67 2.36 } 3C273 }
 '70 2.58 } 0.367 }
 '73 2.827 } 3C48 }

Zammir → QSO: 250
 > galaxy: 150

Slipher '14 $v = 10^3$ km/sec
 Hubble '29 $(z = 0.003)$
 $v = HR$
 Palomar $0.003 \leq z < 0.02$
 0.2

Minkowski '54 Radiogalaxy
 - Vsaee Cyg A $z = 0.056$
 (2 yr 2 1/2 1/2 1/2 1/2 1/2) } 3C 295 $z = 0.4614$
 (1960)

Doppler shift

$$z + 1 = \frac{\sqrt{1 + \frac{v}{c}}}{\sqrt{1 - \frac{v}{c}}} \quad (z = \frac{v}{c} \text{ for } \frac{v}{c} \ll 1)$$

$$\frac{v}{c} = 0.8 \rightarrow z = 2$$

$$= 0.9 \rightarrow z = 3.9$$

膨張宇宙空間

$$ds^2 = dt^2 - a(t)^2 dl^2$$

comoving coordinate

$$ds = 0:$$

$$l = \int_{t_1}^{t_2} \frac{dt}{a(t)}$$

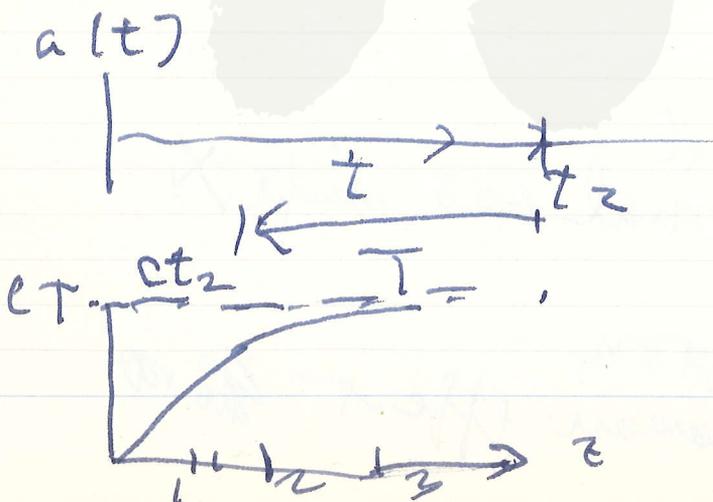
$$= \int_{t_1 + \Delta t_1}^{t_2 + \Delta t_2} \frac{dt}{a(t)}$$

$$\frac{\Delta t_1}{a(t_1)} = \frac{\Delta t_2}{a(t_2)}$$

$$v \propto \frac{1}{\Delta t} \propto \frac{1}{\lambda}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{a(t_2)}{a(t_1)} = 1 + z > 1$$

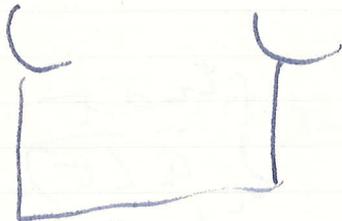
$$a(t) \propto t^{2/3}$$



$$\frac{z_1 + 1}{z_2 + 1} = \left(\frac{t_2}{t_1} \right)^{2/3}$$

宇宙の大きさ
 100億光年 ~ 200億光年

Radio astronomy



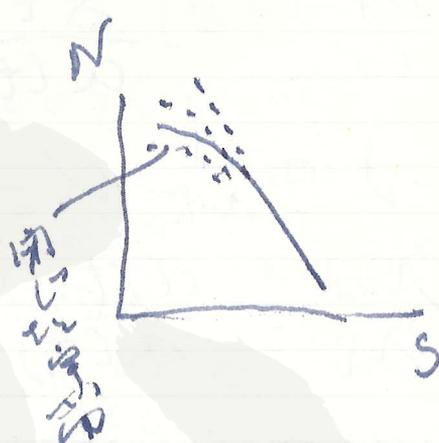
D
 1 mile

Rayle

$$S = \frac{1}{r^2} \frac{1}{(1+z)^2}$$

$$N \propto r^3$$

$$N \propto S^{-3/2}$$



宇宙の膨張
 cosmological
 gravitational red shift X
 Doppler
 (photon-photon
 Raman Raman effect 宇宙の膨張)

Haray [171]
"A2" の2号10月。(野谷氏)

多気体分布と粒子数平均

河津会 1973.6.14

横山君

T-Y
 M-T-Y

1) $\sigma_r = \text{const}$

2) $\mu_r = \text{const}$

3) 多気体

4) 非平衡状態

300-400 MeV/c

$\langle n \rangle \sim S^{3/4}$ or $\log S$

5-10 GeV/c
 多気体

多気体は Poisson 分布に近づく

NA 200 GeV/c

Charlton et al.: pp \rightarrow n charged particles
 Correlation parameter f_2

$$f_2 = \langle n(n-1) \rangle - \langle n \rangle^2 \approx 0$$

$$P_n = e^{-\langle n \rangle} \frac{\langle n \rangle^n}{n!} \quad n \rightarrow n'$$

$$\langle n(n-1) - (n-1+1) \rangle = \langle n \rangle^2$$

$f_2 \approx 0$ at 50 GeV/c



$$P_n = \frac{\sigma_n}{\sigma_{\text{total}}}$$

平均値 \bar{n} の Poisson 分布

f_2 : 2 個の相関

$$\langle n = \alpha \cdot n_c \rangle \quad n_c: \text{cluster の数}$$

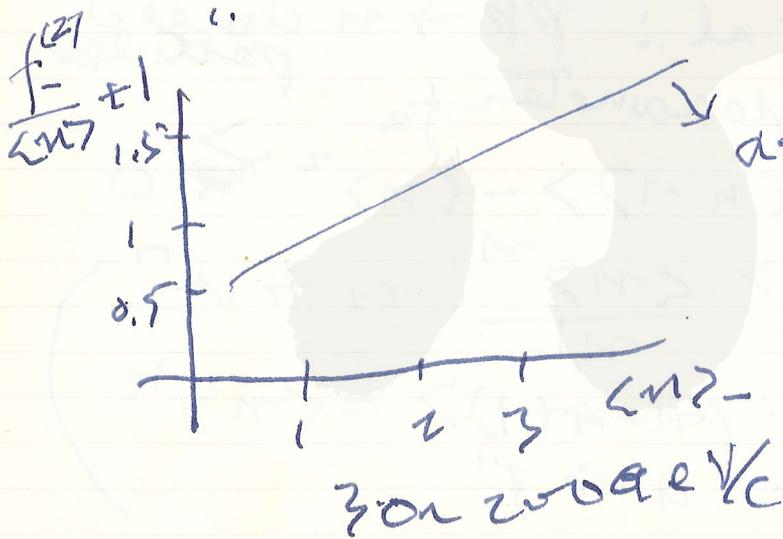
n_c の分布は Poisson 分布 $\lambda = \bar{n}$

$$f_{2c} = \dots \text{etc.}$$

$$f_2 = (\alpha - 1) \langle n \rangle$$

$$f_3 = (\alpha - 1)(\alpha - 2) \langle n \rangle$$

\vdots



Koba-Nielson-Olesen
scaling relation

$$P_n = \frac{\sigma_n}{\sigma_{tot}} \rightarrow P(n) = \frac{L}{\langle n \rangle} \psi\left(\frac{n}{\langle n \rangle}\right) + \frac{1}{\langle n \rangle^2} \psi^{(1)}\left(\frac{n}{\langle n \rangle}\right)$$

z
||

$$\langle n^k \rangle \rightarrow \sum_n n^k P_n \rightarrow \int n^k p(n) dn$$

$$= \langle n \rangle^k \int z^k \psi(z) dz$$

ψ is independent

$$\langle n^k \rangle = \langle n \rangle^k c_k$$

$$50 \text{ GeV} \sim 300 \text{ GeV}/c \quad (\text{NAL})$$

↓

$$f^{(k)} \rightarrow P_n$$

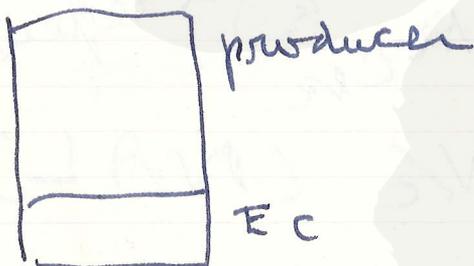
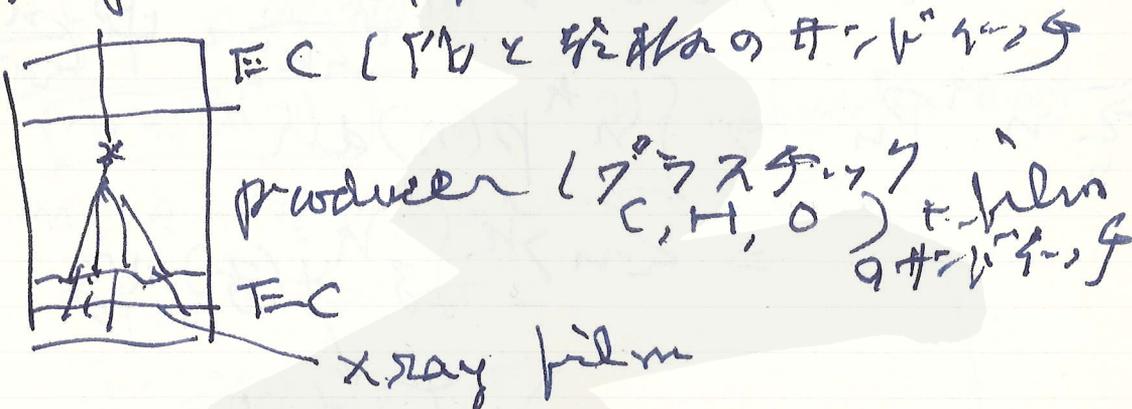
湯川会

AT 湯川 (9th. 期)

Jet Shower Analysis

1973年 7月12日 荻原

X-particle 分析



湯川会 27 Jet Shower 3期

$\gamma \rightarrow e^+e^-$ mass $\eta^0 \rightarrow 3\pi^0$
 $(\rightarrow 6\gamma)$

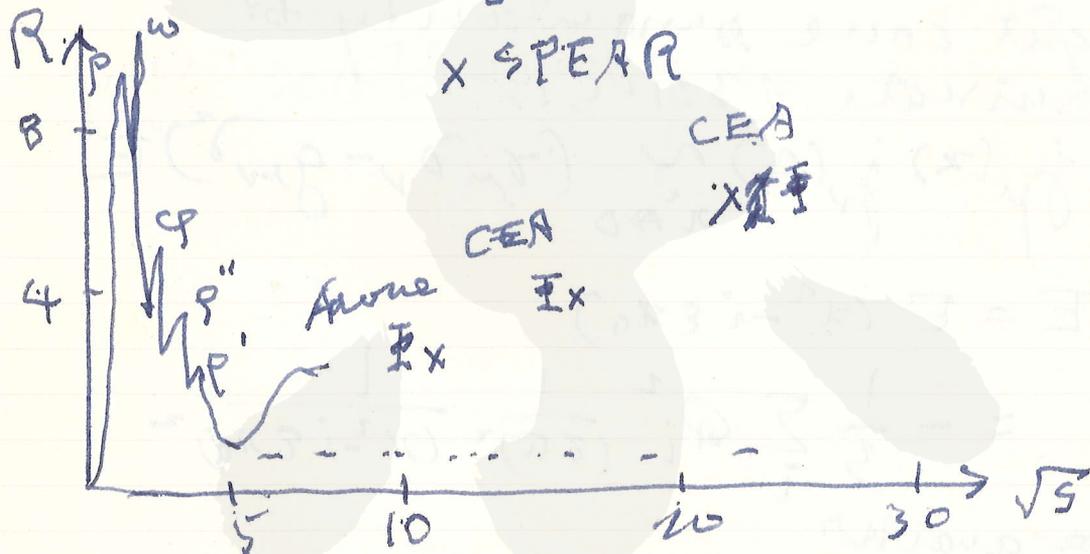
Jet shower AEC-11-C-39

年表: 三

e^-e^+ annihilation と素. 理. 程. の
 概観

加速器 1974 年 10 29 10

加速器	ISR Colliding beam	\sqrt{s}
X ACO (Orsay)	$E_{e^-} + E_{e^+} = 2E_e$	1.0 GeV
VEP-2 (Novosibirsk)		1.5
Anone (Frascati)		3.0
X CEA (Cambridge)		5.0
1973 SPEAR (SLAC)		5.4 (1973) → 9.0
1974 DORIS (DESY)		5.4 (1974) → 8.4
1975 DCI (Orsay)		3.0



$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadron})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \rightarrow \frac{4\pi\alpha^2}{3s}$$



理論

one-photon \leftrightarrow α^2

$$\sigma = \sum_x \left| \text{Diagram} \right|^2 = \frac{8\pi^2 \alpha^2}{s} W(s)$$

$$\left(\frac{R_{\mu\nu} k_\nu}{k^2} - g_{\mu\nu} \right) W(k^2) \equiv \int dx e^{ikx} \langle 0 | j_\mu(x) j_\nu(0) | 0 \rangle$$

$W(k^2)$: dimensionless

$$R = 6\pi W(s)$$

$$\sigma \xrightarrow{s \rightarrow \infty} \frac{1}{s} \rightarrow R = \text{const}$$

Right come singularity for
 dominate $\sim 3\pi i/s$.

$$j_\mu(x) j_\nu(0) \underset{x^2 \neq 0}{\sim} (\gamma_{\mu\nu} \partial_\nu - g_{\mu\nu}) E$$

$$E = E(x^2 - i\epsilon x_0)$$

$$= -\frac{1}{3} \sum_i Q_i^2 \frac{1}{(x_0)^2} \frac{1}{(x^2 - i\epsilon x_0)^2}$$

(free quarks)

$$j_\mu = \sum_i Q_i \bar{\Psi}_i \gamma_\mu \Psi_i$$

$$R \sim \sum_i Q_i^2$$

Asymptotic $i\epsilon$ in \mathcal{S} &
charged scalar field

$$\partial_\mu \langle \phi | \phi^\dagger(x) \partial_\mu \phi(x) \rangle \sim A q_\mu$$

$$R \sim Q^2 \left(1 - \frac{q m^2}{s}\right)^{3/2} [1 + 4 \operatorname{Im} a_1(s)]$$

$$\operatorname{Im} a_1(s) \leq 1$$

$$\leq 5 Q^2 \left(1 - \frac{q m^2}{s}\right)^{3/2}$$

$q-q$ scattering

$p-p$ scattering