Nuclear Structure

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

DATE

NO.

Gamow: Nuclear Spin of Radioactive Elements
(Proc. Roy. Soc. 146, 217, 1934)

There seems at present to be rather definite evidence that the rate of radioactive disintegration, and excitation of the product-nucleus usually connected with it, is largely affected by the values of nuclear angular momenta.

It was shown by the author† that the existence of intense components of α-ray fine-structure indicates that the disintegrating-, and the product-nuclei possess different spins. The angular momenta received in the case by α-particles of the normal group (transitions between normal states of both nuclei) will reduce its probability of escape in favour of other groups corresponding to the formation of the excited product.

From this point of view we have to accept the change of nuclear spins in normal disintegration of radioactive C-products (ThCC''; RaCC''; AcCC'') and also of most members of the actinium-family. Unfortunately the existing formula for decrease of probability does not take into account all the factors influenced by angular momenta of α-particle (only the effect due to increase of potential barrier can be simply evaluated) and therefore the values of spin difference cannot be exactly estimated.

† 'Nature' 129, 470, 1932; 131, 618, 1933
Gamow & Rosenblum, C.R. 197, 1620, 1933.

The role of nuclear spin in the process of β-integration was recently indicated by Fermi[‡] in his theory of β-decay. According to this theory the decay constant λ for β-disintegrating bodies depends on the factor:

$$\left| \int u_n v_n^* \, d\omega \right|^2, \qquad (1)$$

where u_n and v_n are the eigenfns of nuclear neutron and resultant proton. We may notice, however, that this result is not necessarily connected with the special form of Fermi's theory, and will hold for practically every theory treating the β-ray as the transformation of a nuclear neutron into a proton.

The factor (1) must be of the order of magnitude unity if the initial and resultant nuclei possess the same spin (permitted transitions):

$$i = i' \qquad (2)$$

and be reduced to the value

$$\left(\frac{\text{radius of the nucleus}}{\text{wave length of } \beta\text{-particle}} \right)^{2\Delta i} \sim \left(\frac{1}{100} \right)^{\Delta i}$$

if this condition is not fulfilled (non-permitted transitions):

$$i \neq i' \qquad (2')$$

This explains the result found by Sargent[†] that plotting the logarithms of decay constants against the logarithms of maximum energy of β-particles we get the experimental

‡ ~~Proc~~ Ric. Sci.; 2, No. 12, 1933; ZS. 88, 161, 1934

† Proc. 139, 659, 1933

points distributed between two different curves. We must say that the β-spectra belonging to the class I (RaD, UX_1, ThB, AcB, RaB, AcC″, ThC″, UX_2) correspond to $i = i'$ while those of the class II (RaE, MTh_2, ThC, RaC) to $i \neq i'$ (most probably $i = i' \pm 1$). One would also expect the shape of continuous β-spectra to be different for permitted and non-permitted transitions, although the present experimental evidence is not sufficient to prove that.

Remembering that the product-nucleus can be formed in different excited states, with the energy-excess not greater than the total energy-difference between the two nuclei, we must consider the following possibility. The normal state of the product nucleus possesses a spin different from that of the original one ($i_0 \neq i'_0$) but one of its excited states has the same value of spin. ($i_0 = i'_n$) In such case we must expect that the observed β-spectra will will correspond to a permitted transition leading to the formation of an excited nucleus and consequently will be accompanied by a strong γ-line with absolute intensity unity (or by several less strong γ-lines if different radiative transitions from this completely excited level are

~~each~~ equally possible). This has actually been observed by Ellis and Mott ‡, for several β-disintegrating bodies (for example, for ThB C and ThC″D). The fact that there are no β-transformations belonging to the non-permitted class and at the same time possessing γ-lines of absolute intensity unity speaks in favour of the exclusion rules (2) and (2′).

We shall now apply these considerations to the forking regions of three known radioactive families for which the experimental evidence is rather complete.

In fig. 1 is given the scheme of radioactive decay from ThA to ThD (lead). From the absence of intense fine structure in the α-disintegration-row leading to the formation of ThB, we conclude that all these nuclei (including ThB) possess the same value of spin, and one can hardly doubt that this value is zero ($i_0(\mathrm{ThB}) = 0$). Now ThB emits a continuous β-spectrum belonging to the class I and possesses a quadrupole γ-line, $\gamma = 0{,}24 \times 10^6$ e.V., with absolute ~~value~~ intensity unity†. Thus we must say that observed β-spectra (with $E_{max} = 0{,}36 \times 10^6$ e.V.) represents the permitted transformations of ThB-nucleus into the excited state of the ThC-nucleus, and that this later state

‡ Proc. 139, 369, 1933.
† Ibid, 639, 1933.

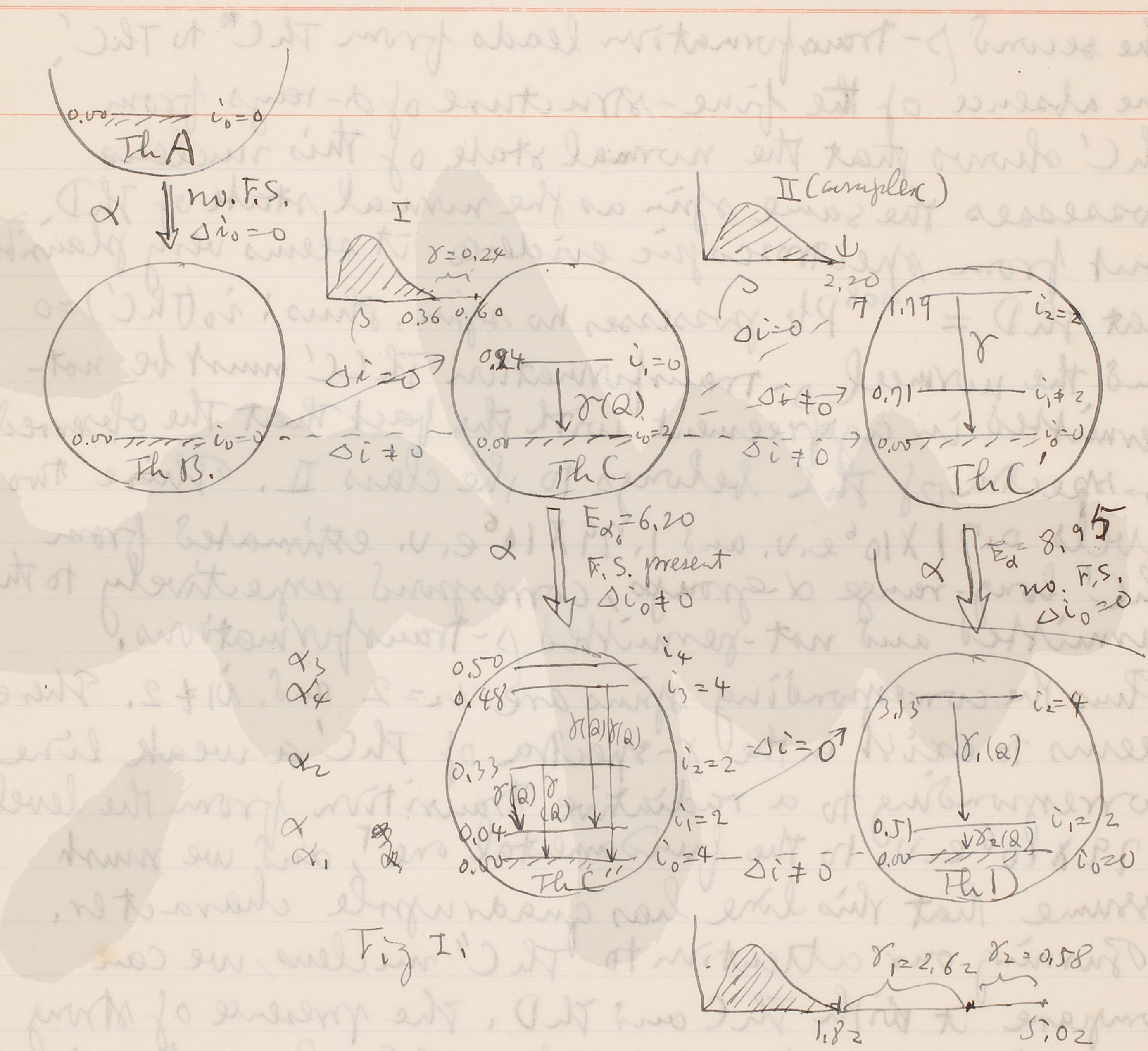
DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

Quadripoleの出すγ-rayの多いのは
どうしてか ?

$\sum_{i=1}^{n} 2e\cdot r_i = \frac{2e}{n} R$

DATE

NO.



possesses the spin $i_0(\text{ThC})=0$. The spin of normal state of ThC must be consequently $i_0(\text{ThC})=2$ as the value 0 would permit the β-transformation between the normal state and the value 2 is excluded by the quadripole character of the γ-ray, 0.24×10^6 e.V.

The second β-transformation leads from ThC^* to ThC'. The absence of the fine-structure of α-rays from ThC' shows that the normal state of this nucleus possesses the same spin as the normal state of ThD. But from spectroscopic evidence it seems very plausible that $ThD = {}^{208}Pb$ possesses no spin. Thus $i_0(ThC') = 0$ and the normal β-transformation $ThC\,C'$ must be not-permitted in agreement with the fact that the observed β-spectra of ThC belongs to the class II. These two levels 0.71×10^6 e.v. and 1.79×10^6 e.v. estimated from ThC' long-range α-groups, correspond respectively to the permitted and not-permitted β-transformations. Thus the corresponding spins are $i_2 = 2$ and $i_1 \neq 2$. There seems to exist in the γ-spectra of ThC' a weak line corresponding to a radiative transition from the level 1.79×10^6 e.v. to the fundamental one†, and we must assume that this line has quadrupole character.

Turning our attention to ThC'' nucleus, we can compare it with ThC and ThD. The presence of strong fine-structure of α-rays from ThC shows that $i_0(ThC'')$ is in any case different from 2. Comparing ThC'' with ThD, we see that the normal β-transition is not-permitted, and consequently $i_0(ThC'') \neq 0$. The observed β-spectra of ThC'' belongs to the class I and leads to the totally excited state of the ThD

† Skobelzyn, C. R. 194, 1486, 1932

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

DATE..................
NO..................

nucleus, for which $i_0(\text{ThC}'') = i_n(\text{ThD})$. The totally excited state of ThD corresponds to an energy excess of 3.20×10^6 e.V. and gives rise to two successive γ-lines, quadrupole line $\gamma_1 = 2.62 \times 10^6$ e.V. and dipole line $\gamma_2 = 0.58 \times 10^6$ e.V., both having the absolute intensity unity. This gives to the level 0.58×10^6 e.V. the spin 1 ~~We want, therefore, to indicate the other possibility~~ and to the upper level 3.20×10^6 e.V. the spin 1 or 3. These values are hardly possible as it would be difficult to account for the observed fine structure of ThC α-rays if the spin-difference between corresponding normal state is only unity.

We want, therefore, to indicate the other possibility for the level scheme of ThD nucleus. From the upper limit energy-balance for the forking

$$\text{ThC} \rightarrow \text{ThC}' \rightarrow \text{ThD}$$
$$\text{ThC} \rightarrow \text{ThC}'' \rightarrow \text{ThD}$$

we obtain for the energy of totally excited ~~state~~ state of ThD $2.20 + 8.95 - 6.20 - 1.82 = 3.13 \times 10^6$ e.V., which is more nearly represented by the sum of $\gamma_1 = 2.62 \times 10^6$ and the ~~γ-ra~~ other γ-line $\gamma_2 = 0.51 \times 10^6$ e.V., corresponding to the quadrupole-transition and having the absolute intensity 0.3. If we accept the γ-line 0.51×10^6 e.V. instead of 0.58×10^6 e.V., we must give to the first level (0.51) the spin 2 and ~~to~~ to the top-level (3.13)

(A) Note added in proof, May 23, 1934, - The detailed investig. of relative intensities of diff. comp. of β-spectra has shown that it is difficult to account for the observed intensity of the normal β-group of ThC accepting the spin-diff. $i_0(\mathrm{ThC}) - i_0(\mathrm{ThC}')$ to be equal to 2 units; it seems necessary to accept for it the value 1. As the value for $i_0(\mathrm{ThC}) = 2$ seems to be rather

---

the spin 0 or 4. As the value 0 is excluded (otherwise the normal β-transformation ThC″D would be permitted), we must accept for the upper level of ThD and consequently for the normal state of ThC″, the spin 4, as is indicated in our diagram. This would give the spin-difference $\Delta i_0 = 4 - 2 = 2$ to the normal α-group of ThC. As on the other hand, the group α_3 seems to possess the same Δi as α_0 † we must accept for the excited state 0.48 of ThC′ nucleus also the value $i_3 = 4$. This fits with the fact that the γ-transition from this level to a fundamental one has quadrupole character. The levels 0.04 and 0.33, connected with 0.00 and 0.48 by quadrupole γ-transitions, may have $i = 2$ or $i = 4$; we choose $i = 2$, as the corresponding fine-structure α-components are relatively strong. The evidence about the level 0.50 is indecisive as the only known γ-transition can be a quadrupole as well as ~~the~~ a dipole one; the extremely high intensity of corresponding α_4-group is also surprising.‡ It must, however, be noticed that the selection-rule for quadrupole transitions used in the article of Ellis and Mott is correct only for a model of a single radiating particle in a central field, and is not necessarily applicable to real nuclei consisting of a great no. of particles. The possibility is not excluded (although not necessarily required) that for real

† G. v R., loc. cit.

‡ (A)

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

DATE ....
NO. ....

definite we must accept for i_0(ThD) the value 2 and explain the abs~~olut~~ence of hyperfine structure in the optical spectra of lead by the vanishing of the corresponding Landé-factor. This will change our arguments concerning this part of diagram and the spins of normal and two excited states of ThD nucleus will now be: 1; 2; 4 (accepting the level 0.58) or 1; 1; 3 (accepting the level 0.51). The first hyp. seems now to be more probable.

nuclei the transition $\Delta i = \pm 1$ (except $0 \to 1$) may be permitted in quadrupole radiation.

The evidence concerning the two other radioactive families is less complete, and we shall mention only the main points. As the β-spectrum of RaB belongs to the class I and also some intense γ-lines are present we must either regard this case as analogues to ThB, and accept the value $i_0(\mathrm{RaC}) \neq 0$ or suppose that the normal and also one or several of the excited states of RaC nucleus possess the spins zero ($i_0(\mathrm{RaC}) = 0$). The second possibility is, however, excluded as RaC′ possesses a transition from the level 1.414 to the fundamental one which is, as well known, not permitted in radiation. This means that the fundamental state of RaC′ nucleus (and also excited state 1.414) possesses the spin zero and consequently, as RaC β-spectra belongs to the class II, $i_0(\mathrm{RaC}) \neq 0$.
The existence of fine structure of RaC α-rays give us also $i_0(\mathrm{RaC}) \neq i_0(\mathrm{RaC''})$. The upper limit of the RaC″ β-spectrum is not measured but can be estimated from upper-limit energy balance and belongs to the class II. Thus $i_0(\mathrm{RaC''}) \neq i_0(\mathrm{RaD})$.
For RaD we probably have a situation analogous to RaB, and for the RaEF transformation we get $i_0(\mathrm{RaE}) \neq i_0(\mathrm{RaF})$.

* The author is very glad to express his thanks to Prof N. Bohr for the kind interest and helpful discussion of this paper.

The actinium family is rather analogous to that of thorium and radium with the difference, however, that both the AcC C′ and AcC″D β-~~transition~~ formations (upper limit of AcC β-spectra being again estimated from energy-balance†) seems here to belong to permitted transitions. One must, however, be very careful with the upper limits of β-spectra estimated in the above way.

In speaking about the energy-distribution in continuous β-spectra we must conclude that β-spectra of ThB and ThC″ must be simple. On the other hand the spectra of ThC must be constructed from different β-components corresponding to the excitation of different ThC′ levels; the same kind of complexity we must also ascribe to continuous β-spectra of RaC leading to strongly excited RaC′ nucleus. Ellis and Mott‡ tried to construct these complex spectra from different components with relative intensities defined by the percentage of excitation of corresponding levels of C′-products. Their constructions, however ~~must~~ may not be quite correct, as they assumed that all β-components have the same shape as the second class spectra of RaE, when it seems that some of these components belong to first class transitions (ThC″ component in the mixture ThC-ThC′ and the slowest components of ThC and RaC spectra) and, consequently, may have different shape.*

† Beck, Nature, 132, 967, 1933

‡ Proc. 141, 502, 1933.

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

DATE
NO.

β

1) Gamow: Mechanism of γ-Excitation by β-Disintegration
(Nature, 131, 5?7, 1933)

nucleusが α, n, 及び p. (0 or 1) から成ると考ふる.
β-disintegrationの結果 nが pにかはったとすると, 一般に higher level にある neutron が proton にかはると考へられるから, 生まれた protonは excess energy を有し, lower level へおちて hard γ-ray を出すであらう. 而して ~~odd~~ odd atomic number の場合の様に残り一個の proton が存在するならば, ~~而る~~ 及び二個の neutron が α-particle を作り, 出てくる γ-ray は更に hard なものになるであらう.
之は β-disintegration に際し (α-disinteg. の場合より) hard な γ-ray で intensity の大なるものを伴ふこと. 又 odd element から出るものの方が更に hard である理由を説明してる.

---

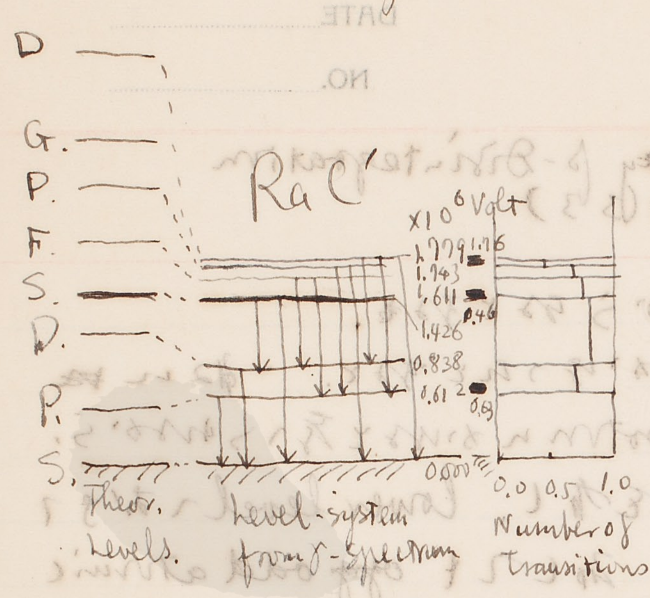
2) Gamow: Nuclear Energy Levels (ibid., 433, ―)

α

Nucleus中の particle の間に働く force は内部では小さく, periphery でのみ大きいと仮定し, 之を flat bottom を有し, infinitely high wall をもった potential で idealise するならば moving particle の energy level は Bessel 函の root で表はされることになる. 実際は wall は無限に高くなく 又急でもないから. upper level は押へ不足したものになる.
之を long range α-particle に関する Rutherford の measurement では RaC' の実験 (Proc. 131, 684, 1931)
Ellis による γ-ray の intensity が ~~から~~ 測定されてる (Proc. 129, 180, 1930)

又 Taylor, Mott (Proc. 138, 665, 1932) による int. conv. coef から dipole, quadrupole transition の区別がわかる。

これから level system を作ると ②の如くなる。



こで見ると level の 1/2 近くが long range α-group に correspond している事がわかる。

	first zero	second zero
$J_{1/2}$	3,1416	6,2832
$J_{3/2}$	~~3,1558~~	
$J_{5/2}$	~~3,1499~~	
$J_{7/2}$	~~3,1441~~	

Janke S. 222 参照。

j が大きければ α-disint の prob は小さい。

(equal energy のもの α-particle の出る prob は S に対し P, D, F, G 夫々 1.3, 4, 16, 105 分の一である)

observe された transition は次の通り。

Constructed $h\nu \times 10^{-6}$ volt	Δj	Observed $h\nu \times 10^{-6}$ volt	Δj
0.588	2	0.589	..
0.612	1	0.612	1
0.773	1	0.773	1
0.838	2	0.839	..
0.941	0	0.941	0; 2
0.999	2	1.000	..
1.311	0	1.130	0; 2
1.167	1	1.168	..
1.426	0→0	1.426	0→0
1.743	1	1.744	..
1.779	2	1.778	0; 2

possible な transition 21 中 11 だけ見つけられた。残りの 10 の中 2 個は F→S。4 個は実験の範囲外。4 個は observe されなかった。(多分 intensity が弱いため)

またそれぞれの level interval を cross する γ-ray の intensity の sum をとると ③の如く 0.226×10⁶ volt の γ-ray (absolute intensity 0.2 位) の存在が要求される。

以上の如く実験とよく合うから上の model は大体正しく、実験と理論の間の deviation から実際の potential distribution が計算される。

3) Gamow: Fundamental state of Nuclear α-particles

(α) α-Disintegration に伴う γ-rays は α-particles が fundamental level に落ちるより小さい energy で出て resultant ~~atom~~ nucleus を excited state に残すためである。(Gamow: Nature 126, 397, 1930)

29 fine structure の relative intensity はこれに対応する energy $E(\alpha_n)$ に対する potential barrier の transparency によって与えられる。

$$J(\alpha_n) = \exp\left[-\frac{4\pi e^2(Z-2)}{h\sqrt{2mE(\alpha_n)}} + \frac{8e\sqrt{m}}{\hbar}\sqrt{Z-2}\sqrt{r_{eff}}\right]$$

$$\sqrt{r_{eff}} = \sqrt{r_0}\,[1-0.02\,j(j+1)] \quad \text{か?}$$

但し j は出て行く α-particle の azimuth q. n.

これからして intensity は Energy と共に減るのみならず j が 0, 1, 2, … となると共に非常に減る。その例は 1.3 4 16 105 位である。これから normal decay (α particle が j = 0) の場合には low energy group は very weak である筈。

所が 異なれる radioactive family の nuclear radii の change の irregularity からして、ある element の normal α-group の j は 0 でないことを知った (Gamow, ibid, 129, 470, ~~Intern~~ 1932)。例へば radium-actinium, actinium emanation 及び恐らくは AcC, RaC, ThC もそれである。之は original nucleus と resultant nucleus が異なる spin を有することを意味する。(之は Bi^{209}, Tl^{205} の如く stable で α-particle 一つ2となるものでも spin は夫々 9/2, 1/2 なることから見ても不思議でない。) この場合には original と同じ spin の resultant excited state への transition に相当する α-particle の intensity が比較的大きくなる。(Rosenblum: …)

実際上記のものでは low energy の強い group をもって居る。其 relative intensity から j をきめられる筈。

Group	α_1	α_0	α_2	α_4	α_3
Calc. for $j=0$	1.0	0.7	0.03	0.005	0.004
Obs. Int.	0.3	1.0	0.03	0.002	0.02
Rel. decrease	17.	3.5	5.0	13.	1.

これから見て. α_1, α_4 ; $j=3$　　α_0, α_2 ; $j=2$

α_3 ; $j=0$ or 1.

これからして ThC と ThC'' の normal state の間 spin diff は 3. で ThC'' の excited level は

$\alpha_1(S)$; $\alpha_0(p)$; $\alpha_2(p)$; $\alpha_4(S)$; $\alpha_3(d$ or $f)$

である.

他のものについても同様な分析が出来るが実験が不充分

尚 AcX と ThC とよく似ていて level system が 40% 異なる丈 之は radius が 20% ちがったためである

$(6.6 \times 10^{-13}, 7.9 \times 10^{-13})$

例へば ~~excite せ~~ された level と RaC′ (β-ray による excite された)
α-ray による
の level とを比較すると 12% の方が energy differences が 4 倍
も大きい。之は β-decay ~~の~~ 後の γ-ray は proton transit.
によるといふ hypothesis とよく合ふ。

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

Kemban
R. Q. M.

DATE.
NO.

~~Bechert: Die Intensitäten von Dublettlinien nach der Diracschen Theorie (Ann. 6, 200, 1930)~~

Gamow et Rosenblum: Les diamètres effectifs des noyaux radioactifs. (C. R. 197, 1620, 1933)

la constante de désintégration: λ, la vitesse de la particules α: v, le numéro atomique Z ~~et~~ 及び le rayon r_0 du noyau の間には

$$\log\lambda = \log\frac{4h}{m r_0^2} - \frac{8\pi^2 e^2}{h}\frac{(Z-2)}{v_{eff}} + \frac{16\pi e\sqrt{m}}{h}\sqrt{(Z-2)} \times\sqrt{r_0}(1-\sigma)$$

なる関係がある。但し $v_{eff} = v\left(1+\frac{m}{M}\right)$

$$\sigma = \frac{h^2}{32\pi^2 m e^2}\frac{j(j+1)}{r_0(Z-2)}$$

j: le nombre azimutal quantique de la particule émise.

$$\text{rayon effectif} = r_0(1-\sigma)^2 = r_{eff} \cong r_0 - \frac{h^2}{16\pi^2 m e^2 (Z-2)} j(j+1)$$

rayon effectif を他の constant から計算してみると regular ~~cov~~ curve からずれてることがある。之は $j \neq 0$ なるためだと考へられる。(Gamow, Nature, 129, 1932, p.470; 131, 1933, p.618)

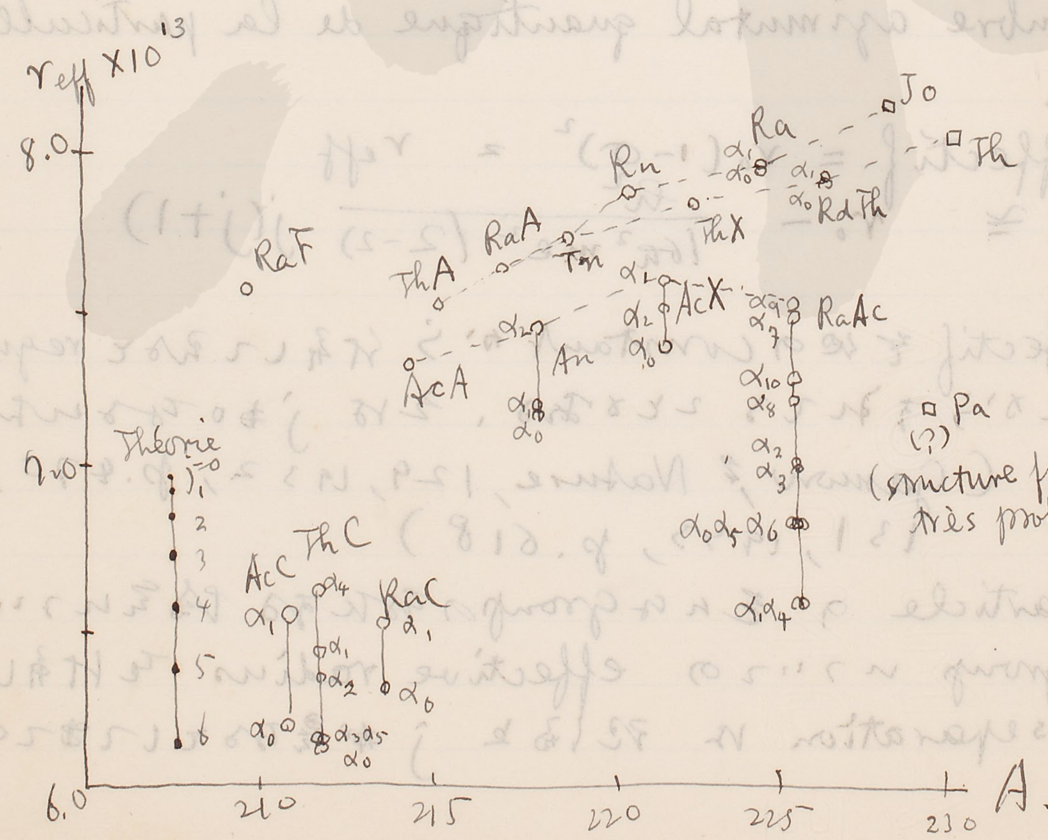
実際. α-particle の色々な group の対応する結果について それぞれの group についての effective radius を計算してみると その separation は 記号と j が異なってでてくる

値と order は合う。
きつちりといかぬのは potential barrier の形が j によつてかはりうるためだらう。

尚 Ra, Th, Ac の三 family に對する r_{eff} の値は夫々 smooth curve の上にある。

$j \neq 0$ の場合には radioactive atom を magnetic field で orientate すれば、でゝくる α-ray の anisotropy の存在を期待しうる

jの値と出てくる
之と γ-ray とを比較すれば、そのせり分の可否を check 出来、又 j の selection rule が nucleus の中でもあてはまるかどうかがわかる。

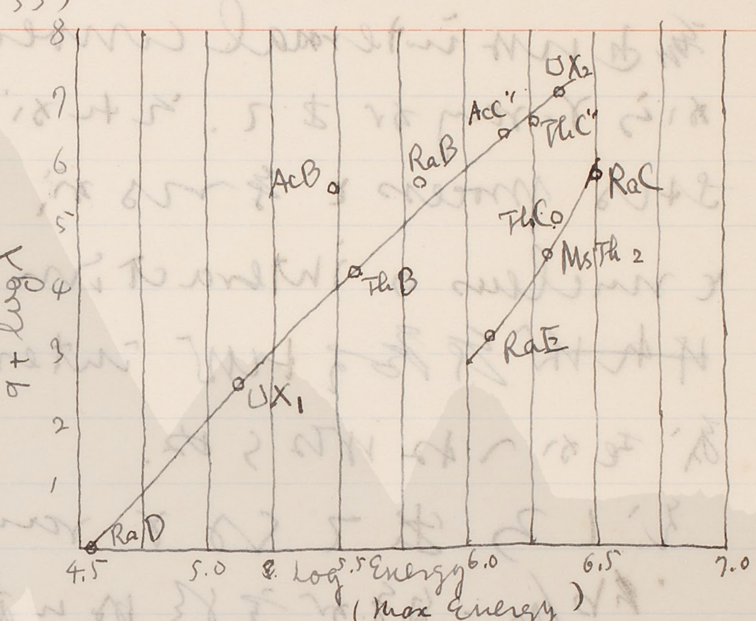


DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

DATE
NO.

(1) Sargent: The Maximum Energy of β-Rays.
(Proc. 139, 659, 1933)

Group I: UX_2を除く
αββα or αβαβ sequence
a first β-ray ~~group~~ member である.
Group II: αββα sequence
a second β-ray member.



(Max. の代りに average energy をとっても同様な grouping が出来る)

(2) Ellis and Mott: The Internal Conversion of the γ-Rays and Nuclear Level Systems of ThB and ThC.
(Proc. 139, 369, 1933)

(3) Beck: Conservation Law and β-Emission (~~Pro~~ Nature 132 967, 1933)

†*: Extrapolated from Sargent's first and second curves.

$RaC \to RaC'$: 3.2×10^6
$RaC' \to RaD$: 7.8×10^6
11.0×10^6

$RaC \to RaC''$: 5.5×10^6
$RaC'' \to RaD$: 5.0×10^6*
10.5×10^6

$AcC \to AcC'$: 0.4×10^6†
$AcC' \to AcD$: 7.5×10^6
7.9×10^6

$AcC \to AcC''$: 6.7×10^6
$AcC'' \to AcD$: 1.5×10^6
8.2×10^6

(4) Taylor & Mott : The Internal Conversion of γ-ray II (Proc. 142, 215, 1933)

普通いはれる internal conversion とは nucleus (outer electron) から γ-ray が出て、それが outer electron に absorb される process と考へられるが、もしも outer (K) electron と nucleus の interaction の影響を neglect ~~しなければ~~ 考慮すれば internal conversion の定義をかへねばならぬ.

むしろ出てくる γ-ray p と β-ray b の数の比 b/p. 自身が意味はない ~~求められ~~ 直接る事である.

(~~p-b~~ $b/p-b$ でなしに)

つまり、γ-ray の intensity が electron ejection によって殆んど intensity を減じない,

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

Radioact.

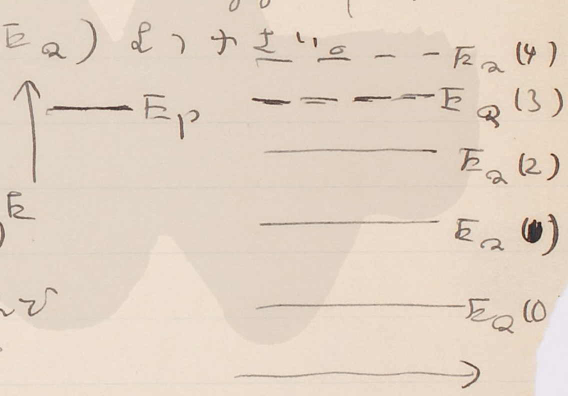
DATE

NO.

Ellis and Mott : Energy Relations in the β-Ray Type of Radioactive Disintegration (Proc Roy Soc. 141, 502 (sep. 1933))

nucleus P が Q に かはる ~~する~~ β-ray を emit して 行く。其れ $E_P > E_Q$ である。$E_P - E_Q$ が β-spectrum の upper limit ~~maximum~~ に相当し。β-ray の energy が $E_P - E_Q$ より小さければ 余分の energy は disappear すると考える。(Pauli の様に mass の小さい neutral particle が出ると考へても ~~よい。~~ β-ray の後に excite されてゐた事が 好ましい) この考へと矛盾せぬ。)

β-ray が出た後で γ-ray が出る場合、その energy は β-spectrum の ~~limit~~ max. energy (上の例でいへば $E_P - E_Q$) より小さい。



ThC''.Pb の γ-ray (strong) 2.62×10^6 volts は強力で β-ray の upper limit は 1.8×10^6 volt (Sargent : 139, 659, 1933)

しかも この state への excitation の起る prob は 殆ての dis 1 である。

~~であるから 0.582 + 1.6~~ 2.6 MeV Ellis and Mott (139, 369 によれば