Formation of Light Nuclei in the Expanding Universe

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Abstract

Building up of light nuclei up to Ne^{20} out of initial protons and neutrons in the early stage of the expanding universe is studied, taking account of the α-capture reactions. If we take the matter density $\rho_m = 10^6 \sim 10^7$ g/cm^3 at $T = 10^{10}$ °K, we can obtain the relative abundance of hydrogen, helium and oxygen consistent with the present observation. The possibility of forming heavier nuclei is discussed.

§ 1. Introduction and Summary

One of the outstanding astrophysical problems is the origin of chemical elements in our universe. Two different hypotheses have been proposed so far. In one the elements are assumed to be formed during the pre-stellar stage of the universe. The so-called α-β-γ theory[1] is a typical one based on this hypothesis. According to this theory the primordial substance from which the elements were formed was a highly compressed neutron gas. As the gas temperature fell as the result of the expansion of the universe, neutrons decayed into protons, and the elements were built up through successive neutron captures and β-decays. The most serious difficulty of the α-β-γ theory is the fact that sufficient amount of carbon and oxygen cannot be formed due to the absence of stable nuclei of mass number 5 and 8.[2)]

The other hypothesis, as suggested by Hoyle[3)], Fowler and Greenstein[4)], is that the elements originated in dense and hot interior of stars, starting from protons only. This theory is supported by the anomalous abundances of the heavy nuclei observed on some stars. It has also the difficulties that the natural α-radioactive nuclei will not be formed through nuclear reactions which proceed in a rather long time scale under the stellar condition, and that we have no observational evidence for the stars composed entirely of hydrogen. It must be admitted, however, that some fraction of the present matter has undergone nuclear reactions in its stellar stage.

At present, we can not assert which idea, pre-stellar or stellar, is more preferable to explain the main feature of the chemical abundance. This problem will be intimately connected with the further development of the evolutional scheme of stars as emphasized by Taketani, Hatanaka and Obi.[5)] In this paper we shall examine whether or not the above mentioned difficulties of the α-β-γ theory can be avoided by considering the α-capture processes such as $3He^4 \rightarrow C^{12}$ which have recently been studied by Hayakawa et al.[6)] based on new experimental data.

It is necessary to take the matter density much higher than in the case of α-β-γ theory for the formation of C^{12}

nuclei by the α-reactions. Under such a condition the expansion of the universe will be faster than the neutron-decay. On this point it is favorable to follow the improvement made by one of the present authors[7)*] that there exist protons several times more abundant than neutrons in the pre-elements stage as the results of their interactions with energetic electrons and neutrinos such as

$$\begin{aligned} n + e^{+} &\rightleftarrows p + \bar{\nu} \\ n + \nu &\rightleftarrows p + e^{-} \\ n &\rightleftarrows p + e^{-} + \bar{\nu} \end{aligned} \tag{1.1}$$

The neutron-proton ratio at the beginning of the formation of light nuclei was found to be nearly 1:6 in the case of low matter densities as used in the α-β-γ theory. Applying the results in I to the case of high matter densities $\rho_m = 10^5 \sim 10^7$ g/cm^3 at the temperature 10^{10} °K, we find that the n-p ratio is $1:2.5 \sim 1:4$ as shown in § 2.

Now we consider the formation of light nuclei, dividing it into two stages. In the first stage, ~~the processes forming~~ very light nuclei up to He^4 are formed out of the above

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* Hereafter we refer to as I. In I, the calculation was performed using wrong value of neutron decay life-time 20 min. instead of 12.8 min. due to uncertain experimental data at that time. Correcting this point, numerical results in I agree with those by Alpher and Follin.[8)]

neutrons and protons and in the second stage carbon, oxygen and heavier nuclei are built up. The first stage begins at $T \lesssim 9\times10^{9}\,{}^{\circ}K$ in the case under consideration. When the expansion of the universe proceeds and its temperature falls to the one corresponding to the nuclear binding-energies, neutrons and protons react with each other to form deuteron, tritons, He^3 and He^4. Among these reactions we choose the following processes with high reaction rates:

$$\begin{aligned} p+n &\rightleftarrows d+\gamma+2.23\ \text{Mev},\\ d+d &\rightleftarrows t+p+4.04\ \text{Mev},\\ d+d &\rightleftarrows He^3+n+3.26\ \text{Mev},\\ d+t &\rightleftarrows He^4+n+17.58\ \text{Mev},\\ d+He^3 &\rightleftarrows He^4+p+18.34\ \text{Mev}. \end{aligned} \qquad (1.2)$$

In the state of such high temperature as $T \gtrsim 9\times10^{9}\,{}^{\circ}K$, we must include the inverse reactions but can neglect the β-decays because of the fast expansion of the universe. When all the nuclear reactions (1.2) is faster than the expansion of the universe at the beginning of this stage, there exists a statistical equilibrium. As the temperature falls, the rates of inverse reactions with large Q-values decrease, which produce a deviation from the equilibrium state. Calculations show that at the end of the first stage, we have

$$n_p : n_{He^4} = 3:1 \sim 6:1,$$

and amounts of neutrons, deuterons, tritons and He^3 nuclei are very small ($< 10^{-7}$ of protons) as shown in Fig. 2a and 2b in § 3. Such a small amount of neutrons will be insufficient to build up the heavy elements according to the scheme of α-β-γ theory. In the later stage, we have only to consider the reactions in which protons and He^4 will play a role.

When the temperature falls to $4 \sim 5\times10^9$ °K, the formation of C^{12} and O^{16} by successive α-capture reactions begins as the second stage. In ~~this~~ our case the matter density is so high that a considerable amount of Be^8 nuclei can be in statistical equilibrium with He^4.[6] These Be^8 nuclei collide with other He^4 to form C^{12} nuclei, which, in turn, capture additional He^4 to form O^{16} nuclei. Taking the matter density $\rho_m = 10^6 \sim 10^7$ g/cm^3 at $T = 10^{10}$ °K, we obtain final O^{16} abundance in accordance with the observational cosmic abundance, but the calculated C^{12}-O^{16} ratio is comparatively smaller than the observational one. As a possibility to avoid this difficulty we can consider the spallation of O^{16} nuclei by non-thermal protons which are accelerated by the turbulence accompanying the expansion of the universe (see in § 4).

Finally, in § 5, we will discuss the results of our calculations and the other possible routes of forming C^{12} and O^{16} out of protons and He^4 nuclei only. As the next stage of the formation of light nuclei, we will have to consider the processes forming Ne^{20} and heavier nuclei from protons,

He^4 and O^{16} nuclei. In this stage, the expansion of the universe is so slow that β-decays, such as $Na^{21} \rightarrow Ne^{21} + e^+$, $Al^{25} \rightarrow Mg^{25} + e^+$, etc., can not be neglected. As the results of (α, n) reactions involving Ne^{21}, Mg^{25}, etc., some amount of neutrons will be produced. Therefore it may be possible that heavy elements are synthesized by the captures of these neutrons.

§ 2. Proton-neutron ratio before the formation of He^4 nuclei

According to the general theory of relativity, the expansion of the universe in the stage of high density is given by

$$\frac{1}{\ell}\frac{d\ell}{dt} = \left(\frac{8\pi}{3} G\rho\right)^{\frac{1}{2}} \qquad (2.1)$$

where ℓ is an arbitrary proper length of a volume containing a given amount of matter, ρ the total mass density and G the gravitational constant. We restrict ourselves in the case where matter density ρ_m is greater than radiational mass density $\rho_r = \frac{aT^4}{c^2}$, that is,

$$\rho_m > 10^5 \text{ g/cm}^3 \quad \text{at} \quad T = 10^{10}\,{}^\circ\text{K}.$$

If $\rho_m < 10^8$ g/cm^3 at $T = 10^{10}\,{}^\circ$K, radiation pressure p_r is greater than matter one p_m. Further, in the case where $\rho_m \lesssim 10^7$ g/cm^3 at $T = 10^{10}\,{}^\circ$K, we can neglect the rather complicated effect of the degeneracy of electrons on the reactions (1.1) as shown in I.* For simplicity we take as

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* According to (9) of I, we find λ, which expresses the degree of degeneracy, is less than 0.2 in the case under consideration.

the range of ρ_m:

$$10^5 \lesssim \rho_m \lesssim 10^7 \ \mathrm{g/cm^3} \quad \text{at} \quad T = 10^{10}\,{}^\circ\mathrm{K}. \tag{2.2}$$

It will be shown in § 4 that an appropriate amount of oxygen can be formed under the condition (2.2). Neglecting the contribution of kinetic energy of matter to ρ, we obtain from the equation of adiabatic expansion,

$$\frac{d(\rho \ell^3)}{d\ell^3} = \frac{d(\rho_r \ell^3)}{d\ell^3} = -\frac{1}{3}\rho_r . \tag{2.3}$$

From (2.3) we have $\rho_r \propto \ell^{-4}$ and $T \propto \ell^{-1}$. As $\rho_m \propto \ell^{-3}$ we can put

$$\rho_m = \rho_0 \left(\frac{T}{10^{10}}\right)^3 , \tag{2.4}$$

with ρ_0 as a constant specifying the universe. We have from (2.1), (2.4) and (2.5)

$$\rho_m = 7.9 \times 10^5 \ t^{-2} \tag{2.5}$$

and

$$\frac{1}{T} = 1.08 \times 10^{-10} \left(\frac{\rho_0}{10^6}\right) t . \tag{2.6}$$

In Table 1 we show the time scale given by (2.6)

Table 1. The relation between t and T

$T(10^9\,{}^\circ\mathrm{K})$		10	4	2	1	0.5	0.2	0.1
t(sec)	$\rho_0 = 10^6$ g/cm^3	0.89	3.5	10	28	80	316	890
	$\rho_0 = 10^7$ g/cm^3	0.28	1.1	3.2	8.9	25	100	280

Under the above conditions (2.2), the proton-neutron ratio in the pre-elements stage can be calculated with slight modifications of I, that is, by using the correct neutron decay life-time 12.8 min. and by taking $\rho = \rho_m$ in (2.1). The calculated proton-neutron ratio is shown in Fig. 1.

Finally, we want to remark that in Newtonian mechanics we can use the same equation as (2.1) in the case when gas with spherically symmetrical distribution expands uniformly and when the magnitudes of its kinetic and potential energies are comparable. It is suggested that the supernovae explosions in their early stage are described approximately with the same time scale.

§ 3. Formation of He^4 nuclei

Suppose the nuclear reactions among four kinds of nuclei A, B, C and D

$$A + B \rightleftarrows C + D + Q, \tag{3.1}$$

where Q stands for the energy release of the reaction. In our case, the nuclear reactions occur at so high temperature that in general their inverse reactions must be taken into account. Taking account of the expansion of the universe the change in the number of nuclei C per unit volume, n_c, is expressed by

$$\frac{d}{dt}\left(\frac{n_c}{\rho}\right) = \frac{1}{\rho}\left[p\, n_A n_B - p' n_c n_D\right] \tag{3.2}$$

with

$$p/p' = n_A^{(0)} n_B^{(0)} / n_C^{(0)} n_D^{(0)} . \tag{3.3}$$

Here $n_A^{(0)}$ is the number of nuclei A in thermodynamical equilibrium at temperatur T and density ρ.

$$n_A^{(0)} = g_A \left(\frac{2\pi HAkT}{\hbar^2} \right)^{\frac{3}{2}} \exp\left[\frac{(A-Z)\mu + Z\lambda - \overset{M_A}{\cancel{HA}}c^2}{kT} \right], \qquad (3.4)$$

where A and Z are the mass number and atomic number of nucleus A, respectively; H is proton mass; λ, μ are the chemical potentials of proton and neutron; $g_A = 1 + 2I_A$ is the statistical weight of the ground state of nucleus A with spin I_A. The thermonuclear rate p in (3.2) is a function of temperature which is expressed as

$$p(T) = \left(\frac{8}{\pi\mu}\right)^{\frac{1}{2}} (kT)^{-\frac{3}{2}} \int_0^\infty \sigma(E)\, e^{-E/kT}\, E\, dE, \qquad (3.5)$$

where $\mu = \frac{AB}{A+B}H$ and $\sigma(E)$ is the reaction cross section with E being the kinetic energy in the center of mass system. It is convenient in what follows to use

$$w_i = \frac{n_i}{n}, \qquad n = \frac{\rho}{H} \qquad (3.6)$$

instead of n_i. Then the time variation of w_C is given by

$$\frac{dw_C}{dt} = p(T)\, n \left\{ w_A w_B - \frac{g_A g_B}{g_C g_D} \left(\frac{AB}{CD}\right)^{\frac{3}{2}} e^{-\frac{Q}{kT}} w_C w_D \right\}. \qquad (3.7a)$$

For the radiative capture, $A + B \rightleftarrows C + \gamma$, we have in a similar way

$$\frac{dw_C}{dt} = p(T)\, n \left\{ w_A w_B - \frac{g_A g_B}{g_C} \left(\frac{AB}{C}\right)^{\frac{3}{2}} \left(\frac{2\pi HkT}{\hbar^2}\right)^{\frac{3}{2}} e^{-\frac{Q}{kT}} \frac{w_C}{n} \right\}. \qquad (3.7b)$$

If the expansion of the universe is slower than all the nuclear reactions in (1.2), the concentrations of all the nuclear species vary in accordance with equilibrium values corresponding to the temperature and density at each stage of the expansion. We can obtain equilibrium values $w_i^{(0)}$'s, as functions of temperature, from (3.4) and the conservation of protons and neutrons

$$w_p + w_d + w_t + 2w_{He^3} + 2w_\alpha = a \qquad (3.8)$$
$$w_n + w_d + 2w_t + w_{He^3} + 2w_\alpha = b$$

where a:b $(a+b=1)$ is the proton-neutron ratio at the beginning of He^4 formation as calculated in § 2. The dotted curves in Fig. 2a and 2b show the variation of $w_i^{(0)}$'s obtained in this way.

As shown in Fig. 2a and 2b, formation of He^4 begins at $T \lesssim 9\times10^9\,{}^\circ K$. In this stage the β-decays can be neglected due to the fast expansion of the universe, as seen from Table 1. We consider only the five reactions shown in (1.2), neglecting the others with much lower reaction rates at the temperatures under consideration. Reaction rates of (1.2) which is calculated from (3.5) with experimental data on the cross section are shown in Table 2. The $T^3(d,n)He^4$ and $He^3(d,p)He^4$ reactions show the effect of resonances and their reaction rates are considerably different from those obtained by Fermi and Turkevich.[2]

Table 2 Reaction rates $p(T)$

Reactions	Reaction Rates: Ours	Reaction Rates: Fermi-Turkevich
$p+n \to d+\gamma$	$p_1=9.3\times10^{-20}$	6.6×10^{-20}
$d+d \to He^3+n$	$p_2=9.00\times10^{-15}\, T_8^{-\frac{2}{3}}\, 10^{-\frac{5.18}{T_8^{1/3}}}$ *	$3.0\times10^{-15}\, T_8^{-\frac{2}{3}}\, 10^{-3.99/T_8^{\frac{1}{3}}}$
$d+d \to T^3+p$	$p_3=9.00\times10^{-15}\, T_8^{-\frac{2}{3}}\, 10^{-\frac{5.18}{T_8^{1/3}}}$	$3.0\times10^{-15}\, T_8^{-\frac{2}{3}}\, 10^{-3.99/T_8^{\frac{1}{3}}}$
$d+T^3 \to He^4+n$	$p_4=5.48\times10^{-13}\, T_8^{-\frac{3}{2}}\, 10^{-3.92/T_8}$	$5.0\times10^{-13}\, T_8^{-\frac{2}{3}}\, 10^{-4.24/T_8^{\frac{1}{3}}}$
$d+He^3 \to He^4+p$	$p_5=4.59\times10^{-13}\, T_8^{-\frac{3}{2}}\, 10^{-13.00/T_8}$	$1.5\times10^{-12}\, T_8^{-\frac{2}{3}}\, 10^{-6.72/T_8^{\frac{1}{3}}}$

* $T_8 = T/10^8$

At the beginning of this stage, protons, neutrons, deuterons, tritons, He^3 and He^4 nuclei are found to be in statistical equilibrium as a whole. As the temperature falls, the inverse reactions of $T^3(d,n)He^4$ and $He^3(d,p)He^4$ become not so essential due to their large Q-values that the above equilibrium begins to break. However, a group of nuclei other than He^4 are closely in equilibrium among themselves due to their small Q-values. Therefore, using (3.4), we can express $\mathcal{W}_d$, $\mathcal{W}_t$ and $\mathcal{W}_{He^3}$ in terms of $\mathcal{W}_n$

and w_p approximately as

$$w_d = \frac{1}{f_1} w_n w_p , \quad w_t = \frac{1}{f_1^2 f_2} w_n^2 w_p \quad \text{and} \quad w_{He^3} = \frac{1}{f_1^2 f_3} w_n w_p^2 \qquad (3.9)$$

where

$$f_1 = 0.471 \left(\frac{2\pi H k T}{\hbar^2} \right)^{\frac{3}{2}} \frac{1}{n} e^{-\frac{Q_1}{kT}} \qquad (Q_1 = 2.23 \text{ Mev}),$$

$$f_2 = 3.46 \, e^{-Q_2/kT} \qquad (Q_2 = 4.04 \text{ Mev}), \qquad (3.10)$$

and

$$f_3 = 3.46 \, e^{-Q_3/kT} \qquad (Q_3 = 3.26 \text{ Mev}).$$

Then, the time variation of w_α is given from (3.7) by

$$\frac{dw_\alpha}{dt} = n \left[P_4(T) \{ w_d w_t - f_4 w_\alpha \} + P_5(T) \{ w_d w_{He^3} - f_5 w_\alpha \} \right] , \qquad (3.11)$$

where

$$f_4 = 5.61 \, e^{-Q_4/kT} \qquad (Q_4 = 17.58 \text{ Mev})$$

$$f_5 = 5.61 \, e^{-Q_5/kT} \qquad (Q_5 = 18.34 \text{ Mev}). \qquad (3.12)$$

The order estimation shows that this approximation is very good for $T \gtrsim 10^9$ °K. Taking an approximation that

$$w_d + w_t + 2 w_{He^3} \ll w_p + 2 w_\alpha ,$$

$$w_d + 2 w_t + w_{He^3} \ll w_n + 2 w_\alpha , \qquad (3.13)$$

using (3.8) and (3.9) and replacing variable t by $T_9 = T/10^9$, we have from (3.11)

$$-\frac{d\varpi_\alpha}{dT_9} = 2.54\times10^{28}\left(\frac{\rho_0}{10^6}\right)^{\frac{1}{2}} T_9^{\frac{1}{2}}\Big[P_4\{b-2\varpi_\alpha\}\Big\{\frac{1}{f_1^3 f_2}(b-2\varpi_\alpha)^2(a-2\varpi_\alpha)^2 - f_4\varpi_\alpha\Big\} \tag{3.14}$$

$$+P_5\{a-2\varpi_\alpha\}\Big\{\frac{1}{f_1^3 f_3}(b-2\varpi_\alpha)^2(a-2\varpi_\alpha)^2 - f_5\varpi_\alpha\Big\}\Big].$$

From (3.8), (3.9), (3.13) and (3.14), all ϖ_i's are calculated, which are shown by the solid curves in Fig. 2a and 2b. As seen from Fig. 2a and 2b, amounts of neutrons, deuterons, tritons and He^3 are very small. Such small amount of neutrons is not sufficient to build up the heavy elements by neutron captures according to the scheme of α-β-γ theory. After this stage, we must consider the reaction involving only protons and He^4 nuclei for the formation of nuclei heavier than He^4.

§ 4. Formation of C^{12} and O^{16} nuclei

C^{12} and O^{16} nuclei are formed by a series of the nuclear reactions

$$2He^4 \rightleftarrows Be^8+\gamma,\quad Be^8+He^4 \rightleftarrows C^{12}+\gamma,\quad C^{12}+He^4 \rightleftarrows O^{16}+\gamma \tag{4.1}$$

from He^4 nuclei which have been formed in the stage considered in § 3. These reaction rate are calculated by Hayakawa et al.[6] recently. Using their results, the change in the relative abundances of C^{12} and O^{16} nuclei , ϖ_c and ϖ_o, are expressed as

$$-\frac{d\varpi_c}{dT_9} = 2.54\times10^{28}\left(\frac{\rho_0}{10^6}\right)^{\frac{1}{2}} T_9^{\frac{1}{2}}\Big[P_{Be\to C}\Big\{\varpi_{Be}\varpi_\alpha - 4.36\times10^{17}\left(\frac{\rho_0}{10^6}\right)^{-1} T_9^{-\frac{3}{2}}\,10^{-\frac{37.18}{T_9}}\varpi_c\Big\} \tag{4.2a}$$

$$-P_{C\to O}\Big\{\varpi_\alpha\varpi_c - 5.20\times10^{17}\left(\frac{10^6}{\rho_0}\right) T_9^{-\frac{3}{2}}\,10^{-\frac{36.04}{T_9}}\varpi_o\Big\}\Big],$$

$$-\frac{dw_O}{dT_9} = 2.54\times10^{28}\left(\frac{\rho_0}{10^6}\right)^{\frac{1}{2}} T_9^{-\frac{1}{2}} \times\left[P_{C\to O}\left\{w_C w_\alpha - 5.20\times10^{7}\left(\frac{10^6}{\rho_0}\right)T_9^{-\frac{3}{2}}\,10^{-\frac{3.604}{T_9}}\,w_O\right\} - P_{O\to Ne}\,w_O w_\alpha\right] \tag{4.2b}$$

where

$$P_{Be\to C} = 1.92\times10^{-23}\,T_9^{-\frac{3}{2}}\,10^{-\frac{1.390}{T_9}}$$

$$P_{C\to O} = 1.34\times10^{-14}\,T_9^{-\frac{2}{3}}\,10^{-14.05/T_9^{\frac{1}{3}}}.$$

Since we can consider that $2He^4 \rightleftarrows Be^8 + \gamma$ is always in equilibrium[6], w_{Be} is expressed in terms of w_α as

$$w_{Be} = 3.58\times10^{-8}\,T_9^{\frac{3}{2}}\,10^{-\frac{.0484}{T_9}}\left(\frac{\rho_0}{10^6}\right) w_\alpha^2. \tag{4.3}$$

The second term in the square brackets of (4.2b) describes the decrease of O^{16} caused by the reaction $O^{16} + He^4 \to Ne^{20} + \gamma$. This term will be neglected in the calculations and its effect on the abundance of O^{16} will be discussed in § 5. We can consider the value of w_O in this approximation as the abundances of O^{16} together with all heavier nuclei. Then, if $w_O \gg w_{Ne}$, w_O gives the very abundance of O^{16}, and if $w_O \ll w_{Ne}$ it gives the abundances of Ne^{20} and heavier nuclei. Using an approximation $w_C \ll w_O$, we solve (4.2) for w_O and then w_C as the function of temperature by numerical integrations. The results are shown in Fig. 2a and 2b. Taking $\rho_0 = 10^6 \sim 10^7$ g/cm^3, we can obtain the value of w_O which is close to the observed proton-oxygen abundance ratio, $1:10^{-4}\sim 10^{-3}$.[9]* However, the carbon-oxygen ratio obtained here is only $1/10 \sim 1/6$ of the observed ratio. A way to overcome the difficulty of low abundance of C^{12} nuclei will be suggested in § 5.

§ 5. Discussions

We will discuss the results of our calculations, the other routes for the formation of C^{12} and O^{16}, the formation of Ne^{20} and heavier nuclei and the difficulties of low abundance of C^{12} shown in § 4.

(i) Other routes to C^{12}

We have considered only 3α-reactions in the formation of C^{12}, but it might be possible that C^{12} nuclei are formed through other reactions. Since at $T = 4\sim 5\times 10^{9}\,{}^{\circ}K$, numbers of neutrons, deuterons, tritons and He^3 nuclei are found to be very small as shown in Fig. 2, we can neglect the possibility of forming C^{12} and heavier nuclei by the reactions involving nuclei other than protons and He^4. Let us consider the routes of formation from protons and He^4 only. Possible routes are

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\begin{aligned}
&Be^8 + p \rightleftarrows B^9 + \gamma - 0.19\ \text{Mev} \\
&B^9 + p \rightleftarrows C^{10} + \gamma + 4.02\ \text{Mev} \qquad (5.1) \\
&B^9 + \alpha \rightleftarrows C^{12} + p + 7.56\ \text{Mev},
\end{aligned}
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* According to Taketani-Hatanaka-Obi Theory,[5] the abundance of elements formed in pre-stellar stage should be compared with those in the stars of population II, in which the ratio appears to be nearly $1:3\times 10^{-4}$.

beside the one: $He^4 + p \rightleftarrows Li^5 + \gamma$ and $Li^5 + \alpha \rightleftarrows Be^8 - p$ producing again Be^8 which is in equilibrium with He^4. At the high matter density under consideration, the reaction $Be^8(p,\gamma)B^9$ is in a state of equilibrium as in the case of $2He^4 \rightleftarrows Be^8$. Then, we have

$$w_{B^9}/w_{Be} \sim 10^{-7}\left(\frac{\rho_0}{10^6}\right) T_9^{\frac{3}{2}}\, 10^{-\frac{0.93}{T_9}}\, w_p. \tag{5.2}$$

The ratio of reaction rates of $Be^8(\alpha,\gamma)C^{12}$ to $B^9(\alpha,p)C^{12}$ is given by

$$\frac{P_{Be\to C}}{P_{B\to C}} \simeq \frac{\Gamma_\gamma(Be^8\to C^{12})}{\Gamma_\alpha(B^9\to C^{12})}\ , \tag{5.3}$$

if the resonance are present in both reactions. From (5.2) and (5.3) it follows that the ratio of the number of C^{12} nuclei formed by $B9(\alpha,p)C^{12}$ to one by $Be^8(\alpha,\gamma)C^{12}$ is only $10^{-3} \sim 10^{-6}$. Therefore, we can neglect this route.

(ii) Another route from C^{12} to O^{16}

As another possible route from C^{12} to O^{16}, we can consider the following reactions.

$$\begin{aligned} &C^{12} + p \rightleftarrows N^{13} + \gamma + 1.95\ \text{Mev}, \\ &N^{13} + \alpha \rightleftarrows O^{16} + p. \end{aligned} \tag{5.4}$$

At $T \lesssim 1.5\times10^{9}\,^\circ K$, the rate of O^{16} formation by the above route is found to be greater than that of the radiative α-capture. If we take account of this route, the w_c/w_o value calculated in § 4 is estimated to decrease remarkably.

(iii) Formation of Ne^{20} nuclei

In calculating w_o in § 4, we neglected the reaction

$O^{16}(\alpha,\gamma)Ne^{20}$. Since the photodisintegration of Ne^{20} is negligible at $T \leqslant 3\times10^{9}\,{}^{o}K$, the relative abundance of Ne^{20}, ϖ_{Ne}, can be calculated using the following equation

$$-\frac{d\varpi_{Ne}}{dT_9} = 2.54\times10^{28}\left(\frac{\rho_0}{10^6}\right)^{\frac{1}{2}} T_9^{-\frac{1}{2}} P_{O\to Ne}\, \varpi_O \varpi_\alpha . \qquad (5.5)$$

At $T \gtrsim 0.6\times10^{9}\,{}^{o}K$ we must use $P_{O\to Ne}$ which is calculated by Hayakawa et al.[6] as the non-resonance case. But the spin and parity of the energy level of Ne^{20} is unknown and Γ_γ in $O^{16}(\alpha,\gamma)Ne^{20}$ is uncertain. If we assume $\Gamma_\gamma = 10^{-3}$ eV, we have $\varpi_O \sim \varpi_{Ne}$ as the final abundances. If Γ_γ is as large as 10^{-2}eV, we have $\varpi_{Ne} \sim 10\varpi_O$, which shows that we cannot neglect this reaction in the calculation of ϖ_O in § 4. In this case O^{16} are used up in the formation of Ne^{20} and ϖ_O will become as small as 1/10 of ϖ_{Ne}.

Let us consider other route of formation of Ne^{20} such as

$$O^{16} + p \rightarrow F^{17} + \gamma + 0.59 \text{ Mev},$$

$$F^{17} + \alpha \rightarrow Ne^{20} + p.$$

This route will compete with the one, $O^{16}(\alpha,\gamma)Ne^{20}$, at $T \lesssim 10^{9}\,{}^{o}K$ because of the low Q-value, 0.59 Mev, but will be neglected in the stage of Ne^{20} formation.

(iv) Formation of heavier nuclei than Ne^{20}

At $T \lesssim 10^{9}\,{}^{o}K$ the number of remaining neutrons become so small that the formation of heavier nuclei than Ne^{20} seems to be possible only by p- and α-reactions. But in this stage the expansion of the universe is so slow that we cannot neglect the β-decays whose life-times are about $1 \sim 10$ sec. There exist the following reactions through which the heavier nuclei will be formed from proton, He^4 and Ne^{20}[10].

$$\left.\begin{array}{l} Ne^{20} + p \rightarrow Na^{21} + \gamma + 2.45 \text{ Mev} \\ \qquad Na^{21} + \alpha \rightarrow Mg^{24} + p + 6.88 \text{ Mev} \\ \qquad Na^{21} \rightarrow Ne^{21} + e^{+} \quad (\tau = 23 \text{ sec}) \\ \qquad\qquad Ne^{21} + \alpha \rightarrow Mg^{24} + n \end{array}\right\} \quad (5.6a)$$

$$\left.\begin{array}{l} Mg^{24} + p \rightarrow Al^{25} + \gamma + 2.29 \text{ Mev} \\ \qquad Al^{25} + \alpha \rightarrow Si^{28} + p + 9.70 \text{ Mev} \\ \qquad Al^{25} \rightarrow Mg^{25} + e^{+} \quad (\tau = 7 \text{ sec}) \\ \qquad\qquad Mg^{25} + \alpha \rightarrow Si^{28} + n \end{array}\right\} \quad (5.6b)$$

$$\left.\begin{array}{l} Si^{28} + p \rightarrow P^{29} + \gamma + 2.72 \text{ Mev} \\ \qquad P^{29} + \alpha \rightarrow S^{32} + p + 4.22 \text{ Mev} \\ \qquad P^{29} \rightarrow Si^{29} + e^{+} \quad (\tau = 4.5 \text{ sec}) \\ \qquad\qquad Si^{29} + \alpha \rightarrow S^{32} + n \end{array}\right\} \quad (5.6c)$$

and so on. The (p,γ) reaction and the competition of β^+-process with (α,p) reaction determine the number of neutrons formed by (α,n) reactions. It is desirable to calculate the amount of neutrons produced in these routes in order to see whether the formation of heavy nuclei is possible or not.

(v) <u>A way to reproduce C^{12}</u>

The calculated value of $\mathcal{W}_d$ in § 3 is much smaller than the observed one, which is $\sim 10^{-4}$.

The relative abundance of C^{12} nuclei, w_C, obtained in § 4 is also smaller than the observed one. To solve this difficulty, we shall suggest the spallation, such as (p,d) (p,α), of some O^{16} nuclei by non-thermal protons. Ginzburg[11], and Hayakawa and Terashima[12] have suggested the existence of non-thermal protons accelerated in the expanding turbulent envelope of supernovae in order to explain the origin of cosmic rays. In our case also, it might be possible that turbulent motions accompanying the expansion of the universe will produce non-thermal protons in a similar way. If these small number of protons could disintegrate some fraction, for example, 1/10 of O^{16} nuclei, the difficulty of low abundance of C^{12} could be solved.

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Fig. 1. Neutron-proton ratios for different values of ρ_0. The case $\rho_r > \rho_m$ is added for comparison.

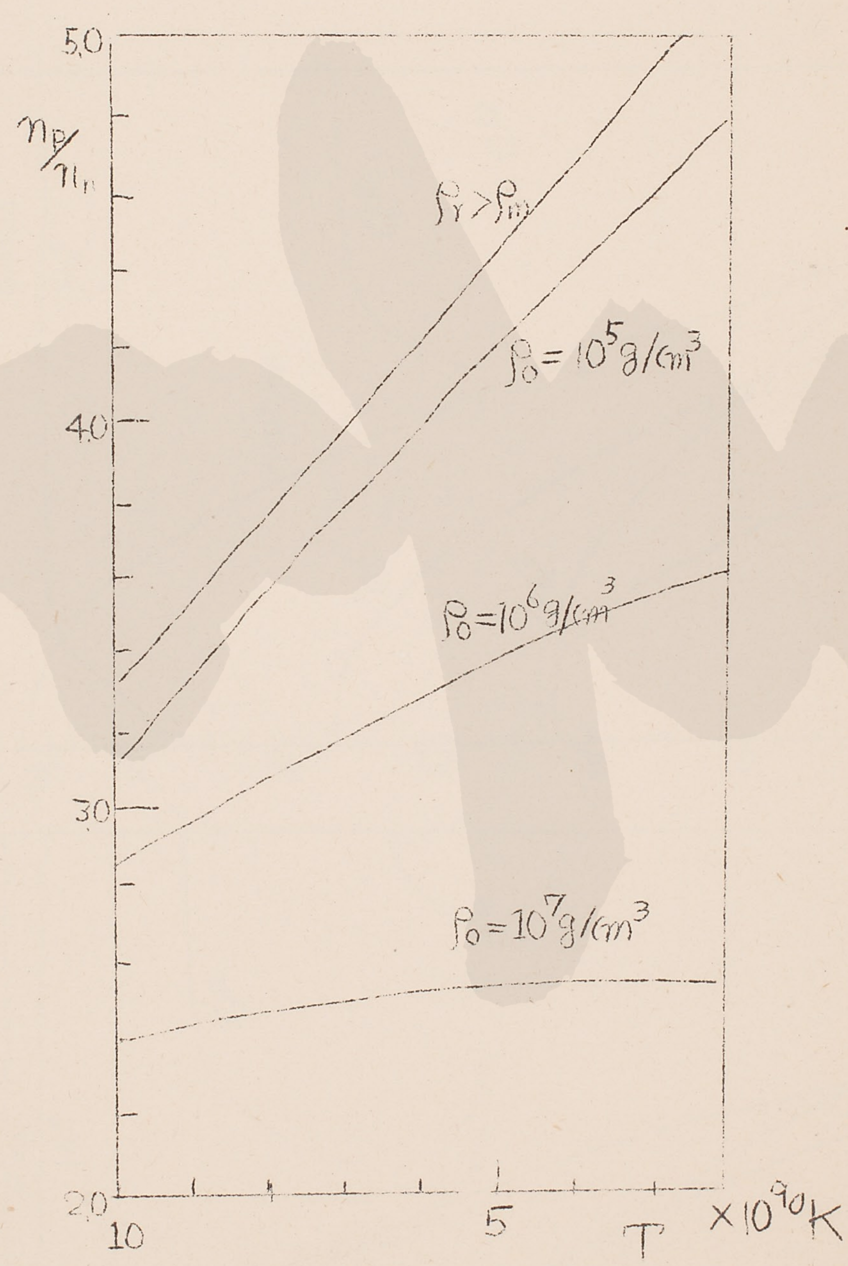

Fig. 2a. Temperature variation of the chemical abundances for $\rho_0 = 10^6 g/cm^3$. Dotted curves represent their equilibrium values.

Fig. 2b. The same variation for $\rho_0 = 10^7 g/cm^3$.