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N. Linear Eq. Sept. 1954 (1)

$$\frac{\partial^2\varphi}{\partial x_\mu \partial x_\mu} - \kappa^2\varphi + \lambda\varphi^2 = 0$$

$$\varphi(x_\mu) = \sum_{n=-\infty}^{+\infty} a_n e^{+in(k_\mu x_\mu)}$$

real ; $\tilde{a}_n = a_{-n}$

$k_\mu k_\mu = \cancel{\kappa}$

$$\sum_{n=-\infty}^{+\infty} (k_\mu k_\mu n^2 + \kappa^2)\, a_n e^{+in(k_\mu x_\mu)}$$

$$= \lambda \sum_{\substack{l,m \\ l+m=n}} a_l a_m e^{i(l+m)k_\mu x_\mu}$$

$$= \lambda \sum_{p:\text{even or } p:\text{odd}} a_{\frac{1}{2}(n+p)} a_{\frac{1}{2}(n-p)} e^{in k_\mu x_\mu}$$

$$\left.\begin{aligned} l &= \tfrac{1}{2}(n+p) \\ m &= \tfrac{1}{2}(n-p) \end{aligned}\right\} \quad \left.\begin{aligned} l+m &= n \\ l-m &= p \end{aligned}\right\}$$

n : even, p : even
n : odd, p : odd

$$(n^2 k_\mu k_\mu + \kappa^2)\, a_n = \lambda \sum_{\substack{p:\text{even for } n:\text{even} \\ p:\text{odd for } n:\text{odd}}} a_{\frac{1}{2}(n+p)} a_{\frac{1}{2}(n-p)}$$

~~$k_\mu k_\mu$ a_1, a_{-1} だけが大きいとすると~~

~~$(k_\mu k_\mu + \kappa^2) a_1 = \lambda a_1$~~

a_n は充分小さい，従って $a^*_n a_m$ は充分 a_n より更にずっと小さいとすると：

第一近似：$(n^2 k_\mu k_\mu + \kappa^2)$ の小さいような n に対して a_n は比較的大きい，

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それならば $k_\mu k_\mu + \kappa^2 = 0$
ならば、a_1, a_{-1} は任意に取れる。

$$(n^2 k_\mu k_\mu + \kappa^2) a_n = \lambda \sum_{p:\text{odd}} a_{\frac{1}{2}(1+p)}\, a_{\frac{1}{2}(1-p)} + \lambda \sum_{p:\text{odd}} a_{-\frac{1}{2}(1-p)}\, a_{-\frac{1}{2}(p+1)}$$

$$= 2\lambda\,(a_1 a_{n-1} + a_{-1} a_{n+1})$$

$$\begin{cases} a_1 = A e^{i\theta} \\ a_{-1} = A e^{-i\theta} \end{cases}$$

$$\kappa^2 a_0 = 2\lambda\, a_1 a_{-1}$$

A^2 ;

$$(4 k_\mu k_\mu + \kappa^2)\, a_2 = \cancel{2\lambda\, a_1 a_0}\ \lambda\, a_1^2 \qquad (4k_\mu k_\mu + \kappa^2 = -3\kappa^2)$$

$$-3\kappa^2 a_{-2} = \lambda\, a_{-1}^2$$

$$a_0 = \frac{2\lambda}{\kappa^2} A^2 + \cdots$$

$$a_2 = -\frac{\lambda}{3\kappa^2} A^2 e^{2i\theta} + \cdots$$

$$a_{-2} = -\frac{\lambda}{3\kappa^2} A e^{-2i\theta} + \cdots$$

A^3 ;

$$-8\kappa^2 a_3 = 2\lambda\, a_2 a_1 + \cdots + \cancel{a_3 a_0} \qquad (a_1 = Ae^{i\theta})$$

$$-8\kappa^2 a_{-3} = 2\lambda\, a_{-1} a_{-2}$$

$$a_3 = \frac{2\lambda}{8\kappa^2} A^3 e^{3i\theta} + \cdots$$

$$a_{-3} = \frac{2\lambda}{8\kappa^2} A^3 e^{-3i\theta} + \cdots$$

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$$a_n = C_n^{(n)} A^n e^{ni\theta}$$

$$a_{n-1} = C_{n-1}^{(n-1)} A^{n-1} e^{(n-1)i\theta} + C_n^{(n-1)} A^{n} e^{ni\theta}$$

$$\vdots$$

$$a_m = C_m^{(m)} A^m e^{mi\theta} + C_{m+1}^{(m)} A^{m+1} e^{(m+1)i\theta} + \cdots + C_n^{(m)} A^n e^{ni\theta}$$

$$\vdots$$

$$a_1 = C_1^{(1)} A e^{i\theta} + \cdots$$

$$\cancel{a_{-1} = C}$$

$$a_0 = C_2^{(0)} A^2 + \cdots$$

$$a_{-1} = C_{-1}^{(-1)} A e^{-i\theta} + \cdots$$

$a_{-2}, a_{-1}, a_0, a_1, a_2$

$$-3\varkappa^2 a_{-2} = \lambda(a_2 a_0 + a_{-1} a_{-1} + a_0 a_{-2})$$

$$0 = \lambda(a_{-2} a_1 + a_{-1} a_0 + a_0 a_{-1} + a_1 a_{-2})$$

$$\varkappa^2 a_0 = \lambda(a_{-2} a_2 + a_{-1} a_1 + a_0^2 + a_1 a_{-1} + a_2 a_{-2})$$

$$0 = \lambda(\qquad)$$

$$-3\varkappa^2 a_2 = \lambda(\qquad)$$

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$$a_{-2} = \frac{-\lambda}{3\kappa^2} A^2 e^{-2i\theta}$$

$$a_{-1} = A e^{-i\theta} + \cancel{C_{-1} A^2} + C_{-1}' A^3$$

$$\cancel{C_{-1}} = 0$$

$$a_0 = \frac{2\lambda}{\kappa^2} A^2$$

$$a_1 = A e^{i\theta} + \cancel{C_1 A^2} + C_1' A^3$$

$$\cancel{C_1} = 0$$

$$a_2 = -\frac{\lambda}{3\kappa^2} A^2 e^{2i\theta}$$

$$\cancel{a_{-1}^2 = A^2 e^{-2i\theta} + (C_1 e^{-i\theta} + C_{-1} e^{i\theta}}$$

$\underline{A^4}$

$$-\frac{\lambda}{3\kappa^2} \cdot \frac{2\lambda}{\kappa^2} (e^{-2i\theta} + e^{2i\theta}) + C_1' C_{-1}' = 0$$

$$|C_1'|^2 = \frac{2\lambda^2}{3\kappa^4} (e^{2i\theta} + e^{-2i\theta})$$

$$= \frac{\lambda^2}{3\kappa^4} |\cos 2\theta|$$

$\underline{A^3}$

$$-\frac{\lambda}{3\kappa^2} \cdot e^{-i\theta} + \frac{2\lambda}{\kappa^2} e^{-i\theta} + \frac{2\lambda}{\kappa^2} e^{i\theta}$$

$$+ \frac{\lambda}{3\kappa^2} e^{-i\theta} \neq 0$$

$$(n^2K+\varkappa^2)a_n = \lambda \sum_{l+m=n} a_l a_m$$

$$(4K+\varkappa^2)a_2 = \lambda(a_2a_0 + a_1a_1 + a_0a_2)$$

$$(K+\varkappa^2)a_1 = \lambda(a_2a_{-1} + a_1a_0 + a_0a_1 + a_{-1}a_2)$$

$$\varkappa^2 a_0 = \lambda(a_2a_{-2} + a_1a_{-1} + a_0^2 + a_{-1}a_1 + a_{-2}a_2)$$

$$a_2 = b_2A + C_2A^2 + d_2A^3 + \cdots$$

$$a_1 = b_1A + c_1A^2 + d_1A^3 + \cdots$$

$$a_0 = b_0A + c_0A^2 + d_0A^3 + \cdots$$

A: $(4K+\varkappa^2)b_2 = \lambda$

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$$(K+x^2)a_1 = \lambda(a_1 a_0 + a_0 a_1)$$

$$x^2 a_0 = \lambda(a_1 a_{-1} + a_0^2 + a_{-1} a_1)$$

$$a_1 = A_1 e^{i\theta}$$

$$a_0 = \frac{K+x^2}{2\lambda}$$

$$\frac{K+x^2}{2\lambda}\left(x^2 - \frac{K+x^2}{\cancel{2}}\right) = 2\lambda A^2$$

$$A_1 = \frac{1}{(2\lambda)}\sqrt{(K+x^2)\frac{(x^2-K)}{2}}$$

$$(4K+x^2-2\lambda a_0)A_2 e^{2i\theta} = A_1^2 e^{2i\theta}$$

$$\begin{cases} (K+x^2)A_1 = 2\lambda\left(\dfrac{A_1^3}{4K+x^2-2\lambda A_0} + A_1 A_0\right) \\ x^2 A_0 = 2\lambda\left(\dfrac{A_1^4}{4K+x^2-2\lambda A_0} + A_1^2 + A_0^2\right) \end{cases}$$

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phase factor
$a_n \to e^{in\theta}$

$$(n^2K+\varkappa^2)a_n = \lambda \sum_{l+m=n} a_l a_m$$

n = real,
$a_n = a_{-n}$.

$$\begin{cases} (n^2K+\varkappa^2-2\lambda a_0)a_n = \lambda \sum\limits_{\substack{l+m=n \\ l,m\neq 0}} a_l a_m & n\neq 0 \\ (\varkappa^2-\lambda a_0)a_0 = \sum\limits_{l=1}^{\infty} a_l^2 \end{cases}$$