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Kyoto University

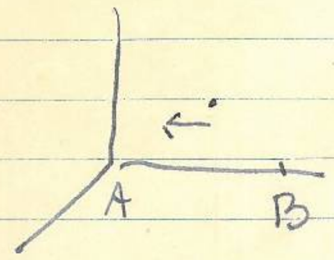
Causality 1

April 1955

林忠

C.1.

A. Einstein, Ann. d. Phys. 23 (1907) 371~384

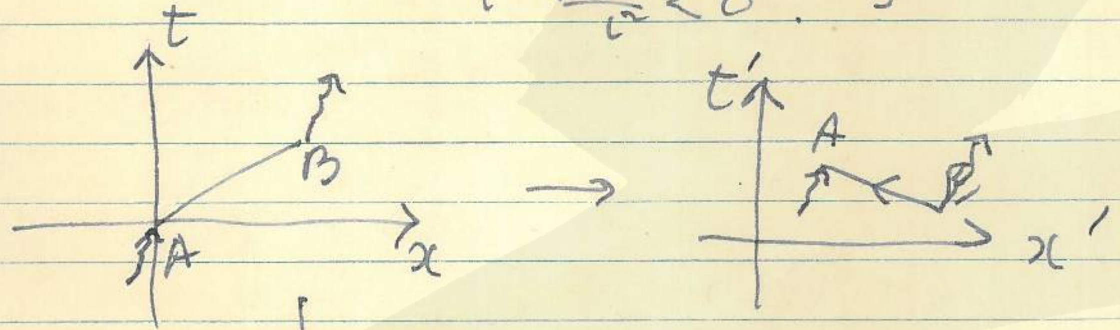


$$\frac{W-v}{1-\frac{Wv}{c^2}} \qquad v<c$$

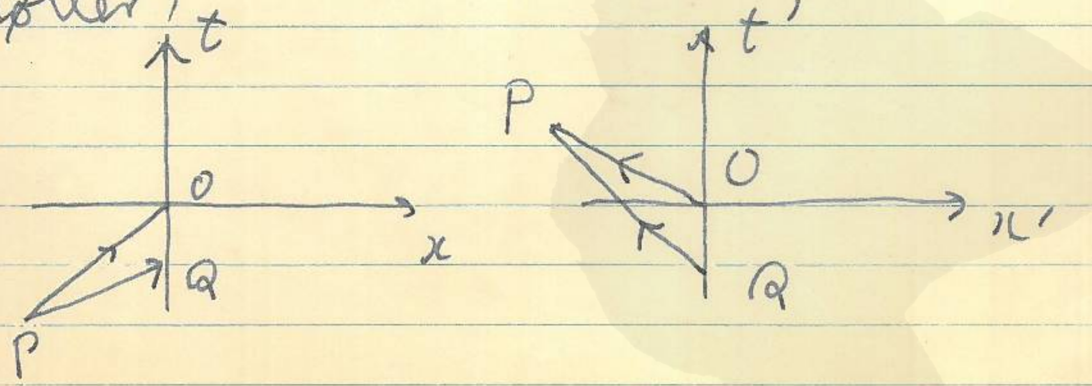
$$T = l\,\frac{1-\frac{Wv}{c^2}}{W-v}$$

$W>c$: $T<0$ となりうる

$$\left.\begin{array}{l} W-v>0 \\ 1-\frac{Wv}{c^2}<0 . \end{array}\right\}$$



(Møller)



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田中一氏:

van Kampen. (3)(4)(5)(5′)

当面の設定　　$E_{in}(t) \to E_{out}(t)$

$\psi_{in}(t) \to \psi_{out}(t)$

scattering centre ; spherically symmetric

i) $\begin{cases} E_{in} : \quad \ldots \quad \int A(k) \dfrac{e^{-ik(r+t)}}{r} dk \\ E_{out} : \quad \ldots \quad \int B(k) \dfrac{e^{ik(r-t)}}{r} dk \end{cases}$

$$B(p) = S(p) A(p)$$

エネルギー、確率、の保存

$$\int_0^\infty |A(p)|^2 dp = \int_0^\infty |B(p)|^2 dp$$

$$S(p) S^*(p) = 1$$

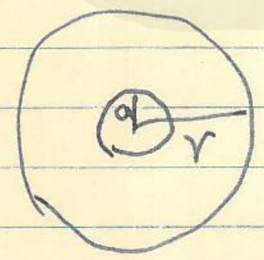
Causality Condition

i) incid. wave packet

$= 0 \qquad t < -(r-a)$

outr wave packet

$= 0 \qquad t < r - a$



ii) $\int_a^\infty 4\pi r^2 |\psi_{in}(r, -\infty)|^2 dr$

$= 1$

$r > a$

$$\to \int_a^\infty 4\pi r^2 |\psi_{in}(r, 0) + \psi_{out}(r, 0)|^2 dr \leq 1$$

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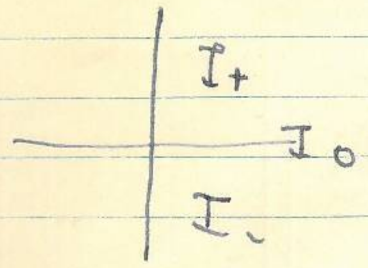
analytical continuation

$S(p) \to S(\lambda)$



i) $S(\lambda)$ regular $\lambda \in I_+$

$|S(\lambda)| < 1$ $\quad \lambda \in I_+$

ii) $S(p) \to S(\lambda)$

$S_a(\lambda) = e^{2i\lambda a} S(\lambda)$

$S_a(\lambda)$ regular in 1st quadrant (including real axis)

$\mathrm{Im}\, S_a(\lambda) < 1$.

bounded

iii) $S(-\lambda) = S^*(\lambda)$

$f(r+t) = \int A(k)\, e^{ik(r+t)}\, dk$

$f(x) = 0$ for $x < 0$

i) $A(\lambda)$ $\quad \lambda = k + i\gamma$ $\qquad A(\lambda) \xrightarrow[\gamma \to 0]{} A(k)$

ii) $A(\lambda)$ reg. $\in I_+$

iii) $\int_{-\infty}^{+\infty} |A(k+i\gamma)|^2\, dk < M \quad \gamma \geq 0$

M: γ indep.

$S(\lambda) = S(\lambda^*)^{-1}$

meromorphic fn

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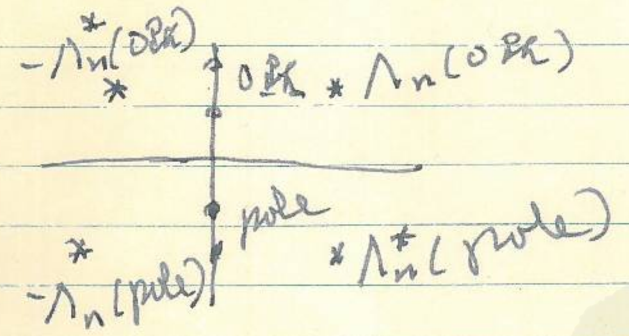
L. 4

$$S(\lambda) = \pm e^{-2i\alpha\lambda} \prod_n \frac{(\lambda-\Lambda_n)(\lambda+\Lambda_n^*)}{(\lambda-\Lambda_n^*)(\lambda+\Lambda_n)} \prod_m \frac{iL_m-\lambda}{iL_m+\lambda}$$

$L_m \geq 0$



Gacy fn

$$e^{\sum_{n=0}^{\infty} a_n z^n} = e^{2i\alpha_1\lambda + \alpha_2}$$

$$S(\lambda) = \prod \frac{1-\lambda^2/\lambda_n^2}{1-\lambda^2/\lambda_n^{*2}} \exp\left\{ i\,\alpha'\lambda^2 - i\int_0^\infty \frac{s^2\lambda^2-1}{s^2+\lambda^2}\, d\alpha(s)\right\}$$

$\alpha(s)$: 有界増加;

$\alpha'(s)$: 連続函数 $+ \sum_n \delta(s-c_n)K_n$

Sym. Rel. $\quad L_m \lesseqgtr 0$

11/12 Ph: (6) Gell-Mann 等.
(7) Karplus 等

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4月30日 　　　　　　　　　　　　　　　　　C. 5.

長谷川

Y. Nambu: On the Structure of the Scattering Matrix

i) Lorentz invariance

ii) causality

$$H_{int} = -\sum_{\alpha=1}^{3} g i \bar{\psi} \gamma_5 \tau_\alpha \psi \varphi_\alpha$$

$$M_{\alpha\beta}(l,k) = \int e^{ikx} e^{-ilx'} dx\, dx' M_{\alpha\beta}(x,x')$$

$$M_{\alpha\beta}(x,x') = (\Phi_f, P(g\bar{\psi}\gamma_5\tau_\alpha\psi(x), g\bar{\psi}\gamma_5\tau_\beta\psi(x'))\Phi_\alpha)$$

Φ_α: Heisenberg initial state
Φ_f: 　　〃　　final 　〃

$\psi, \bar{\psi}, \varphi$: Heisenberg operator

$$M = \langle q, t | A_\alpha(x), A_\beta(x') | p, s \rangle$$

A_α: Heisenberg operator

$|p,s\rangle$: H. state

1. translation 　$x_\mu \to y_\mu = x_\mu + a_\mu$
　　　　　　　$x'_\mu \to y'_\mu = x'_\mu + a_\mu$

$$A_\alpha(x) \to A_\alpha(y) = e^{ip_\mu a_\mu} A_\alpha e^{-ip_\mu a_\mu}$$

$$M \to M' = e^{-i(p-q)a_\mu} M$$

2. h. l. t. 　$R: x_\mu \to y_\mu = \sum_\nu C_{\mu\nu} x_\nu$

$$|p,s\rangle \to R|p,s\rangle = \sum_t \varepsilon C_{st} | C_{\mu\nu} p_\nu \rangle = \varepsilon | Rp, Rt \rangle$$

$$A_\alpha(x) \to R A_\alpha(x) R^{-1} = \varepsilon' A_{R\alpha}(Rx)$$

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$$M(z) \to M'(z) = \pm M(Rz) = M(z)$$

3. charge conj. gage inv.
4. Causality → micro-causality

$$[\quad] = 0 \qquad \{\quad\} = 0 \qquad \text{space-like}$$

$$M_1 = \langle q, t | \{A_\alpha(x), A_\beta(x')\} | p, s \rangle$$

$$M_2 = \langle q, t | [A_\alpha(x), A_\beta(x')] | p, s \rangle$$

$$M_3 = \varepsilon(x - x') M_2$$

$$M_4 = \langle q, t | P(A_\alpha(x), A_\beta(x')) | p, s \rangle$$

$$= \frac{1}{2} M_1 + \frac{1}{2} M_3$$

$$M_i(k, l)\, \delta^4(k - l + p - q)$$

$$\equiv \frac{1}{(2\pi)^4} \iint e^{-ilx'} e^{kx} M_i(x, x')\, dx\, dx'$$

$$P = \frac{1}{2}(p + q), \qquad \alpha = \frac{1}{2}(p - q) = \frac{1}{2}(l - k)$$

$$K = \frac{1}{2}(k + l)$$

$$M_i(P, \alpha, K)$$

$$M_3(K) = \frac{1}{\pi i} \int \frac{1}{K_0 - K_0'} \delta^3(K - K') M_2(K')\, dK'$$

$$M_\pm = M_2 \pm M_3$$

$M_+ \neq 0$ only in future

$M_- \neq 0$ only in past

$$G_\pm(x) = e^{iq_\mu x_\mu}\left(1 + \frac{1}{i} b_\mu \frac{\partial}{\partial x_\mu}\right) \Delta_\pm^{(m^2)}(x)$$

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$$(\square^2 - m^2)\Delta_\pm^{(m)} = -\delta^4(x)$$

$$\Delta_\pm(0) = 0 \qquad \frac{\partial \Delta_\pm(0)}{\partial t} = \pm\delta^3(\vec{r})$$

$$\Delta_\pm(x) = 0 \quad \text{for} \quad \begin{cases} t > 0 \\ t < 0 \end{cases}$$

$$G_\pm(k) = \frac{1 + (b, k)}{(k + a \pm i\eta)^2 + m^2}$$

$$G(k_0) = O(1/k_0)$$

$\mathfrak{M}_+$: set of fns → linear comb. of G_+

$$G_\pm(k) = \int \frac{1}{(k + a + i\eta)^2 + m^2}\,\rho(m^2, a_\mu)\, dm^2 (da)$$

$$+ \int \frac{k_\mu}{(k + a + i\eta)^2 + m^2}\,\rho_\mu(m^2, a_\mu)\, dm^2 (da)$$

$$M_\pm(K) = \iiint \frac{\rho_1(m^2, \alpha, \beta) + \rho_2(m^2, \alpha, \beta)(K, P) + \rho_3(m^2, \alpha, \beta)(K, Q)}{(K + \alpha P + \beta(Q \pm i\eta))^2 + m^2}\, dm^2 d\alpha d\beta$$

$$M_2(K) = \pi \iiint \varepsilon(K + \alpha P + \beta Q)\,\delta\{(K + \alpha P + \beta Q)^2 + m^2\}\{\rho_1 + \rho_2(K, P) + \rho_3(K, Q)\}\, dm^2 d\alpha d\beta$$

$$M_3(K) = \pi \iiint P \frac{1}{(K + \alpha P + \beta Q)^2 + m^2}(\rho_1 + \cdots)$$

$$M_1 = \pi \iiint \delta((K + \alpha P + \beta Q)^2 + m^2)(\rho_1 + \rho_2(K, P) + \rho_3(K, Q))\, dm^2 d\alpha d\beta$$

$$M_4 = -i \iiint \frac{1}{(K + \alpha P + \beta Q)^2 + m^2 - i\eta}(\rho_1 \cdots$$

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dispersion relation (Kramers')
renormalization

湯川 Ph.

Fierz, Diskussion der D_c-Funktion.
1950

time — energy

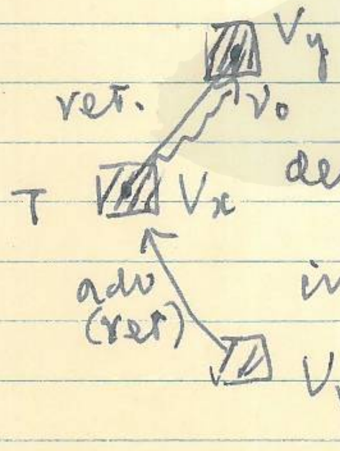
$D_c(x-x_1) = D^{\pm}$ $\quad t_2 \gtrless t_1$

$\pm$: pos. (neg) freq.

1. positive energy quantum
2. negative energy quantum (Heisenberg)

$$\int_{V_x}(dx)^4\int_{V_y}(dy)^4\,\bar\psi(y)\gamma_\mu\psi(y)D_c(y-x)\bar\psi(x)\gamma_\mu\psi(x_\mu)$$

$\Delta\nu > 1/T$ } $\nu_0 T \gg 1$
$\Delta\nu \ll \nu_0$



decrease in energy in V_x

increase in energy in V_x

$$\psi(x) = u_\mu(\vec{x})e^{-i\nu t}a(\nu) + v_\mu^*(\vec{x})e^{i\nu' t}a_1^*(\nu')$$

$$\bar\psi = \psi^*\gamma^4$$

$$\psi^* = \quad e^{+i}$$

$$\bar\psi\gamma\psi \sim a_1 a_2^*\rho_\mu(\vec{x})e^{-i\nu_0 t - \frac{t^2}{T^2}} \quad \nu_0 = (\nu_1 - \nu_2) > 0$$

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c. 9.

$$D_c = D^{+}_{ret} + D^{-}_{adv}.$$

$$\boxed{|\vec{\gamma}| \cdot \nu_0 \gg 1}$$

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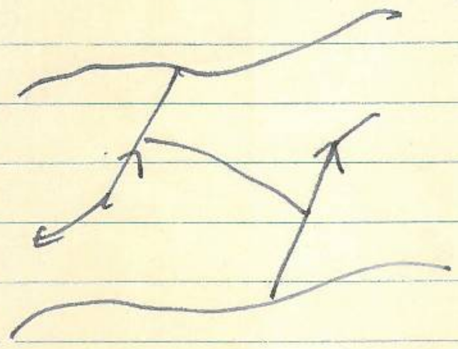
藤本 Pr: Stueckelberg

(2) A propos des divergences en théorie des champs quantifiés

① (1) Causalité et structure de la matière.

classical



$\xi''_{(k)}, \pi''_{(k)}$ at τ''

$\xi'_{(i)}, \pi'_{(i)}$ at τ'

$$\xi''_{(k)} = \frac{1}{m_{(k)}} \int^{\tau''} d\lambda \, \pi_{k}(\lambda)$$

$$\begin{cases} \xi''_{(k)} = F_{(k)}[\tau'', \tau'; \dots \pi'_{(i)}, \xi'_{(i)} \dots] \\ \pi''_{(k)} = G_{(k)}[\qquad ' \qquad ' \qquad] \end{cases}$$

$\longleftarrow$

l'action causale $D^{(ret)}(x/y)$

quantum:

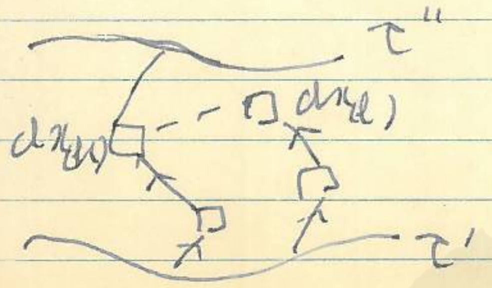
$S[\tau''; \dots \xi''_{(k)} \dots \xi''_{(k)} / \tau'; \dots \xi'_{(i)} \dots \xi'_{(k)}]$

$D^{+}(x/y)$ $\quad x^4 > y^4$

$\Gamma_{(i)}$ $\quad x_{(i)}$ $\quad$ elementary process

$D^{c}(x_{(k)}/y_{(l)})$ $\quad$ l'action causale

$$D^{c} = D^{s} + \frac{i}{2} D' = D^{+}_{ret} + D^{-}_{av} \quad x^4 \gtrless y^4$$



$D^{+}(\xi''_{k}/x_{(k)}) \Gamma_{k} dx_{(k)} D^{c}(x_{(k)}/x_{(l)})$

$\Gamma_{(l)} dx_{(l)} \cdot \dots$

$\equiv dS[\dots \xi''_{k} \dots / x_{(l)} \dots x_{(k)} \dots / \xi'_{(i)}]$

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C. U.

$$U^{+}(u_{k_1}'' \cdots u_{k_n}'' / \xi_k')\, S\{\ \xi_k'' / \xi_k'\}\, U(\xi_k' \cdots /)$$
$$= S\{\ u_{k}'' / u_k'\ \}$$

加合系： Stueckelberg (2), (3)

$${}_pD^c(x-z,\ y-z,\ \cdots)$$
$$\lim_{p\to\infty} {}_pD^c \qquad {}_pD^c(x-z) = (2\pi)^{-4}\int_p dp\, e^{ip(x-z)} D^c(p)$$

後関系： (1) Bloch
(2) Chrétien and Peierls

Bloch: $L_i = g\int dx'dx''dx'''\, F(x',x'',x''')\,\psi^{+}(x')\,u(x'')\,\psi(x''')$

local : $\delta(x'-x'')\,\delta(x'-x''')$

non-local : $(2\pi)^{-4}\int dk\, e^{ik\left(\frac{x'+x'''}{2}-x''\right)}\,\delta(kr)$
$\times\ \delta(r^2-\lambda^2) \qquad r = x'-x'''$

i) Lorentz invariance
$s = (x''-x''')^2$
$t = (x'''-x')^2$
$u = (x'-x'')^2$

ii) $F\neq 0$ only for small s, t, u
$r_0 \sim 3\cdot 10^{-13}$ cm

iii) Hermite $F(x''',x'',x') = F^{*}(x',x'',x''')$

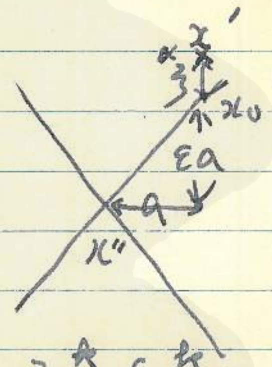
$$F(x', x'', x''') = \delta(\alpha' x' + \alpha'' x'' + \alpha''' x''')\, G(x', x'', x''')$$

$$\alpha' + \alpha'' + \alpha''' = 0$$

$$s = (x' - x'')^2 \longrightarrow G(s)$$

$$\bar{f}(x'') = \int dx'\, G(s) f(x')$$

$$\xi = \varepsilon (x_0^4 - x'^4)$$



$\varepsilon = \pm 1$

$$s = 2a\xi - \xi^2$$

i) $f(x') = f(\vec{x}_0, x_0^4 - \varepsilon\xi) \cong \sum_k \xi^k f_0^k$

$$f_0^k = \left(\frac{\partial}{\partial x^4}\right)^k f(x')\Big|_{x' = x_0}$$

ii) $dx'^4 = \dfrac{ds}{2(a-\xi)} \cong \dfrac{ds}{a} \sum_0^\infty \left(\dfrac{\xi}{a}\right)^m$

iii) $\xi = a - (a^2 - s)^{1/2} \cong \dfrac{s}{a} \sum_0^\infty \left(\dfrac{s}{a^2}\right)^n$

$$\bar{f}(x'') \cong \int d\vec{x}' \sum_0^\infty M_{m+k} \frac{f_0^k}{a^{2m+k+1}}$$

$$M_n = \int_{-\infty}^{\infty} ds\, s^n G(s)$$

$$M_0 = M_1 = \cdots = M_{p-1} = 0$$

$m + k = p \qquad 2m + k + 1 = p + 1 + m$

$$\sim \frac{1}{a^{p+1}}$$

$$d\vec{x}' \propto a^2\, da$$

$p + 1 > 3, \qquad p > 2$

conv. for $p \geq 3$

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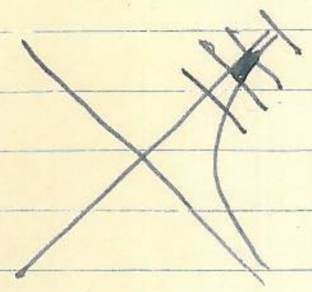
C, 13

$f=1.$ $\int G(s)\,dx'$: conv for $\{p \geq 2\}$



$s>0$ $\quad G(s)$: one value

$s\leq 0$ $\quad G(s)$ depends on the sign of x^4

G_+ even

G_- odd $\qquad s>0 \quad G_-(s)=0$

$$G(x)=\int dk\, e^{ikx} g(k)$$

$$g(k)=g(q) \qquad q=k^2$$

$$G_+ \to g_+ : \quad G_+ = -i\pi \int_{-\infty}^{+\infty} d\alpha\, dq\, \varepsilon(\alpha)\, e^{\frac{i}{2}(\alpha s + \frac{q}{\alpha})} g_+(q)$$

$$G_- \to g_- : \quad G_- = -i\pi \int_{-\infty}^{+\infty} d\alpha\, dq\, e^{\frac{i}{2}(\alpha s + \frac{q}{\alpha})} g_-(q)$$

Fourier - Bessel

$G(s)$: (1) continuous

(2) $|s|\to\infty \qquad G(s)\to 0 \sim \frac{1}{|s|^k} \quad k\geq 3$

$$\int G(s)\,dx' : \text{conv.}$$

(3) $M_n = 0 \qquad n<p$

$\left.\begin{matrix} f=1 \\ k=0 \end{matrix}\right\}$ $\quad \sum M_{m+\cancel{k}} \frac{f_0^{k}}{a^{2m+\cancel{k}+1}} \qquad m=p$

$$\int^{p} d\vec{x}' \sim a^2 da \qquad p\geq 2$$

$g(q)$ (1) $g(q)$: conti. $\quad 2k$ 回連續導函数

(2) $g^{(n)}(q) \sim \left(\frac{1}{q}\right)^{k+1} \quad q\to\pm\infty \quad (n=0,\cdots 2k)$

(3) $\int_{-\infty}^{+\infty} dq\, g(q)=0$

$$\int_{-\infty}^{+\infty} dq\, q\, g(q)=0$$

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regulator xの問

$$G_+(s) = (2\pi)^3 \int_{-\infty}^{+\infty} dq\, g_+(q) D^{(1)}(s,q)$$

$\sqrt{-q} = m$

$$G_-(s) = \quad " \quad " \quad D(s,q)$$

$$\begin{cases} \Delta_R = \int_{-\infty}^{+\infty} dx\, \rho(x) \Delta(x;x) \\ \int dx\, \rho(x) = 0 \\ \int dx\, x\rho(x) = 0 \end{cases}$$

(2) Peierls ; Crétien 1953

$\bar{f} = \int G f$

1) f : $t=0$ で最大,

2) $K_n = \int d^4x\, |f(x)|^2 t^{2n} < \infty$

空間方向 $< \frac{1}{r^2}$

時間方向 $\frac{1}{t^\infty}$

3) f : smooth

4) $K_{nl} = \int d^4x \left| \frac{\partial^l f(x)}{\partial t^l} \right|^2 t^{2n} < \infty$

$f = e^{-\alpha r^2 - \beta t^2}$

g の條件 : $\left| \frac{d^\nu g(q)}{dq^\nu} \right| < b_\nu \quad \nu \le N$

$\bar{f}$: t^{-N}

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McManus:

$$g(q) = \frac{(2\pi)^{-2}\lambda^4}{\lambda^4 + (p^2+m^2)^2} \quad \left(\begin{smallmatrix}\lambda \\ m\end{smallmatrix} : \text{const}\right)$$

三點の場合 Bloch

$$M_{nn'}(\mu q) = 0 \qquad n+n' \leq p$$

$$\propto \frac{1}{a^{p+2}}$$

Peierls: $g : p^2, q^2 (p \pm q)^2$

$$\left\{ \begin{array}{l} \dfrac{\partial^{\alpha\beta} g}{\partial(p^2)^{\alpha}\partial(p+q)^{2\beta}} < b_{\alpha\beta} \qquad \alpha+\beta \leq m \\ \quad \cdot\cdot \\ \quad \cdot\cdot \end{array} \right.$$

林氏: Causal Behavior of Field Theories with Non-localizable Interactions
M. E. Ebel Danske 29 No. 2. (1954)

May 28, 1955:

片山氏: Stueckelberg et Wanders: Acausalité de l'interaction non-locale

Lagrangian
$\downarrow$ causal condition (macro-causality)
unitary matrix

$$D^c = \underbrace{\mathcal{D}^c}_{\text{macro}} + \Delta$$