I. Renormalization in Q.E.D., group

木庭代: Bogoljubov and Širkov DAN. 103 (55)　　Dec. 22, 1955　　(1)

$G_1 \to G_2 = Z_2 G_1$

$D_1 \to D_2 = z_3 D_1$　　　$Z_1 = Z_2$

$e_1 \to e_2 = z_3^{1/2} e_1$　　　Ward's identity

$$G(k) = i\,\frac{a(k^2)\hat{k} + b(k^2)m}{k^2 - m^2}$$

$$D_{mn}(k) = \frac{i}{k^2}\left(g^{mn} - \frac{k^m k^n}{k^2}\right) d(k^2)$$

$$d\left(\frac{k^2}{\lambda_2^2}, \frac{m^2}{\lambda_1^2}, e_1^2\right) = z_3\, d\left(\frac{k^2}{\lambda_1^2}, \frac{m^2}{\lambda_1^2}, e_1^2\right)$$

$$e_2^2 = z_3^{-1} e_1^2$$

$$k^2 = \lambda_1^2 : \cdots \quad z_3 = d\left(\frac{\lambda_1^2}{\lambda_2^2}, \frac{m^2}{\lambda_1^2}, e_2^2\right)$$

$$d = d\left(\frac{k^2}{m^2}, e_0^2\right)$$

$$e^2 = e_0^2\, d\left(\frac{\Lambda^2}{m^2}, e_0^2\right)$$

$a, b \to s$

$$s\left(\frac{k^2}{\lambda_2^2}, \frac{m^2}{\lambda_2^2}, e_2^2\right) s\left(\frac{\lambda_2^2}{\lambda_1^2}, \frac{m^2}{\lambda_1^2}, e_1^2\right)$$

$$= s\left(1, \frac{m^2}{\lambda_2^2}, e_2^2\right) s\left(\frac{k^2}{\lambda_1^2}, \frac{m^2}{\lambda_1^2}, e_1^2\right)$$

$$k^2/\lambda_2^2 = x, \quad m^2/\lambda_2^2 = y, \quad \lambda_1^2/\lambda_2^2 = t$$

$$e^2 d(x, y, e^2) = e^2 d(t, y, e^2)\, d\left(\frac{x}{t}, \frac{y}{t}, e^2 d(t, y, e^2)\right)$$

$$\ln s(x, y, e^2) + \ln s(1, y/t, e^2 d(t, y, e^2))$$

$$= \ln s\left(\frac{x}{t}, \frac{y}{t}, e^2 d(t, y, e^2)\right) + \ln s(t, y, e^2)$$

$$\frac{\partial\, e^2 d(x, y, e^2)}{\partial x} = \frac{e^2 d(x, y, e^2)}{x}\left[\frac{\partial}{\partial \xi} d\left(\xi, \frac{y}{x}, e^2 d(x, y, e^2)\right)\right]_{\xi=1}$$

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(2.)

$$\frac{\partial \ln s(x, y, e^2)}{\partial x} = \frac{1}{x}\left[\frac{\partial}{\partial \xi} s\left(\xi, \frac{y}{x}, e^2 d(x, y, e^2)\right)\right]_{\xi=1}$$

$$\frac{k^2}{\lambda^2} \sim 1$$

renormalization group

II. Revision of Perturbation Theory in Q.E.D.

$m^2/|k^2| \ll 1$; $y = 0$

$$\frac{\partial\, e^2 d(x, 0, e^2)}{\partial x} = \frac{e^2 d}{x}\varphi(0, e^2 d)$$

$$\ln \frac{s(x, 0, e^2)}{s(x_0, 0, e^2)} = \int_{x_0}^{x} \frac{dz}{z}\left[\frac{\partial}{\partial \xi} \ln s(\xi, 0, e^2 d)\right]_{\xi=1}$$

$$\int_{e^2}^{e^2 d} \frac{dz}{z\,\varphi(0, z)} = \ln x$$

$$\ln \frac{s(x, 0, e^2)}{s(x_0, 0, e^2)} = \int_{e^2 d(x_0)}^{e^2 d(x)} \frac{dz}{z\,\varphi(0, z)}\left[\frac{\partial}{\partial \xi} \ln s(\xi, 0, z)\right]_{\xi=1}$$

$k^2 \sim \lambda^2 \gg m^2$

$$d^{-1}(\xi, 0, e^2) = 1 - \frac{e^2}{3\pi}\ln \xi - \frac{e^4}{4\pi^2}\ln \xi + \cdots$$

$$\frac{1}{\varphi(0, z)} = \frac{3\pi}{z}\left\{1 - \frac{3z}{4\pi} + c_1 z^2 + \cdots\right\}$$

~~$e^2 d$~~ x: 任意で $e^2 d < M$ ということは有り得ない.

$$\because \ln x < 3\pi \int_{e^2}^{e^2 d} \frac{dt}{t^2}(1+c) < \frac{3\pi}{e^2}(1+c)$$

Landau $\left(< 3\pi\left(\frac{1}{e^2} - \frac{1}{e^2 d}\right)\right)$

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$$\int_{e^2}^{\infty} \frac{d\tau}{2\varphi(0,z)} < \infty$$

なら Landau がいうように Q.E.D. は不満足.

$$d^{-1}(x,0,e^2) = 1 - \frac{e^2}{3\pi}\ln x + \frac{3}{4\pi}e^2 \ln\left(1 - \frac{e^2}{3\pi}\ln x\right)$$
$$+ \cdots$$

$$a(\xi,0,e^2) = 1 + c_2 e^4 \ln\xi + \cdots$$
$$b(\xi,0,e^2) = b_0\left[1 - \frac{3e^2}{4\pi}\ln\xi + e^4 \cdots\right]$$

Infrared: $k^2 \sim m^2$.

ps. meson
Renormal. group:
$$\left.\begin{array}{l} S' = z_1 S, \quad \Gamma' = z_2^{-1}\Gamma \quad D' = z_3 D \\ g'^2 = z_1^3 z_2^{-2} z_3^{-1} g^2 \end{array}\right\}$$
$$\delta = S^2 \Gamma^2 D.$$

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I. Removal of Infinities in Q.E.D.
point interaction ⟶ ?
perturbation にせめる.

$$f + \lambda I(f) = f_0$$

λ:小　$I(f_0) = \infty$

という問題がある.

ところ　$\lim_{\lambda \to 0} \lambda I(f) \neq 0$

($f + \dfrac{\lambda}{f - f_0} = f_0$

$(f - f_0)^2 = -\lambda$　?)

G, $D_{\mu\nu}$, Γ に関する結合方程式をつくる.

II. Asymptotic Expansion of Green Fn for Electrons in Q.E.D.

III. asymptotic expansion of G. F. for photons in Q.E.D.

$$D_{\mu\nu}(k) = \overset{\circ}{D}_{\mu\nu}(k) - \frac{e_1^2}{\pi i} D_{\mu\sigma}(k)$$
$$\times Sp\left[\int G(p)\Gamma_\sigma(p, p-k; k)G(p-k)\gamma_\tau d^4p\right]$$
$$\times \overset{\circ}{D}_{\tau\nu}(k)$$

$\frac{1}{137}$

$k > m$　$d_t(k^2) = \dfrac{e^2}{e_1^2}\left\{1 - \dfrac{e^2}{3\pi}\ln\left(-\dfrac{k^2}{m^2}\right)\right\}^{-1}$

$$\frac{e^2}{e_1^2} = 1 - \frac{e^2}{3\pi}\ln\frac{1}{a^2m^2}$$

(5)

electron-electron scattering (Møller scatt.)

$$e_1^2 \delta^2(0)\, \rho^2(0)\, d\tau^0$$

$$e^2 = \frac{e_1^2}{1 + \frac{e_1^2}{3\pi} L}$$

$$e_1^2 = \frac{e^2}{1 - \frac{e^2}{3\pi} L}$$

$L \sim \frac{3\pi}{e^2}$ となって、$e_1^2 \ll 1$ と矛盾する.

$\xi = L > \frac{3\pi}{e^2}$ では考えない

gravitation interaction が e.m. int と同じ order になるのは

$$p^2 \sim e^2/\kappa \rightarrow p \sim \cancel{[illegible]}\, 10^{27}\, eV$$

$L \sim 100$