

Field Group

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田中一正: On Quantum Field Theories
 Danske 29(1955)

R. Haag

自由な場と束縛場の違い...

$f_R(r)$ $R=1, 2, \dots$
 自由な場 \rightarrow 相互作用のない場の可換性
 (Hilbert空間の次元)

束縛 \rightarrow 連続な場

$\Psi(v_1, v_2, \dots)$
 自由な場 \leftrightarrow 可換性
 束縛場 \leftrightarrow 連続性

(0) $\langle \dots \rangle$ field theory is well defined
 math. scheme?

素数の存在と相互作用の存在?

(1) Postulate

- A. Quantum Physics
- B. Lorentz Invariance
- C. Partial Postulate

A. (a) state $\rightarrow \Psi \in \mathcal{H}_2$
 (b) $w = |\langle \Psi_2, \Psi_1 \rangle|^2$

B. $\alpha'_\mu = a_{\nu\mu} \alpha_\nu - b_\mu$
 \mathcal{H}_j : Darstellungsraum der h.T.
 $D(L_1)D(L_2) = \pm D(L_1 L_2)$
 $\Psi' = D(L)\Psi$

C. ~~event~~ event is \mathcal{R}^4 (local)
 Δd resolving time $\ll d/c$
 counter \leftrightarrow counter

(C₁) one particle state; Lorentz group
 の irreducible repres.
 particles \mathcal{H}_j $j \in \mathcal{H}_j$

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(C2) Partial state $\Psi(x, t)$
 $\lim_{t \rightarrow \pm\infty} \Psi(x, t) = \text{one particle state}$

(C3) boson or fermion
 $D(L) = 1 + \sum_j D_j(L) + \sum_j \{D_j(L) \times D_j(L)\} + \sum_{j \neq j'} D_j(L) \times D_{j'}(L)$

(C4) L.T. is SL_2 invariant or state or
 状態 \rightarrow vacuum state Φ_0

Formalism of field theory

a) $U_k^{(j)} \Phi_j = \lambda \Phi_{j+k}$ $\xi = (\xi_j, \xi_{j+k}, \dots)$
 $U_k^{(j)}$: complete operator system

b) 交換関係

c) $U_k^{(j)} \Phi_0 = 0$

incoming field $\varphi^{(j)}(x)$
 outgoing $\varphi^{(j')}(\bar{x})$
 actual $\Psi^{(j)}(x)$ \rightarrow $\varphi^{(j)}(x)$
 \downarrow $\varphi^{(j')}(\bar{x})$

a) Basic Quantity.

b) 交換関係

c) $P_\mu, M_{\mu\nu}$: Ψ の生成

d) 交換性

Mathematical Consideration

$U_k, U_{k'}$ $\Psi(x)$ $\pi(x)$
 同値な表現: 基底の表現 $(0, \dots, 0)$

(3)

\mathcal{H}_1 : countable
 $\mathcal{H}_1 \rightarrow \mathcal{H}_2$ Φ_0 state,
 ↓
 reducible

$U_R \rightarrow U_R$

$$U_R = \cosh \epsilon \cdot U_k + \sinh \epsilon U_k^\dagger$$

$$U_R^\dagger = \sinh \epsilon \cdot U_k + \cosh \epsilon U_k^\dagger$$

$$T = \frac{i}{2} \sum (U_k^\dagger U_k^\dagger - U_k U_k)$$

$$e^{i\epsilon T}$$

T: $\mathcal{H}_1 \rightarrow \mathcal{H}_2$ proper ca.)

T Ψ a norm in ∞

$$\Psi_1, \Psi_2 \in \mathcal{H}_1 \quad (\Psi_1, e^{i\epsilon T} \Psi_2) = 0$$

$$\Phi' = e^{i\epsilon T} \Phi$$

Application

Causality $\left\{ \begin{aligned} \varphi(\vec{x}) &= R \varphi(\vec{x}') R^\dagger \\ \pi(\vec{x}) &= R \pi(\vec{x}') R^\dagger \end{aligned} \right.$

$$[\varphi(\vec{x}), \varphi(\vec{x}')] = 0 \quad \text{space-like}$$

$$\rightarrow \langle 0 | [\pi(\vec{p}'), \varphi(\vec{p})] | 0 \rangle = c' \delta(\vec{p}' - \vec{p})$$

c or c' is R is finite ca.)