Research Institute for Fundamental Physics
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L.(1)

H. Lehmann, Über Eigenschaften von Ausbreitungsfn und Renormierungs-konstanten quantisierter Felder (Nuovo Cimento XI (1954), (1° Aprile), 342)

a) Skalare Felder:

$$\left.\begin{array}{l}\dfrac{\partial A}{\partial x_\mu} = i[A(x), P_\mu] \\ [P_\mu, P_\nu] = 0\end{array}\right\}$$

$$P_\mu \Phi_k = k_\mu \Phi_k \qquad (k_0 \geq 0)$$

$$\left.\begin{array}{l}(\Phi_0, A(x)A(x')\Phi_0) = \langle A(x)A(x')\rangle_0 \\ \quad = i\Delta^{(+)\prime}(x-x') \\ \langle A(x')A(x)\rangle_0 = -i\Delta^{(-)\prime}(x-x') \\ \langle [A(x), A(x')]\rangle_0 = i\Delta'(x-x') \\ \quad = -2i\varepsilon(x_0-x_0')\bar{\Delta}'(x-x') \\ \langle \{A(x), A(x')\}\rangle_0 = \Delta^{(1)\prime}(x-x') \\ \langle T A(x)A(x')\rangle_0 = \tfrac{1}{2}\Delta_F'(x-x')\end{array}\right\}$$

$$\Delta_F'(x) = 2i[\theta(x_0)\Delta^{(+)\prime}(x) - \theta(-x_0)\Delta^{(-)\prime}(x)]$$

$$= \Delta^{(1)\prime}(x) - 2i\bar{\Delta}'(x)$$

$$\langle A(x)A(x')\rangle_0 = \sum_k (\Phi_0, A(x)\Phi_k)(\Phi_k, A(x')\Phi_0)$$

$$= \sum_k A_{0k}(x)A_{k0}(x')$$

$$= \sum a_{0k}a^*_{0k}\exp[ik(x-x')]$$

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$$(\Phi_0, A(x)\Phi_k) = A_{0k}(x)$$
$$= a_{0k} \exp(ikx)$$
$$\left(\because \frac{\partial A_{0k}(x)}{\partial x_\mu} = i(\Phi_0, [A(x), P_\mu]\Phi_k) = i k_\mu A_{0k}(x) \right)$$
$$\rho(-k^2) = (2\pi)^3 \sum a_{0k} a_{0k}^* \quad \text{(isotropy of vacuum)}$$
$$\Delta^{(1)\prime}(x) = \int_0^\infty \Delta^{(1)}(x; \varkappa^2)\rho(\varkappa^2)d(\varkappa^2)$$
$$\rho(-k^2) = \int_0^\infty \rho(\varkappa^2)\delta(k^2+\varkappa^2)d(\varkappa^2)$$
$$\rho(\varkappa^2) \geq 0$$
$$\Delta'(x) = \int_0^\infty \Delta(x; \varkappa^2)\rho(\varkappa^2)d(\varkappa^2)$$
$$\langle [A(\mathbf{x},t), A(\mathbf{x}',t)] \rangle_0 = 0$$
$$\langle [\dot{A}(\mathbf{x},t), A(\mathbf{x}',t)] \rangle = -i\delta(\mathbf{x}-\mathbf{x}') \times \int_0^\infty \rho(\varkappa^2)d(\varkappa^2)$$

b) Spinor Felder:
$$\langle \{\psi_\alpha(\mathbf{x},t), \bar{\psi}_\beta(\mathbf{x}',t)\} \rangle_0 = \gamma^4_{\alpha\beta}\delta(\mathbf{x}-\mathbf{x}') \times \int_0^\infty \rho_1(\varkappa^2)d(\varkappa^2)$$

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$$(\Psi_{k}, A(x) A(x') \Psi_l) = \sum_j A_{kj}(x) A_{jl}(x')$$

$$= \sum a_{kj} a^*_{lj} \exp i(k_{kj} x - k_{lj} x')$$

$$k_{kj} = k_j - k_k$$

$$(\Psi_k, A(x) A(x') \Psi_k) = \sum_j a_{kj} a^*_{kj} \exp i k_{kj}(x-x')$$

$$= \exp -i(k_k - k_0)(x-x')$$

$$\times \sum_j \underbrace{a_{kj} a^*_{kj}}_{\rho_{kj}} \exp i k_{0j} (x-x')$$

(possible anisotropy of the state Ψ_k)

Linear theory:

$a_{0k} = a^*_{k0} \neq 0$ only for k: one particle state

$a_{kl} = a^*_{lk} \neq 0$ only for the case $n_k + 1 = n_l$.

and $a_{kl} a^*_{lk} = n_l$

$a^*_{lk} a_{kl} = n_k$

$$a_{kl} a^*_{lj} = a^*_{lj} a_{kl}$$

$$\{k_\mu k_\mu + \kappa^2\}\, a_{k_\mu}\, \Phi(\{n_{k_\mu}\}) = 0$$

$$\Phi = P_\kappa \Phi$$

$$\Phi = \begin{pmatrix} \Phi(0, \overset{\kappa}{\text{non-zero}}, 0) \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\sum_{k_\mu} e^{ik_\mu x_\mu}\{k_\mu k_\mu + \kappa^2\}\, a_{k_\mu}\, \Phi = 0$$

$$\left\{-\frac{\partial^2}{\partial x_\mu \partial x_\mu} + \kappa^2\right\}\varphi \cdot \Phi = 0$$

$$\left.\begin{aligned} \Phi' &= a_{k}^{*}\Phi \\ \Phi'' &= \varphi^{*}\Phi \end{aligned}\right\}$$ は Φ-空間の外にある。

$$\int e^{-ik_\mu x_\mu}\left\{-\frac{\partial^2}{\partial x_\mu \partial x_\mu} + \kappa^2\right\}\varphi\,(dx)^4\, \Phi = 0$$

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