Hund, Materie als Feld, 1954, S. 367 ~ 369 (1)

I. One-field eq. Zusammenhalt of Materie

$$\left(\frac{\partial^2}{\partial x_\mu \partial x^\mu} - \kappa^2\right) u = -\eta u^2$$

$$L = -\frac{\sigma}{2}\left(\frac{\partial u}{\partial x_\mu}\frac{\partial u}{\partial x^\mu} + \kappa^2 u^2\right) + \frac{\sigma\eta}{3}u^3$$

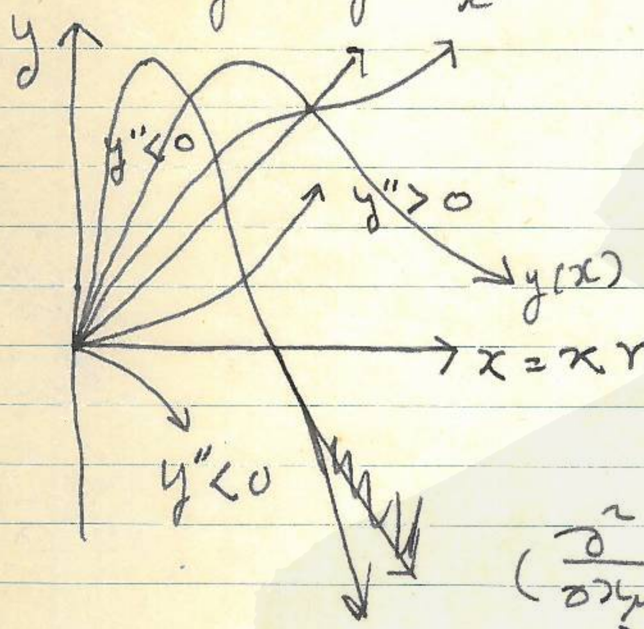
$u(r)$: $\left(\frac{d^2}{dr^2} - \kappa^2\right)(ru) + \eta r u^2 = 0$

$-6B = (A-1)A$

$r = x/\kappa$, $u = \frac{\kappa^2}{\eta}\frac{y}{x}$

$6Bx = (A-1)(Ax+Bx^2)$

$y'' - y + \frac{y^2}{x} = 0$ $\quad y'' = y\left(1 - \frac{y}{x}\right)$



There is a solution $y(x)$ which is zero at $x=0$ and $x=\infty$.

$$u = \frac{\kappa}{\eta}\frac{y(\kappa r)}{r}$$

$\kappa \to 0$ $\quad u \to 0$

$y(x) \cong Ax + Bx^2$ for $x \ll 1$ $\quad B<0$, $A>1$

$\cong Ce^{-x}$ for $x \gg 1$

$$\left(\frac{\partial^2}{\partial x_\mu \partial x^\mu} - \kappa^2\right) v = -\eta v^* v$$

$u(r,t) = v(r)e^{-i\omega t}$ $\quad$ v: real とおく

$$\left(\frac{d^2}{dr^2} + \omega^2 - \kappa^2\right)(rv) + \eta r v^2 = 0$$

$\omega^2 - \kappa^2 < 0$, $\quad \kappa' = \sqrt{\kappa^2 - \omega^2}$

$\omega' = \sqrt{\omega^2 - \kappa^2}$ $\quad v = \frac{\kappa'}{\eta}\frac{y(\kappa' r)}{r}$

$\omega^2 - \kappa^2 > 0$ $\quad \left(\frac{d^2}{dr^2} + \omega'^2\right)(rv) + \eta r v^2 = 0$

$r = x/\omega'$, $u = \frac{\omega'^2}{\eta} y/x$

$$y'' = -y\left(1 + \frac{y}{x}\right)$$

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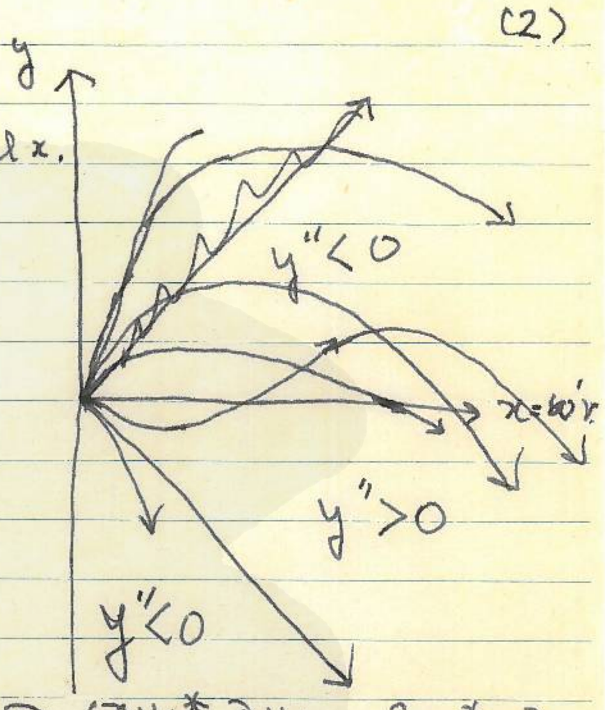
Oscillating solutions
for $\{ y \to x,\ y' > -1. \}$ for small x.
$y = 0$ for $x = 0$



II. Two-field Equations

$$L = -\frac{\varepsilon_0}{2}\left(\frac{\partial u}{\partial x_\mu}\frac{\partial u}{\partial x^\mu} + \lambda^2 u^2\right) - \frac{\sigma}{2\kappa}\left(\frac{\partial \psi^*}{\partial x_\mu}\frac{\partial \psi}{\partial x^\mu} + \kappa^2 \psi^*\psi\right) + g u \psi^*\psi$$

for $g = 0$: $\psi = A e^{-ic\kappa t}$

$$\frac{\partial^2 \psi}{\partial x_\mu \partial x^\mu} - \kappa^2 \psi = -\frac{2\kappa g}{\sigma} u \psi$$

$$\frac{\partial^2 u}{\partial x_\mu \partial x^\mu} - \lambda^2 u = -\frac{g}{\varepsilon_0}\psi^*\psi$$

$$\psi = A e^{-i\omega t} \qquad u = \frac{g}{\varepsilon_0 \lambda^2} A^* A$$

$$\frac{\omega^2}{c^2} = \kappa^2 - \frac{2g^2}{\varepsilon_0 \sigma}\frac{\kappa}{\lambda^2} A^* A$$

$$\frac{\omega}{c} - \kappa \cong -\frac{g^2}{\varepsilon_0 \sigma}\frac{1}{\lambda^2} A^* A$$

$$E = \hbar(\omega - \kappa) \cong -\frac{g^2}{\varepsilon_0 \hbar c}\frac{\hbar^3}{m_u^2 c} n$$

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i) 1st kind solutions

~~$\psi = A_1 e^{-i\omega_1 t} + A_2 e^{-i\omega_2 t}$~~

$$\psi = A \exp i(kr - \omega t)$$

$$-k^2 + \omega^2/c^2 - \kappa^2 = -\frac{2\kappa g}{\sigma} u$$

$$-\lambda^2 u = ~~A~~ - \frac{g}{\varepsilon_0} A^* A$$

$$u = \frac{g}{\varepsilon_0 \lambda^2} A^* A$$

$$\boxed{\frac{\omega^2}{c^2} = \kappa^2 + k^2 - \frac{2g^2}{\varepsilon_0 \sigma} \frac{\kappa}{\lambda^2} A^* A}$$

$$\omega - \kappa c \cong \frac{c}{2\kappa} k^2 - \frac{g^2}{\varepsilon_0 \sigma} \frac{1}{\lambda^2} A^* A$$

$$E \cong \frac{1}{2m_\psi}(\hbar k)^2 - \frac{g^2}{\varepsilon_0 \hbar c} \frac{\hbar^3}{m_u^2 c} n$$

ii) 2nd kind solutions

$\psi = 0$: $\psi^* \psi = 0$

$$u = B \exp i(kr - \omega t)$$

$$\boxed{\frac{\omega^2}{c^2} = \lambda^2 + k^2}$$

iii) mixture $\quad u = B_0 + B_1 \exp i\{(k_1 - k_2) r - (\omega_1 - \omega_2) t\} + \tilde{B}_2 \exp -i(\ )$

$$\psi = A_1 \exp i(k_1 r - \omega_1 t) + A_2 \exp i(k_2 r - \omega_2 t)$$

$$\left(\frac{\omega_1^2}{c^2} - k_1^2 - \kappa^2\right) A_1 = (B_0 A_1 + B_1 A_2)\left(-\frac{2\kappa g}{\sigma}\right)$$

$$\left(\frac{\omega_2^2}{c^2} - k_2^2 - \kappa^2\right) A_2 = (B_0 A_2 + \tilde{B}_1 A_1)\left(\frac{2\kappa g}{\sigma}\right)$$

$$0 = B_1 A_1 \exp i\{(2k_1 - k_2) r + (2\omega_1 - \omega_2) t\}$$

$$0 = \tilde{B}_1 A_2 \exp i\{\ \} \ \cdots$$

$$-\lambda^2 B_0 = -\frac{g}{\varepsilon_0}(A_1^* A_1 + A_2^* A_2)$$

$$(\qquad \lambda^2) B_1 = A_2^* A_1$$

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このようにして [illegible]

i) one nucleon ~~(no meson)~~ solution
(with ~~meson field~~ uniform meson field)
(or meson ~~wi~~ field corresponding to zero momentum-energy (and zero mass) mesons)

$$\psi = A \exp i(kr - \omega t)$$
$$u = \frac{g}{\varepsilon_0 \lambda^2} A^* A$$
$$\frac{\omega^2}{c^2} = \kappa^2 + k^2 - \frac{2g^2}{\varepsilon_0 \sigma} \frac{\kappa}{\lambda^2} A^* A$$

Thus, for very large $|A|$, $\omega^2 < 0$ or the mass of the nucleon becomes imaginary.

ii) one meson solution (with no nucleon)

$$\psi = 0$$
$$u = B \exp i(kr - \omega t)$$
$$\frac{\omega^2}{c^2} = \lambda^2 + k^2 .$$

Superposition of the solutions of second kind is always a solution of field equations, while the mere superposition of the solutions of first kind is, in general, not a solution of field eq.

Fourier Transform:

$$u(x_\mu) = \int u(k_\mu) \exp(i k_\mu x_\mu)\, d^4k$$
$$\psi(x_\mu) = \int \psi(p_\mu) \exp(i p_\mu x_\mu)\, d^4p$$
$$u\psi(x_\mu) = \int u(P_\mu - p_\mu)\, \psi(p_\mu) \exp(i P_\mu x_\mu)\, dp\, dP$$

$$\begin{matrix} & k_\mu & p_\mu \\ P_\mu & 1 & 1 \\ p_\mu & 0 & 1 \end{matrix} \qquad p_\mu + k_\mu = P_\mu$$

$$p_\mu = p_\mu \qquad u\psi(P_\mu) = \int u(P_\mu - p_\mu)\, \psi(p_\mu)\, d^4p$$

$$\begin{cases} (-P_\mu P_\mu - \varkappa^2)\psi(P_\mu) = -\frac{2\varkappa g}{\sigma}\int u(P_\mu - p_\mu)\psi(p_\mu)d^4p \\ (-k_\mu k_\mu - \lambda^2)u(k_\mu) = -\frac{g}{\varepsilon_0}\int \psi^*(k_\mu - p_\mu)\psi(p_\mu)d^4p \end{cases}$$

$$\psi(P_\mu) = \psi^{(0)}\delta(P_\mu - P_\mu^{(0)}),\quad \psi^* = \psi^{(0)*}\delta(P_\mu + P_\mu^{(0)})$$

$$(+P_\mu^{(0)}P_\mu^{(0)} + \varkappa^2)\psi^{(0)}\delta(P_\mu - P_\mu^{(0)}) = +\frac{2\varkappa g}{\sigma}u(P_\mu - P_\mu^{(0)})$$

$$u(P_\mu - P_\mu^{(0)}) = u^{(0)}\cdot\delta(P_\mu - P_\mu^{(0)})$$

$$u(k_\mu) = u^{(0)}\delta(k_\mu)$$

$$(~~k_\mu~~ k_\mu + \lambda^2)u^0\delta(k_\mu) = -\frac{g}{\varepsilon_0}\psi^{0*}\psi^0\int\delta(k_\mu - p_\mu + P_\mu^{(0)})$$

$$\times\delta(p_\mu - P_\mu^{(0)})\,d^4p$$

$$\lambda^2 u^0\delta(k_\mu) = -\frac{g}{\varepsilon_0}\psi^{0*}\psi^0\delta(k_\mu)$$

$$\psi^*(p_\mu) = \sum_j \psi^{(j)}\delta(p_\mu - p_\mu^{(j)})$$

$$\sum_j (p_\mu^{(j)}p_\mu^{(j)} + \varkappa^2)\psi^{(j)}\delta(p_\mu - p_\mu^{(j)})$$

$$= +\frac{2\varkappa g}{\sigma}\sum_j u(p_\mu - p_\mu^{(j)})\psi^{(j)}$$

$$u(k_\mu) = \sum_\ell u^{(\ell)}\delta(k_\mu - k_\mu^{(\ell)})$$

$$\sum_j\left\{(p_\mu^{(j)}p_\mu^{(j)} + \varkappa^2)\psi^{(j)}\delta(p_\mu - p_\mu^{(j)}) - \frac{2\varkappa g}{\sigma}\sum_\ell u^{(\ell)}\delta(p_\mu - k_\mu^{(\ell)} - p_\mu^{(j)})\psi^{(j)}\right\} = 0$$

$l=0$: $\left.\begin{array}{l}(p_\mu^{(j)} p_\mu^{(j)} + \varkappa^2) - \frac{2\kappa g}{\sigma} u^{(0)} = 0 \\ k_\mu^{(0)} = 0\end{array}\right\}$

$l \neq 0$: $k_\mu^{(l)} \neq 0$:

$$\sum_j u^{(l)} \psi^{(j)} = 0$$

$$\sum_l (k_\mu^{(l)} k_\mu^{(l)} + \lambda^2) u^{(l)} \delta(k_\mu - k_\mu^{(l)})$$

$$= \frac{g}{\varepsilon_0} \sum_{j,m} \psi^{(m)*} \psi^{(j)} \int \delta(k_\mu - p_\mu + p_\mu^{(m)}) \delta(p_\mu - p_\mu^{(j)}) d^4p$$

$$= \frac{g}{\varepsilon_0} \sum_{j,m} \psi^{(m)*} \psi^{(j)} \delta(k_\mu - p_\mu^{(j)} + p_\mu^{(m)})$$

(i) $k_\mu^{(l)} k_\mu^{(l)} + \lambda^2 = 0$ $\qquad u^{(l)} \neq 0$.

(ii) $k_\mu^{(l)} k_\mu^{(l)} + \lambda^2 \neq 0$

$$k_\mu^{(l)} = p_\mu^{(j)} - p_\mu^{(m)}.$$

$$(k_\mu^{(l)} k_\mu^{(l)} + \lambda^2) u^{(l)} = \frac{g}{\varepsilon_0} \psi^{(m)*} \psi^{(j)}$$

$$\sum_m u^{(l)} \psi^{(m)} = \frac{g}{\varepsilon_0} \sum_m \frac{\psi^{(m)*} \psi^{(m)}}{k_\mu^{(l)} k_\mu^{(l)} + \lambda^2} \psi^{(j)}$$

$$= 0$$

$$\sum_m \frac{\psi^{(m)*} \psi^{(m)}}{(p_\mu^{(j)} - p_\mu^{(m)})(p_\mu^{(j)} - p_\mu^{(m)}) + \lambda^2} = 0$$

$p_\mu^{(j)} - p_\mu^{(m)}$: mixture of time-like and space-like vectors.

$m, j = 1, 2, 3$

ψ　　$p_\mu^{(1)}$　$p_\mu^{(2)}$

u　　0, $k_\mu^{(1)}$　$-k_\mu^{(1)}$

$u\psi$　　$p_\mu^{(1)}$,　$p_\mu^{(2)}$

$k_\mu^{(1)}+p_\mu^{(1)}$, $-k_\mu^{(1)}+p_\mu^{(1)}$

$k_\mu^{(1)}+p_\mu^{(2)}$, $-k_\mu^{(1)}+p_\mu^{(2)}$

$\psi^*\psi$　0, $-p_\mu^{(2)}+p_\mu^{(1)}$　$-p_\mu^{(1)}+p_\mu^{(2)}$

$k_\mu^{(1)}=p_\mu^{(1)}-p_\mu^{(2)}$

~~k_μ~~ $u\psi$:　$p_\mu^{(1)}$, $p_\mu^{(2)}$

$2p_\mu^{(1)}-p_\mu^{(2)}$,　$p_\mu^{(2)}$

$p_\mu^{(1)}$　　$2p_\mu^{(2)}-p_\mu^{(1)}$

ψ :　$p_\mu^{(1)}$　$p_\mu^{(2)}$　$p_\mu^{(3)}$

u :　0　$k_\mu^{(1)}$　$-k_\mu^{(1)}$

~~$u\psi$~~:

ψ : 0, $\pm p_\mu^{(1)}$, $\pm 2p_\mu^{(1)}$, $\pm 3p_\mu^{(1)}$, ...

u :　0, $p_\mu^{(1)}$, $-p_\mu^{(1)}$. $2p_\mu^{(1)}$, $-2p_\mu^{(1)}$, . .

$u\psi$: 0, $\pm p_\mu^{(1)}$, $\pm 2p_\mu^{(1)}$, $\pm 3p_\mu^{(1)}$, $\pm 4p_\mu^{(1)}$ . . .

$\psi^*\psi$: 0, $\pm p_\mu^{(1)}$, $\pm 2p_\mu^{(1)}$, ...

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$$\Psi(p_\mu) = \sum_{j=-\infty}^{+\infty} \psi^{(j)} \delta(p_\mu - j p_\mu^{(0)})$$

$$\Psi^*(p_\mu) = \sum_{j=-\infty}^{+\infty} \psi^{(j)*} \delta(p_\mu + j p_\mu^{(0)})$$

$$u(k_\mu) = \sum_{j=-\infty}^{+\infty} u^{(j)} \delta(k_\mu - j p_\mu^{(0)})$$

$$u^{(j)*} = u^{(-j)}$$

$$(j^2 p_\mu^{(0)} p_\mu^{(0)} + \kappa^2)\psi^{(j)} = \frac{2\kappa g}{\sigma} \sum_l u^{(l)} \psi^{(j-l)}$$

$$\because \int u(P_\mu - p_\mu)\psi(p_\mu) d^4p \qquad (d^4p)$$

$$= \sum_{l,j} u^{(l)} \int \delta(P_\mu - p_\mu - l p_\mu^{(0)}) \psi^{(j)} \delta(p_\mu - j p_\mu^{(0)})$$

$$= \sum_{l,j} u^{(l)} \psi^{(j)} \delta(P_\mu - (j+l) p_\mu^{(0)})$$

$$= \sum_j \sum_l u^{(l)} \psi^{(j-l)} \delta(P_\mu - j p_\mu^{(0)})$$

$$(j^2 p_\mu^{(0)} p_\mu^{(0)} + \lambda^2) u^{(j)} = \frac{g}{2\sigma} \sum \psi^{*(l)} \psi^{(j-l)}$$