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Quantum Field Theory
and
Gravitation

$2\kappa = 1$

$l_0 = \sqrt{2\kappa \hbar c}$

$\kappa = \dfrac{8\pi\gamma}{c^4}$

$= \sqrt{\dfrac{4\gamma\hbar}{c^3}}$

$\approx 10^{-32}$ cm

Schwarzschild

① $\dfrac{2\gamma M}{c^2 r} > 1$

$M \sim \dfrac{\hbar}{cr}$

$r \gtrsim l_0$

Landau?
Deser
Laurent
Misner

$S = \int (L_D + g) d^4x \to e^{iS}$

$g_{\mu\nu} \to \gamma^\mu$

$\gamma^\mu p_\mu + m$

Lehmann non-tensor tensor

$i \dfrac{\partial \phi}{\partial x^\mu} = [\phi, P_\mu]$

$\to \dfrac{\partial A_\mu}{\partial x^\nu} = [P_\nu, A_\mu] \to$ contrad.

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Zur fünfdimensionalen Darstellung der Relativitätstheorie (1)
(ZS. f. Phys. 46 (1927), 188)

1. Bewegungsgl. und Erhaltung der Ladung:

$$\frac{dx^i}{d\tau} = \frac{\partial H_0}{\partial p_i}, \quad \frac{dp_i}{d\tau} = -\frac{\partial H_0}{\partial x^i}$$

$$p_i = m g_{ik}\frac{dx^k}{d\tau}, \quad H_0 = \frac{1}{2m} g^{ik} p_i p_k$$

$$x^0 \rightarrow p_0 = e/\beta c$$

$$\frac{dx^0}{d\tau} = \frac{\partial H_0}{\partial p_0} = 0, \quad \frac{dp_0}{d\tau} = -\frac{\partial H_0}{\partial p_0} = 0$$

$$H = \frac{1}{2m} g^{ik}\left(p_i - \frac{e}{c}\varphi_i\right)\left(p_k - \frac{e}{c}\varphi_k\right) + \text{const}\left(= \frac{1}{2m}p_0^2\right)$$

$$H = \frac{1}{2m}\left\{g^{ik}p_i p_k - 2\beta\varphi^k p_k p_0 + (1+\beta^2\varphi_k\varphi^k)p_0^2\right\}$$

$$= \frac{1}{2m}\sum \gamma^{ik} p_i p_k.$$

$$\gamma^{ik} = g^{ik}, \quad \gamma^{i0} = -\beta\varphi^i, \quad \gamma^{00} = 1+\beta^2\varphi_k\varphi^k$$

$$p_i = m\sum \gamma_{ik}\frac{dx^k}{d\tau}$$

$$\begin{cases} m\dfrac{dx^0}{d\tau} = (1+\beta^2\varphi_k\varphi^k)p_0 - \beta\varphi^k p_k \\ m\dfrac{dx^i}{d\tau} = g^{ik}p_k - \beta\varphi^i p_0 \end{cases}$$

$$\begin{cases} d\sigma^2 = d\vartheta^2 + ds^2 \\ d\vartheta = dx^0 + \beta\varphi_i dx^i, \quad ds^2 = g_{ik}dx^i dx^k \end{cases}$$

$$\frac{d}{d\tau}\left(\frac{\partial L}{\partial \frac{dx^i}{d\tau}}\right) = \frac{\partial L}{\partial x^i}$$

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$$L = \frac{1}{2}m\left(\frac{ds}{d\tau}\right)^2 = \frac{m}{2}\left\{ g_{ik}\frac{dx^i}{d\tau}\frac{dx^k}{d\tau} + \left(\frac{dx^0}{d\tau} + \beta\varphi_i\frac{dx^i}{d\tau}\right)^2\right\}$$

$$\left(\quad L_0 = \frac{m}{2} g_{ik}\frac{dx^i}{d\tau}\frac{dx^k}{d\tau} + \beta p_0 \varphi_i \frac{dx^i}{d\tau}\right.$$

2. Invarianzeigenschaften

$$x^{0\prime} = \psi(x^0, x^1, x^2, x^3, x^4)$$

$$x^{i\prime} = \psi_i(x^1, x^2, x^3, x^4)$$

$\frac{\partial\psi}{\partial x^i}$: unabhängig von x^0

$$\frac{\partial}{\partial x^0}\left(\frac{\partial\psi}{\partial x^i}\right) = \frac{\partial}{\partial x^i}\left(\frac{\partial\psi}{\partial x^0}\right) = 0 \rightarrow \frac{\partial\psi}{\partial x^0} = \text{const.} = 1$$

$$x^{0\prime} = x^0 + \psi_0(x^1, x^2, x^3, x^4)$$

$\Theta^{\mu\nu}$: Θ_{00} : invariant

$\Theta_{0i}\, dx^i$: invariant

$$T_{ik} = g_{i\mu}g_{k\nu}\Theta^{\mu\nu} = \Theta_{ik} - \beta(\varphi_i\Theta_{0k} + \varphi_k\Theta_{0i}) + \beta^2\varphi_i\varphi_k\Theta_{00}$$

$$\gamma^{-1} = |\gamma_{ik}| = g^{-1}$$

$$\bar{x}^i = x^i + \varepsilon\xi^i(x^1, x^2, x^3, x^4) \qquad i = 0, 1, \cdots 4$$

$$\left.\begin{cases} \dfrac{\partial\sqrt{-g}\, s^k}{\partial x^k} = 0 \\ \mathrm{Div}_i T = F_{ik}s^k \end{cases}\right) \rightarrow \mathrm{Div}_i\Theta = 0$$

Erhaltungssätze

$$\square U = \frac{1}{\sqrt{-\gamma}}\sum\frac{\partial}{\partial x^i}\left(\sqrt{-\gamma}\,\gamma^{ik}\frac{\partial U}{\partial x^k}\right)$$

(3)

$$\beta = \sqrt{2\kappa}$$

$$G_{ik} = R_{ik} - \frac{1}{2} g_{ik} R = -\kappa (T^{ik} + S^{ik})$$

$$G^{ik} =$$

Existenz des elektrischen Elementarquantums:

$$\gamma_{ik} = \begin{pmatrix} \alpha_{ik} & \beta_{ik} e^{\frac{2\pi i}{h} p_0 x^0} \\ \beta^*_{ik} e^{-\frac{2\pi i}{h} p_0 x^0} & \alpha^*_{ik} \end{pmatrix}$$

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