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H. Yukawa

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Sept. 18 9~12.00
Yukawa (30); Pais, Watanabe (Nara) (Enatsu)
(Nakano) (20)
Moller (30), Bloch, Pais (15) Utiyama
~~2~3~~.
Peierls (30)

Sept. 18. 2~3.30
~~Yukawa (20)~~; Umezawa, Katayama,

Abstract Book

~~Utiyama. (30)~~

~~Sept. 19~~

Wentzel, Tomonaga

Section A

~~Peierls, Hayakawa~~

Kyoto and Tokyo, September 1953

INTERNATIONAL CONFERENCE ON THEORETICAL PHYSICS

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Abstract Book

Section A

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SECTION A

An Attempt at a Unified Theory of Elementary Particles

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The introduction of non-local interaction between local fields is likely to be an important step toward the construction of a consistent field theory free from divergence difficulties. However, a further step in the same direction seems to be necessary in order to approach nearer to a unified theory of elementary particles. The concept of non-local field is introduced for this purpose. A non-local field describes relativistically a system which is elementary in the sense that it could no longer be decomposed into more elementary constituents, but was so substantial that it contains impicitly a great variety of particles with different masses, spins and other intrinsic properties[1]. For instance, a non-local scalar field is defined as a scalar function depending on two sets of space-time parameters and can be written as

$$(x_\mu'|\varphi|x_\mu'') \equiv \varphi(X_\mu, r_\mu)$$

where

$$X_\mu = \frac{x_\mu' + x_\mu''}{2}, \qquad r_\mu = x_\mu' - x_\mu''.$$

The most general equation for the free field is of the form

$$F\left(\frac{\partial}{\partial X_\mu}, r_\mu, \frac{\partial}{\partial r_\mu}\right)\varphi(X_\mu, r_\mu) = 0 \tag{1}$$

where the operator F is a certain function of $\frac{\partial}{\partial X_\mu}$, r_μ and $\frac{\partial}{\partial r_\mu}$ which is invariant under any inhomogeneous Lorentz transformation. If we assume that F is linear in $\frac{\partial^2}{\partial X_\mu \partial X_\mu}$ and separable, i.e.,

$$F \equiv -\frac{\partial^2}{\partial X_\mu \partial X_\mu} + F^{(r)}\left(r_\mu r_\mu, \frac{\partial^2}{\partial r_\mu \partial r_\mu}, r_\mu \frac{\partial}{\partial r_\mu}\right), \tag{2}$$

then we have eigen-solutions of the form $\varphi \equiv u(X)\chi(r)$, where u and χ satisfy

$$\left(\frac{\partial^2}{\partial X_\mu \partial X_\mu} - \mu\right)u(X) = (F^{(r)} - \mu)\chi(r) = 0 \tag{3}$$

μ being the separation constant. Thus, the masses of the free particles, which are associated with the non-local field φ, are given as the eigenvalues of the square root

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of the operator $F^{(r)}$ which characterizes the so-to-speak internal structure of the elementary non-local system. One can choose the operator $F^{(r)}$ such that the eigenvalues $\sqrt{\mu_n}\equiv m_n$ are all positive and discrete. In that case, one can expand an arbitrary non-local field into series of the corresponding internal eigenfunctions $\chi_n(r)$:

$$\varphi(X,r)=\sum_n u_n(X)\chi_n(r) \tag{4}$$

Now, when a non-local scalar field φ interacts with a local spinor field $\psi(x_\mu)$, for instance, one can reduce the problem to that of the interaction between the spinor field ψ and the infinitely many local Boson fields, which are defined respectively by $u_1(x_\mu)$, $u_2(x_\mu)$, ... The field equations become

$$\left(\frac{\partial^2}{\partial x_\mu''\partial x_\mu''}-m_n^2\right)u_n(x_\mu'')=\int\Phi_n(x',x'',x''')\bar{\psi}(x')\psi(''')dx'dx''' \tag{5}$$

$$\left(\gamma_\mu\frac{\partial}{\partial x_\mu'}+M\right)\psi(x')=-\sum_n\int\Phi_n(x',x'',x''')u_n(x'')\psi(x''')dx''dx''' \tag{6}$$

where

$$\Phi_n(x',x'',x''')\equiv g\tilde{\chi}_n(x'-x''')\delta\left(\frac{x'+x'''}{2}-x''\right) \tag{7}$$

and M is the mass of the spinor particle. If we compare these equations with the field equations in the case of non-local interaction between local fields, we notice that the internal eigenfunction $\chi_n(r)$ characterizes the form function for the particle with the mass m_n. The essential difference between the theory of non-local field and that of non-local interaction is that, in the former case, we have to take into account simultaneously all the particles with different masses m_1, m_2, ... which are derived from an eigenvalue problem. Furthermore, the form function for each of these particles is uniquely determined by the same eigenvalue problem. In the above example, in which we started from a non-local scalar or pseudoscalar field, all these particles have integer spins. On the contrary, particles with half integer spins would be obtained, if we would have started from a non-local spinor field.

In this connection, it is to be remarked that Watanabe suggested recently a possible relation between the non-local Boson field and de Broglie's fusion theory[2]. Namely, a pair of spinor particle and anti-particle could be regarded as a non-local field which describes a great variety of Bose particles.

In any case, the choice of the invariant operator F remains to be arbitrary, until a new principle for its determination would be revealed. At the present stage, we should be satisfied with considering simple examples in order to understand the general situation which we may face in non-local field theory. Thus, for the sake of illustration, one can assume a very simple form for F:

$$F\equiv-\frac{\partial^2}{\partial X_\mu\partial X_\mu}+\frac{\lambda^2}{4}\left(-\frac{\partial^2}{\partial r_\mu\partial r_\mu}+\frac{1}{\lambda^4}r_\mu r_\mu\right)^2 \tag{8}$$

where λ is a small constant with the dimension of length. One may call this the four dimensional oscillator model for the structure of elementary particles which

was considered first by Born in connection with his idea of self-reciprocity[3]. One can easily see that the mass spectrum in this case is discrete and is given by

$$m_{n_1,n_2,n_3,n_0}=\frac{1}{\lambda}|n_1+n_2+n_3-n_0+1| \tag{9}$$

where n_1, n_2, n_3, n_0 are zero or positive integers. The main trouble with the four dimensional eigenvalue problem in general is the infinite degeneracy. If we try to get rid of this difficulty, the theory may well become more complicated.

In connection with the problem of the decomposition of a non-local field into irreducible part, a rotator model for the structure of elementary particles was suggested by Hara[4]. A rigid sphere model as suggested by Nakano indicates another possibility. A modification of the problem of the self-energy in the ordinary local field theory as proposed by Enatsu could also be regarded as a different way of approach to the problem of determination of the structure of elementary particles.

There are a number of points which are to be investigated in order to see whether a consistent theory could be constructed if we proceed in this direction. One serious limitation of non-local theories is that so far we have to make use of the weak coupling approximation. Although we doubt the validity of such an approximation in connection with the problems of mesons and nuclear forces, we cannot depart from it easily, simply because we do not have as yet any thoroughly relativistic formulation of field theory which is free from the assumption of weak coupling.

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On the Problem of Convergence in the Non-Local Field Theories

Christian MØLLER

(Abstract : see p. 97)

Field Theories with Non-Local Interactions

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The paper will report joint work with M. Chretien investigating the conditions to be satisfied by an acceptable field theory with non-local interactions. We require the theory to be Lorentz invariant and gauge invarinat, to contain no non-causal effects, by which we mean, effects propagated backwards in time over time intervals greater than the extent of the form factor inherent in the theory and to contain no infinites.

The requirement for gauge invariance is put in order to ensure that the mass of a photon will be zero in all orders of approximation.

A particular form of the theory is investigated in which the invariance requirements and the causality are automatically ensured. The classical (non-quantized) equations resulting from such a theory are satisfactory. Their rigorous quantization presents complications, but it is at any rate possible to carry through the quantization within the framework of a series expansion of powers of the coupling constant. It then appears that the theory gives divergent answers, both for the self-energy of a particle, and for the polarization of the vacuum.

These results apply independently of the precise nature of the form factor which appears in the theory. A number of possible modifications are discussed, but give the impression that none of them will remove the infinities without violating the requirements of causality and invariance. One is therefore tempted to draw the conclusion that non-local interactions do not open up a possibility of constructing a theory that is free of singularities, at any rate without introducing some very new element.

Review of the Works of Non-Local Theories in Japan

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(Abstract not yet received, Aug. 20, 1953)

The Theory of the Structure of Elementary Particles (*Comment*)

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Although recent progress in the theory of elementary particles has gained brilliant success on one hand, the essential limitation of the theory itself seems to

have become clear especially in these two or three years. It is in connection, for example, with a remarkable regularity between the rest mass of elementary particles as first pointed out by Nambu,[1] with curious nature of V particles which seems hard to be understood within the framework of the current theory, or with divergent difficulties inherent in the quantum field theory from the day of its birth. The most serious weak point of the current theory is that the assignment of the spin or the rest mass value to elementary particles, or the introduction of the mutual interaction between them must be done entirely *ad hoc*.

Under this circumstance, it would be of great interest to investigate the structure of elementary particles, and thus try to elucidate the intrinsic correlation that may lie behind them. Recently H. Yukawa[2] and one of the authors[3] suggested to regard the elementary particles as corresponding to various states of the internal motion of a kind of Urmaterie, and pointed out that the non-local field would be nothing but the one that would describe the Urmaterie. Some development of this idea will be discussed.

Following this idea, the first task is to find the constants of internal motion which are responsible to the structure of elementary particles. Such quantities are found by starting with Fierz's[4] remark that the angular momentum of the internal motion just corresponds to spin.

Thus, separating the square of the infinitesimal rotational operator of the internal coordinate in four dimensional space $R_{\mu\nu}R^{\mu\nu}$ into the spin part S^2 and its counterpart M^2

$$R_{\mu\nu}R^{\mu\nu}=S^2+M^2$$

the mass operator M^2 is found in a relation with the internal motion. The ground to interpret M^2 as the mass operator is firstly that it is a scalar, secondly that it is equal to an expectation value of the energy of the internal motion in the rest system of the center of mass, and thirdly that it determines together with S^2 the behaviour of the wave function under Lorentz transformations. Thus, eigenvalue equations for S^2 and M^2 determine the spin and mass spectrum of elementary particles. Eigenfunctions of these equations compose form factors when interaction is introduced, and by solving the equations under the boundary condition that the form factors thus composed should serve as effective cutoff factors, the mass spectrum of elementary particles that can exist in nature is unambiguously given.

A remarkable feature encountered when we apply this theory to spinor field is the appearence of a new structure constant that classifies the Fermi particles into two families. This is due to the fact that, in the case of the spinor, the spin and mass operator do not compose a complete set of mutually commuting operators with respect to internal coordinates. Hence, in this case, the specification of elementary particles with the spin and rest mass is incomplete, which means that the elementary particles possess a new structure constant θ other than them. It is easily shown that θ has eigenvalues 1, and classifying all Fermi particles into two families according to these eigenvalues, they determine the minimum values of the rest mass appearing in these two families. These two correspond to the nucleon and lepton family respectively if constants appearing in the theory are suitably chosen, and therefore θ is interpreted as expressing the intrinsic difference of the nucleon and lepton families. θ is further shown to relate closely to mesic charge, and by assuming the invariance of the theory under a kind of gauge transformation analogous to that in electrodynamics, the conservation of heavy particles follows as an immediate consequence.

To construct the theory of the interaction of the Urmaterie is a very complicated problem, and satisfactory answer has not yet been given. Here we only remark that in our theory the form of factors is uniquely determined as eigenfunctions of the spin and mass operators, and that, by introducing the interaction in the form of the interaction between Urmaterie, various relations are suggested among interaction constants of the local theory. An example of such relations we know is the universal Fermi interaction among Fermi particles.

From these discussions we might say that our theory shows an example of overcoming, at least partly, the limitation of the current theory pointed out at the beginning, and we hope that our theory might serve as a first step toward the line of this approach, even if it might not be correct at the ultimate stage.

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Four Dimensional Eigenvalue Problem *(Comment)*

Hiroshi ENATSU

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Previous calculations have yielded divergent expressions for the mass type self-energy. It seems that the failure results from the fact that, although in the technical aspects covariant formulations preserve relativistic fertures at all stages, neither method appears to be completely relativistic in the conceptual aspects. In fact, the infinite mass corrections, together with proper masses, must be considered as eigenvalues of mass operators in the four dimensional coordinates space. In order to remove these difficulties, a new method will be developed. We begin by discussing the proper time formulation proposed by Schwinger. In the canonical form, where we have dynamical variables varying with the proper time, the Hamiltonian is not the energy but the mass. Therefore, the familiar Hamiltonian form of quantum mechanics can be generalized for the mass problem. As an example we consider a Dirac particle which is in interaction with scalar photons. In this case the negative self-energy of the Dirac particle is regarded as the self-cohesive potential which is a function of the proper time in the space-like direction. Introducing semiclassical approximation and some simplification, we discuss possibilities of obtaining finite mass values.

Application of Schwinger's Principle to the System of Local Fields with Non-Local Interaction *(Comment)*

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In the system of local fields with non-local interaction, it is customarily believed that we can not set up a state vector $\Psi(t)$ which satisfies Schrödinger Eq. (or Tomonaga-Schwinger Eq.)

$$i\frac{d\Psi(t)}{dt}=\mathscr{H}(t)\Psi(t). \tag{1.1}$$

Accordingly it may be impossible to construct the S-matrix by chronologically piling up the effects of interaction. Thus it is desirable to have a new equation in place of (1.1) which holds in both cases of local and non-local interactions.

For this purpose Schwinger's principle seems to be most suitable. Let us consider a system of local-fields with local interaction, Lagrangian density of which is given by

$$\mathscr{L}(x,g)=\mathscr{L}_0(x)+gL(x).$$

Following Schwinger's principle, we have

$$\frac{d}{dg}<\varphi_0=\varphi';\infty|S|\varphi_0=\varphi'';-\infty>=i<\varphi_0=\varphi';\infty|SW|\varphi_0=\varphi'';-\infty> \tag{1}$$

$$\frac{d}{dg}<\varphi_0=\varphi';-\infty|\varphi(x)|\varphi_0=\varphi'';-\infty>$$
$$=i\int_{-\infty}^{\infty}<\varphi_0=\varphi';-\infty|[\varphi(x),L(x')]_\theta|\varphi_0=\varphi'';-\infty>d^4x', \tag{2}$$

where φ_0 is the operator of free fields, $[\quad]_\theta$ means Bloch's notation, and W is given by

$$W=\int_{-\infty}^{\infty}L(x)d^4x.$$

These equations can be easily derived by considering an infinitesimal variation of the coupling constant g.

In case of non-local interactions, it may be also permissible to assume the existence of the state vector only at $t=\pm\infty$, if the form factor satisfies Bloch's condition of "Normal class", accordingly the adiabatic procedure of switching on and off of interactions at remote past and future is allowable.

In this normal case, the S-matrix can be also constructed by starting from the field equations. It can be proved that the S-matrix thus obtained satisfies Eq. (1), and further

$$\frac{d}{dg}<\varphi_0=\varphi';-\infty\,|\,\varphi^{\rm out}(x)\,|\,\varphi_0=\varphi'';-\infty> \\ =i<\varphi_0=\varphi';-\infty\,|\,[\varphi^{\rm out}(x),W]\,|\,\varphi_0=\varphi'';-\infty> \tag{3}$$

holds instead of Eq. (2). Of course Eq. (3) is satisfied also in case of local interactions.

Thus we get the conclusion that in case of non-local interactions Schwinger's principle should be modified in the following way:

i) the transformation functions between the states are assumed to exist only at infinitely remote past and future,

ii) any operator can be only defined by matrix-elements between the states mentioned above,

iii) the fundamental equation which the S-matrix should satisfy is

$$\frac{d}{dg}<\varphi_0=\varphi';\infty\,|\,S\,|\,\varphi_0=\varphi'';-\infty>=i<\varphi_0=\varphi';\infty\,|\,SW\,|\,\varphi_0=\varphi'';-\infty>$$

where

$$<\varphi=\varphi';\infty\,|\,\varphi_0=\varphi'';-\infty>\equiv<\varphi_0=\varphi';\infty\,|\,S\,|\,\varphi_0=\varphi'';-\infty>$$

φ=the Heisenberg operator of fields,

iv) the out-going operator φ^{out} satisfies the following equation

$$\frac{d}{dg}\varphi^{out}(x)=i[\varphi^{out}(x),W],$$

where

$$\varphi^{out}(x)=S^{-1}\varphi_0(x)S.$$

Realistic and Historical View Point in the Theory of Elementary Particles *(Comment)*

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The applicability of the present theory of elementary particles is discussed not only from the mathematical view point but also from the philosophical one, which we used to call as a realistic and historical view point and which has played an important role in the course of the recent developments of the theoretical physics in Japan.

The Correspondence-Theoretic Approach to the Non-Local Theory *(Comment)*

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In quantum electrodynamics the renormalization theory based on the perturbation method has been successfully applied to obtain the finite answers to be compared with experiments. But, the present quantum field theory in general allows the interactions which lead to unrenormalizable divergences. In particular, we see that the large spin field ($s\geq 1$) with the non-vanishing mass can never have renormalizable interactions (excepting the neutral vector field). In accordance with Heisenberg we have called interactions the first and the second kind according as they are renormalizable or not[1].

If only the interactions of the first kind exist in the nature, we can construct the closed non-singular theory within the framework of the renormalization theory[1]. Since, however, there are some evidences in favour of those of the second kind, we are faced with the problem how to treat this kind of interactions. From the correspondence-theoretic view-point, we can expect that the solution of this problem may be obtained by extending the renormalization method in some way.

The interactions of the second kind have many characteristic features. When subtracting the divergences encountered there in a similar way as in the case of the first kind, we are forced to introduce non-local and non-linear interactions as the counter terms[1] (see below). This is due to the fact that in this case a new dimensional constant other than the mass enters the theory i.e., the coupling constant with the dimension. Physically this circumstance leads to the strong field reactions, both inductive and resistive. We have found[2] that by taking into account the effect of strong reactions and thus deviating from the usual perturbation procedure we can construct the non-singular theory for the interaction of the second kind.

This is achieved by using the modified Feynman functions Δ'_F instead of the usual one Δ_F. To obtain the non-singular theory it is further required in some cases to introduce appropriate auxiliary fields or interactions. Since, in general, these interactions belong to the second kind we may say that our procedure assumes the similar behavior as the theory of the non-local action. When applied to the interaction of the first kind this method usually improves the theory, i.e., eliminates some divergences from the renormalization constants. (This possibility was previously noticed by W. Thirring[3], too).

By use of modified vertices we can expect that the similar results can be obtained. But, we have succeeded in doing so only for a case[4], i.e., for the interaction $f\bar{\psi}_1\psi_2\varphi_1\varphi_2$ (meson pair theory). In this case, we find that it becomes necessary to introduce the non-local counter terms such as*

$$\psi_1\bar{\phi}_1\frac{(D(\partial)+\mathfrak{d})}{1-(D(\partial)+\mathfrak{d})\,(D'(\partial)+\mathfrak{d}')}\psi_2\phi_2, \qquad \text{etc.,}$$

where D's and $\mathfrak{d}$'s are linearly and logarithmically diverging constants, respectively and D's linear functions of the differentiation operator ∂. Recently, Edwards[5] has made a similar attempt by starting with integral equations for vertex factors. He suggested a possibility of constructing the divergence free quantum electrodynamics, but could not treat the interaction of the second kind. In effect, our method amounts to begin with the non-local interactions.

Our treatment employs the integral equations with non-local form, and so it is equivalent to start at the outset with the non-local action (modified Feynman function) or non-local interaction (modified vertex). Thus, through such an investigation the correspondence-theoretic approach would be made to the non-local theory in general; for instance it may give some principles for determining the form factors. Further, it becomes possible to classify the non-local interactions in a corresponding way as the case of local interactions. It is desirable to find a more general method which can treat unambiguously both kinds of interactions in an unified way as suggested by Edwards[5]. Contrary to the argument by Edwards, however, such a treatment, if any, would show the essential difference between both kinds of interactions.

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H. Umezawa, ibid. **7** (1952), 579.

*) This is the only example, so far published, of the non-local counter term explicitly given for the interaction of the second kind.

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Theory of Quantization of the General Fields with General Interactions (*Comment*)

Yasuhisa KATAYAMA, Hiroomi UMEZAWA and Chushiro HAYASHI

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An attempt is made to generalize the current covariant Hamiltonian formalism in the quantized field theory to the cases where fields have arbitrary spins and their interactions contain any higher derivatives of the field quantities, including the non-local interactions. For this purpose, we define the particles described by free fields as having the character of harmonic oscillators, that is to say, each free field has two independent field quantities, and we assume that it is possible to introduce the interactions without altering this character essentially.

If the Lagrangian contains higher derivatives of field quantities, the number of independent variables becomes in general larger than that in the free fields, so that there exists no unitary transfoamation which connects the free fields with the interacting ones. To avoid this difficulty, we give up the description of the motion of fields as a whole and seek for solutions which retain the character of the harmonic oscillators of free fields, that is, solutions which are continuous with those of free fields in the limit of the vanishing coupling constant. In other words, we put proper relations among the initial values of the dynamical system and choose the particular paths of motion on which the number of independent variables is suppressed to that in free fields.

To carry out the above program we generalize the Yang-Feldman's integral equation by the addition of appropriate terms satisfying free field equations, which corresponds to choosing appropriate boundary conditions, so that unitary transformations $U(\sigma, \sigma')$ exist which connect field quantities $\psi(x, \sigma)$, etc., with $\psi(x, \sigma')$, etc., each satisfying free equations and free commutation relations. It is shown that the interaction Hamilonians thus obtained play the role of time displacement operator and satisfy the integrability condition.

These Hamiltonians are in general expressed as power series in the coupling constant. The series are finite only in the customary cases: scalar meson with scalar or vector couplings, pseudscalar meson with pseudscalar or pseudvector couplings, and vector meson with vector or tensor couplings. When the degree of derivatives in the interaction is higher than the above, spin of the free field is $^3/_2$ or higher, and especially the interaction is of non-local character, the series become infinite. The interaction of the Konopinski-Uhlenbeck's type is an example of the latter case.

The Hamitonians depend on the boundary conditions imposed on the field quantities, that is, depend on the choice of the additional terms in the Yang-Feldman's equation. However, this makes no new difficulty since the representations having different Hamiltonians are connected with each other by unitary transformations. It is also remarked that these Hamiltonians are equal to those which are obtained in accordance with the Heisenberg-Pauli's method by choosing the independent field quantities, having the same degree of freedom as the free fields, which are given as functionals of the interacting field quantities in the Heisenberg representation. It can be said that the generalized Yang-Feldman's method is to seek for these field quantities.

When the Hamiltonian is obtained and the transformation function is determined, several differential conservation laws are obtained with an aid of the transformation theory. The S-matrix defined as the transformation function between $-\infty$ and $+\infty$ is shown to coincide with the result obtained by several authors, for example, by Möller-Kristensen and Bloch in the case of non-local interaction. If we perform the transformation from the interaction representation to the Schrödinger one, we can treat the stationary problems including bound states in the non-local interaction, for instance, by using the Tamm-Dancoff's method.

The defect in our theory is that Hamiltonians are obtained as the infinite series according to the weak coupling hypothesis so that their convergency becomes a new problem to be investigated. There remain unsettled the problems how we should restrict the form of the interactions or the form factor in the non-local interaction in order to obtain the convergent theories.

Many-Body Problem in Quantum Field Theory

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The connection between Salpeter-Bethe wave functions and the state vectors is investigated. Further, the extention of the Feynman-Dyson theory so as to include the scattering involving bound states is attempted. We shall begin with the discussion of the first problem. This problem has previously been discussed by Namiki and Suzuki.

The state vector Ψ_s in the Heisenberg picture, for instance, for a one nucleon system, will be expressed as

$$\Psi_s=\left[\int f_s(x)\boldsymbol{\psi}^*(x)d^3x+\ldots\ldots\right]\Psi_0, \tag{1}$$

where Gothic letters are the Heisenberg operators, and Ψ_0 is the true vacuum. In order to represent the state completely, we need in the integrand of (1) various combinations of operators such as

$$\boldsymbol{\psi}^*(x),\ \boldsymbol{\psi}^*(x)\boldsymbol{\phi}(\xi),\ \boldsymbol{\psi}^*(x)\dot{\boldsymbol{\phi}}(\xi),\ \ldots\ldots, \tag{2}$$

corresponding to the meson cloud. Here $\boldsymbol{\psi}$ and $\boldsymbol{\phi}$ mean the wave functions of nucleon and meson.

It is readily proved that combinations of $\boldsymbol{\psi}$, $\boldsymbol{\psi}^*$, $\boldsymbol{\phi}$ and $\dot{\boldsymbol{\phi}}$ are necessary and sufficient to completely represent a state.

When the product of several operators appears, we must fix their ordering, and for this purpose Wick's T-product is convenient. The T-product for Heisenberg operators is well defined. Hence an arbitrary state may be expanded as

$$\Psi_s=\left[\int f_s(x)\boldsymbol{\psi}^*(x)d^3x+\int f_s(x,\xi)T[\boldsymbol{\psi}^*(x)\boldsymbol{\phi}(\xi)]d^3x\,d^3\xi\right.$$
$$\left.+\int f_s'(x,\xi)T[\boldsymbol{\psi}^*(x)\dot{\boldsymbol{\phi}}(\xi)]d^3xd^3\xi+\ldots\ldots\right]\Psi_0, \tag{3}$$

where all arguments refer to the same time.

Rigorously speaking, the S-product or the normal product is more convenient than the T-product. The S-product for Heisenberg oprators is defined by

$$:ABC\cdots Z:=T(ABC\cdots Z)-\sum A^{\cdot}B^{\cdot}\,T(C\cdots Z)$$
$$+\sum A^{\cdot}B^{\cdot}C^{\cdot\cdot}D^{\cdot\cdot}\,T(E\cdots Z)-\cdots\cdots, \tag{4}$$

where the dotts are the contraction symbols defined by

$$A^{\cdot}B^{\cdot}=(\Psi_0,\,T(AB)\Psi_0). \tag{5}$$

Now with the aid of the present S-product, we can express a state vector Ψ_s as

$$\Psi_s=\Big[\int f_s(x)\psi^*(x)\,d^3x+\int f_s(x,\xi):\,\psi^*(x)\phi(\xi):d^3xd^3\xi+\int f_s'(x,\xi):\psi^*(x)\dot{\phi}(\xi):d^3xd^3\xi$$
$$+\cdots\cdots\Big]\Psi_0. \tag{6}$$

We call the coefficients of these operators

$$f_s(x),\; f_s(x,\xi),\; f_s'(x,\xi),\;\cdots\cdots \tag{7}$$

as the contravariant components of the state vector Ψ_s.

On the other hand, functions like

$$g_s(x)=(\Psi_0,\psi(x)\Psi_s),\; g_s(x,\xi)=(\Psi_0,:\psi(x)\phi(\xi):\Psi_s),\;\cdots\cdots \tag{8}$$

are called as the covariant components, which are just the S-B wave functions. These nomenclatures are suggested by the close analogy of the present formalism with the vector analysis in an oblique coordinates system. In what follows we denote such quantities as ψ, ψ^*, ϕ and $\dot{\phi}$ indiscriminately by e, then the co- and contra- variant components g's and f's are given respectively by

$$g_{s,N}=(\Psi_0,:e_1\cdots\cdots e_n:\Psi_s),\qquad N=(1,2,\cdots\cdots,n)$$
$$\Psi_s=\sum_N\int f_{s,N}:e_n^+\cdots\cdots e_1^+:\Psi_0,\qquad (+:\ \text{Hermitian conjugate}) \tag{9}$$

where $\int$ corresponds to the spatial integrations in (6).

If we choose $t_1=\cdots\cdots=t_n$ in each product, we readily have

$$(\Psi_r,\Psi_s)=\sum_N f_{r,N}{}^*g_{s,N}=\sum_N\int f_{s,N}g_{r,N}{}^*. \tag{10}$$

The connection between these two components are given, if we choose $t_1=\cdots\cdots=t_n>t_1'=\cdots\cdots=t_n'$, by

$$g_{s,N}=\sum_{N'}\int\mathfrak{K}(N|N')f_{s,N'}, \tag{11}$$

where

$$\mathfrak{K}(N|N')=(\Psi_0,T[:e_1\cdots e_n::e_{n'}{}^+,\cdots e_{1'}{}^+:]\Psi_0). \tag{12}$$

This mixed T-product can easily be reduced into a linear combination of simple T-product kernels like

$$K(N,N')=(\Psi_0,T[e_1\cdots e_n\;e_{n'}{}^+\cdots e_{1'}{}^+]\Psi_0), \tag{13}$$

by means of the definition (4). The functions K's are just the Feynman kernels. After the settlement of both components, expectation values of physical quantities in a given state are calculable.

Next we shall study the Feynman kernels or Green functions (13). We employ the following interaction Hamiltonian

$$H_{\rm int}=i\eta\bar{\psi}O_\alpha\psi\cdot\phi_\alpha+Q_\alpha\phi_\alpha-H_{\rm self}\,, \tag{14}$$

where $H_{\rm self}$ is the counter term to cancel divergences like self energies about which, however, we shall not discuss here.

The c-number quantity Q_α represents the external source introduced formally for later convenience.

We define the transformation function U by

$$i\frac{\delta}{\delta\sigma(x)}U(\sigma,\sigma_A)=H_{\rm int}(x)U(\sigma,\sigma_A),\ \text{with}\ U(\sigma_A,\sigma_A)=1. \tag{15}$$

We utilize the following abbreviation:

$$<A(1)B(2)\cdots Z(n)>=\frac{(\Phi_0,T[U(\infty,-\infty)A(1)B(2)\cdots Z(n)]\Phi_0)}{(\Phi_0,U(\infty,-\infty)\Phi_0)}, \tag{16}$$

wher Φ_0 is the free vacuum.

In the presence of external sources, the kernels are defined by

$$<\psi(1)\bar{\psi}(2)>=S_F'(1,2),\;<\phi_a(1)\phi_b(2)>=\Delta_F'(1,2)_{ab},$$
$$<\psi(1)\overline{\psi}(2)\phi_a(0)>=K_a(12;0),\ \text{etc.}. \tag{17}$$

The utility of introducing the external source Q_α consists in the identity

$$i\frac{\delta}{\delta Q_a(0)}(\Phi_0,T[U(\infty,-\infty)A(1)B(2)\cdots Z(n)]\Phi_0)$$
$$=(\Phi_0,T[U(\infty,-\infty)\phi_a(0)A(1)B(2)\cdots Z(n)]\Phi_0). \tag{18}$$

Now S_F' satisfies the equation

$$S_F'(1,2)=S_F(1-2)+\eta\int d\omega_3S_F(1-3)O_a<\phi_a(3)>S_F'(3,2)$$
$$+\int d\omega_3d\omega_4S_F(1-3)\sum{}^*(3,4)S_F'(4,2). \tag{19}$$

On differentiating the above equation with respect to $Q_a(0)$, we have from (16), (17), and (18)

$$K_a(12;0)=<\phi_a(0)>S_F(1-2)$$
$$+\eta\int d\omega_3 S_F(1-3)\,O_b[\Delta_F'(0,3)_{ab}S_F'(3,2)+<\phi_b(3)>K_a(32;0)]$$
$$+\int d\omega_3 d\omega_4 S_F(1-3)\Big[i\frac{\delta\Sigma^*(3,4)}{\delta Q_a(0)}\cdot S_F'(4,2)+\Sigma^*(3,4)K_a(42;0)\Big]. \tag{20}$$

Comparing this integral equation for K_a to (19), we obtain a formal solution

$$K_a(12;0)=<\phi_a(0)>S_F'(1,2)+\eta\int d\omega_3 S_F'(1,3)\,O_b S_F'(3,2)\cdot\Delta_F'(0,3)_{ab}$$
$$+\int d\omega_3 d\omega_4 S_F'(1,3)\Big[i\frac{\delta\Sigma^*(3,4)}{\delta Q_a(0)}\Big]S_F'(4,2). \tag{21}$$

If we let Q_a vanish, then we may replace the kernels defined in (17) by

$$(\Psi_0, T[\psi(1)\bar\psi(2)]\Psi_0)=S_F'(1-2),\ (\Psi_0, T[\phi_a(1)\phi_b(2)]\Psi_0)=\delta_{ab}\Delta_F'(1-2), \tag{22}$$

etc.

in virtue of the relation found by Gell-Mann and Low

$$(\Psi_0, T[A(1)\cdots Z(n)]\Psi_0)=\frac{(\Phi_0, T[U(\infty,-\infty)A(1)\cdots Z(n)]\Phi_0)}{(\Phi_0, U(\infty,-\infty)\Phi_0)}. \tag{23}$$

The rigorous proof of the relation between Ψ_0 and Φ_0 is given by Tanaka with some generalizations.

Hence, in the absence of external fields, eq. (21) may be read as

$$K_a(12;0)=<\phi_a(0)>_{Q=0}S_F'(1-2)+\eta\int d\omega_3 S_F'(1-3)\,O_a S_F'(3-2)\Delta_F'(0-3)$$
$$+\int d\omega_3 d\omega_4 S_F'(1-3)\Big[i\frac{\delta\Sigma^*(3,4)}{\delta Q_a(0)}\Big]_{Q=0}S_F'(4-2). \tag{21'}$$

The above procedure provides us with a general means to connect Fermion kernels with Fermion-Boson kernels.

In the two nucleon system, we have similar formulas starting from the equation

$$K(12;34)=S_F'(12;34)+\int d\omega_5\cdots d\omega_8 S_F'(1,5)S_F'(2,6)G(56;78)K(78;34), \tag{24}$$

where

$$K(12;34)=-(\Phi_0, T[U(\infty,-\infty)\psi(1)\psi(2)\bar\psi(3)\bar\psi(4)]\Phi_0)/(\Phi_0, U(\infty,-\infty)\Phi_0),$$

and

$$S_F'(12;34)=S_F'(1,3)S_F'(2,4)-S_F'(1,4)S_F'(2,3).$$

The application of the limiting procedure of Gell-Mann and Low to these integral equations furnishes us with those for the covariant components. From (24) we have

$$g_a(12)=g_a^0(12)+\int d\omega_3\cdots d\omega_6 S_F'(1-3)S_F'(2-4)G(34;56)_{Q=0}\,g_a(56), \tag{25}$$

where

$$g_a^0(12)=g_{a'}(1)g_{a''}(2)-g_{a''}(1)g_{a'}(2),$$

and $g_{a'}(1)=(\Psi_0, \psi(1)\Psi_{a'})$, $\Psi_{a'}$ being a one nucleon state designated by the quantum number a'.

The formal solution of this equation is given by

$$g_a(12)=g_a^0(12)+\int d\omega_3\cdots d\omega_6[K(12;34)G(34;56)]_{Q=0}\,g_{a'}(5)g_{a''}(6)$$
$$=g_a^0(12)+\int d\omega_3\cdots d\omega_6[S_F'(12;34)G(34;56)]_{Q=0}\,g_{a'}(5)g_{a''}(6)$$
$$+\frac{1}{2}\int d\omega_3\cdots d\omega_{10}[S_F'(12;34)G(34;56)K(56;78)G(78;9\,10)]_{Q=0}$$
$$\times g_{a'}(9)g_{a''}(10). \tag{26}$$

Next we construct the S matrix.

The S matrix is defined by

$$g_a(12)\sim\sum_b S_{ba}g_b^0(12), \qquad \text{(asymptotically)} \tag{27}$$

where the superscript zero in general refers to the direct product of the covariant components of the individual incident particles.

Since S_F' satisfies the outgoing wave condition, it must contain the δ_+ function of energy in the momentum representation.

The asymptotic form is obtained if we replace the δ_+ function by the δ function. This is readily achieved if we only remember

$$\lim_{t\to+\infty}e^{-i\omega t}\delta_+(\omega)=\delta(\omega), \tag{28}$$

and

$$\lim_{t_1\to+\infty}S_F'(1-2)=\sum_b g_b(1)\bar g_b(2),\ (t_1\to\infty).\quad \text{cf. (22)}. \tag{29}$$

Combining (26), (27), (28) and (29), we have the S matrix for an elastic nucleon-nucleon collision

$$S_{ba}=\delta_{ba}+\int d\omega_3\cdots d\omega_6\,\bar g_b^0(3;4)G(34;56)_{Q=0}\,g_{a'}(5)g_{a''}(6)$$
$$+\frac{1}{2}\int d\omega_3\cdots d\omega_{10}\bar g_b^0(34)[G(34;56)K(56;78)G(78;9\,10)]_{Q=0}\,g_{a'}(9)g_{a''}(10)$$
$$=\frac{1}{2}\int d\omega_1\cdots d\omega_4\bar g_b^0(12)G(12;34)_{Q=0}\,g_a(34). \tag{30}$$

Collisions like

$$p+p\to d+\pi^+ \tag{31}$$

can be treated in a similar way. In this case we should refer to

$$\lim_{t_1,t_2\to+\infty}K(12;34)_{Q=0}=\sum_b g_b(12)\bar g_b(34),\ (t_1,t_2\to\infty). \tag{32}$$

instead of (29). Starting from an equation for the two-body kernel just similar to (21'), the S matrix for (31) is given by

$$S_{ba}=\frac{1}{2}\int d\omega_0\cdots d\omega_4\bar{g}_a(12)\,\bar{g}_{\beta\alpha}(0)\,\mathscr{O}_\alpha(12\,;34\,;0)\,g_a(34)$$
$$+\frac{1}{2}\,i\eta\int d\omega_0\cdots d\omega_8\bar{g}_a(12)\,\bar{g}_{\beta\alpha}(0)\,\mathscr{O}_\beta(23\,;0)\,\mathscr{D}(14)[S_F'(34\,;56)\,G(56\,;78)]_{Q=0}$$
$$\times g_a(78)\,. \tag{33}$$

where

$$g_{\beta\alpha}(0)=(\Psi_0,\phi_\beta(0)\Psi_\alpha),\ \Psi_\alpha:\text{ one }\pi^+\text{ state,}$$
$$g_a(12)=(\Psi_0,T(\psi(1)\psi(2))\Psi_a),\ \Psi_a:\text{ one deuteron state,}$$

and $\mathscr{O}$'s are defined as

$$\left[i\frac{\delta\Sigma^*(12)}{\delta Q_a(0)}\right]_{Q=0}=\eta\int d\omega_3\,\mathscr{O}_a(12\,;3)\,\Delta_F'(3-0)-\eta O_a\delta(1-2)\,\Delta_F'(1-0)\,, \tag{34}$$
$$\left[i\frac{\delta G(12\,;34)}{\delta Q_a(0)}\right]_{Q=0}=\int\delta\omega_5\,\mathscr{O}_a(12\,;34\,;5)\,\Delta_F'(5-0)\,, \tag{35}$$

and $\mathscr{D}(12)$ is given by

$$\mathscr{D}(12)=(\gamma\partial_1-\kappa-i\eta O_a\langle\phi_a(1)\rangle_{Q=0})\,\delta(1-2)-i\Sigma^*(12)_{Q=0}\,.$$

On a Treatment of Many-Body Problems in Quantum Field Theory (*Comment*)

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The usual representation, employed in perturbation theoretical as well as in Tamm-Dancoff's treatment of many-body problems in quantum field theoy, is one which is labelled by the occupation numbers of bare particles. Since the observed particles are not bare but dressed in a sense that they accompany clouds of virtual particles, it seems desirable to use an alternative representation labelled by the occupation numbers of dressed particles. We define, namely, a set of basic states $\{\Omega_n\}$, such that each of them can be interpreted as representing a prescribed number of freely travelling dressed particles and is in one to one correspondence to each eigenstate Φ_n of the free Hamiltonian. Among them, the dressed or true vacuum state Ω_0 and the dressed (either fermion or boson) one-particle state Ω_p with momentum p are naturally defined as the eigenstates of the total Hamiltonian H corresponding to the bare vacuum state Φ_0 and the bare one-particle state Φ_p, respectively. They will be expressed as

$$\Omega_0=D_v{}^*\Phi_0=e^{\Delta}e^{d_v*}\Phi_0\,,$$

and

$$\Omega_p=A_p{}^*D_v{}^*\Phi_0=A_p{}^*\Omega_0\,,$$

where $D_v{}^*$ and $A_p{}^*$ are certain infinite sums of products of free creation operators, i.e., those in the interaction representation defined by the free Hamiltonian H_0 (including the mass-renormalization operators), and represent the "dresses" of the vacuum and the particle, respectively. e^{Δ} is the constant normalization factor arising from the vacuum closed graphs. We define the state of two dressed particles with momenta p and q, say Ω_{pq}, by

$$\Omega_{pq}=A_p{}^*A_q{}^*D_v{}^*\Phi_0=A_p{}^*A_q{}^*\Omega_0\,.$$

The two particles may be called free, since in the above expression, the virtual clouds of the two particles, $A_p{}^*$ and $A_q{}^*$, co-exist without being deformed by the presence of each other. The states of more than two dressed particles can be defined in the same way. Thus to each eigenstate Φ_n of H_0, we can define a corresponding state Ω_n, forming a complete set altogether. Unfortunately, the set is a non-orthogonal one. It will be needed to define the "reciprocal lattice" $\{\breve{\Omega}^n\}$, which satisfies the relations

$$(\breve{\Omega}^n,\Omega_m)=\delta_{nm}\,.$$

The method of constructing the set $\{\breve{\Omega}_n\}$ can be established easily.

Corresponding to the mutually reciprocal bases $\{\breve{\Omega}_n\}$ and $\{\Omega_n\}$, a state-vector Ψ is represented by either the co- or contra-variant components, say η_n or ξ^n.

The total Hamiltonian H operating on Ω_n, gives, for instance,

$$H\Omega_{pq}=E_v\Omega_{pq}+(\mathcal{E}_p+\mathcal{E}_q)\,\Omega_{pq}+H_i^{\rightarrow}\Omega^{\cdot}{}_{pq}\,,$$

where $\mathcal{E}_p=\sqrt{p^2+m^2}$ (with m representing the renormalized mass) is the energy of the dressed particle with the momentum p and E_v the energy of the vacuum. H_i is the interaction Hamiltonian and the symbol $H_i^{\rightarrow}\Omega^{\cdot}{}_{pq}$ means those terms out of $H_i\Omega_{pq}$ that arise from terms of H_i with more than two annihilation operators contracting simultaneously with the creation operators in both $A_p{}^*$ and $A_q{}^*$, or in $A_p{}^*$, $A_q{}^*$ and $D_v{}^*$. (The other terms of $H_i\Omega_{pq}$ together with $H_0\Omega_{pq}$ give the first and the second term.) Thus, the last term may be called the interaction between the two dressed particles. Using this relation, the Schrödinger equation

$$(E-H_0)\Psi=H_i\Psi$$

is represented by contra-variant components ξ^n, as

$$(\mathcal{E}-\mathcal{E}_n)\,\xi^n=\sum_m(\breve{\Omega}^n,H_i^{\rightarrow}\Omega^{\cdot}{}_m)\,\xi^m,$$

and in co-variant components η_n, as

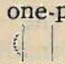
$$(\mathcal{E}-\mathcal{E}_n)\,\eta_n=\sum_m(H_i^{\rightarrow}\Omega^{\cdot}{}_n,\breve{\Omega}^m)\,\eta_m\,,$$

with $\mathcal{E}=E-E_v$. $\mathcal{E}_n$ is the sum of the particle energies corresponding to the state n. They can also be written as

$$(\mathcal{E}-\mathfrak{H}_0)\,\xi=H\,\xi\quad\text{and}\quad(\mathcal{E}-H_0)\,\eta=\mathfrak{H}^+\eta\,, \tag{I}$$

with

$$(\mathfrak{H}_0)_{nm}=\mathcal{E}_n\delta_{nm},\quad(\mathfrak{H}_i)_{nm}=(\breve{\Omega}^n,H_i^{\rightarrow}\Omega_{\cdot m})\,.$$

The advantages of the equation (I) over the usual one are that (i) the energy of the system appears as the difference from the vacuum energy, $\mathcal{E}=E-E_v$, which is just the observed quantity: (ii) neither the vacuum energy divergence nor the independent one-particle self-energy divergences corresponding to such Feynman diagrams as  arise from $\mathfrak{H}_i$: (iii) the theoretical or experimental informations about the one-particle state Ω_p, or A_p^* can be directly applied to many-particle problems. In connection with the last point, it may be remarked that there are close relations between the terms of d_v^* and A_p^*, which enable us to determine A_p^* out of d_v^* or *vice versa.*

On the other hand, the disadvantages of our equations are that (i) $\mathfrak{H}_i$ is not given in an explicit form: (ii) $\mathfrak{H}_i$ is not Hermitian, which makes it impossible in general to apply the equations for the eigenvalue problem.

The second point can be overcome if we employ an alternative equation deduced from the equations (I), namely,

$$(\mathcal{E}G-\mathfrak{F}_0)\xi=\mathfrak{F}_i\xi\,,$$

where G, $\mathfrak{F}_o$ and $\mathfrak{F}_i$ are the Hermitian operators with the matrix elements

$$G_{nm}=(\Omega_n\,,\Omega_m)\,,$$

$$(\mathfrak{F}_0)_{nm}=\frac{\mathcal{E}_n+\mathcal{E}_m}{2}(\Omega_n,\Omega_m)\,,$$

$$(\mathfrak{F}_i)_{nm}=\frac{1}{2}\{(\Omega_n,H_i^{\rightarrow}\Omega^{\cdot}{}_m)+(H_i^{\rightarrow}\Omega_n,\Omega_m)\}.$$

This equation guarantees the real property of the energy eigenvalue, and besides, it needs no explicit knowledge of the reciprocal lattice $\{\breve{\Omega}^n\}$.

(1) This was achieved from a different approach by Dyson. F.J. Dyson, Phys. Rev. **90**, (1953) 994.

Exact Theory of Line Breadth

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This work is concerned with a theory of line breadth phenomena that permits the calculation of the line shape by suitable expansions, in principle to any desired order of approximation.* The usual expansions of the S-matrix can, of course, not be applied here. The chief defficulty is a proper definition of an excited atomic state, in the sense of quantum electrodynamics. For a state with finite life time the virtual admixtures depend on the way in which the atom was excited. The elmination of the virtual field is unambiguous only for the ground state (or metastable levels, regarded as stable).

*) This is an extension of the recent work by Arnous and Zienau, Arnous and Bleuler, Arnous, Helv. Phys. Act. 1951-52.

To eliminate the virtual field, a canonical transformation T is carried out which is left to some extent arbitrary but is determined so that the ground state of the atom (G), vacuum (V), and the state of a single free particle or photon (K) become stable, i.e.

$$S_{G|G}(t)=S_{\nu|\nu}(t)=S_{k|k}(t)=1 \tag{1}$$

for any time t. The Hamiltonian H_0+H is transformed

$$T^{-1}(H_0+H)T=H_0+K,\ K=K_1+K_2+\cdots\cdots$$

The expansion of K is according to e. By (1), $T|G>$ etc. are determined, but $T|z>$ ($z=$ excited state etc.) is undetermined. Also $T|G+K>$ (ground state + incident photon) is determined. Quantities referring to an excited state must be regarded as depending on the excitation conditions, if they depend explicitly on T. On the other hand, it will turn out that the level *width* is independent of T and has therefore an unambigous meaning.

For the solution of the wave equation a formalism is used that exhibits directly the characteristic resonance denominators which must occur. (The formalism is similar to that first used by Heitler and Ma). The wave equation, with an initial condition at t_0,

$$i\dot{\psi}=(H_0+K)\psi,\ (t>t_0)$$

is solved by

$$\psi(t)=R(t)\psi(t_0),\ S(t)=e^{itH_0}R(t)e^{-it_0H_0}$$

$$R(t)=\frac{i}{2\pi}\int_{-\infty}^{+\infty}dE\,e^{-i(t-t_0)E}\xi(E-H_0-K) \tag{2}$$

$$\xi(x)=\frac{P}{x}-i\pi\delta(x).$$

(2) fulfils the initial condition $R(t_0)=1$. $R(t<t_0)=0$, but no physical meaning is attached to R when $t<t_0$. We denote quantities which are diagonal simultaneously with H_0 by a subscript d. ($nd=$ non diagonal), and split off the diagonal part of $\xi(E-H_0-K)$.

$$\begin{aligned}\xi(E-H_0-K)&=N(E)+N(E)U^s(E)N(E)\\ N(E)&\equiv\xi(E-H_0-K)_d,\ U^s\equiv U_{nd}^s.\end{aligned} \tag{3}$$

N can be written

$$N=\left(E-H_0+\frac{i}{2}\Gamma(E)\right)^{-1},\Gamma\equiv\Gamma_d. \tag{4}$$

and s the characteristic resonance denominator. R (Γ) will be the line breadth. For Γ and U^s expansions can be derived in which the denominators N occur explicitly

$$\frac{1}{2i}\Gamma(E) = (K + KN(E)K_{nd} + \cdots\cdots)_d$$
$$U^s(E) = (K + K_{nd}NK_{nd} + \cdots\cdots)_{nd}. \tag{5}$$

These expansions are carried up to the fourth order. For K the expansion (1) is inserted. Then Γ and U^s are expressed by H, but T also occurs. It is advisable to rewrite Γ and N.

In all orders a part can be split off from Γ which is independent of T and E, and the rest is proportional to $E-H_0$. The latter is combined with the $E-H_0$ of N. Then N can be put into the form (for a state 0)

$$N_{0|0}(E) = \frac{1+\angle_{0|0}(E)}{E-H_0+\frac{i}{2}\bar{\Gamma}_{0|0}(E_0)} \equiv (1+\Delta(E))\bar{N}(E). \tag{6}$$

$\bar{\Gamma}$ is independent of E and T. Its real part is the level width of 0 and this is independent of the excitation. The numerator is re-written

$$\bar{U}(E) = (1+\Delta(E))U^s(E)(1+\Delta(E)) \tag{7}$$

This depends in general on E and T.

The "unperturbed" energy H_0 is chosen so that the mass and charge renormalization as well as the level shifts are included. The eigenvalues of H_0 are the displaced levels with the experimental mass. The level displacements are determined by the condition

$$\mathfrak{J}_m \bar{\Gamma}_{0|0}(E_0) = 0 \tag{8}$$

(in all orders). (8) gives the usual level shift in second order, but in the fourth order there is a very minor difference as compared with the definition of the level sift used elsewhere. The difference is a term $\gamma_2^2/4\ K_0$ (K_0 = excitation energy) $\gamma_2 \equiv \mathrm{R}\bar{\Gamma}_{2\,0|0}(E_0)$. This is the analogue of the classical frequency shift of a damped oscillator. The term is exceedingly small.

When $t\to\infty$, the atom will be found in the ground state (+ photons). Then N degenerates into a ξ-function (as $\mathrm{R}\bar{\Gamma}\to 0$) and the integration (2) can be carried out. One finds

$$S_{FA}(\infty) = \bar{U}(E_F)_{F|A}\,\xi(E_F-E_A)\,e^{it_0(E_F-E_A)}. \tag{9}$$

In general $E_F \neq E_A$. For finite times $\bar{U}(E)$ is needed for all E. (9) gives the probability amplitudes after a long time. Unambiguous results for U or S (i.e. independent of the excitation conditions) are obtained only when the atom is also initially in the ground state. The simplest case is the resonance scattering $G+K \to G+K'$. In the first non-vanishing approximation the Weisskopf-Wigner formula is obtained, except for the fact that the numerator is not quite constant. If the incident light wave covers a continuous spectrum $I(K)\,dk$ (number of photons) with random phases the shape of the emission line is const. $K'I(K')/((K'-K_0)^2+\gamma^2/4)$, $\gamma\equiv\mathrm{R}\bar{\Gamma}(E_0)$. The factor $K'I(K')$ can, of course, also be deduced from elementary theory. The distortion of the line (including a shift of the maximum) due to the K'-dependent factor can be quite appreciable for radiofrequency transitions.

Radiative corrections exist for $\bar{\Gamma}$ and $\bar{U}$. They are estimated to be 137^{-3}-times smaller than the lowest order. (with a large numerical factor of order 20). The angular dependence of the corrections to $\bar{U}$ is, of course, not the same as for the first term. If the incident wave is a wave packet with strict phase relations the corrections to $\bar{U}$ can be much larger.

See abstract of H. Suura on P. 18

Convergence of the Perturbation Method in the Quantum Field Theory *(Comment)*

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In the preceding paper of T. Imamura and the present author, they proved that the S-matrix expanded as a power-series in the coupling constant g is divergent though each term of this series is finite by virtue of the relativistic cut-off (non-local interaction). This divergence results from the fact that the number of Feynman-graphs increases rapidly with increase in the order n of powers of the coupling constant.

In the present paper we make the Green-function (Feynman's kernel) converge by making some modifications on the fundamental Equations.

Let us consider a system of nucleon-meson fields with non-local interaction.

Using formally Schwinger's new formalism we can easily derive the Green-function of nucleon field G, that of meson field $\mathfrak{G}$, and the system of equations stasisfied by these quantities. Among these equations

$$\Gamma = -\frac{1}{g}\frac{\delta G^{-1}}{\delta<\phi>} \tag{1}$$

will be of importance in the following discussions of the present paper.

The divergence of the power-series representing the S-matrix is essentially caused by Eq. (1).

Accordingly let us adopt the new equation

$$\Gamma = U - ig^2\, UG\Gamma G\Gamma\mathfrak{G} \tag{1$'$}$$

as a definition of the vertex operotar instead of (1). Here U is a non-local form factor belonging to the so-called "Normal Class".

As a result of this modification, the number of graphs with the irreducible proper self-energy type with $2n$ vertices is reduced to

$$\frac{(2n-2)!\,(n-2)!}{n!\,(n-1)!} \tag{2}$$

from the value

$$(2n-1)!\,2^{n-1}(n-1)! \tag{2$'$}$$

as derived from Eq. (1).

In case of (2), we can actually prove the convergence of the Green-functions $\mathfrak{G}$ and G etc. by using the method of majorant series.

The remaining problems on which we should make a study are (i) to give the physical foundation to the modified Eq. (1)′, (ii) to confirm if (1)′ is sufficient to make all the elements of the S-matrix converge, (iii) to investigate whether the consequences of our theory are physically acceptable or not.

Cosmotron Experiments on V-Particles

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(Abstract not yet received Aug. 20, 1953)

On the Multiple Meson Production

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It is shown that the production of mesons and nucleons in very energetic nucleon-nucleon collisions depends essentially on the extent to which the observed rest mass of a nucleon is due to its meson field and the spacial distribution of the energy in this field. The production of showers of mesons and nucleons is considered, assuming that a fraction ε of the observed rest mass of the nucleon is carried by the field, this rest mass being distribution according to some arbitrary function f of distance from the centre of the nucleon. The remaining fraction of the rest mass is assumed to be concentrated in a sphere of radius ρ round the centre of the nucleon this radius being of the order of the Compton wave length of the nucleon or less. It is shown how the experimental study of meson nucleon production in high energy encounters provides a method of obtaining information on the fundamental question of the field energy of a nucleon and the localization of this energy in its meson field.

Interpretation of Cosmic Ray Jets.

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Cosmic Ray jets are penetrating showers distinguished by a very small angular spread of the jet particles which indicates an extremely high primary energy $E(>100$ Bev, say). The most remarkable feature is the fact that the number of heavy tracks N_H is rather small, 0–5, say, in contrast to the p.s. at lower energies where N_H is usually larger.

The following interpretation is based on a picture first used by Roesler and McCusker. The primary nucleon penetrates through the nucleus sweeping out all nuclear matter along its path together with the mesons produced and thus creates a more or less clear-cut penetration tunnel. We estimate (i) the energy transfer U to the residue nucleus and hence the number of evaporation tracks and (ii) the number of shower particles n_s.

I. *Energy transfer.* U consists of 2 parts. (i) The direct energy transfer U_1 to the nucleons of the residue nucleus. (friction). If all jet nucleons have energies $\gg Mc^2$, U_1 is very small, ≤ 15 Mev, even in a diametrical passage through an Ag-nucleus. This is no longer true if the jet nucleons, near the end of the tunnel, have energies of only a few Mc^2. (ii) *Surface energy* of the tunnel. If we assume that the surface energy per unit area is the same as for a spherical nucleus one obtains for a maximum tunnel length in Ag an energy transfer $U_2 \sim 105$ Mev. This estimate, of course, is very crude. The number of evaporation tracks can now be calculated from the theory of Le Couteur. To obtain an average of $N_H \simeq 3$ (as seems observed), one requires an energy of 140 Mev, in reasonable agreement with the above estimate. The fluctuations of N_H turn out to be rather large ($\Delta N_H \simeq 1.7$ when $N_H=3$), and are due to several causes. Thus there is no difficulty in understanding the small value of N_H observed and even the fact that jets with $N_H=0$ are quite frequent.

When the impact parameter increases, U decreases, more or less in proportion to the tunnel length, but falls off sharply when the impact parameter is so large that no tunnel is created but merely a piece of the nucleus is knocked off. (near-glancing collisions). In this case $N_H=0,1$ is to be expected.

II. *Number of shower particles.* This depends, of course, on the model used for meson production. Recent experiments show that multiple production is comparatively rare for primary energies up to 30 Bev (McCusker, Porter and Wilson), in disagreement with all theories which predict multiple processes as the normal event. (Heisenberg's theory, for example, predicts an average of 9 mesons per collision at 30 Bev). It follows that the great majority of penetrating showers must be interpreted in accordance with the plural theory. For the jets in question nothing

definite is known yet. We therefore use two models: (i) a plural model (ii) a mixed plural-multiple model.

(i) *Plural model.* It is assumed that if n fast nucleons hit a nucleon at rest *simultaneously* in a $n+1$ body collision, then n mesons are produced (Heitler and Janossy). At these extreme energies we need not distinguish between π, κ . . . etc. mesons. If d is the tunnel length in units of the average distance between two collisions (assuming the cross section to be geometric) one obtains for the number of charged jet particles

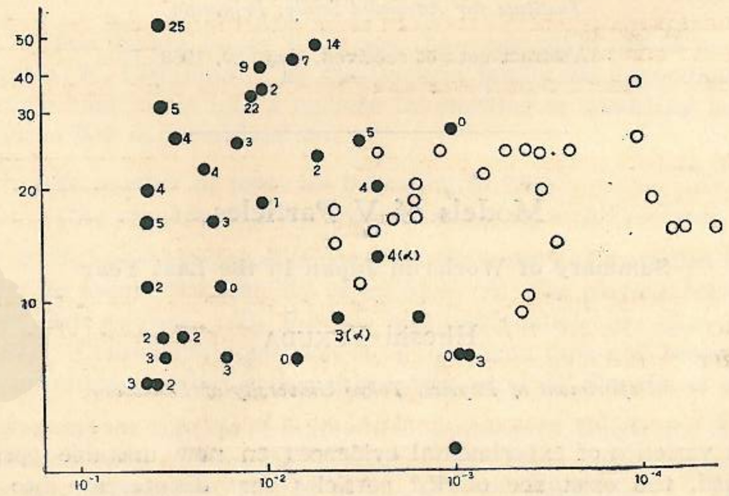
$$n_s = \frac{1}{3}d^2 + \frac{5}{6}d + \frac{1}{2} \tag{1}$$

(if 1/3 of the mesons and 1/2 of the nucleons are neutral). For Ag a maximum tunnel length, $d=7.2$. This gives $n_s=23$. Apart from fluctuations, this should be the maximum number of shower particles, and this is independent of the energy. However, the fluctuations are very large. A rough estimate shows that for Ag, $n_s=23\pm12$. For Cu, $n_s=17.5\pm10$. Fig. 1 shows that, apart from a few events of fairly low energies (see below) an upper limit really exists, that is constant over a wide range of energies (factor $\sim$100) and this limit agrees quite well with the above estimate.

(ii) *Mixed plural-multiple model.* A simple model is this (Roesler *et al*): In a jet-nucleon collision c_M mesons are produced. c_M is independent of the energy of the jet (=primary energy E). This gives $n_s=c_M \cdot d$, and to obtain the correct upper limit in Fig. 1 we have to put $c_M=4$.

Evidently, many different (and more complicated) models can explain the observed number n_s.

Fig. 1 snows the individual events observed by Daniel, Davies, Mulvey and Perkins ($AgBr$, full circles) and by Kaplon and Ritson (Cu, open circles), against the angle $\bar{\phi}^2$.



The numbers attached to the full circles give N_H. In the second experiment N_H is not measured. The events in the neighbourhood of the upper boundery are interpreted as due to a diametrical passage through the nucleus, events with smaller n_s are due to larger impact parameters. Owing to the large fluctuations and the small statistical material available (especially concerning N_H) no more quantitative comparison between theory and experiment for small n_s is possible at present. One might think that the lower boundary of Fig. 1 gives the true multiplicities in a nucleon-nucleon (or glancing) collision. This would give an average multiplicity 4–6. However, showers with N_H=0,1 and n_s=2–5 are likely to be missed and it is therefore impossible to say where the true lower boundary lies*).

The above considerations cease to hold, when the emerging nucleons from an angle so large that the tunnel is no longer cylindrical. This limit lies at $\bar{\phi^2} \sim 3\cdot 10^{-2}$. In this case the emerging nucleons also have energies of only a few Mc^2. Then U_1 is much larger and also U_2 increases. Moreover, it must then happen sometimes that a jet nucleon forms an angle substantially larger than the average. In this case more nuclear matter is drawn into the process (increase of the effective d). Since n_s is sensitive to d some showers with considerably larger n_s and N_H must exist in the neighbourhood of the above limit. This explains qualitatively the few events in the upper left corner of Fig. 1.

*) Evidently, a strong bias against small n_s exists in the Kaplon *et al.* experiment. In the region where both experiments overlap no showers n_s<10 are observed in the former experiment, whereas several events n_s=4–10 are found in the Daniel *et al.* experiment.

Baryon – Meson – Photon System

(Title not yet known)

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(Abstract not yet received, Aug. 20, 1953)

Models of V Particles.

Summary of Works in Japan in the Last Year

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Although varieties of experimental evidences on new unstable particles have been presented, the existance of V_1^0 particles that disintegrate into proton and

negative pion seems to be established. We are, therefore, concerned primarily with the V_1^0 particle and give only a brief account of other kinds of particles. A tentative assumption is made that these particles are elementary, though the possibility is left that some of them are composite.

With the aid of selection rules the types of particles may be asigned. The absolute selection rules follow from the invariances under (i) space rotation, (ii) space reflection and (iii) charge conjugation or time reversal. (i) and (ii) are well known and allow one to classify bosons as scalar (S), pseudoscalar (Ps), vector (V) and pseudovector (Pv). (iii) gives a sort of even-odd alteratives, scalar (s), pseudoscalar (ps) and pseudvector (pv) couplings belongings to even alternative, while vector (v) and tensor (t) couplings to odd one. There are relative selection rules on the isotopic spin, (iv) charge symmetry and (v) charge independence. The theorem of Fukuda and Miyamoto results from (iii) and (iv). There may be other quantum numbers, such as the v-spin of Peaslee. The quantum number should not be adopted arbitrarily, however, unless the corresponding operation exists.

The selection rules are easily applied to the following examples.

$$V_2^0 \to \pi^+ + \pi^- \quad (1)$$

allow us to rule out V_2^0 being Ps and Pv due to (i) and (ii) and $S(v, \tau_4=1)$, $S(s, \tau_3)$, $V(\tau_4=1)$ due to (iii) and (iv). Similarly

$$\tau^\pm \to \pi^\pm + \pi^+ + \pi^- \quad (2)$$

excludes $\tau^\pm$ being S due to (i) and (ii) and V and $Pv(t)$ due to (iii) and (iv).

For fermion (iii) together with (i) serves to discriminate Dirac and Majonara types. The stability of nuclei requires the Dirac type of nucleon. In any nuclear transformation the number of nucleons is conserved, or more rigorously, (the number of nucleons) – (the number of anti-nucleons) = const. This conservation law no longer holds in the presence of V_1^0 particles, however, because

$$V_1^0 \to p + \pi^- \quad (3)$$

is observed. Thus we are forced to extend the consrvation to the nucleon family (NF) (S. Oneda, K. Nishijima). By the nucleon family we understand to include such a particle that turns into a nucleon by emitting or absorbing bosons. Then the conservation law is generalized as

(the number of particles belonging to NF)
– (the number of anti-particles belonging to NF) = const. (5)

Further conditions have to be imposed on the nature of particles belonging to NF, in order to ensure the stability of nuclei. (A) *The particle belonging to NF is not lighter than nucleon.* (B) *Particle and anti-particle are discriminated for such a particle.* Consequently fermion is of the Dirac type and boson is described by complex operators whether it is charged or neutral (S. Tanaka *et al.*).

We may extend the concept of family to all fermions (S. Oneda and H. Umezawa). (C) *If a fermion turns into another fermion by emitting or absorbing real or virtual bosons, these two fermions are defined to belong to the same family.* There may be the lepton family, to which electron (e), nautrino (ν) and muon (μ) belong. To this κ particle may belong, provided its decay takes place as

$$\kappa \rightarrow \mu+\nu+\nu' \tag{5}$$

where ν' could be neutrino or a counter particle of κ. The wave functions of fermions belonging to the same family must anticommute with each other. The existence of the lepton family different from NF will be established, if

$$V_1^0 \rightarrow p+\mu \tag{6}$$

does not occur, provided μ is fermion. The occurence of (6) contradicts with the stability of nuclei any way, unless V_1^0 is described by a complex boson field.

The discussions thus far made should hold independent of the life time. Once the life time of V_1^0 is taken into consideration and the transformation (3) is interpreted in current terms of field theory, the magnitude of the coupling constant for (3), assuming the interaction as direct and pv, is estimated as ,$10^{-13}\sim10^{-14}$ which results in far too small production cross section for $\pi^-+p\rightarrow V_1^0$. This figure is just the same order as the magnitude of coupling constant for $\pi\rightarrow\mu+\nu$. The behaviour of V_1^0 would, therefore, be similar to that of μ, if no other interaction existed than required from (3) (S. Ogawa).

It is worth remarking that one has to be cautious in estimating the magnitude of coupling constants for pseudoscalar thėory when processes involve nucleon loops. For example, the life-time for $\pi^0\rightarrow2\gamma$ that is probably $\sim10^{-15}$ sec. according to recent experiments is hardly explained in terms of the lowest order perturbation theory that gives $\sim10^{-18}$ sec. to the life-time. The discrepancy is too large to be accounted for by the aid of conventional higher order approximation, though a slight improvement is attained for symmetrical meson theory. The discrepancy is found to be due mainly to the damping of the virtual pair creation which has been shown by Brueckner *et al.* to be quite large for the scattering of pions (T. Nakano and K. Nishijima). Our success to account for the life-time for $\pi^0\rightarrow2\gamma$ is instructive for any similar decay process, particlarly for $\tau\rightarrow3\pi$, in which the damping effect may change perturbation result by several order. Hence we think it sensible to avoid the processes with nucleon loops in any quantitative discussion of new particles, unless the damping effect is properly evaluated.

Taking the above into account we are going to construct reasonable models of V_1^0 particles. Models are classified on account of the mechanisms of V production into (I) direct production and (II) indirect production. (I) is suddivided into (Ia) single production (Ib) pair production.

(Ia) The single direct production is represented by

$$\left.\begin{matrix}\pi\\N\end{matrix}\right\} +N\rightarrow V_1^0+a+\cdots\cdots \tag{7}$$

Since the interaction should be strong in order to realize the copious production, we would expect a fast decay as

$$V_1^0\rightarrow N+\left.\begin{matrix}\pi\\N\end{matrix}\right\}+\bar{a}+\cdots\cdots\rightarrow N+\left.\begin{matrix}\pi\\\gamma\end{matrix}\right\}, \tag{8}$$

unless a selection rule makes (8) slow. It has been thought that V_1^0 might have very large spin and/or isotopic spin. An elementary particle of spin larger than 1 is not welcome, however, since we do not intend to work outside the present quantum field theory (Y. Katayama). This a ssumption will also fail for a composite V_1^0, because there may be a number of those excited states of smaller spins, to which the radiative transition can take place much faster than the observed life time. The high isotopic spin results in numerous charge states with roughly same energy, in contradition with observations.

(Ib) The direct pair production was proposed by Pais, but is not favoured by experiments. We shall, however, show that one has not to abandon the entire idea of Pais but can modify it, avoiding the contradiction with observations.

(II) The indirect production of V_1^0 via pair production is now proposed (K. Nishijima). The indirect production via single production has succeeded to explain the behaviour of muons, but the same idea is not workable in our case. The difficulty lies in that the decay takes place through the inverse process too quickly. The inverse process can be made to forbid, if a pair of particles are produced at an initial act

$$N+N\rightarrow X+X, \tag{9}$$

as has been assumed by Pais. The interaction (9) is so strong as to be consistent with the abundance of V_1^0 that the coupling is classified as even,

$$g^2_{even}/4\pi\sim1. \tag{10}$$

Since X should not be observed in cloud chambers and the decay products of V_1^0 should be coplaner with the origin of the initial interaction, the life time of X has to be shorter than 10^{-12} sec. In order that few pairs are observed, only a fraction of X decays into V_1^0 in competition with other modes, say

$$X\rightarrow\begin{cases}V_1^0+\cdots\cdots(g_V)\\N\ +\cdots\cdots(g_N)\end{cases} \tag{11}$$

The magnitude of coupling constants, g_V and g_N, are determined as follows. g_N is slightly higher than g_V, resulting in the single V_1^0 with great probability. Both g_N and g_V have to be as small as the decay

$$V_1^0\xrightarrow{(g_V)}X+\cdots\cdots\xrightarrow{(g_N)}N+\cdots\cdots \tag{12}$$

takes place slower than (3). Since the interaction (3) is described by the odd coupling of Pais,

$$g^2_{odd}/4\pi\sim10^{-12}, \tag{13}$$

the competition between (3) and (12) requires

$$(g_N^2/4\pi)\,(g^2_V/4\pi)<g_{odd}^2/4\pi, \tag{14}$$

hence, on account of (13)

$$10^{-6}>g_N^2/4\pi\gtrsim g^2_V/4\pi>10^{-10} \tag{15}$$

Thus the coupling constants, g_N and g_V, are of intermediate magnitude between g_{even} and g_{odd}.

Our model allows various modifications without altering the essential scheme discussed above.

Influence of New Unstable Particles on the Temperature Effect of Cosmic Rays Underground (*Comment*)

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A possible influence of kappa particles on the temperature effect of cosmic rays far underground has been studied extensively by Barrett *et al.*[1]. and the result is compared with their own experiment. The influence makes the temperature coefficient smaller than that expected from the time delay only due to pi-mu decays, because the delay due to kappa-mu decays is shorter. Their experimental result has, however, shown a larger temperature coefficient that is, $0.79 \pm 0.20\%$ per °C than expected exclusively from pi-mu decays. Although there exist inevitable ambiguities in deriving the temperature coefficient, this too large figure leads us to suggest the influence of other unstable particles. Among various kind of new particles all but kappa particle disintegrate into pions. Recent experiments performed by M. I. T. group[2] have revealed that most of charged V particles which had once thought to be kappa particles seem to be chi particles decaying into pions. Hence a greater part of new particles may be a source of pions at high altitudes. Yet no reliable evidence has been obtained on the abandance of such new particles compared with that of pions produced directly by nucleon-nucleon collisions, but the ratio seems to amount the order of ten percent or more[3]. It seems, therefore, highly probable that the new particles contribute to the time delay through two or more steps of disintegrations. And such a delay certainly increases the temperature coefficient by an appreciable amount. It may, therefore, be worth while to discuss the increase of the temperature coefficient as a function of energies.

Since few knowledges are available on the new particles, it may be better to concentrate ourselves to an essential feature and to leave out details for later investigations. Hence we take several simplified assumptions. We add another assumption added to those taken by Barrett *et al.* in Appendix in their paper[1]. That is to equate the absorption mean free paths of all particles but muons under consideration. This is adopted not only due to the analytical convenience, but also to the plausibility.

Due to the temperature change at atmospheric depth x in unit of the mean fee path

$$T(x) \to T(x)\,(1+\xi(x)), \tag{1}$$

the intensity of muons with energy E exclusively from pi-mu decays increases by

$$\delta I(\pi-\mu) \propto \bar{\xi}(E) - (B_\pi/E)\bar{\xi}_\pi(E), \tag{2}$$

where B_π is the same as B in the quoted paper of Barrett *et al.* And

$$\bar{\xi}(E) = \int_0^{x_0} \xi(x')\, e^{-x'} dx', \; \bar{\xi}_\pi(E) = \int_0^{x_0} \xi_\pi(x')\, e^{-x'} \frac{dx'}{x'}, \tag{3}$$

with

$$\xi_\pi(x, E) = \int_0^{x} \xi(x') \left(\frac{x'}{x}\right)^{B\pi/E} dx'. \tag{4}$$

For the muons coming from new particles, say V, the intensity increases by

$$\delta I(V-\pi-\mu) = \bar{\xi}(E) - (B_\pi/E)\bar{\xi}_\pi(E) + (1+B_\pi/E)\bar{\xi}_V(E), \tag{5}$$

where ξ_V is obtained from ξ_V by replacing B_π by B_V, namely replacing the life time of pions by that of V particles. If the production rate of V particles is f part of that of pions, the intensity of muons due to V particles is $fB_V/(E+B_V)$ times that due to pions produced directly.

For all muons the intensity increase is given by

$$\delta I = \left(\bar{\xi}(E) - \frac{B_\pi}{E}\bar{\xi}_\pi(E)\right) + \frac{fB_V}{(1+f)B_V+E}\left(1+\frac{B_\pi}{E}\right)\bar{\xi}_V(E). \tag{6}$$

The first term in (6) is the same as that without new particles. The second term makes a positive contribution in which we are interested.

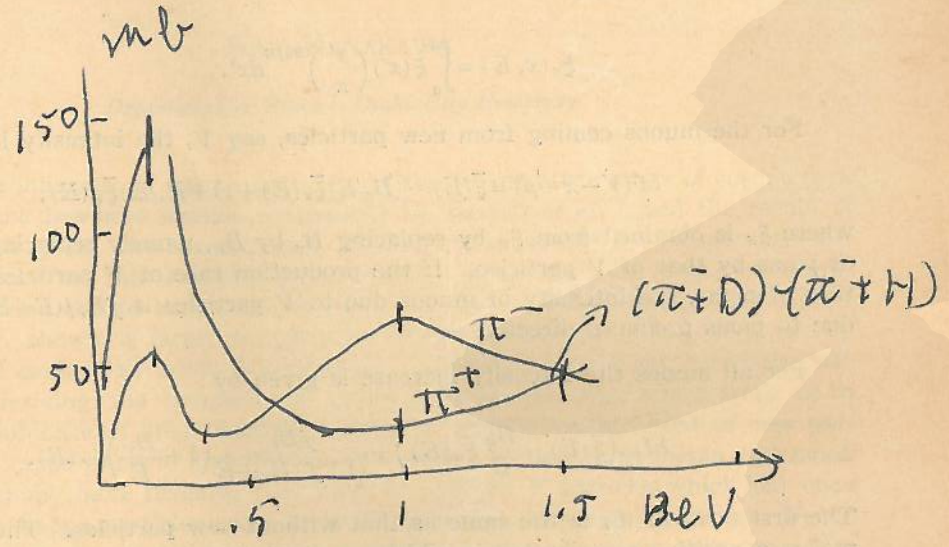
In order to estimate the contribution of this term, we first notice that B_π is negligibly small compared with E, because $B_\pi \sim 10^{11} eV$ and $E \sim 10^{12} eV$. We do not have any precise value of B_V, but it may be greater than B_π and smaller than $10^{13} eV$. Further B_V may not be the same for all new particles. If B_V is close to E, as it is probable, the last term of (6) is roughly equal to f. The magnitudes of $\bar{\xi}$ depend upon the structure of the atmosphere but all may be of the same order and possibly $\bar{\xi} < \bar{\xi}_\pi < \bar{\xi}_V$. Hence the production ratio f approximately reflects itself in the temperature coefficient. We can not draw any conclusion from the above analysis in the present stage of our knowledges, but we can expect an appreciable increase of the temperature coefficient so as to explain the experimental result.

The influence of new particles can not be neglected in accounting for the angular distribution of penetrating particles underground. The distribution is expected to be steeper that evaluated on the pi-mu decay only. The experiment of

Barret *et al.* indicates an opposite direction, however. Further experimental material is waited for arguing our suggestion.

References

1) P. H. Barrett *et al*: Rev. Mod. Phys. 24 (1952), 133.
2) H. S. Bridge *et al*: Phys. Rev. 90 (1953), 921.
3) Daniel *et al*: Phil. Mag. 43 (1952), 753.



219 MeV $\left(\frac{d\sigma}{d\Omega}\right)_{--} = 5.8\,Y_0 + 1.4\,Y_1 + 1.9\,Y_2 - 0.18\,Y_3 - 0.1\,Y_4$

20～21

Experimental Work at Chicago

Gregor WENTZEL

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(Abstract not yet received, Aug. 20, 1953)

Brookhaven report

(Title not yet known) *(Comment)*

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(Abstract not yet received, Aug. 20, 1953)

A Remark on the Relation between Pion Nucleon Scattering and Photomeson Production *(Comment)*

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If pion nucleon scattering is an elastic one, its S matrix is unitary, but really it is accompanied by the inverse process of photomeson production, so that only the total S matrix involving the both processes is unitary. But experimental cross section of photomeson is smaller than that of pion scattering in the appropriate energy range, and one can approximate this unitarity condition according to the largeness of their S matrix, and obtain a relation between them. This relation is most simply expressed in terms of phase shift analysis, that is, in terms of

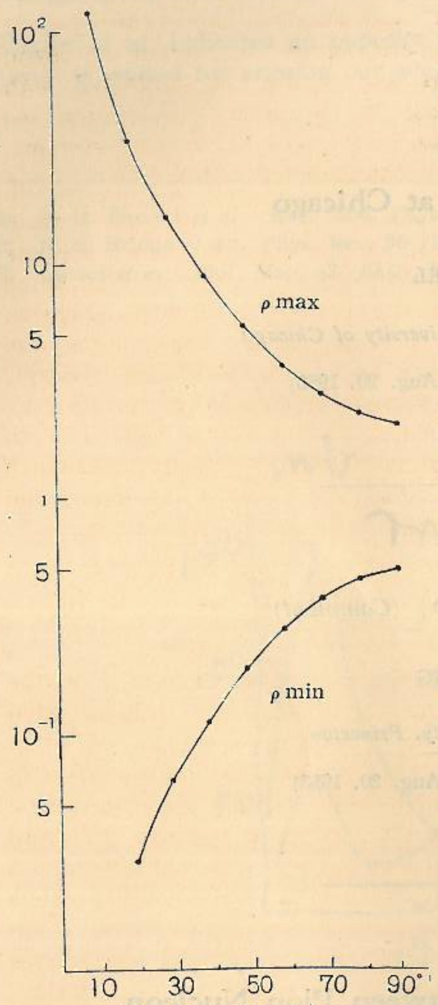

Fig. 1 $\Delta = |\delta_3 - \delta_1|$

eigenvalues of S matrix. Let R be eigenvalue of scattering ($=S$ matrix -1) of photomeson and S that of S matrix of pion scattering, we have

$$R^*/R = -S^*$$

(* means complex conjugate)

Thus when the phase shifts of pion scattering are known, one can get some information about photomeson almost without any special dynamical assumptions. For example, the ratio ρ of contributions of partial waves of definite nature to two processes $\gamma + p \to n + \pi^+$ and $\gamma + p \to p + \pi^0$ (excluding interference terms) is limited in a finite range. Let Δ be the difference of charge singlet and triplet phase shifts of same nature in poin scattering, we have

$$\frac{3}{4}\frac{3+A}{\sin^2\Delta} - 1 \geqq \rho \geqq \frac{3}{4}\frac{3-A}{\sin^2\Delta} - 1,$$

where $\qquad A = (1 + 8\cos^2\Delta)^{1/2}$;

see Fig. 1. Thus one may hope to decide whether Fermi or Yang's phase shifts are correct, but the present data on photo meson are too rough and both solutions are consistent with the above condition, though, if anything, Yang's solution is a little favored.

Phenomenological Theory of the Pion-Nucleon Interaction

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The experiments on pion-nucleon scattering at Chicago and Columbia in the energy region 60–135 Mev. have been analyzed by their authors on the basis of the charge-independence hypothesis and on the assumption that d and higher phase shifts can be disregarded. The following results have been obtained: (1) the only

p phase sift of importance is α_{33} (isospin $T={}^3/_2$ and angular momentum $J={}^3/_2$); (2) both s phase shifts $\alpha_1(T={}^1/_2)$ and $\alpha_3(T={}^3/_2)$ are required to explain the asymmetric angular distributions; (3) the signs of α_1 and α_{33} are the same and both are opposite to the sign of α_3; (4) in the energy region under consideration, α_{33} increases approximately as q^3 (q is the momentum of the pion in the c.m. system), α_3 increases even more rapidly than α_{33} while α_1 changes very little.

Result (1) does not necessarily imply a resonance with the $T={}^3/_2$, $J={}^3/_2$ state of the pion-nucleon system since even weak coupling pseudoscalar theory predicts an attractive α_{33} phase shift contrary to the repulsive character of the other three p phase shifts; moreover, all improvements of the weak coupling approximation enhance the magnitude of α_{33} and diminish the three repulsive p phase shifts (*e.g.* Dyson's work with the $PS(PS)$ theory and Chew's work with the $PS(PV)$ theory). Result (2) indicates that the dominant p interaction predicted by the $PS(PV)$ theory in the non-relativistic region (for the nucleons) is insufficient and that an s interaction must also be present. In this respect, the $PS(PS)$ theory looks much more promising since a well known canonical transformation leads to an interaction Hamiltonian of the following form (to order g^2/M^2):

$$\frac{g}{2M}\int\psi^*\boldsymbol{\sigma}\cdot\nabla\boldsymbol{\tau}\cdot\boldsymbol{\phi}\,\psi + \frac{g^2}{2M}\int\psi^*\boldsymbol{\phi}\cdot\boldsymbol{\phi}\psi + \left(\frac{g}{2M}\right)^2\int\psi^*\boldsymbol{\tau}\cdot\boldsymbol{\phi}\times\boldsymbol{\pi}\psi, \qquad \text{(I)}$$

where ψ and ϕ are the nucleon and pion wave functions respectively, π is the canonically conjugate momentum to $\boldsymbol{\phi}$, $\boldsymbol{\tau}$, $\boldsymbol{\sigma}$ are the isospin and ordinary spin three-vectors, g is the coupling constant, M is the nucleon mass and $\hbar = c = \mu = 1$ (μ is the pion mass). The first term of (I) is the p interaction which is dominant in the $PS(PV)$ theory whereas the second and third terms constitute the s interaction (the second terms is the pair term considered by many authors whereas the third term has only recently been considered by Drell and Henley). Result (3) implies that the second term of (I) can not explain the experimental data since it predicts the same sign for α_1 and α_3 and no charge-exchange s scattering: moreover, Wentzel has shown by means of an exact calculation that the higher order corrections to the pair term strongly reduce its contribution to π^+ and ordinary π^- scattering. On the other hand, the third term of (I) leads to a positive sign for α_1 and to the opposite sign for α_3 since it is manifestly a spin-orbit coupling term in isospin space. However, it still remains to be demonstrated that the damping of the isospin-orbit term by higher order corrections is less important than the damping of the pair term. With this qualification, the $PS(PS)$ interaction Hamiltonian can account qualitatively for results (1)–(3) although it is not clear that it can fully account for result (4); perhaps, the pion-pion interaction term required for renormalization purposes will also be helpful in this respect.

Result (4) can easily be explained by the author's phenomenological hypothesis that an effective attractive potential with a repulsive core exists between the pion and nucleon in the $T={}^3/_2\,s$ state. If to this hypothesis are added the phenomenological assumptions that a pure attractive potential exists between the two members of the pion-nucleon system in the $T={}^1/_2\,s$ state and that α_{33} obeys a q^3-law (at least in the energy range 0–135 Mev.), then the six parameters of the phenomenological theory (the radius of the repulsive core, the ranges and depths of the two attractive wells and the coefficient in the q^3—law) can be determined from the Chicago and Columbia pion-nucleon scattering experiments and from Panofsky's equivalent experiment on charge-exchange scattering at zero energy. Within the accuracy of these experimental data, different potential wells are of course possible and the various possibilities will be discussed.

The phenomenological theory has been employed to predict the differential and total cross-sections for π^+ and π^- scattering (both direct and charge-exchange) by protons in the energy range 20–40 Mev. where experiments have recently been performed at Rochester. The agreement between theory and experiment in all details is remarkably good provided that the Coulomb interference is taken into account for π^+ and π^- ordinary scattering. As a matter of fact, the Coulomb effects are so large in this low energy region that the signs of the phase shifts are unequivocally determined to be in agreement with the theoretical choice. Furthermore, the variety of agreement is so great—with the backward maximum in the differential π^+ scattering at 40 Mev., with the nearly equal total π^+ and π^- cross-sections at 37 Mev., with the forward maximum in the differential ordinary π^- scattering at 35 Mev., with the backward maximum in the differential charge-excange scattering at 38 Mev., and with the linear variation of the total charge-exchange cross-section from 20 to 40 Mev.—that one can accept with considerable confidence the validity of the charge-independence hypothesis and the necessity for the correct field theory to predict for this low energy region essentially zero values for the three p phase shifts other than α_{33}, a very small value for α_3, and positive signs for α_1 and α_{33}.

(Title not yet known) (*Comment*)

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(Abstract not yet received, Aug. 20, 1953)

Theoretical Analysis of Pion and Nucleon Scattering

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Recent experiments on pion and nucleon scattering together with their phase-shifts analysis exhibit the resonance phenomena in the $T=3/2$, $J=3/2$ state, and

the steep energy dependence of s-wave phase shifts. The resonance-like scattering may be interpreted by taking into account the reaction of the meson field in some way quite different from the perturbational estimation. In fact, Dyson and Bethe showed this aspect relativistically using the Tamm-Dancoff method. As to the steep rise of the s-wave cross sections and the characteristic sign of the $T=3/2$ and $T=1/2$ states of the isotopic spin, Wentzel and Marshak remarked that the pseudo-scalar meson theory with pseudo-scalar coupling contains terms, besides the core term, which are found to bring the signs in the correct direction.

We have also made investigations on the pion nucleon scattering by trying to attack above points in a slightly different way. In order to make an approximation we made use of the Tamm-Dancoff method without pair formation and neglecting nucleon recoil. The reason why we have adopted this method is that, as Wentzel first pointed out, the large core term in the Hamiltonian resulting from the Dyson-transformation reduces the effect of core term, namely the s-wave contribution, to a large extent, so that the pseudo-scalar coupling and pseudo-vector one become in many respects to have the same features, *i.e.* the term of p-wave interactions are almost the same in both cases, at least in the non-relativistic region. But according to Drell and Henley, if one considers the higher order contributions to the hard core term, the coupling constant of core interaction may become even attractive for a sufficiently large coupling constant, which makes it difficult to understand the s-wave phase shift. K. Sawada and S. Tani have been investigating the effectiveness of the Dyson-transformation in the case of a large coupling constant from the same standpoint as that of Drell and Henley, *i.e.* all diverging quantities are retained, their effects being estimated with the cut-off of virtual meson momentum near the nucleon rest energy. They showed that the Dyson-transformation is very crude in this large coupling case, and in fact, the Foldy-transformation is more appropriate for the purpose of eliminating the pseudo-scalar interaction. The reduction factors of all the terms (pseudo-vector coupling, core-term, mass-term and the term named H_π by Drell and Henley) were evaluated by T. Akiba numerically and it was shown that every term is reduced by about the same amount; especially the pseudo-vector coupling term is reduced more strongly than the core term, and the sign change in the latter one does not appear in the range of the coupling constant running from 10 to 40. If, therefore, one adjusts the pseudo-vector coupling term so as to explain the p-wave scattering (or nuclear force), the core term remains even in a larger proportion than that without the inclusion of higher order corrections. Thus, the s-wave damping of Wentzel *et al.* would also work in our case. But since H_π contains the vector product of ϕ and π, the damping of H_π does not hold and in fact they remain about in the same magnitude. Therefore the charge dependence of the s-wave is effective in the same direction as Wentzel has already pointed out, which is convenient to explain the experimental results. But according to N. Fukuda, S. Goto and K. Sawada, the magnitude is not sufficient and the steep rise of cross sections cannot be explained if one uses the interaction without reduction factors with a moderate coupling constant. In this connection N. Fukuda *et al.* evaluated the s- and d-wave phase shifts in the pseudo-vector coupling with the Born approximation, and the results show that the signs of s-wave phase shifts are quite wrong and that d-wave phase shifts are only about 5 degrees at 200 Mev.; the wrong signs of these s-wave phase shifts seem fatal in this case. We shall report the results when one uses the reduction factors evaluated above in the ps–ps theory.

As to the p-wave scattering, it remains to be solved how the self-energy should be overcome even in the non-relativistic Tamm-Dancoff treatment. We have tried to subtract this self-energy by taking into account the fact that in the Tamm-Dancoff treatment the reaction of the meson field in every configuration is different since the maximum number of mesons which are present at the same time is restricted; (for example, if one considers only up to the two-meson configurations, the self-energy of the zero-meson configuration is affected by the reaction up to two mesons, but that of the one-meson configuration only from two-meson state). Hence we should add different counter terms to the Hamiltonian and determine these counter terms by solving the eigen-value problem. It should be demanded in this case that the incident meson wave of normal mass is permitted and that the eigen-value zero (no-meson one-nucleon state) is allowed. Fixing these counter terms, we can solve the scattering problem with one incident meson, obtaining the wave matrix in the form

$$(\boldsymbol{k}\,|\,K(\varepsilon_{k0})\,|\,\boldsymbol{k}_0)+\frac{\sum_{l}\dfrac{(\boldsymbol{k}\,|\,K(\varepsilon_{k0})\,|\,\boldsymbol{l})}{\varepsilon_{k0}-\varepsilon_{l}}H_{l}{}^{*}\cdot\sum_{l}(\boldsymbol{l}\,|\,K(\varepsilon_{k0})\,|\,\boldsymbol{k}_0)}{\varepsilon_{k0}+\varDelta_0(0)-\varDelta_0(\varepsilon_{k0})}$$

and one needs only to solve the integral equation which has just the same form of kernel as Dyson and Bethe treated in their relativistic treatment. We shall compare the phase shifts obtained in this way, especially the phase shifts of the 1/2, 1/2 state, with the experimental results. These investigations are based on the Tamm-Dancoff treatment including up to two mesons. The modifications coming from higher configurations were investigated recently by Chew in the Born approximation and it was shown that the contribution would be at most 20%. We have also checked the correction in the Tamm-Dancoff treatment, and the results show that a fairly large numerical factor appears from the higher configurations; it seems, however, to confirm the conclusion of Chew.

Incidentally, we can obtain the state functional of the physical nucleon, and by treating the electromagnetic interaction of the nucleon and meson as perturbation, we have been able to evaluate the anomalous magnetic moment of the nucleon and the cross section of photo-meson production (in pseudo-vector coupling theory with non-relativistic approximation); the results will be reported.

The main results thus far obtained in the case of the pseudo-vector coupling theory have been acquired non-relativistically and by the cut-off treatment. K. Nishijima is developing the relativistic theory of bound state interaction, which may be applied also to pion deuteron scattering. It will be discussed elsewhere.

Perturbational evaluation of the $\gamma-2\pi$ reaction was performed by T. Kotani and T. Kobayashi and the possibility of determination of the coupling type between the meson and nucleon may be obtained by observing the angular distribution of the negative pion.

Electro-magnetic Phenomena of Pion Nucleon System *(Comment)*

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We have evaluated both the anomalous magnetic moment of the nucleon and the photo-meson production by the Tamm-Dancoff method for the case of the pseudo-scalar meson with the pseudo-vector coupling: the nucleon was treated in the non-relativistic approximation and its recoil effect was neglected entirely. In this case, the difficulty of the self-energy problem remains, its renormalization being very hard. We added the different counter self-energy terms to every configuration and determined their magnitudes in such a way that the eigen-value problem gives the renormalized energy. Then we solved again the scattering problem; one-meson being present when the interaction is switched off. The state functional obtained in this way was used to evaluate the electromagnetic interaction, which was treated in the first Born approximation. The numerical evaluation is now being performed by S. Okubo, S. Matsuyama *et al.* and the result will be discussed.

On the Scattering Problem in the Intermediate Strength of Coupling *(Comment)*

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Recent theoretical research on the scattering problem, in the case of the intermediate strength of coupling, has been done mainly by Tomonaga's intermediate coupling theory and the Tamm-Dancoff approximation. These two theories are formally very convenient in order to understand the physical meaning of the evaluation, but there are some difficulties in their mathematical treatment. On the other hand, the method of the canonical transformation proposed by the author is very convenient to treat, though the formal proof that it could be used in the case of the intermediate strength of coupling has not been done. A discussion will be given here for the applicability of this method.

For a comparison, we took the same model as discussed by S. Tomonaga in the intermediate coupling theory (charged vector longitudinal meson). The interaction

Hamiltonian is

$$H=-\frac{f}{\kappa}\int\frac{k}{\sqrt{K}}dk\left\{(a_k^*+b_k)\tau_{NP}+(a_k+b_k^*)\tau_{PN}\right\}.$$

Performing the canonical transformation

$$\Psi=e^{-\lambda G}\Psi_1; \qquad [G,H_0]_-=-H,$$

we have the transformed Hamiltonian as follows:

$$H_0+\xi(\lambda)H+\eta(\lambda)[G,H]_-+\eta_G(\lambda)K_{00}GG+\cdots\cdots.$$

The parameter λ is introduced, since the original form of the interaction reappeares in the higher order terms. ξ's are expressed by convergent series for any value of λ, for example,

$$\xi(\lambda)=\lim_{n\to\infty}\xi_n(\lambda)$$

$$\xi_n(\lambda)=(1-\lambda)+(-2V_2)\left(\frac{\lambda^2}{2!}-\frac{\lambda^3}{3!}\right)+\cdots\cdots+(-1)^nV_2^n2^{2n-1}n!\left(\frac{\lambda^{2n}}{(2n)!}-\frac{\lambda^{2n+1}}{(2n+1)!}\right),$$

where $V_2=\left(\frac{f}{\kappa}\right)^2\int\frac{k^2}{K^3}dk$ is the measure of the coupling strength. The series $\xi_n(\lambda)$ converges very strongly, and in fact it becomes stationary at about $\xi_7(\lambda)$ for $V_2=1$, $\lambda=1$. The primary interaction can be eliminated completely by taking $\xi(\lambda)=0$. The 2nd order canonical transformation can be obtained by putting $\lambda=1$ from the beginning, and so the term $-\frac{2}{3}V_2H$ reappeares in the above expression, which has just the opposite sign and comparable magnitude as the original one for $V_2=1$, but the condition $\xi(\lambda)=0$ can eliminate completely the primary interaction. If the τ-spin is the classical quantity, there leaves no interaction by the above canonical transformation with $\lambda=1$, and the distribution of mesons thus obtained is the Poisson one. But owing to the quantum effect of the τ-spin, the correlation occurs in the meson cloud, giving rise to a change in the distributions given by $\lambda=1$. As the value of λ we have:

V_2	1	$1/2$	$1/3$
λ	0.728	0.813	0.856

being evaluated from $\xi_{10}(\lambda)=0$. These λ-values give almost the same distributions of the meson and dissociation probability as those given by S. Tomonaga using Ritz's variational method.

Further approximation can be obtained quite easily by going on to the Tamm-Dancoff treatment of the resulting interaction Hamiltonian. For the purpose of comparison, we write the equation in the one-meson configuration, using Ω_{10} and α defined by S. Tomonaga,

$$(K-E)\Phi(k)=(2\alpha-\Omega_{10}-1)\varphi_0(k)\int\varphi_0(k')\Phi(k')dk'$$
$$+(1-\alpha)\left\{\varphi_0(k)\int\varphi_0(k')K'\Phi(k')dk'+\varphi_0(k)K\int\varphi_0(k')\Phi(k')dk'\right\}.$$

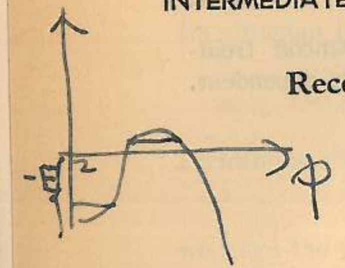
When the α and Ω_{10} are evaluated for the scattering by the Tamm-Dancoff treatment restricting the meson-number to two, they turn out to be energy dependent, but for the comparison we put $E=0$; then

	$V_2=1$	$V_2=1/2$	$V_2=1/3$
$\alpha(0)$	0.791	0.861	0.892
α	0.724	0.835	0.869
$\Omega_{10}(0)$	0.668	0.762	0.856
Ω_{10}	0.611	0.738	0.800

The values of Ω_{10} and α in the intermediate coupling theory were evatuated by Z. Maki and M. Sato.

We have compared the intermediate coupling treatment by S. Tomonaga with our canonical transformation method up to the strength $V_2=1$, and obtained nearly the same results. The latter method is favorable for the inclusion of recoil effect, and we are now performing the evaluation in pseudo-vector symmetrical coupling theory. The characteristic features of these treatments are that no renormalization is performed and all quantities are evaluated numerically by the cut-off method.

$\sim(E-H)^2+g^2\phi^2 \qquad H=\frac{1}{2}\pi^2+\frac{1}{2}\omega^2\phi^2 + V$?



Recoil Corrections in Strong Coupling Meson Theory

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The essence of the "strong coupling method" is the attempt to reduce the problem to that of small vibrations of the field oscillators about certain equilibrium positions. This has been successful in various meson theories, in a static approximation. In the pseudoscalar theory with pseudoscalar coupling, the usual way to arrive at a static approximation is via a Dyson or Foldy transformation. This procedure, however, is cumbersome because of the complicated structure of some the terms contained in the transformed Hamiltonians.

A more favourable starting point, for the one-nucleon problem, is the equation

$$\left[\gamma_\nu \frac{\partial}{\partial x_\nu} + m + ig\gamma_5\tau_\alpha\phi_\alpha(x)\right]\psi = 0, \tag{1}$$

$E^2+\frac{1}{2}\pi^2+\frac{1}{2}\omega^2\phi^2 + g^2\phi^2 + \cdots$

or the resulting second order equation

$$\left[\frac{-\partial^2}{\partial x_\nu^2} + m^2 + g^2\phi_\alpha^2(x) + ig\gamma_5\gamma_\nu\tau_\alpha \frac{\partial\phi_\alpha(x)}{\partial x_\nu}\right]\psi = 0. \tag{2}$$

It the field operator $\phi_\alpha(x)=\phi_\alpha(\vec{x}, t)$ is interpreted according to the old-style quantum theory of fields, this theory still requires vacuum subtractions. But, alternatively, Eq. (1) may be interpreted as the equation of Goldberger and Gell-Mann, which is derived from Feynman's theory[1] and therefore disposes of all vacuum effects automatically. Interpreted in this way, $\phi_\alpha(x)$ operates on a four dimensional set of meson states. It is also possible to introduce a relativistic "cut-off" factor ϕ_α.

In either interpretation, a static approximation is readily defined, either by splitting Eq. (1) into two-component equations, or, more immediately, by neglecting certain terms (*e.g.* $\gamma_5\gamma_4$) in Eq. (2). In this static limit, the mesons in s and p states are uncoupled, and the s mesons behave according to (pseudo-) scalar pair theory, p mesons according to pseudovector-coupling Yukawa theory. The well-known strong-coupling methods can be applied, and it is also easy to go beyond the static approximation by treating the effects of nucleon recoils as small (non-relativistic) corrections. These cause meson-nucleon scattering to occur also in d states, with a characteristic spin and isotopic spin dependence. The s scattering does not become strongly dependent on the isotopic spin (contrary to observation).

Of course, the strong coupling approach requires a certain minium value of g^2, depending on the cut-off. On the other hand, if g^2 becomes too large, the relativistic corrections cause a breakdown of the system in that certain normal modes of the small vibrations become unstable. Only for large values of the

1) Neglecting closed loop diagrams.

nucleon to meson mass ratio are the two conditions well compatible. Actually, with the mass ratio 6.7, a rather strong cut-off (at less than half the nucleon mass) becomes necessary. This presumably indicates that the strong coupling results cannot be trusted quantitatively.

Questions of a more basic nature remain open. The small vibrations, even if they are stable in the sense of classical mechanics, are prone to decay by passage through a potential barrier, leading apparently to an entirely unstable situation (perhaps describable as "creation of infinitely many mesons"). This is true even for weaker coupling.[2] It is conceivable that an apropriate relativistic cut-off (applied to both meson and nucleon momenta) stabilizes the system. But without such a tool, the existence of physically meaningful relativistic solutions seems highly doubtful, except in the trivial case of infinitesimally weak coupling.

2) Methods which cut out states containing more than 2 (or n) mesons are apt to conceal this difficulty.

Fundamental Formalism of Intermediate Coupling Theory

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Thirteen years ago the author proposed an approximation method in the meson theory suitable to the nuclear interaction of intermediate strength. The theory was applied only to static problems. Now we extend the theory to scattering problems. Similar attempts have been done by Watson and Hart and also by Lee and Christian, but our way of approach seems more to correspond to the proper sense of the intermediate coupling theory. The difference of our method from theirs will be seen in what follows.

As an illustration we take the vector longitudinal charged meson interacting with an infinitely heavy nucleon. The Hamiltonian for this system is given by

$$H=\int K\{a^*(k)a(k)+b^*(k)b(k)\}dk - \frac{f}{\kappa}\int\frac{k}{\sqrt{K}}[\{a(k)+b^*(k)\}Q+\{a^*(k)+b(k)\}Q^*]dk, \tag{1}$$

where $a(k)$ and $b(k)$ represent the destruction operators of positive and negative mesons with momentum $\mathbf{k}$, respectively, κ the meson mass in unit of $\hbar=c=1$, and f the coupling constant. Other quantities are defined as

$$k=|\mathbf{k}|,\quad K=\sqrt{k^2+\kappa^2},\quad Q=\frac{1}{2}(\tau_1+i\tau_2).$$

For our purpose $a(k)$ and $b(k)$ are decomposed into two parts, one describing the cloud part of mesons (o-mesons) that tend to zero at infinite distances from the fixed nucleon and the other the scattering part (s-meson) (Z. Maki, M. Satō and S. Tomonaga). The cloud part should consist in such a part of the Hamiltonian that does not contain $\boldsymbol{k}$ explicitely. This can be done by taking $\varphi_0(k)$ proportional to the k-dependent coefficient in the last term of (1) in the same way as in my previous work,

$$\varphi_0(\boldsymbol{k}) = k/K^{3/2}K_2 \quad \text{with} \quad K_2{}^2 = \int dk\,(k^2/K^3). \tag{2}$$

A divergent quantity K_2 is introduced as a normalization factor and may be evaluated by the cut-off method.

Taking $\varphi_0(k)$ as a member of a complete orthonormal set, $a(k)$ and $b(k)$ are expanded as

$$\left.\begin{aligned} a(\boldsymbol{k}) &= A_0\varphi_0(\boldsymbol{k}) + \int A_s\varphi_s(\boldsymbol{k})\,ds\,, \\ b(\boldsymbol{k}) &= B_0\varphi_0(\boldsymbol{k}) + \int B_s\varphi_s(\boldsymbol{k})\,ds\,. \end{aligned}\right\} \tag{3}$$

$\varphi_s(k)$ consist in an orthonormal set with continuous eigen values and describe waves going to infinity. A's and B's and their complex conjugates satisfy the following commutation relations:

$$\left.\begin{aligned} [A_0, A_0{}^*] &= [B_0, B_0{}^*] = 1, \\ [A_s, A_{s'}{}^*] &= [B_s, B_{s'}{}^*] = \delta(s-s')\,. \end{aligned}\right\} \tag{4}$$

Introducing (3) into (1) the total Hamiltonian is reduced to three parts, each corresponding to o-mesons, s-mesons and the interaction between them. In this reduced form the s-meson part is not diagonal yet but may be diagonalized with respect to φ_s by their appropriate choice as

$$\int \varphi_s{}^*(\boldsymbol{k})K\varphi_{s'}(\boldsymbol{k})\,dk \equiv S\delta(s-s')\,. \tag{5}$$

This choice of φ_s makes it unnecessary to refer to individual φ_s, so that s-mesons may be described by a single operator as (H. Hasegawa and S. Hayakawa)

$$\left.\begin{aligned} a(\boldsymbol{k}) &= A_0\varphi_0(\boldsymbol{k}) + A(\boldsymbol{k}), \\ b(\boldsymbol{k}) &= B_0\varphi_0(\boldsymbol{k}) + B(\boldsymbol{k})\,. \end{aligned}\right\} \tag{6}$$

$A(k)$ and $B(k)$ are the destruction operators of positive and negative s-mesons and obey the commutation relations

$$\begin{aligned} [A(\boldsymbol{k}), A^*(\boldsymbol{k}')] &= [B(\boldsymbol{k}), B^*(\boldsymbol{k}')] \\ &= \delta(\boldsymbol{k}-\boldsymbol{k}') - \varphi_0(\boldsymbol{k})\varphi_0(\boldsymbol{k}') \equiv \delta'(\boldsymbol{k}-\boldsymbol{k}')\,. \end{aligned} \tag{7}$$

On account of (6) (1) is reduced to

$$H = H_0 + H_s + H_{0s} \tag{8}$$

with

$$\begin{aligned} H_0 &= K_0[A_0{}^*A_0 + B_0{}^*B_0 - V\{(A_0+B_0{}^*)Q + (A_0{}^*+B_0)Q^*\}] \\ &\equiv K_0\Omega\,, \end{aligned} \tag{8a}$$

$$H_s = \int K[A^*(\boldsymbol{k})A(\boldsymbol{k}) + B^*(\boldsymbol{k})B(\boldsymbol{k})]\,d\boldsymbol{k}\,, \tag{8b}$$

$$\begin{aligned} H_{0s} = &\int \varphi_0{}^*(\boldsymbol{k})K[(A_0{}^*-VQ)A(\boldsymbol{k}) + (B_0{}^*-VQ^*)B(\boldsymbol{k})]\,d\boldsymbol{k} \\ &+ \int [A^*(\boldsymbol{k})(A_0-VQ^*) + B^*(\boldsymbol{k})(B_0-VQ)]K\varphi_0(\boldsymbol{k})\,d\boldsymbol{k}\,. \end{aligned} \tag{8c}$$

$$\left(K_0 = \int \varphi_0{}^*(\boldsymbol{k})K\varphi_0(\boldsymbol{k})\,d\boldsymbol{k}, \quad V = fK_2/\kappa\right)$$

In my previous theory only the cloud part H_0 was taken into consideration and the static solution was obtained by means of the variational method. The solution may be obtained by any mean as

$$(\Omega - \Omega_\nu)\chi_\nu = 0\,. \tag{9}$$

With the aid of a set of eigen functions χ_ν, the state vector of the total system is expanded as

$$\Psi = \sum_\nu \Phi_\nu \chi_\nu\,. \tag{10}$$

Φ_ν is the probability amplitude that the cloud state is ν and there are any number of s-mesons.

The essential point of our intermediate coupling theory is that no restriction should not be imposed on the number of o-mesons but the s-meson part may be solved approximately. The o-meson part was solved by means of the variation method with the trial function of the Hartree approximation. Then the amplitude for the no s-meson configuration (proton state) was given by

$$\begin{aligned} \Phi^{(0)} = &\sum_n \frac{c_n{}^{(0)}}{\sqrt{n!\,n!}} \sum_{ki} f_1{}^+(\boldsymbol{k}_1{}^+)\cdots f_n{}^+(\boldsymbol{k}_n{}^+) f_1{}^-(\boldsymbol{k}_1{}^-)\cdots f_n{}^-(\boldsymbol{k}_n{}^-)\,|p, \boldsymbol{k}_1{}^+\cdots\boldsymbol{k}_n{}^+, \boldsymbol{k}_1{}^-\cdots\boldsymbol{k}_n{}^-> \\ &+ \sum_n \frac{d_n{}^{(0)}}{\sqrt{n!\,(n+1)!}} \sum_{ki} f_1{}^+(\boldsymbol{k}_1{}^+)\cdots f_{n+1}{}^+(\boldsymbol{k}_{n+1}{}^+) f_1{}^-(\boldsymbol{k}_1{}^-)\cdots f_n{}^-(\boldsymbol{k}_n{}^-) \\ &\quad |n, \boldsymbol{k}_1{}^+\cdots\boldsymbol{k}_{n+1}{}^+, \boldsymbol{k}_1{}^-\cdots\boldsymbol{k}_n{}^->, \end{aligned} \tag{11}$$

where

$$f_1{}^+(\boldsymbol{k}) = \cdots = f_n{}^-(\boldsymbol{k}) = \varphi_0(\boldsymbol{k})\,. \tag{12}$$

When there is a pnsitive s-meson with momentum $\boldsymbol{k}_0$, the amplitude of this configuration is given, corresponding to (11), by (H. Hasegawa and M. Nogami)

$$\begin{aligned} \Phi^{(1)} = &\sum_n \frac{c_n{}^{(1)}}{\sqrt{n!n!}} \sum_{ki} f_1{}^+\cdots f_n{}^-\delta'_{kk_0}\,|p, \boldsymbol{k}_1{}^+\cdots\boldsymbol{k}_n{}^-, \boldsymbol{k}^+> \\ &+ \sum \frac{d_n{}^{(1)}}{\sqrt{n!\,(n+1)!}} \sum_{ki} f_1{}^+\cdots f_n{}^-\delta_{k'k_0}\,|n, \boldsymbol{k}_1{}^+\cdots\boldsymbol{k}_n{}^-, \boldsymbol{k}^+>. \end{aligned} \tag{13}$$

$c_n{}^{(1)}$ and $d_n{}^{(1)}$ can be obtained by means of the variational method together with the eigen energy of the system as was done for $\Phi^{(0)}$. The energy is expressed as

$$E = E_0 + K(k_0) + \Delta E\,, \tag{14}$$

where E_0 means the energy of a nucleon with o-mesons around it and $K(k_0)$ represents the total energy of the meson at infinity where it is free. Energy shift ΔE is shown to be expressed in terms of the phase shift $\delta(k_0)$ for the scattering (K. Sawada) as

$$\Delta E = (\delta(k_0)/R)\,(k_0/K), \tag{15}$$

where ΔE is evaluated as the s-meson is kept inside a sphere with radius R. A more refined relation between ΔE and δ makes us possible to derive the scattering amplitude that will be derived by another method in what follows.

It is important to note that no symmetrization is needed for s- and o-mesons in (13), because δ' and f are diagonal with one another. Hence $\Phi^{(0)}$ and $\Phi^{(1)}$ may be represented in Fock space as

$$\Phi_\nu = \begin{pmatrix} \Phi_\nu{}^{(1)} \\ \Phi_\nu{}^{(1)} \end{pmatrix}. \tag{16}$$

Introducing (10) and (16) into $(H-E)\Psi=0$, with H given in ($8a$–$8c$), we obtain a set of equations

$$(K_0\Omega_\nu - E)\,\Phi_\nu{}^{(0)} + \sum_{\nu'} A_{\nu\nu'} \int dk' \varphi_0(k')\, K'\, \Phi_{\nu'}{}^{(1)}(k') = 0, \tag{17a}$$

$$(K_0\Omega_\nu + K - E)\,\Phi_\nu{}^{(1)}(k) - \varphi_0(k) \int dk' K' \varphi_0(k')\, \Phi_\nu{}^{(1)}(k') + \sum_{\nu'} A_{\nu\nu'}{}^*\,(K-K_0)\,\varphi_0(k)\,\Phi^{(0)} = 0, \tag{17b}$$

restricting the number of s-mesons to one only. The operator

$$A = A_0 - VQ^*, \quad A^* = A_0 - VQ \tag{18}$$

have matrix elements corresponding to changes of cloud states accompanying the absorption and emission of a s-meson, rsepectively. It is obvious, therefore, that the last terms in (17a) and (17b) represent such processes. The second term of (17b) corresponds to a sort of potential scattering due to the artificial separation of meson operators in (6).

If further approximation is made that only one excited cloud state is effective, (17a) and (17b) are readily solved as

$$\Phi_0{}^{(1)}(k) = \delta(k-k_0) + \left(\frac{1}{E_0-K} - i\pi\delta\,(E_0-K)\right) T(E_0, K)\,\varphi_0(k)\,\varphi_0(k_0), \tag{19}$$

where

$$T(E_0, K) = \frac{E_0 F(E_0, K)}{(E_0 - K_0\Omega_{10}) - \int dk' K' \varphi^2(k')\, F(E_0, K') \left(\frac{1}{E_0-K'} - i\pi\delta\,(E_0-K')\right)} \tag{20}$$

with

$$E_0 = E - K_0\Omega_0, \quad \Omega_{10} = \Omega_1 - \Omega_0$$

and

$$F(E_0, K) = (K_0\Omega_{10} - E_0) + A_{01}{}^* A_{10}\,(K-K_0).$$

From this result it is easily seen that terms with A and A^* in (17) give rise to a resoance scattering and a remaining term results in $K_0\Omega_{10}-E_0$ in $F(E_0, K)$, contributing to the potential scattering. For $V=0$ these two kinds of scattering cancel with one another and T vanishes, as it should be.

Thus our problem is reduced to obtain the matrix elements $A_{\nu\nu'}$, etc. This is

now carried out by solving cloud levels numerically. The levels thus obtained are shown in Fig. 1 for a number of V-values. Since we know Ω_ν as above, the matrix elements are given by

$$A_{\nu'\nu} = -\frac{V(\Omega_{\nu'} - \Omega_\nu)}{1 + (\Omega_{\nu'} - \Omega_\nu)}\, Q^*{}_{\nu'\nu}, \tag{21}$$

on account of the following relation

$$A_0\Omega - \Omega A_0 = A_0 - VQ^*.$$

Some of the numerical values are given in Table 1.

Now we are in position to discuss the relations of our method to other approximations.

Perturbation method. It is easy to see that

$\varphi_0(k)$
$= H'/$(energy denominator).

Thus our method already uses a part of solutions of the perturbation method.

Sawada approximation. Sawada invented to treat the pion-nucleon scattering by deriving the second order effective potential for pion nucleon interactions. This is shown to be equivalent to solve the cloud part by the second order perturbation.

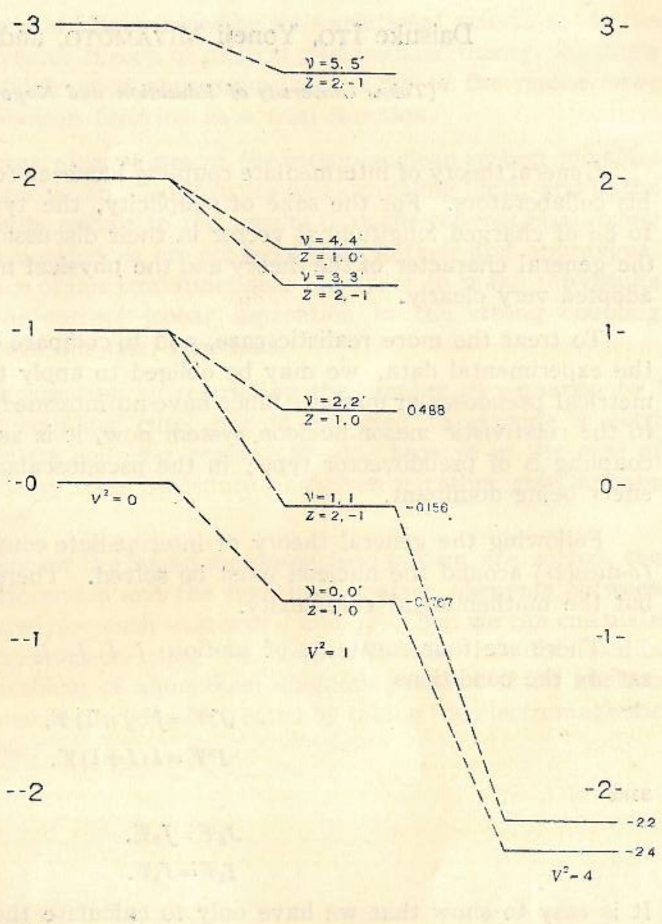

Fig. 1. Energy levels of cloud parts Ω_ν.

Tamm-Dancoff approximation. In the T–D approximation the probability amplitude is represented in Fock space and up to two mesons are taken into consideration irrespective to whether they belong to cloud or s-part. We can show that this is the case by introducing (3) into (8) and restricting the number of both o- and s-mesons to two.

Stroug coupliug approximation. This will be discussed in the following abstract.

Table 1.

$(1\vert A^*\vert 0) = 0.724$	$(2'\vert B^*\vert 0) = -1.02$
$(3\vert A^*\vert 0) = -0.293$	$(2'\vert A\vert 0) = -0.10$
$(5\vert A^*\vert 0) = -0.298$	$(1\vert B\vert 0) = -0.17$

Numerical values of matrix elements given in (21).

Application to Meson-Nucleon Scattering (*Comment*)

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General theory of intermediate coupling has been formulated by S. Tomonaga and his collaborators. For the sake of simplicity, the type of the meson was assumed to be of charged longitudinal vector in their discussions. Their example illustrates the general character of the theory and the physical meaning of the approximations adopted very clearly.

To treat the more realistic case, and to compare the theoretical predictions with the experimental data, we may be obliged to apply the theory to the case of symmetrical pseudoscalar meson. Since have no intermediate coupling theory applicable to the relativistic meson-nucleon system now, it is assumed that the meson-nucleon coupling is of pseudovector type; in the pseudoscalar coupling case the relativistic effect being dominant.

Following the general theory of intermediate coupling, the state of meson cloud (o-meson) around the nucleon must be solved. There is no fundamental difficulty but the mathematical complexity.

There are four constants of motion J, I, J_3, I_3. The wave function Ψ must satisfy the conditions

$$J^2\Psi = J(J+1)\Psi,\quad I^2\Psi = I(I+1)\Psi, \tag{1}$$

and

$$J_3\Psi = J_3\Psi,\quad I_3\Psi = I_3\Psi. \tag{2}$$

It is easy to show that we have only to calculate the state for which $J_3 = J$ and $I_3 = I$ (we call these states "top" states); other states being constructed by operating the angular momentum operator $J^{(-)}$ and $I^{(-)}$ repeatedly. For the top states we may impose the conditions

$$J^{(+)}\Psi_{top} = I^{(+)}\Psi_{top} = 0, \tag{3}$$

instead of (1). In the following we shall consider the conditions (2) and (3) instead of (1) and (2). This procedure simplifies the elimination of the angular part of the meson field variables considerably. This is equivalent to Tomonaga-Miyazima's procdure and is more convenient than it. For that purpose the method using the generating function will also be considered.

After the elimination is performed, Schrödinger equation can be solved by employing any suitable approximation methods, e. g. using variational procedure or restricting the number of o-mesons.

In the variational method, the accuracy depends primarily upon the choice of trial functions. But, tentatively, the square root of the Poisson's distribution function

$$\psi(n) = exp\left(-\frac{1}{2}\alpha^2\right)\frac{\alpha^n}{(n!)^{1/2}} \tag{4}$$

is chosen and its parameter α's are determined by the variational principle. In the case of charged longitudinal vector theory or neutral pseudoscalar theory, Poisson's distribution of o-mesons is a fairly good approximations. This is the reason why we choose the Poisson's distribution function as a trial function.

For the ground state, energy eigenvalues of the meson-nucleon system coincides with the results from the perturbation theory in its weak coupling limit, and in the strong coupling limit, our results reduce to the results from the conventional strong coupling theory. But these coincidences are achieved only in their first approximation, because of the crude choice of our trial functions. In order to make the results so accurate as to reproduce the correct isobar separation in the strong coupling limit, we would have to improve our trial functions.

To solve the Schröedinger equation by restricting the number of o-mesons to 2 or 3, is invalid in the strong coupling case, but is expected to provide a good approximation in the intermediate coupling region ($V^2 \sim 1$), because in the case of charged longitudinal vector theory, this procedure has given a rather good answer for the low lying isobaric levels.

If the Shrödinger equations for the bound meson-nucleon system are solved, the excitation energies of isobaric states and the transition matrix elements between these states are easily computed for each values of I and J. Then we can calculate the scattering of meson by the nucleon using the same method as that discussed in the preceding lecture. The problem of anomalous magnetic moment of the nucleon and photoproduction of the meson can also be treated by taking the electromagnetic interactions as the perturbation.

Strong Coupling Limit of Intermediete Coupling Theory (*Comment*)

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The present author[1] has formulated a new approximate method to treat the pion-nucleon scattering when the coupling is of the intermediate strength. According to this method, we shall now discuss the pion-nucleon scattering under the assumption of the strong coupling. As was shown by Z. Maki and M. Sato the results obtained, agree with those derived by Wentzel[2] and S. Tomonaga[3]. Our method, however, will make clear not only the underlying ideas of the usual strong coupling theory from the different standpoints, but also how the approximations, which are applicable to the case of intermediate coupling theory, fail in strong coupling limit.

As an illustration, we take the longitudinal charged vector meson interacting with one heavy nucleon. Then, assuming the wave function of the self-field around the nucleon as

$$\Phi=\begin{pmatrix}\varphi(x,y)\\ \psi(x,y)\end{pmatrix}=\frac{1}{\sqrt{2\pi}\sqrt{r}}\begin{pmatrix}f(r)e^{i(m-1/2)\theta}\\ g(r)e^{i(m+1/2)\theta}\end{pmatrix}, \tag{1}$$

we obtain the following simultaneous differential equations for the radial functions $f(r)$ and $g(r)$,

$$\left.\begin{aligned}&\left\{-\frac{d^2}{dr^2}+\frac{m(m-1)}{r^2}+r^2-2(\Omega_0+1)\right\}f(r)-2Vrg(r)=0\\ &\left\{-\frac{d^2}{dr^2}+\frac{m(m+1)}{r^2}+r^2-2(\Omega_0+1)\right\}g(r)-2Vrf(r)=0\end{aligned}\right\}. \tag{2}$$

The energy levels given by (1.2) are roughly classified into three groups. The first group of the states is lying at $\Omega_0=-V^2/2$, and we shall call it 'low level' group. The second group designated by the 'high level' group is situated above the low level group by the amount $2V^2$ and the third in the intermediate region of the above two. If we assume that the coupling is strong, i.e., $V\gg 1$, it is easily shown that only the low and the high energy levels are important. From now on, we, therefore, neglect the terms of the order $1/V$. Then we obtain

$$\begin{aligned}&f(r)=g(r)=(1/\sqrt{2})H_n(r-V)\exp\{-(1/2)(r-V)^2\},\\ &\Omega_0=-(1/2)V^2+n-(1/2)+m^2/2V^2,\end{aligned} \tag{3}$$

in low levels and

$$f(r)=-g(r),\qquad \Omega_0=3V^2/2,$$

in high levels. H_n stands for the normalized hermitian function. Now, it is convenient to divide A, Q^* etc. into two parts in the representation, in which Ω is diagonal, as

$$A=\bar{A}+A',\qquad Q^*=Q^*+Q^{*\prime}\qquad \text{etc.} \tag{4}$$

$\bar{A}$, $\bar{Q}^*$ etc. have non-vanishing matrix elements only for the transitions among high levels or among low levels, while A', Q'^* etc. have non-vanishing matrix elements only for the transitions between low and high levels. Then, by putting $r=\bar{r}+V$, A' and $\bar{A}$ etc. are written as

$$A'=-VQ^{*\prime},\quad \bar{A}=\frac{e^{-i\theta}}{2}\left(\bar{r}+\frac{\partial}{\partial\bar{r}}\right)-\frac{e^{-i\theta}}{2}\frac{1}{\bar{r}+V}\frac{\partial}{\partial\theta}+V\left(\frac{e^{-i\theta}}{2}-\bar{Q}^*\right). \tag{5}$$

We now consider that both the initial and the final states in the scattering process are in the low levels, and that we express the cross section using the non-vanishing terms in the strong coupling limit, $1/V\to 0$. It is easily deduced from a simple consideration that the necessary matrix elements are the types of matrix elements $(l'\,|\,\bar{A}\,|\,l)$ and $\sum_{h'}(l'\,|\,A'\,|\,h')(h'\,|\,A'^*\,|\,l')$. Substituting $\bar{A}+A'$, $\bar{Q}^*+Q^{*\prime}$ for A, Q^* in

the hamiltonian, it is found that the transitions of bound parts from high to low levels or the inverse transitions accompanied by the emission or the absorption of s-mesons can be caused only by the term H' which contains the large coefficient V. That is, H is rewritten as

$$\left.\begin{aligned}&H=K_0\Omega+H_0+H_s+H',\qquad \Omega=A^*A+B^*B-V^2\\ &H_0=\int K_{0s}(\bar{A}^*a_s+\bar{B}^*b_s)\,ds+c.c.,\quad H_s=\int S(a_s^*a_s+b_s^*b_s)\,ds\\ &H'=-V\int K_{0s}[Q'(a_s+b_s^*)+Q'^*(a_s^*+b_s)]ds\end{aligned}\right\}. \tag{6}$$

Accordingly, if we could eliminate the term H' by performing a canonical transformation and further the transformed hamiltonian could be rewritten in a closed form in the approximation in which we neglect the order $1/V$, the scattering cross section could be calculated without taking the high levels into account. Now, such a transformation is actually possible and the transformation function is given by

$$\bar{H}=\exp(G)\cdot H\cdot\exp(-G)$$

$$G=-\frac{Q_1'-Q_2'}{2VK_0}\int K_{0s}(a_s+b_s^*)\,ds+\frac{Q_1'^*-Q_2'^*}{2VK_0}\int K_{0s}(a_s^*+b_s)\,ds, \tag{7}$$

where

$$Q'=Q_1'+Q_2',\qquad (l|Q_1'|h)\neq 0,\qquad (h|Q_2'|l)\neq 0,$$
$$(h|Q_1'|l)=(l|Q_2'|h)=0.$$

Next carrying out the following variable transformations,

$$\sqrt{2}\exp(i\theta)\cdot\bar{A}=\sqrt{2}\exp(-i\theta)\cdot\bar{B}\equiv\alpha,\ \sqrt{2}\exp(-i\theta)\bar{A}^*=\sqrt{2}\exp(i\theta)\bar{B}^*=\alpha^* \tag{8}$$

$$a_s=(1/\sqrt{2})\exp(-i\theta)\cdot(x_s-iy_s),\quad b_s=(1/\sqrt{2})\exp(i\theta)\cdot(x_s+iy_s) \tag{9}$$

and

$$\left.\begin{aligned}&x(k)=\alpha\varphi_0(k)+\int x_s\varphi_s(k)\,ds,\quad x^*(k)=\alpha^*\varphi_0(k)+\int x_s^*\varphi_s(k)\,ds\\ &y(k)=\int y_s\varphi_s(k)\,ds,\quad y^*(k)=\int y_s^*\varphi_s(k)\,ds,\end{aligned}\right\} \tag{10}$$

we can write $\bar{H}$ as $\bar{H}=H_x+H_y$, where

$$H_x=\int Kx^*(k)x(k)\,dk,\quad [x(k),x^*(k')]=\delta(k-k').$$

This shows that the x-meson is not scattered by the nucleon. We assume now that the incident meson is a positive one. This state of positive charge can be regarded as a superposition of the state of x-meson and that of y-meson with an equal weight. Then we see that just a half of the incident mesons will be scattered and that only a half of the scattered mesons are positive. As for H_y, we obtain

$$H_y=\frac{1}{2}\int(p^2(k)+K^2q^2(k))\,dk-\frac{1}{2K_0}\left(\int\varphi_0(k)K\sqrt{K}q(k)\,dk\right)^2 \tag{11}$$

by the variable transformations,

$$p(k)=(1/i)\sqrt{K/2}(y(k)-y^*(k)),\quad q(k)=1/\sqrt{2K}(y(k)+y^*(k)). \tag{12}$$

From (11), we have only to find out the normal modes of vibration in the form

$$\psi(\boldsymbol{k})=\delta(\boldsymbol{k}-\boldsymbol{p})-\left\{\frac{1}{K-E}+i\pi\delta(K-E)\right\}R(\boldsymbol{k},\boldsymbol{p}) \tag{13}$$

to get the scattering cross section. The scattering amplitude $R(p,p)\ (|\boldsymbol{p}|=|\boldsymbol{p}'|=p)$ is given by

$$R(p,p)=\varphi_0^2(p)\Big/\left[\int\frac{\varphi_0^2(k)}{K+E}kd+\int\frac{\varphi_0^2(k)}{K-E}dk+i\pi\int\delta(K-E)\varphi_0^2(k)\,dk\right] \tag{14}$$

and agrees with the old result.[3]

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Lattice-Space Quantization of a Nonlinear Field Theory *(Comment)*

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A method for the approximate diagonalization of certain types of quantum field hamiltonians is developed which is not limited to weakly nonlinear systems. It consists in omitting the gradient terms in zero order, ane diagonalizing the resulting hamiltonian by replacing the field defined in a continuum syace by a field defined in a lattice syace. This unperturbed system is equivalent to a countable infinite number of uncoupled nonlinear oscillators, which are then coupled together when the gradient terms are included as a perturbation. The method is applied to the quantization of the classical nonlinear meson theory that was introduced in an earlier paper to provide a qualitative explanation of the saturation of nuclear forces, according to which a positive ϕ^4 term is added to the field hamiltonian. Although the quantized theory is manifestly noncovariant, it is found that a single-particle solution exists that has an aqproximately relativistic relation between energy, momentum and rest mass. It turns out to be essential that the lattice constant be kept finite, as all computed physical quantities become meaning ess in the continuum limit (in which the lattice constant approaches zero). It is shown that these particles obey Einstein-Bose statistics, and that they scatter from each other. Nucleons are introduced as classical sources for the meson field, and calculations are made on the nucleon isobaric state, interaction of mesons with nucleons and heavy nuclei, and nucleon-nucleon interaction. Most of the results of the earlier classical theory have close counterparts in the present quantized theory. The possibility of extending the method to the quantixation of both meson and nucleon fields when they are strongly coupled together is discussed briefly.

See the abstract of K. Sawada on P. 41.

p-p forces Jastrow
polarization : $\sigma = \sigma_0\,[1+\epsilon\cos\phi_2]$
~~Jastrow~~ 25%

Recent Works in United States

singular tensor force.

Robert E. MARSHAK

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core only for S-state

(Abstract not yet received, Aug. 20, 1953)

p-d : 50% polarization

Peierls
n-p scattering
135 MeV
Harwell
more backward scattering

General Opinions

Gregory BREIT

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(Abstract not yet received, Aug. 20, 1953)

Studies on Nuclear Forces in Japan

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A new method for investigating the nuclear force was proposed in 1950~1951 by M. Taketani, S. Nakamura, and M. Sasaki. The idea is as follows: The high singularities occurring near the origin of the potential, which was derived from the meson theory, gave so essential effects to the numerical results that we had no means so far to compare it with the non-singular phenomenological potentials such as square-well, exponential-well, and Yukawa-type ones. We suppose, however, that these singularities would have no physical meaning at least inside the nucleon Compton wave length, since the idea of the adiabatic potential will lose its validity in case the two nucleons approach so close together; very complicated circumstances might occur such as recoil effects, creation of virtual nucleon pairs and the effects of heavy mesons, some of which would be related to the self-field of the nucleon to be renormalized properly. These effects may have considerable influences on the main features of the nuclear forces far beyond the nucleon Compton wave length. In view of the above considerations, the method proposed by us is to use the theoretical nuclear potential near and on the outside of the range of the nuclear force and to cut off this potential inside about the one-half of the range, replacing it in this region by other phenomenological parameters (e. g. square-well). As to these parameters, we consider them to be dependent on the relative velocity of the nucleons or other state variables; retardation, recoil and other effects are expected to be included together in these parameters.

In this way, the theoretical adiabatic potential will have its own meaning as potential up to the energy of 100 Mev and we are allowed to compare the theoretical results with experimental data in this energy region. This was performed in 1951 by M. Taketani, S. Machida, and S. Onuma. It was at that time that we found the fact that the 4th order potential is more effective than the 2nd order one near the force range (Machida Effect), and that the pseudo-scalar meson theory with pseudo-vector coupling fits rather well with the low energy experimental data.

It will be natural to ask whether the above situation is characteristic only for the 4th order potential, or the effect becomes larger and larger if one goes over to the 6th, the 8th . . . orders and give the divergent series. This problem was treated by N. Fukuda, H. Fukuda and K. Sawada in the general form and the conclusion was obtained that the higher order terms do not diverge. Also S. Machida and K. Senba evaluated the 6th order nuclar force and found that the 6th order potential do not change the main features of the 4th order terms. Concerning the difference between pseudo-scalar coupling and pseudo-vector coupling, K. Sawada showed that the core term appearing in the pseudo-scalar coupling is strongly damped by the inertia effect, hence the 4th order term being nearly the same as that of pseudo-vector coupling. It has also become clear that the non-adiabalic potential is not so different from the adiabatic one, so far as the scattering of the nucleon up to the energy of 100 Mev is concerned.

From these considerations, we reached the conclusion that the potential which we have obtained from the 2nd and 4th order adiabatic treatment can be regarded as the standard of the theoretical nuclear potential. Using this potential, S. Otuki, W. Watari and S. Onuma performed the calculation of neutron-proton scattering at 100 Mev. Their results show that this potential has its own characteristic features comparing to other potentials used so far; for example, they give a little smaller total cross-sections compared with the experimental data, contrary to the slightly larger cross-sections predicted by the usual potentials. Owing to these characteristic aspects, this potential is very important as a standard of the theoretical nuclear force. It is true of cource that this potential is not sufficient to understand all the experimental data; but it is very probable that we may compare the theoretical potential with the experimental data by adding some small corrections to this potential.

Another very important effect concerning the nuclear force is to answer the

ps. - pv.
ps. - ps.

question to what extent the isobar state will affect the nuclear force; the existence of the isobaric resonance is a very probable fact from the scattering experiments of the pion and nucleon. It is to be expected that for the phenomena up to on energy 100 Mev the nuclear force only near and on the outside of the force range is effective, the virtual mesons important in these region necessarily being of low energy ones, and so the effect of the isobaric resonance would be very small. Thus, the meson potential will not suffer any appreciable change near the force range by the existence of isobaric resonance.

M. Sugawara investigated the effect of isobar by using the Rarita-Schwinger theory and introducing the spin 3/2 states and found that the 4th order potential thus derived gives a very great isobaric effect. This seems to be a very peculiar fact. T. Kikuta also estimated the effect using the square-well potential by referring to Pauli-Kusaka's strong coupling theory and found a large effect of isobar near the force range.

The evaluation of nuclear force by Tomonaga's intermediate coupling formalism has been performed by H. Hasegawa. The results will be discussed.

The recoil effect on the 4th order nuclear force was performed by I. Sato. The result showed that the recoil effect changes the adiabatic potential appreciably inside the force range. But since from the general consideration we suppose that the nuclear force near the force range will not be altered to any extent, this result seems rather peculiar; more accurate treatment is now being performed.

One of the important conclusions derived from our former investigations is that the theoretical adiabatic potentical of the pseudo-vector coupling predicts a very large D-state probability for the deuteron (9%). And it is necessary to acquire more accurate information on the D-state probability or the deuteron wave function from the experimental data. The D-state probility of 4% used up to date is not so definite. Concerning this point, S. Machida obtained the D-state probility to be of 5% to 10% from the hyperfine structure of the deuteron ground state, this result being considered very remarkable.

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ps. ps.

(Title not yet known) *(Comment)*

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(Abstract not yet received, Aug. 20, 1953)

Brueckner

Nucleon-Nucleon Scattering by the ps(pv) Meson Potential *(Comment)*

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It is necessary to make clear how far we can explain experimental results about nuclear forces assuming the present meson theory. Our purpose here is to investigate the characteristic features of the meson potential in the intermediate and high energy regions.

As the potential assumed in the outside region, i.e., we have adopted the symmetrical *ps* meson potential of the second and fourth order perturbation calculation with *pv* coupling in the static approximation calculated by M. Taketani *et al*[1]. We consider that near the range of nuclear force, the sixth and higher order contributions do not alter the features of the potential so severely. This fact has actually been shown by S. Machida for the sixth order contribution. In the inside region where the static meson potential becomes meaningless, we have adopted phenomenological potential, i.e., hard core or square well, which may well be energy dependent, but what has been adopted here fits the low energy data[1]. So these results are not the final one but serve only as a standard.

The coupling constant $g^2/4\pi$ is taken as 0.08.

Phase shifts of $n-p$ scattering at 90 Mev are shown in Table I.

${}^3\delta_3{}^2$ and ${}^3\delta_2{}^1$ are calculated using the equivalent potentials by the *W-K-B* approximation[2]. Calculations are perfomed by the numerical integration.

Table I n–p phase shifts at 90 Mev by $ps(pv)$ meson potential.

triplet state ${}^3(L)_J$	phase shift ${}^3(\delta)_J{}^L$	inside region
$J=1$ even α-wave	${}^3\delta_1{}^\alpha=\ 20°58'$	zero cut off which fits low energy data.
γ-wave	${}^3\delta_1{}^\gamma=-16°24'$	
amount of admixture	$\eta_1=0.2695$	
${}^3D_2$	${}^3\delta_2{}^2=\ 15°44'$	
${}^3D_3$	${}^3\delta_3{}^2=-\ 1°\ 0'$	
${}^3P_0$	${}^3\delta_0{}^1=\ 21°36'$	hard core cut off at $x=0.3329$.
${}^3P_1$	${}^3\delta_1{}^1=-10°40'$	
${}^3P_2$	${}^3\delta_2{}^1=\ 1°37'$	
singlet state ${}^1(L)_J$	**phase shift ${}^1(\delta)_{J=L}$**	
${}^1S_0$	${}^1\delta_0=\ 47°18'$	hard core cut off which fits low energy data.
${}^1D_2$	${}^1\delta_2=\ 2°44'$	
${}^1P_1$	${}^1\delta_1=-14°31'$	hard core cut off at $x=0.3329$.

(x are measured in the unit of the meson Compton wave length.)

Because the central force of the triplet even state is repulsive, ${}^3\delta_1{}^\alpha$ is comparatively small and ${}^3\delta_1{}^\gamma$ large in its absolute value. The singlet even state is so singular that ${}^1\delta_0$ is comparatively large whereas ${}^1\delta_2$ is comparatively small.

The total cross section is $5.7\times10^{-26}\text{cm}^2$ which is smaller than the experimental value $7.6\times10^{-26}\text{cm}^2$ whereas the total cross sections by the usual phenomenological potentials are larger by about 20～30%.[2,3]

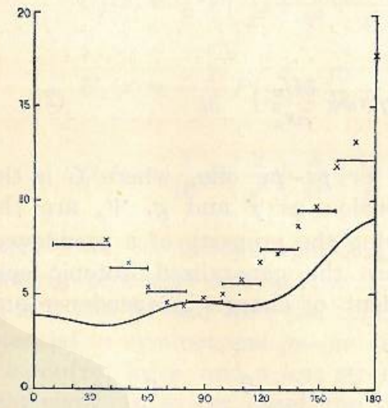

Fig. 1. $n-p$ Scattering by the $ps\ (pv)$ meson potential at 90 Mev. The crosses are the counter data; the horizontal lines are the cloud chamber data (see reference 2).

The angular distribution in the center of mass system is shown in Fig. 1. The dips at 40° and 110° are due to the interference of ${}^3\delta_1{}^\alpha$ and ${}^3\delta_1{}^\gamma$, and of ${}^3\delta_1{}^\gamma$ and ${}^3\delta_2{}^2$. The angular distribution is not so symmetric about 90° mainly because of the repulsive force in the singlet odd state.

Both these results and the sixth order potential derived by Machida make us expect that meson theory of nuclear force can explain the main features of nucleon-nucleon scattering at intermediate and high energy regions.

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Phenomenological Approach to Nucleon Isobars and the Effects upon Static Nuclear Potential *(Comment)*

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In order to treat nucleon isobars phenomenologically, we assume that the perturbation calculations are good approximations if we introduce as the results of the due accounts of higher order corrections the effective interaction Hamiltonian density which permits the virtual transitions of nucleons to or from the excited states of spin and τ-spin 3/2 accompanied with one meson emission or absorption;

$$\frac{G}{\mu}\bar{\Psi}_\mu T_\alpha \psi \frac{\partial U_\alpha}{\partial x_\mu} + \frac{G}{\mu}\bar{\psi} T_\alpha^* \Psi_\mu \frac{\partial U_\alpha}{\partial x_\mu}, \tag{1}$$

in addition to the usual one

$$f\bar{\psi} i\gamma_5 \tau_\alpha \psi U_\alpha \quad \text{or} \quad \frac{g}{\mu}\bar{\psi} i\gamma_5\gamma_\nu \tau_\alpha \psi \frac{\partial U_\alpha}{\partial x_\mu}, \tag{2}$$

corresponding to the symmetrical $ps-ps$ theory or $ps-pv$ one, where G is the effective coupling constant having the same dimension as f and g, Ψ_μ are the field quantities of isobars, $(\bar{\Psi}_\mu\psi)$ and $(\bar{\psi}\Psi_\mu)$ having the property of a pseudovector, and T_α and the hermite conjugates T_α^* are the generalized isotopic spin matrices which are determined by the requirement of charge independence and have the expressions

$$T_1=\begin{pmatrix} 1 & 0 \\ 0 & -1/\sqrt{3} \\ -1/\sqrt{3} & 0 \\ 0 & 1 \end{pmatrix},\ T_2=\begin{pmatrix} -i & 0 \\ 0 & i/\sqrt{3} \\ -i/\sqrt{3} & 0 \\ 0 & i \end{pmatrix},\ T_3=\begin{pmatrix} 0 & 0 \\ 2/\sqrt{3} & 0 \\ 0 & -2/\sqrt{3} \\ 0 & 0 \end{pmatrix}, \tag{3}$$

if ψ and Ψ_μ have the corresponding expressions

$$\psi=\begin{pmatrix} \psi_P \\ \psi_N \end{pmatrix}, \quad \Psi_\mu=\begin{pmatrix} \Psi_\mu^{++} \\ \Psi_\mu^{+} \\ \Psi_\mu^{0} \\ \Psi_\mu^{-} \end{pmatrix}. \tag{4}$$

We assume here tentatively that the Ψ_μ's are determined by exactly the same equations of motion of spin 3/2 particles as given by Rarita and Schwinger. As to the $ps-ps$ coupling term, we replace it using Dyson transformation by

$$\frac{f^2}{2\kappa\hbar c}\bar{\psi}\psi U_\alpha^2 + \frac{f}{2\kappa}\bar{\psi} i\gamma_5\gamma_\nu \tau_\alpha \psi \frac{\partial U_\alpha}{\partial x_\nu}, \tag{5}$$

which are exact up to the fourth order, where κ is the Compton wave length of a nucleon. The effective two coupling constants G and the excitation energy $\Delta E = m_\pi c^2 \Delta\varepsilon$ of isobars are determined very sharply by pion-nucleon scattering cross sections, which are obtained by the second order perturbation calculations, using the approximations of nucleons at rest. The best agreement with data is obtained if $G^2/4\pi\hbar c = 0.1 \pm 0.05$ and $\Delta\varepsilon = 2.4 \sim 2.3$, where $f^2/4\pi\hbar c = 9$ and $(f^5/4\pi\hbar c)_{pair} = 1.5$ for $ps-ps$ theory and $g^2/4\pi\hbar c = 0.07$ for $ps-pv$ theory.

The contributions of the above introduced effective interaction Hamiltonian (1) to the static nuclear potential are calculated up to the fourth order (which is the

lowest order for isobar effect) in the approximations of nucleons at rest. In $ps-pv$ case, the largest contributions come from the virtual processes in which no mesons are present and only one of the nucleons is excited to its isobar states and have the expression $(x=\mu r)$

$$V_{pv}(x) \approx -(m_\pi c^2)(g^2/4\pi\hbar c)(G^2/4\pi\hbar c)[U_1(x) + (\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)})U_2(x) + S_{12}U_3(x)],$$

$$\left.\begin{aligned} U_1(x) &\approx \frac{3}{8}P\left(\frac{1}{x^2}+\frac{4}{x^3}+\frac{10}{x^4}+\frac{12}{x^5}+\frac{6}{x^6}\right)e^{-2x}, \\ U_2(x) &\approx -\frac{1}{16}P\left(\frac{4}{x^3}+\frac{10}{x^4}+\frac{12}{x^5}+\frac{6}{x^6}\right)e^{-2x}, \\ U_3(x) &\approx \frac{1}{16}P\left(\frac{2}{x^3}+\frac{8}{x^4}+\frac{12}{x^5}+\frac{6}{x^6}\right)e^{-2x}, \end{aligned}\right\} \tag{6}$$

where $P=8$, for isotopic spin singlet and $P=5$, for isotopic spin triplet. The remaining processes are of the analogous forms but of much smaller magintude. Thus it can be said that the contributions of nucleon isobars upon static nuclear potential in symmetrical $ps-pv$ theory consist of a very strong and singular attractive central force and a less strong (but absolutely large) singular tensor force of right sign. In $ps-ps$ case, the contributions of nucleon isobars consist of two parts, one of which comes from the second term of (5) and has the same form as (6) and the other of which comes from the first term of (5). The whole expression is

$$\begin{aligned} V_{ps}(x) \approx\ & (m_\pi c^2)(f^2/4\pi\hbar c)(G^2/4\pi\hbar c)(\mu/2\kappa)\left(\frac{2}{x^2}+\frac{4}{x^3}+\frac{2}{x^4}\right)e^{-2x} \\ & -(m_\pi c^2)(f^2/4\pi\hbar c)(G^2/4\pi\hbar c)(\mu/2\kappa)^2[U_1(x) + \boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)})U_2(x) + S_{12}U_3(x)]. \end{aligned} \tag{7}$$

The first term of $V_{ps}(x)$ is of the smaller magnitude than the second term if we take into accounts of the strong damping of meson pair term. So the conclusions are in this case almost the same as in $ps-pv$ case. Numerically the corrections (6) and (7) are approximately of the same magnitude as the usual fourth order nuclear potential in $ps-pv$ case. It can, therefore, be said that the over-all effects of nucleon isobars seem to improve the situations very much in nuclear force problems in symmetrical ps meson theory, thus providing wide possibilities for both types of couplings to enable the deuteron bind correctly. The numerical investigations of deuteron problems are now in progress using the exact correction terms of nucleon isobars and the above values of parameters determined by pion-nucleon scatterings.

Finally it must be remarked that the strong divergences due to the derivatives in (1) can be compensated by the strong damping of meson propagation function which arises from the strong coupling of mesons with nucleon isobars as given by equation (1).

Effects of Nucleon Isobar States in Low Energy Two Nucleon System (*Comment*)

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Effects of nucleon isobar states in low energy two nucleon system are roughly estimated. Nucleon isober states are present when two nucleons approach each other within the range of nuclear forces, as the interaction energy we borrow "Pauli-Kusaka" potential which is derived from pv coupling interaction of ps symmetrical meson theory in the strong coupling limit. In simplicity, radial parts of interaction potential are assumed to be square well type (both central and tensor force range are equal), and its range and depth are adjusted to agree with constants of singlet low energy scattering and deuteron state. We take the lowest energy of nucleon excitation is 300 Mev. It seemed to be too high isobar energy to effect in the low energy phenomena, however, it is shown that existence of isobar states give considerable additional effective potential to the usual ground state nucleon potential, and its ratio are about $1/2 \sim 1/3$ for both singlet and triplet S–waves. Moreover the effects in singlet is greater some extent than in triplet state. The isobar wave have little effects on the potential tail and isobar D–wave (D–quintet state in usual singlet S–state and D–septed state in triplet S–state are predominant among many other isobar states) also have scarecely effects in the inner region of nuclear forces. We see quadrupole moment of deuteron and effective range of scattering are not so altered considerably. Although the transition matrix elements to isobar states in the strong coupling theory may be rather large than in the true situation, in any case it it is necessary to take account of the isobar effects when we adjust the potential parameter to the low energy data.

Tomonaga Approximations in the Nuclear Force (*Comment*)

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In the meson theory of nuclear forces, the method of intermediate coupling theory enables us to understand the physical meaning of this subject intuitively as well as to compare various approximation methods.

Common to all such theories, we assume nucleon to be at rest since they are much heavier than meson.

The meson field treated here is of the type of the charged longitudial vector. According to the general procedure of the intermediate coupling theory, the meson field is decomposed into o and s–meson fields. Roughly speaking, the former corresponds to the bound part and the latter to the unbound one.

In the case of a two-nucleon system, the o–fields of two nucleons are symmetrically and antisymmetrically combined so as to form an orthogonal eigenfields. This procedure is essentially important and comes into our question only in the case of two or more nucleon system. Thus, for instance

$$a(k) = \varphi_{os}(k) A_s + \varphi_{oa}(k) A_a + \alpha(k),$$

where A_s and A_a are o–fields and $\alpha(k)$ is s–fieleds. And

$$\varphi_{os}(k) = (1/v_s)\,(G(k)/K)\cos(\boldsymbol{kr}/2),$$
$$\varphi_{oa}(k) = (1/v_a)\,(G(K)/K)\, i\sin(\boldsymbol{kr}/2).$$

$\boldsymbol{r}$ is the relative coordinate of two nucleons. Then the o–fields approximately give the nucleon self-energy and the static potential.

It is generally considered that the static potential is meaningfull only in the case of the distant separation of two nucleons. We are confined our problem only to this case. Then it is convenient to perform the transformation

$$A_1 = \frac{1}{\sqrt{2}}(A_s + A_a), \quad A_2 = \frac{1}{\sqrt{2}}(A_s - A_a), \quad \text{etc.}$$

This transformation brings the problem to the form to be solved if we have already known the one nucleon problem, which was precisely investigated by S. Tomonaga *et al.*

Choosing the representation which diagonalizes the operator

$$\Omega = A^*A + B^*B - gV_0[(A+B^*)\tau_+ + (A^*+B)\tau_-],$$

the o–field Hamiltonian has the diagonal matrix elements and also the non-diagonal ones.

Especially the diagonal element for the ground state of the charge 1 (corresponding to the $n-p$ system) gives the nuclear potential in the order of $e^{-\mu r}$ which agrees with the results of the conventional theories in the limits of weak coupling and of strong coupling.

Non-diagonal elements cause the isobaric transition, in which some special transition elements are only large.

Effect of the s–field to the nuclear potential can also be explored. To perform this consistently, we expand all the quantity in the power of $e^{-\mu r}$. This method corresponds to the expansion of the number of mesons exchanged (in the sense of the perturbation theory) between two nucleons. Then the isobaric effect gives the potential of the order of $e^{-2\mu r}$ or higher. Contribution of s-field is two-fold. The one is to change slightly the coefficient of $e^{-\mu r}$ potential. The other is to introduce the new $e^{-2\mu r}$ or higher order potentials.

Higher Order Corrections to the Nuclear Force *(Comment)*

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and
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M. Taketani *et al.* showed some years ago that the fourth order adiabatic nuclear potential of the pseudo-scalar meson with pseudo-vector coupling was far more important than the second order one and that almost all the low energy data of the two-nucleon system, especially deuteron, could be explained by using the sum of both potentials. It was not clear, however, whether the sixth and higher order effects could be ingored or not, near the range of the nuclear force. Moreover, it is not decided which of the pseudo-scalar coupling and pseudo-vector one should be chosen. But fortunately, as Wentzel and other authers have demonstrated, the core term of the former coupling is strongly damped and it seems sufficient, at least in the low energy region, to consider only the pseudo-vector coupling. In view of such circumstances, we have investigated, choosing this coupling, to what extent the higher order corrections would contribute to the nuclear potential.

To begin with, H. Fukuda and N. Fukuda have estimated the general n-th order effects by using the recent Brueckner-Watson's formulation of non-adiabatic potential. It was shown that, in case of large n, their effects are negligibly small even inside the force range. J. Osada and Y. Yamamoto supported us in many respects. Secondly, K. Sawada and A. Sugie have studied the Tamm-Dancoff approximation (including two mesons), especially the effects of virtual meson scattering. It was verified that the factor of the usual fourth order potential should slightly be changed. Thirdly, S. Machida and K. Semba have calculated in detail the sixth order adiabatic potential and showed that it is almost negligible near the force range in spite of the large number of Feynman graphs and high singularities.

I. Effects of many-meson exchanges

(N. FUKUDA and H. FUKUDA)

We have been investigating the higher order corrections resulting from the exchanges of many virtual mesons. Our calculations are based on Brueckner-Watson's classification of higher order corrections and only contain terms which

are of the 2^m-order in coupling constant. Our preliminary conclusion is that if the pair formation is neglected the power series thus obtained converges sufficiently strongly. The effect of the omitted terms will not affect the main features.

To perform the calculation, we have assumed; (i) the recoil of the nucleon is omitted (ii) the nucleon pair formation is neglected (iii) the energy denominator is approximated by a suitable form. The approximation of energy denominator is as follows:

$$\frac{1}{\mathcal{E}_1(\mathcal{E}_1+\mathcal{E}_2)\cdots\cdots(\mathcal{E}_1+\mathcal{E}_2+\cdots\cdots+\mathcal{E}_n)} \doteqdot \frac{1}{n!\,\mathcal{E}_1\mathcal{E}_2\cdots\cdots\mathcal{E}_n}, \qquad (1)$$

where $\mathcal{E}_j$ is the energy of the meson with momentum and charge j. This approximation seems not so bad for the discussion of nuclear forces near the range, since in this region the momentum of the meson contributing to the nuclear force is not so large. K. Aizu will show that the singularity of the potential would not be altered by this replacement. In the neutral meson theory we would have the exact potentials by this approximation; the symmetrical theory now being investigated. In the former theory, we have

$$V_{eff}^{2N} = \sum_{n=0}^{N} V_0^{2N} V_0^{2(N-n)} \qquad (2)$$

where V_0^{2N} is the effective potential corresponding to the graph where one nucleon emits n-mesons and the other subsequently absorbs these mesons. V_0^{2N} has the form (Yukawa pot.) $\times (K_0(\mu r))^{n-1}$ apart from differential operations in front; but for V_{eff}, we have to replace one of the (Yukawa pot.) appearing in the two V_0's in (2) by $K_0(\mu r)$:

$$V_0^{2(2m)} = \frac{1}{(2m)!^2} \sum_{i=0}^{2m} \dot{K}_0^i \ddot{K}_0^{2m-i} \frac{(2m-i)!}{i!} \left((\nabla\nabla')^i r^{2m} r'^{2m}\right)_{r=r'} \qquad (3)$$

$$V_0^{2(2m+1)} = \frac{1}{(2m+1)!^2} \sum_{i=0}^{2m+1} \frac{(2m+1-i)!}{i!} \dot{K}_0^i \ddot{K}_0^{2m+1-i} \left((\nabla\nabla')^i r^{2m} r'^{2m} (\overset{(1)}{\sigma}\gamma)(\overset{(2)}{\sigma}\gamma')\right)_{r=r'} \qquad (4)$$

$$\dot{K}_0 = \frac{1}{r}\frac{\partial}{\partial r} K_0(\mu r), \qquad \ddot{K} = \frac{1}{r}\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r} K_0(\mu r). \qquad (5)$$

(We have to replace one of $K_0(\mu r)$ by $Y(\mu r)$ (Yakawa pot.) to obtain (2)). The differentiation can be performed in the following way:

$$V_0^{2(2m)} = \frac{1}{(2m)!^2}\left(Y\frac{\partial}{\partial K_0}\right)$$
$$\times\left\{\sum_{i=0}^{m} \dot{K}_0^{2i}(r^2\ddot{K}_0)^{2m-2i}\frac{(2m-2i)!^2(m!)^2}{(4m-4i+1)!}\sum_{j=0}\frac{2^{4j}(4m-2i-2j+1)!}{(2j)!\,(i-j)!\,(m-i-j)!\,(2m-i-j)!}\right.$$
$$+\sum_{i=0}^{m-1}\dot{K}_0^{2i+1}(r^2\ddot{K}_0)^{2m-2i-1}\frac{(2m-2i-1)!\,(2m-2i)!\,m!^2}{(4m-4i)!}\cdot$$
$$\cdot\sum_j\frac{2^{4j+1}(4m-2i-2j)!}{(i-j)!\,(2j+1)!\,(m-i-j-1)!\,(2m-i-j)!}$$

$\sum_j'$ can be estimated by the "Sattel-Punkt-Method"; the maximum being at $j=\frac{i(m-i)}{m+i}$, i.e. near the intermediary region of the boundary of j. The width of the distribution is about $\frac{mi}{(m+i)^2}\sqrt{m-i}$ and hence the approximation is fairly good.

$$V_0^{2(2m)} \doteqdot \left(2\frac{\partial^2}{\partial r^2}K_0(\mu r)\right)^{2m} \Big/ (2m)!$$

$$V_0^{2(2m+1)} \doteqdot \left(2\frac{\partial^2}{\partial r^2}K_0(\mu r)\right)^{2m+1} \Big/ (2m+1)!\,\frac{(\overset{(1)}{\boldsymbol{\sigma}}\boldsymbol{r})(\overset{(2)}{\boldsymbol{\sigma}}\boldsymbol{r})}{r^2}$$

we thus obtain approximately

$$V_{eff}^{2(2N)} \doteqdot \frac{1}{(2N+1)!}\left(\frac{\partial^2}{\partial r^2}2K_0(\mu r)\right)^{2N-1}\left(\frac{\partial^2}{\partial r^2}Y(\mu r)\right)$$

The series of nuclear potentials, therefore, classified by Brueckner and Watson converges sufficiently strongly even inside the range. We shall discuss the problem in the symmetrical theory later soon.

II. Effect of the inertia of meson field

(K. SAWADA)

According to the recent experiments on the pion-nucleon scattering, the meson cloud around the nucleon differs significantly from that expected from the Born approximaiton; especially it seems to exist a "resonance"-like phenomena in the scattering. As many authors have shown, these results can be explained by the Tamm-Dancoff approximation or the strong and intermediate coupling theories. In case of the nuclear force, however, since the energy of the total system is very low in the region where the "nuclear potential" has its proper meaning, the resonance phenomena cannot do effect so much; nevertheless it can be shown that in the virtual meson scattering the cross-section calculated from the Tamm-Dancoff approximation differs significantly from the Born approximation even at the low energy, due to the "inertia" effects of the bound mesons. So we have studied to what extent the usual adiabatic potential should be modified by taking into account the effects of virtual meson scatterings.

We have chosen the $ps-pv$ theory, since in the $ps-ps$ theory the large coupling constant would give rise to large inertia effect and reduce considerably the main contributions of the $ps-ps$ theory. Hence it is to be expected that both theories would give almost the same results at least in the low energy region. Unfortunately, however, the inertia effect can not be so clearly separated from many diverging quantities that we considered only the following processes adopting the Tamm-Dancoff method.

In the ordinary calculation of the fourth-order nuclear force, the circled part of Fig. 1 contains many states in some proportions which have different total angular momentum and isotopic spin. But as stated above, since in the pion-nucleon scattering the Born approximation seems to be useless, we should replace this circled part by its iteration as in Fig. 2. Separating the graphs corresponding

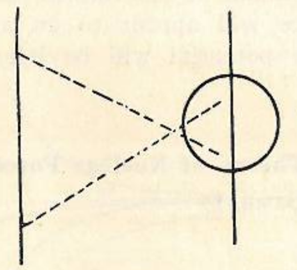

Fig. 1

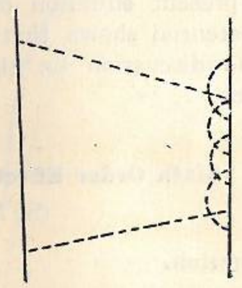

Fig. 2

to Fig. 2 from the T–D-eqs., solving the contribution of these graphs exactiy (as far as possible), and adding other graphs in the Born approximation, we have as the fourth-order potential including the "inertia" effects:

$$\begin{aligned}V^{(4)} = -\left(\frac{g}{\mu}\right)^4\frac{1}{2}\Big[&4\big(2(\nabla\nabla')^2(\overset{(1)}{\boldsymbol{\tau}}\overset{(2)}{\boldsymbol{\tau}})+3(\overset{(1)}{\boldsymbol{\sigma}}\cdot\nabla\times\nabla')(\overset{(2)}{\boldsymbol{\sigma}}\cdot\nabla\times\nabla')\big)G_1(r,r')\\
&+\Big\{\left(\frac{16}{3}J(4)-\frac{8}{3}J(-2)+\frac{1}{3}J(1)-3\right)(\nabla\nabla')^2\\
&+\left(\frac{16}{9}J(4)+\frac{4}{9}J(-2)-\frac{2}{9}J(1)+2\right)(\nabla\nabla')^2(\overset{(1)}{\boldsymbol{\tau}}\overset{(2)}{\boldsymbol{\tau}})\\
&+\left(\frac{8}{3}J(4)+\frac{2}{3}J(-2)-\frac{1}{3}J(1)+3\right)(\overset{(1)}{\boldsymbol{\sigma}}\cdot\nabla\times\nabla')(\overset{(2)}{\boldsymbol{\sigma}}\cdot\nabla\times\nabla')\\
&+\left(\frac{8}{9}J(4)+\frac{8}{9}J(-2)+\frac{2}{9}J(1)-2\right)(\overset{(1)}{\boldsymbol{\sigma}}\cdot\nabla\times\nabla')(\overset{(2)}{\boldsymbol{\sigma}}\cdot\nabla\times\nabla')(\overset{(1)}{\boldsymbol{\tau}}\overset{(2)}{\boldsymbol{\tau}})\Big\}G_2(r,r')\Big]_{r=r'}\end{aligned}$$

where

$$G_1=\frac{1}{2rr'}\left(2F(r+r')-2e^{-\mu r'}F(r)\right)$$

$$G_2=\frac{1}{2rr'}\left(-2F(r+r')+2e^{-\mu r}F(r')+2e^{-\mu r}F(r)\right)$$

$$F(r)=\frac{r}{(2\pi)^3}K_0(\mu r)$$

In these expressions $J(4)$, $J(-2)$ and $J(1)$ come from the states corresponding to $(I=J={}^3/_2)$, $({}^3/_2, {}^1/_2)$ and $({}^1/_2, {}^1/_2)$ respectively; especially $J(4)$ corresponds to the resonance of the pion-nucleon scattering. The results by Born approximation are obtained by putting $J'_s=1$ and coincides with Nishijima's results. (We have taken into account properly partial cancellation of the velocity dependent forces discussed recently by A. Klein, because the results thus obtained should only be compared with that of the canonical transformation.)

In the present situation of the theory J'_s cannot be determined exactly, and the above potential shows that the central force will appear to an appreciable extent. The discussion on the nature of the potential will be left for later considerations.

III. Sixth Order Effects in the Meson Theory of Nuclear Forces

(S. Machida and K. Semba)

§1. Introduction.

Some time ago, M. Taketani, S. Nakamura and M. Sasaki[1] proposed for the analysis of nuclear forces that the problem was to be treated substantialistically in the outside region, while in the inside region phenomenologically. Following this method, it was shown that the second plus fourth-order adiabatic nuclear potentials deduced from symmetrical pseudoscalar π-meson theory with pseudovector coupling were able to explain all low energy phenomena, and might also be able to account for high energy nucleon nucleon scattering experiments to some extent[2]. In the case of pseudoscalar coupling, we would obtain similar results with the case of pseudovector coupling, if we would take into account the reactions of meson field. According to K. Sawada, this effect will not be large for the case of pseudovector coupling[3]. Although it was shown that the non-adiabatic effects were not so large in the pseudovector coupling case[4], there remains a question, whether the second plus fourth order nuclear potentials obtained by the weak coupling perturbation expansion might be considered to be approximately correct or not, when the fourth order terms are comparable with the second order ones.

To investigate this question, we have calculated the sixth order adiabatic nuclear potentials in the symmetrical pseudoscalar π-meson theory with pseudovector coupling. Form the results of calculations of the fourth order non-adiabatic effects, it seems that errors caused by adiabatic approximation would not alter the qualitative features of adiabatic nuclear potentials.

If the sixth order nuclear potentials were large, we might conclude that expansion in the coupling constant was meaningless even in asymptotically and if they were small, there would be a possibility that it was an asymptotic expansion in the case of calculating nuclear potentials.

§2. Sixth order nuclear potentials.

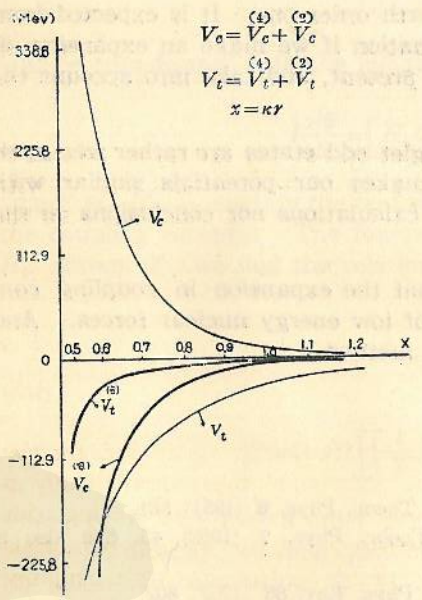

Fig. 1. Triplet even states ($g^2/4\pi=0.08$)

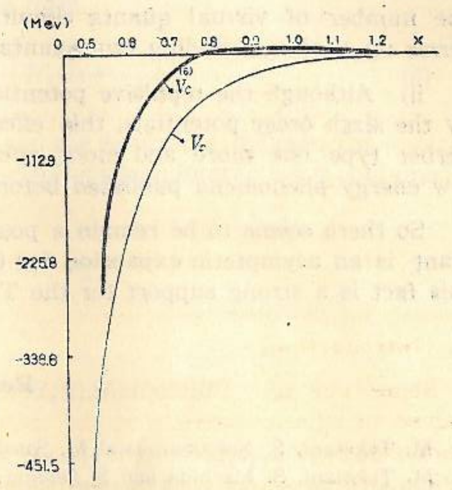

Fig. 2. Singlet even state ($g^2/4\pi=0.08$)

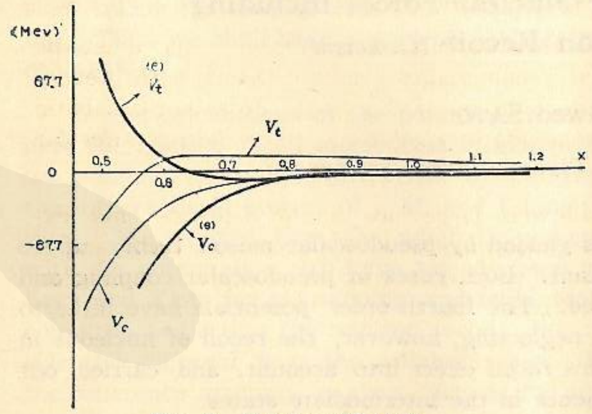

Fig. 3. Triplet odd state ($g^2/4\pi=0.08$)

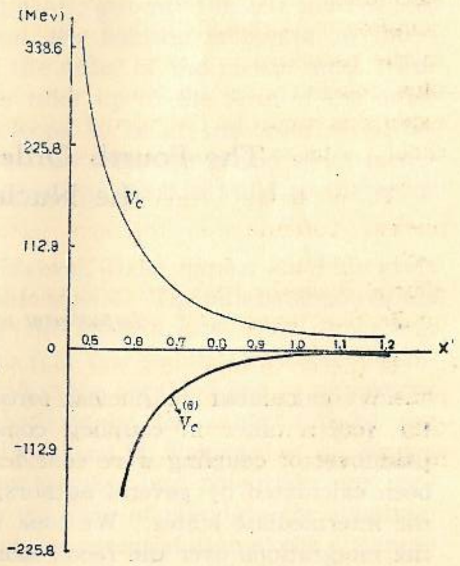

Fig. 4. Singlet odd state ($g^2/4\pi=0.08$)

§ 3. Conclusions.

i) In the region near the nuclear force range, the sixth order potentials are rather small compared with the second plus fourth order one. It is expected from this result that we may obtain a good approximation if we make an expansion in the number of virtual quanta simultaneously present, and take into account the terms up to that including two quanta.

ii) Although the repulsive potentials in singlet odd states are rather weakened by the sixth order potentials, this effect only makes our potentials similar with Serber type one more and more, affecting no calculations nor conclusions on the low energy phenomena published before[2].

So there seems to be remain a possibility that the expansion in coupling constant is an asymptotic expansion for the case of low energy nuclear forces. And this fact is a strong support for the Taketani's method.

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The Fourth Order Nuclear Forces Including the Nucleon Recoil *(Comment)*

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We calculated the nuclear forces yielded by pseudoscalar meson theory up to the fourth order in coupling constant. Both cases of pseudoscalar coupling and pseudovector coupling were considered. The fourth order potentials have hitherto been calculated by several authors, neglecting, however, the recoil of nucleons in the intermediate states. We took this recoil effect into account, and carried out the integrations over the recoil momenta in the intermediate states.

The outline of the procedure is as follows: By a suitable canonical transformation, the Schrödinger equation in the interaction representation

$$i\frac{d\Psi}{dt}=H(t)\Psi \tag{1}$$

will be transformed into the equation of the form

$$i\frac{d\Phi}{dt}=\left[H_2(t)+H_3(t)+H_4(t)+\cdots\cdots\right]\Phi \tag{2}$$

where $H_2, H_3, \cdots\cdots$ contain the real processes, and the subscripts indicate the order in the coupling constant. The fourth order nucleon-nucleon interaction is contained in H_4. From (2), we find the relation

$$-i\int H_4(t)\,dt=S_4-\Sigma \tag{3}$$

with

$$\Sigma=(-i)^2\iint\frac{1}{2}[1+\varepsilon(t-t')]H_2(t)H_2(t')\,dt\,dt' \tag{4}$$

where S_4 is the fourth order part of the S matrix. As it is very tedious to calculate H_4 directly, we first calculate $S_4-\Sigma$, and then defined H_4 so as to equation (3) holds. In calculating $S_4-\Sigma$, we divide it into several parts which correspond to different Feynman graphs, and expand these parts in powers of v/c assuming that $v/c\sim\mu/M$, where v is the initial or final nucleon velocity. For the case of pseudoscalar coupling, we take the leading term and the term of the relative order v/c in $S_4-\Sigma$. For the case of pseudovector coupling, we must take up to the term of the order of $(v/c)^2$, in order to obtain the adiabatic potential in the limit where $M/\mu\rightarrow\infty$. But, in deriving the adiabatic potential, the nucleon momenta in intermediate states are treated as if they were of the order of the meson mass. For the case of pseudovector coupling, we therefore take up to the term of the order of $(v/c)^2$, considering the recoil momenta of nucleons to be of the order of μ, besides taking the leading term and the term of relative order v/c in the strict meaning. Then we shall have a nucleon-nucleon interaction which is valid to the order of $(v/c)^2$.

In the calculations of the potentials thus obtained, there appear the integrals over the nucleon recoil momentum in intermediate states. The adiabatic potentials are obtained from these potentials by carrying the integrations after expanding the integrands in powers of μ/M, and taking the first few terms from these expansions. We verified that the adiabatic potentials thus obtained are in agreement with the potentials which have already been calculated by the other authors (by Levy and Klein for *ps.* coupling, and by K. Nishijima and M. Taketani *et al.* for *pv.* coupling) neglecting the nucleon recoil. But numerically, our potentials are considerably different from the adiabatic ones. In the case of pseudovector coupling, the difference amounts to about 30% of the adiabatic potential even at the distances comparable to the meson Compton wave length, while the general features are same for both the potentials. In the case of pseudoscalar coupling, our potential is much different from the adiabatic one even in the general features, and the former is more advantageous than the latter, because the former gives strong attractive force in the singlet even state.

The remarkable differences between our potentials and the adiabatic ones can be understood by considering the following fact: The expansions of the integrands above mentioned are valid only for a restricted region of the integration variable, so the integrations after the expansions do not always give the convergent results.

In view of the importance of the nucleon recoil in intermediate states, there arises a question: Can we neglect the recoil in the initial and the final states? At first sight, the neglection of this kind of recoil effect seems plausible, as far as one is to do with only low energy problems. However, more detailed studies are necessary to answer completely to this question.

On the D State Probability of Deuteron (*Comment*)

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From the experimental values of the magnetic moments of proton, neutron, and deuteron, we can only say, about the D state probability of deuteron (hereafter denoted as P_D), that

$$0\% \lesssim P_D \lesssim 10\%. \tag{1}$$

This large uncertainty comes from the possible existence of about 6% uncertain corrections to the non-relativistically calculated value (4%), 2% being due to the relativistic corrections[1] and 4% to the dependence of the magnetic moment of one nucleon on the proximity of another[2].

It is the object of this note to show that we can obtain more knowledge about P_D utilising the experimental ratio of the hyperfine structure separations of the ground states of deuterium and hydrogen besides the experimental value of the magnetic moments. The discrepancy between the experimental ratio of the hyperfine structure in deuterium and hydrogen and that obtained from Fermi's formula is expressed by

$$\Delta = 1 - \frac{\left(\frac{\nu_D}{\nu_H}\right)_{exp}}{\left(\frac{\nu_D}{\nu_H}\right)_F}, \tag{2}$$

$$\left(\frac{\nu_D}{\nu_H}\right)_F = \frac{3}{4}\left(\frac{m_D}{m_H}\right)^3 \frac{\mu_D}{\mu_P} \tag{3}$$

m_D and m_H being reduced masses of deuterium and hydrogen atoms, μ_D and μ_P being the magnetic moments of deuteron and proton respectively. From the experimental values for ν_D/ν_H[3], μ_D/μ_P[4], and m_D/m_H, we obtain

$$\Delta = (1.703 \pm 0.008) \times 10^{-4}. \tag{4}$$

Although the major part of this discrepancy has been explained by A. Bohr[5] and more in detail by Low[6] as the effect of the structure of deuteron, the experimental value taking into account the relativistic effects for nucleons and the radiative corrections[7],

$$\Delta_{theor} = (2.30 \pm 0.40) \times 10^{-4}\ {}^{8)}, \tag{5}$$

is well outside the range of experimental uncertainties.

The value (5) obtained for Δ was based on the assumption that (a) each nucleon behaves as a point particle with both its electric charge and magnetic moment concentrated in a point and (b) P_D is just 4%. Since it is meaningless to adhere to the value 4% for P_D, it seems to be the most reasonable approach to derive a reliable restriction to P_D from the experimental value (4) for Δ, taking into account the probable effect of the structure of nucleons.

We have calculated Δ_{theor} varying P_D and assuming point nucleons. From the results given in Fig. 1., we obtain a restriction to P_D, i. e. $7\% \lesssim P_D \lesssim 15\%$. If the anomalous magnetic moment of a nucleon is assumed to be spread uniformly over the order of a certain finite volume, the radius of which being of the order of the π meson Compton wave length, this restriction becomes to $5\% \lesssim P_D \lesssim 13\%$. So we obtain, from the experimental value (4), a restriction to P_D,

$$5\% \lesssim P_D \lesssim 15\%. \tag{6}$$

Combining this value with (1), obtained from the magnetic moments, we arrive at the most reliable value of P_D,

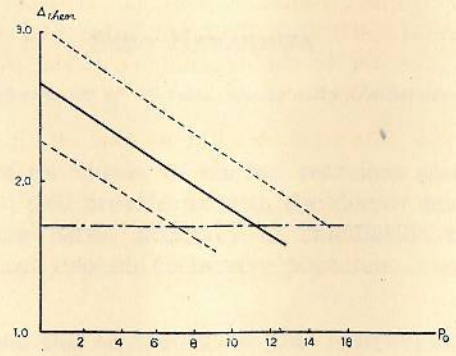

Fig. 1. Δ_{theor} calculated assuming a point nucleon. —— and ...... represent theoretical values and uncertainties, and (—·—·—·—) represent experimental value respectively.

$$5\% \lesssim P_D \lesssim 10\%. \tag{7}$$

It seems worth while to note that the adiabatic nuclear forces derived from the pseudoscalar meson theory with pseudovector coupling up to fourth order in coupling constant are able to reproduce the all low energy data for neutron-proton system including the above obtained value for P_D[9].

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clouded crystal ball

V　EV　ε=1
19 MeV
(attraction)
p-p

Reports on Experiments

Birmingham Conf.

~~Robert Serber~~　Peierls

Department of Physics, Columbia University

(Abstract not yet received, Aug. 20, 1953)

Collective Model for Nuclei

John A. Wheeler

Palmer Physical Laboratory, Princeton University

(Abstract not yet received, Aug. 20, 1953)

Formal Theory of Nuclear Reactions

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The reformulation of the theory of nuclear reactions along the method of Lippman and Schwinger (*LS*) will provide us with the deeper understanding of nuclear phenomena as well as the better accuracy of calculation than current theories. Our formalism is general enough to involve photoreactions. Hence it is valid in the field theory, too.

First of all we extend the *LS* theory to the rearrangement collision like *d-p* reaction. (Y. Fujimoto, S. Hayakawa and K. Nishijima) The total Hamiltonian is decomposed into free and interaction parts in different ways for initial (*a*) and final (*b*) states

$$H=H_a^{(0)}+H_a'=H_b^{(0)}+H_b'. \tag{1}$$

The eigen fuctions of $H_a^{(0)}$ and $H_b^{(0)}$, obeying

$$(H_a^{(0)}-E_a)\Phi_a=0, \qquad (H_b^{(0)}-E_b)\Phi_b=0, \tag{2}$$

consist in two different sets. The outgoing wave is given by

$$\begin{aligned}\Psi_a^{(+)}&=\Phi_a+\xi(E_a-H_a^{(0)})H_a'\Psi_a^{(+)}\\&=\Phi_a+\xi(E_a-H)H_a'\Phi_a\\&=\Phi_a+\xi(E_a-H_b^{(0)})(H_a'+H_b'\xi(E_a-H)H_a')\Phi_a,\end{aligned} \tag{3}$$

where

$$\xi(x)=\frac{1}{x+i\varepsilon}=P\frac{1}{x}-i\pi\delta(x). \tag{4}$$

The transition matrix is obtained by taking the amplitude in Φ_b out of the asymptotic form of $\Psi_a^{(+)}$ as

$$T_{ba}^{(+)}=(\Phi_b,H_a'\Phi_a)+(\Phi_b,H_b'\xi(E_a-H)H_a'\Phi_a)=(\Psi_b^{(-)},H_a'\Phi_a). \tag{5}$$

The first term in the second hand side is nothing but the result is the Born approximation. In the theory of d–p reactions by Bulter only a term of neutron-nucleus interaction in that term is taken into account. The second term may be interpreted as the effect of which the reaction takes place through the collision complex.

The reciprocity, which is expressed by $T^{(+)}=T^{(-)\dagger}$ is proved on account of

$$(\Phi_b,H_a'\Phi_a)-(\Phi_a,H_b'\Phi_b)^*=(E_b-E_a)(\Phi_b,\Phi_a) \tag{6}$$

with

$$E_b=E_a.$$

If a number of like particles take part in reactions, the (anti–) symmetrization is carried out of $\Psi^{(+)}$ in (3). This is shown to be equivalent to (anti–) symmetrize both initial and final states, as it should be. (M. Kawaguchi and K. Nishijima).

Following the current manner in nuclear physics, we may refer to standing waves and get the reaction matrix instead of $T^{(+)}$. Corresponding to (5) we have

$$K_{ba}=(\Phi_b,H_a'\Phi_a)+\left(\Phi_b,H_b'\frac{1}{E_a-H}H_a'\Phi_a\right). \tag{7}$$

The last term of the right hand side can be expanded, by introducing a set of eigenfunctions for the collision complex,

$$(H-E_\lambda)\chi_\lambda=0,$$

as

$$\left(\Phi_b,H_b'\frac{1}{E_a-H}H_a'\Phi_a\right)=\sum_\lambda(\Phi_b,H_b'\chi_\lambda)\frac{1}{E_a-E_\lambda}(\chi_\lambda,H_a'\Phi_a). \tag{9}$$

Combining (9) with $(\Phi_b,H_a'\Phi_a)$ in (7) we obtain

$$K_{ba}=\sum_\lambda\frac{\omega_{b\lambda}\omega_{\lambda a}}{E_a-E_\lambda}, \tag{10}$$

where

$$\omega_{\lambda a}=(\chi_\lambda,H\Phi_a)-(\Phi_a,H\chi_\lambda). \tag{11}$$

(10) is a familiar dispersion formula with resonance energies E_λ. (7) and (10) are two alternative expressions and which of them should be employed is a matter of convenience.

It is not difficult to take the potential scattering into account that is due to the artificial separation of the configuration space into inner and outer parts. The idea is similar to that given in my intermediate coupling theory.

In extending our theory to photo-reactions we need consult with the theory of line breadth. We have only to modify the initial state so as to represent the incident wave.

Our method not only gives the foundation of current dispersion formulas, but also provides practical techniques to discuss the properties of collision complex.

Deuteron Reactions (*Comment*)

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We are going to discuss the (d, p) reaction as an application of the general theory of nuclear reactions[1].

Y. Fujimoto, S. Hayakawa and K. Nishijima[2] made a foundation of Butler's theory[3] of stripping reactions on the basis of our general theory and examined the validity of its approximations. The Hamiltonian of the total system is

$$H=K+H_\xi+H_{np}+H_{n\xi}+H_{p\xi},$$

where ξ, n and p indicate target nucleus, neutron and proton, respectively. K is the kinetic energy of a neutron and a proton and H_ξ the internal energy of a nucleus ξ. H_{np}, $H_{n\xi}$ and $H_{p\xi}$ are the interaction energies between n and p, n and ξ and p and ξ, respectively. The wave function of the initial state (an incident deuteron and a target nucleus) is represented by Φ_i, and that of the final state (an outgoing proton and residual nucleus) by Φ_f. Butler's theory results from the transition matrix element T_{fi}

$$T_{fi}\simeq(\Phi_f,H_{n\xi}\Phi_i)=(\Phi_f,H_{np}\Phi_i), \tag{1}$$

which corresponds to the Born approximation for $H_{n\xi}$ or H_{np}[4]. The exact expression for T_{fi} can be obtained in our theory as

$$T_{fi}=(\Phi_f,(H_{n\xi}+H_{p\xi})\Phi_i)+\left(\Phi_f,(H_{p\xi}+H_{np})\frac{1}{E-H}(H_{n\xi}+H_{p\xi})\Phi_i\right). \tag{2}$$

The correction for Butler's theory is obtained by comparing (2) with (1). Some of the important correction terms in (2) will be discussed.

Y. Fujimoto, Y. Ichikawa and Y. Yamaguchi[5] examined the term

$$(\Phi_f,\ H_{p\xi}\Phi_i)$$

which will be called as the potential term hereafter. It consists of the scattering amplitude of a proton due to nuclear forces and the overlapping integral for a neutron that consists in a deuteron in the initial state and is trapped by a nucleus in the final state. It is found that the potential term gives rise to an angular dependence similar to the stripping term (1) and they are of comparable magnitude.

The other corrections, i.e. the second term in (2), correspond to higher order terms of the Born approximation. A rough estimate shows that they result in roughly isotropic angular dependence and their absolute magnitudes are not negligible, too.

A large part of these correction terms can be expressed as a reaction through a compound nucleus, corresponding to the case where both a neutron and a proton get in a target nucleus. For this purpose we introduce a channel radius R and assume the compound nucleus model in the internal region ($<R$). The formal solution (2) is decomposed into two parts; one is external part ($>R$) and the other is internal part ($<R$). The result can be expressed in terms of a resonance formula. K. Izumo, H. Ui and T. Yoshimura[6] made a phenomenological analysis of Be^9 (d, p) Be^{10} reactions with the compound nucleus model.

As already pointed out[7], the stripping process is very useful for the analysis of properties of residual nuclear states. S. Yoshida[8] obtained an expression for the cross-section of (d, p) reactions in terms of neutron reduced widths γ_n, by generalizing the concept of the reduced width to the case of a bound neutron state. The widths determined in reference to experimental data are compared with the known values of widths based upon neutron scattering and also with the theoretically expected ones[9]. These comparisons give us a measure on the validity of the single particle model.

When the incident energy of a deuteron is as low as the Coulomb barrier energy, the Coulomb field of a target nucleus is no longer negligible. The Coulomb field causes the polarization of the deuteron due to the difference between its centers of mass and of charge. Y. Fujimoto calculated this polarization with the perturbation theory and examined its effect on the angular distribution of emitted protons.

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Interpretation of Electron Scattering Experiments (*Comment*)

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Experiments on the elastic scattering of fast electrons by several elements, reported by Hofstadter, Fechter and McIntyre in the preceding paper, are interpreted with the help on the first Born approximation. These experiments imply nuclear charge distributions that are peaked at the center and taper off smoothly. The root-meansquare radii of the charge distributions, and the nuclear Coulomb energies, are however in approximate agreement with those computed from the usual uniform charge distribution. The effects of radiation loss and nuclear excitation are discussed qualitatively, and the effect of a nuclear electric quadrupole moment is considered quantitatively. It is concluded that none of these can account for the discrepancy between the observed scattering cross section which decreases monotonically with increasing angle, and the diffraction minima and maxima expected from a uniform charge distribution with a sharp or moderately rounded edge. It seems likely that higher order corrections to the first Born approximation will not make a qualitative difference in the computed cross section; theoretical improvements are now being undertaken.

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Polarizations of d-d Reactions (*Comment*)

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Review of Shell Structure

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The shell model of nuclear structure is formed in complete analogy to the Bohr-Sommerfeld theory of the atom. The nuclear model assumes that one can, to a certain degree of approximation, describe the interaction of the neutrons and protons by an average field acting on the nuclear constituents. In such a field the quantized orbits of the nucleons are designated by a quantum number of orbital angular momentum l and a total quantum number n. These orbits are successively filled by neutrons and protons to the extent to which the Pauli principle allows.

The form of the potential should be about that of a square well, and, if we neglect the Coulomb energy, should be the same for neutrons and protons. Experimentally, it has been found that nuclei with certain "magic numbers" of neutrons *or* protons are unusually stable. If one wishes to explain this stability by the fact that at these points a level is completely filled, and the next level is appreciably higher, one has to add one other assumption, namely that of a strong coupling between the intrinsic spin of the nucleon and its orbital angular momentum. A level is then no longer completely characterized by n and l, but also by $j = l + 1/2$ or $j = l - 1/2$. If the sign of this interaction is such as to lower the energy of the level with parallel orientation, $j = l + 1/2$, one can explain indeed the gaps in the energy of levels at the magic numbers.

The model is able to account for the observed spins of the nuclei of odd A by the assumption of certain rules of coupling. The normal coupling, most frequently observed can be explained very simply. One experimental fact to which no exception is known is that the spins of the groundstates of all even-even nuclei are zero. Thus, a nucleus of, say, even N and odd Z can be described as an even-even core of N neutrons and $Z-1$ protons, about which revolves the odd proton in the orbit with angular momentum j. This simple picture, first introduced by Schmidt, predsict that the groundstate of the nucleus has a spin $y = j$, namely that of the orbit of the odd nucleon.

Calculations about the interaction between nucleons have, to some extent, been able to support this qualitative picture.

Comparison of the assumed level scheme and observed nuclear spins shows that the 101 measured nuclear spins are, with very few exceptions, in accord with shell model predictions. There are a few cases in which a proton level is occupied by an odd number of nucleons, more than one particle and less than corresponding to one hole, in which one finds a groundstate spin of one unit less than j. These cases will be discussed later.

The simple Schmidt picture is able to explain qualitatively, although not quantitavely, the magnetic moments of the nuclei of odd A. In the model adopted, the magnetic moment of the nucleus is simply that of the odd nucleon. Thus, for each value of j, one can calculate two magnetic moments, namely that for $l = j + 1/2$ and $l = j - 1/2$. These values are usually referred to as the "Schmidt" values. Since a proton has a charge, and thus has a contribution to the magnetic moment from its orbital angular momentum, while a neutron is uncharged, the calculated values are qualitatively very different for nuclei with odd neutron numbers and with odd proton number.

Experimentally, the magnetic moments of all but a few light nuclei lie in between the Schmidt limits. They follow the trend of the Schmidt values with j and fall fairly well into two distinct groups. Thus the magnetic moments give information about the orbital angular momentum l. Again, there is agreement with the shell model in the vast majority of cases.

The exact numerical agreement between the calculated and measured magnetic moments is very poor. For light nuclei, magnetic moments can be calculated in a more refined manner. This has been done, for instance, by Umezawa and Mizushima. The agreement is in many cases much improved. Miyazawa has given an explanation of the deviations of the magnetic moments in terms of quenching of the intrinsic magnetic moments of the nucleons. A. Bohr has connected the deviations with quadrupole moments.

Nuclei with spins larger than 1/2 have quadrupole moments. Since the shell model specifies the dependence of the eigenfunction on the angles, it is possible to calculate the quadrupole moment divided by $< r^2 >$, the expectation value of r^2. This latter quantity should be approximately eqnal to the square of the nuclear radius R^2. The calculated quadrupole moment depends on the number of particles in a level. It is negative for a single proton outside of closed shells, positive for a proton hole. In general, for a nucleus with an odd number, λ, of protons in a level of given j, when $y = j$, the calculated quadrupole moment is

$$\frac{Q}{< r^2 >} = \frac{2j+1-2\lambda}{2j+2} \tag{1}$$

The magnitude of this expression is always less than one. A nucleus with odd N, even Z, would have a quadrupole moment zero.

Actually, experimental nuclear quadrupole moments are quite large, both for nuclei with odd Z, even N and even Z, odd N. Q/R^2 for Lutecium is about ten. Thus, the magnitude of the quadrupole moments indicates an appreciable distortion of the nuclear core from spherical symmetry. This distortion should be in the same direction as the calculated quadrupole moments. Thus formula (1) should give at least the sign of the quadrupole moments, and the relative magnitude of quadrupole moments of adjacent nuclei. The observed signs of the quadrupole moments thus determine the occupation number of a level. These experimental occupation numbers can always be brought into agreement with those of the shell model. The fact that the quadrupole moment $_{51}Sb^{123}$, with one proton outside the closed shell of 50, and spin 7/2, configuration $(g_{7/2})^1$, has a quadrupole moment which is about three times larger in magnitude than that of $_{53}I^{129}$, with a configuration $(g_{7/2})^3$, seems a nice confirmation of (1).

Nuclei with a magic number of neutrons show practically no distortion of the core. The distortion, energies can be connected to the trend of the energies of isomeric levels, and to the occurence of the unusual coupling in odd nuclei in which the total spin y is one less than that of the odd nucleon.

For heavy nuclei, the distortions are so large that they require a modification of the simple shell model. Such modifications have been considered mainly by A. Bohr, Mottelson and Wheeler.

Review of the Works on the Shell Structure and the Beta-Decay in Japan

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The problems and the main results on the theory of the nuclear shell structure and the beta-decay, recently studied in this country, are briefly summarized.

1. The Nuclear Shell Structure

The spin-orbit coupling shell model of nuclei proposed by M. G. Mayer[1], and Haxel, Jensen, and Suess[2] has been extended by Mizushima, M. Umezawa[3], and Horie[4], to the charge symmetric *j-j* coupling theory where the neutrons and the protons in the unfilled shells are treated simultaneously, using the isotopic spin quantum number. Thus they have introduced the more accurate method in explaining the spins, the magnetic moments and the beta-decay probabilities, especially for the light odd-mass nuclei. Also the mathematical means for the theory have been greatly refined, by M. Umezawa[5] and Flowers[6], employing the group theoretical method of Racah[7] and Yahn[8]. Under the assumption that the total isotopic spin is conserved and also under some subsidiary conditions, the discrepancies of the shell model in the prediction of the spins and the magnetic moments of the light, odd-mass nuclei have been fairly solved. The extended *j-j* coupling theory also accounted for the fact that the first excited state of even-even nuclei have predominantly spin $J=2$ and even parity[9].

Recently, M. Umezawa (see *Comment*) analysed the magnetic moments of the asymmetric odd-odd nuclei, the first excited state of light nuclei and the nuclear quadrupole moment in his theory. It is shown that, of the possible primitive structures, (s, t) = (4, 1) is exclusively preferrable in explaining the observed magnetic moments of Na^{24} and K^{42}. On the other hand, the problem of the quadrupole

moments are beyond the scope of the *j-j* coupling model, even in this extended formulation.

Since a number of features became apparent which contradict to the perfect *j-j* coupling for the light nuclei, a modification of the theory with respect to the internucleonic interactions in the unfilled shells seems to be the first consideration. The consideration of the exchange moment proposed by Miyazawa[10], may be helpful in removing some of the contradiction.

On the other hand, a broadening of the scope of the *j-j* coupling model has been introduced by Horie and Yoshida[11]. They have succeeded to give the magnetic and quadrupole moments of some light nuclei by using the configuration interactions with higher nucleon configurations. Later, Komoda and Sasaki[12] removed the descrepancies of the *j-j* coupling model in the magnetic moment of Li^6 by assuming tensor interaction between $2p$ proton and $2p$ neutron, which resulted in the introduction of the admixture of *D*-state for the *S*-ground state. Horie *et al.* (see *Comment*) have recently calculated the energy levels of the light nuclei by taking into account the contribution of the tensor force successfully. This gives us a new impetus to proceed further in this direction, before the over-all interactions of the nucleons with the whole core[13] will be taken into account.

2. Beta-Decay

We have treated the following two problems on the theory of beta-decay:

I) Field theory of beta-decay. Sakata and his collaborators in Nagoya University who have developed the systematic survey of interactions of the elementary particles in general with a very creative spirit, investigated the phenomenological Fermi interaction of beta-decay as a singular agent which can not be renormalized (See *Comment by S. Sakata and H. Umezana* P. 18). Recently, Tanikawa and H. Umezawa *et al.* (See *Comment*) invented the formalism of beta-decay which seems consistent with some of the evidences of forbidden spectra and which meets the philosophy of renormalisation through the virtual emission and reabsorption of a boson which couples a nucleon and a lepton with the scalar and pseudoscalar interactions, simultaneously. On the other hand, J. Yukawa *et al.* (See *Comment*) pointed out that greater generality of the Fermi interactions might be conceivable if one takes the coupling constants to be complex.

II) Analysis of experiments. Taketani *et al*[14]. advanced the analysis of beta-decay on the ground of the phenomenological Fermi theory in his substantialistic method. Thus Kotani[15] *et al.* calculated the proton momentum spectrum and the proton-electron angular correlation of the beta-decay of the neutron by taking five types of the Fermi interaction into consideration. Here emphasis was laid on the peculiar contribution of the pseudoscalar interaction. On the other hand, the puzzling deviation of the RaE spectrum from the Fermi distribution have motiviated us to carry out the expression for the spectrum including higher order processes. It turned out, as a result, that neither radiative[16] nor mesonic[17] corrections can disturb the usual formula.

The recent development of the shell model[18] and the full arrangement of the *ft* tables[19] has helped enormously in the classification of beta-decay. It was surprising to note that the ft values found the for allowed, first forbidden, and second forbidden transitions differ each other by a factor of as much as $10^4 \sim 10^5$.

On the other hand, interpretations of beta-decay matrix elements, forbidden spectra[20], electron-neutrino and electron-gamma angular correlation and the related phenomena in terms of linear combinations of five forms of the Fermi interaction *i.e.*, scalar (S), vector (V), tensor (T), axial vector (A), and pseudo scalar (P), have been widely attacked. The most stimulating conclusions so far obtained may be summarized as follow:*

1) In addition to the Gamow-Teller selection rules, the Fermi selection rules should be necessary. Evidences: Allowed beta-decay of O^{14}, C^{10} ($o+ \to o+$), and the matrix elements for light nuclei[21].

2) The Fermi term is of S and not of V interaction.
Evidences: First forbidden spectra which have the allowed shape:[22] Pr^{143}, Pm^{147} *etc.*

3) The Gamow-Teller term is predominantly of T and not of A interaction.
Evidence: The electron-neutrino angular correlation[23] of He^6.

4) The linear combination of T and P interactions is required.
Evidence: RaE spectrum[24].

Recently we have attempted to reinvestigate these remarkable results by improving the theory in the following several points:

a) The finite de Broglie wave length effect[25], and modifications of the spectrum by matrix elements have been taken into account. b) The finite size effect given by Rose *et al.*[26] has been improved by Takebe to modify each Dirac component separately. c) Electron-neutrino and electron-gamma angular correlations in the forbidden transitions given by Yamada and Morita[27] are used if the relevant experiments are available. d) Frequent references to the systematic survey of ft values by the real life-time forbidden correction factors by Taketani[28] *et al.* are made.

We limit ourselves to the following problems for the present investigation:

1) Odd mass nuclei, Rb^{87} (the third forbidden), Cs^{137}, Tc^{99}, Fe^{59} (the second forbidden), Pr^{143}, Pm^{147} (the first forbidden).

In explaining these spectra in terms of the Fermi theory in general, trouble is the uncertain ratios of the nuclear matrix elements involved. We first utilize the following simple non-relativistic approximations[29] including the effects of the Coulomb force and the specific nuclear force:

$$Qn(\beta\alpha,\ r) = (\alpha Z/2\rho)(A/n)\ Qn(\beta\sigma \times r,\ r) \tag{1}$$
$$Qn(\alpha,\ r) = (\alpha Z/2\Lambda)(A/n)\ Qn(r,\ r) \tag{2}$$

Here Λ and Λ' will be taken as unknown parameters to be detemined by ex-

*) Fierz condition prefers, of the possible linear combinations of S, V, T, A, P, exclusively (S, A, P), (S, T, P), (V, A, P) and (V, T, P), (see ref. 22)

periments. Qn's are the nuclear matrix elements defined by Greuling[30]. Relations obtained by the independent particle approximation and those improved by the extended j-j coupling model are also used.

It is a striking feature that, in some cases (Rb^{87} and Cs^{137}) the same algebraic sign for S and T is definitly ruled out by Yamada, Morita and Fujita's[33] analysis., if we assume (S, T, P).

2) Even mass nuclei: The reinvestigation of RaE has been worked out by Yamada and Takebe (See *Commet.*) The beta-gamma angular correlation of Sb^{124} and electron-neutrino angular correlation of He^6 and P^{32} have been analysed by Morita (See *Comment*). They have succeeded in part to narrow down the choice of linear combinations of the Fermi interactions, but the final decision may be postponed the more accurate knowledge concerning nuclear matrix elements, and also the spins, in the cases of RaE, Sb^{124}, and P^{32}, could be obtained. (See Table 1).

In conclusion, a favorable law of beta-decay is determined by the analysis of the forbidden spectra:

$$H = g_s S + g_t T + g_p P,\quad -1 \leq g_s/g_t \leq -\frac{1}{2}$$
$$-59 < g_p/g_t < 0,\ \text{or}\ 0 < g_p/g_t < 19$$

However, the ratios of several nuclear matrix elements should be taken as quite a different value than those obtained by Ahrens, Feenberg.[29] These results are more to be deemed as the error in the non relativistic approximation in computing them than that originating from using (S, T, P) group. The relativistic effects[34] in the nucleon motion, and the collective motions[35),36] in nuclei may be important in this problem.

TABLE 1.

i) The law of beta-decay

Elements	Selection rule	Favorable combination	Excluded combination
RaE	$1- \to 0+$	(S, T), (V, A)*	(V, T), (S, A)
	$0- \to 0+$	(T, P)	
Sb^{124}	$3- \to 2+$	(S. T)**	
	$4+ \to 2+$		(S, T)
P^{32}	$1+ \to 0+$	(S, T), (V, A)	(V, A) (S, A)
	$0+ \to 0+$	(V, T), (V, A)	(S, T), (S, A)

* Contribution of the finite de Broglie wave length etc. change the earlier conclusion[24] (Yamada, *comment.*

** Contribution of B_{ij}. term is required for the β—γ angular correlation.

*** $|g_a/g_t| < 0.18$ (Morita, *Comment*).

ii) A favorable combination of Fermi interactions

$$H = g_s S + g_t T + g_p P$$

Elements	Selection rule	Necessary conditions
O^{10}, C^{14}	$0+ \rightarrow 0+$	$1/2 \leqq \lvert g_s/g_t \rvert \leqq 1$* } $-1 \leqq g_s/g_t \leqq -1/2$****
Rb^{87} etc.	$p_{3/2} \rightarrow g_{9/2}$	$g_s/g_t < 0$**
RaE	$0- \rightarrow 0+$	$g_p/g_t < 0$,*** $0 < g_p/g_t < 52$
He^6, B^{12}	$1+ \rightarrow 0+$	$-52 < g_p/g_t < 19$*****

* The condition covers the best value region[31) 32)] for the matrix elements of mirror nuclei.

** The ratios of matrix elements Λ do not agree with non-relativistic approximations, by Ahrens and Feenberg and Yamada.[29)]

*** $g_p/g_t < 0$ requires $\int \beta\gamma_5/i \int \boldsymbol{\sigma}\cdot\boldsymbol{r} > 0.09$ which violates the non-relativistic approximations, by Ahrens, Feenberg, and Primakoff. (P. R. 87,663 (1952)). (Takebe, *Comment.*)

**** Our necessary condition include the combination $S - T + P$ derived from the meson theory of the beta-decay by Tanikawa, (*Comment*).

***** The P interaction should not disturb the allowed spectra (Fujita and Yamada).

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Note on the Beta-Decay

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(Abstract not yet received, Aug. 20, 1953)

On the Interaction Forms of the Beta-Decay

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In formulating the interaction term of the beta-decay, we might expect several possibilities of interaction forms; namely the Fermi (F) types of direct interactions between nucleons and leptons, the Yukawa (Y) theory of beta-decay in which a Bose field is introduced as an intermediary agent of nuclear beta-decay. The extensive investigations have been aimed at narrowing down the choise of this interaction form. In this paper, we choose a formulation of the beta-decay on the basis of the following postulates:

(a) The primary interactions between nucleons, leptons and bosons, by which the beta-decay interactions are derived, are renormalizable.

(b) The Fermi and the Gamow-Teller (G-T) interactions are approximately equally present in the beta-decay matrix elements.

Neither F- nor Y-theory satisfies these postulates. It was recognized that the F direct interactions, the Y tensor interaction of vector meson and the pseudovector interaction of pseudoscalar meson with nucleons or leptons, giving rise to the G-T selection rule, are not renormalizable.

An alternative theory of beta-decay would satisfy the postulates. The basic idea of the theory consists in such assumptions that as sources of a Bose field, there exist not usual pairs of nucleons or leptons, but pairs of a nucleon and a lepton.

The primary interactions of scalar or pseudoscalar field with nucleons and leptons are given by

$$H_S' = g(\bar{\psi}_P\psi_\nu)\varphi^* + f(\bar{\psi}_e\psi_N)\varphi + \text{compl. conj.} \qquad (1)$$

or

$$H_{Ps}' = g'(\bar{\psi}_P\gamma_5\psi_\nu)\varphi^{*\prime} + f'(\bar{\psi}_e\gamma_5\psi_N)\varphi' + \text{compl. conj.} \qquad (2)$$

with the usual notations. These interactions are renormalizable. The interaction (1) and (2) leads to the beta-decay Hamiltonian

$$H_\beta = G\iint(\bar{\psi}_P(r_1)O\psi_\nu(r_1))F(r)(\bar{\psi}_e(r_2)O\psi_N(r_2))\,d\,r_1\,d\,r_2 \qquad (3)$$

where $r = r_1 - r_2$, $O = 1$ or γ_5, $G = gf$ or $g'f'$ for (1) and (2) respectively. $F(r)$ is an approximate Green function of Bose field. The nucleon stability requires larger mass of the boson than the nucleon mass. If we assume the mass as large as 2000 electron mass, $F(r)$ may be replaced by const. $\times\,\delta(r_1 - r_2)$, then (3) becomes a definite combination of the five F interactions.

It is well known that $(S+V)$ or $(T+A)$ combinations of the F type interactions give rise to the Fierz interference factor which is excluded by the recent experimental analyses. In our theory, however, the parts of V and A interactions can be canceled out, if we assume a mixed theory of scalar and pseudoscalar interaction (1) and (2), with the condition

$$gf = g'f' \qquad (4)$$

and the equal masses of scalar and pseudoscalar boson. Thus our theory becomes to be equivalent to the F theory of $(S+T+P)$ interactions with equal weights.

The transformation property under a space inversion of fermion wave functions leads to a possible interaction form other than (1) or (2). Following Yang and Tiomno, if it turns out that protons, neutrons and electrons are all of type A, while the neutrino is of type B, we have the following interactions instead of (1) and (2)

$$H_S' = g(\bar{\psi}_P\gamma_5\psi_\nu)\varphi^* + f(\bar{\psi}_e\psi_N)\varphi + \text{compl. conj.}, \qquad (5)$$

or

$$H_{Ps}' = g'(\bar{\psi}_P\psi_\nu)\varphi^{*\prime} + f'(\bar{\psi}_e\gamma_5\psi_N)\varphi' + \text{compl. conj.} \qquad (6)$$

On the same footing mentioned, we assume a mixed theory of (5) and (6), then the Fierz factor vanishes. The beta-decay Hamiltonian H_β corresponds to a definite combination of the Yang-Tiomno direct interaction $(S'+P'-T')$. S', P' and T' are obtained by replacing the neutrino wave function ψ_ν in the F-type interactions S, P and T by $\gamma_5\psi_\nu$.

The recent experimental analyses seem to suggest the $(S\pm T+P)$ combination. Our theory might give some reasoning of this combination.

This work resulted from discussions with H. Umezawa, S. Morita, S. Tanaka and communications with S. Nakamura.

Nuclear Moments and j-j Coupling Shell Model (*Comment*)

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Recently, it is shown that nuclear magnetic moments can be accounted for by the charge symmetric j-j coupling shell model in which the internuclear inteaction is assumed to be charge symmetric and the seniority number is a good quantum number. This seniority number can be defined in three different ways, as Racah has shown in his famous paper, that is, in tensor form or by introducing simplectic subgroup or by making use of following operator Q

$$Q=\sum q_{ij},\ (j^2JM|q_{ij}|j^2JM)=(2j+1)\delta(J,O)$$

Thus, it is very important to investigate if the nuclear force conserves the seniority number. As shown by Racah, the seniority number is a good quantum number in the case in which the internuclear interaction is ordinary and extreme short range force, that is, $\delta(r_1-r_2)$ interaction. However, judging from recent experimental evidence on the nuclear force, the nuclear interaction seems not to be ordinary one. At present, our knowledge on the internuclear interaction is far from accurate, so it is very difficult problem to see theoretically if the seniority number should be a good quatum number.

In spite of this difficulty it can be shown here that the magnetic moments of unsymmetric odd-odd nuclei are accounted for by the *j-j* coupling shell model. The magnetic moments of Na^{24} and K^{42} which have been measured recently, can be calculated easily by the method shown in reference 1, according to the *j-j* coupling shell model. The results are follows.

	Configuration	J	T	T	Primitive structure	μ_{cal}	μ_{exp}
N^{24}	$(d_{5/2})^{-4}$	4	1	1	(4, 1)	1.80	1.688
					(2, 1)	3.21	
K^{42}	$(d_{3/2})^{-1}(f_{7/2})^3$	2	2	2	(4, 1)	−1.0	−1.137
					(2, 1)	−1.73	

In spite of the success in explaining the nuclear magnetic moments, it is impossible to explain the nuclear quadrupole moment by the pure *j-j* coupling shell model.

According to the *j j* coupling shell model, the quadrupole moments must have following two features.

1) The quadrupole moments of the configuration $(j)^n$ and $(j)^{-n}$ have the same magnitude but opposite sign.

2) Closed shell+one (−one) nucleon nuclei has a negative (positive) quadrupole moment.

The middle and heavy nuclei have large positive experimental quadrupole moments in many cases but have small negative quadrupole moments only in the vicinity of closed shell nuclei and have no large negative quadrupole moment. However according to the *j-j* coupling shell model, if some nuclei have large positive quadrupole moments there must be some other nuclei which have large negative quadrupole moments.

Moreover it seems that these large quadrupole moment is too large to be explainable by the *j-j* coupling shell model. For example Lu^{175} has the following large quadrupole moment.

$$Q=5.9\times10^{-26}$$

This is about six times as large as the maximum value of the quadrupole

moment of one nucleon in the orbit of radius $r=A^{1/3}r_0$. The various kind of nuclear wave function was tried to calculate the quadrupole moment. However, it was found no nuclear wave function has the quadrupole moment several times as large as the maximum quadrupole moment of one nucleon. Thus it seems that the quadrupole moment of middle and heavy nuclei can not be explained by the *j-j* coupling shell model.

In the case of light nuclei, experimental value of the nuclear quadrupole moment is nearly the same as that predicted from the *j-j* coupling shell model. However, Li^7 and K^{39} has the quadrupole moment of the opposite sign from the calculated value. Moreover, it was shown by Towns and Moszkowski that Na^{23}, which has five nucleons in unclosed outermost orbit, has rather large quadrupole moment than others in the neighbourhood, and the calculated quadrupole moment is about ten times smaller than others in the neighbourhood in contradiction with the experiment. Then it appears to be necessary to take into account something other than pure *j-j* coupling shell model.

It is a very interesting problem if it is possible to explain the nuclear excited state by the *j-j* coupling shell model. This problem can not be solved, unless the precise internuclear interaction is known. Therefore it is only possible to investigate the lower excited state semi-phenomenologicaly.

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Effects of the Tensor Forces on the Energy Levels of Light Nuclei (*Comment*)

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From the various evidences about two-nucleon data, the existence of the tensor interactions between two nucleons seems to be almost decisive. We have tried calculations of the energy levels for several light nuclei taking the tensor forces into account, especially for nuclei with p^n configurations as well as some of odd-odd nuclei, provided the independent particle shell model is valid for them.

The interaction between two nucleons is assumed to be generally of the form

$$V_{12}=(W+BP_\sigma-HP_\tau-MP_\sigma P_\tau)V^c(r_{12})+(W^t-H^tP_\tau)S_{12}V^t(r_{12}),$$

where $V^c(r_{12})$ and $V^t(r_{12})$ are central potentials which depend only on the relative distance r_{12} of the two nucleons, and the tensor operator S_{12} is given by

$$S_{12}=3(\boldsymbol{\sigma}_1\cdot\boldsymbol{r}_{12})(\boldsymbol{\sigma}_2\cdot\boldsymbol{r}_{12})r_{12}^{-2}-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$$

the other qnantities have usual meanings.

For the wave functions of the individual nucleons in incmplete shells, the functions

$$R_l(r)=Nr^l\exp(-\nu r^2/2)$$

with normalization

$$\int_0^\infty R_l^2(r)r^2dr=1$$

are employed throughout the whole work, while the Yukawa and square well potentials are alternatively assumed as the radial dependences of the central and the tensor potentials, $V^c(r_{12})$ and $V^t(r_{12})$. The depths and ranges of these potentials as well as the ratio between the four exchange-type forces were determined from the results of Pease and Feshbach[1] for the former case (Yukawa potential) and from those of Padfield[2] for the latter case (square well potential).

M. Sasaki and T. Komoda[3] have also carried out similar calculations for Li^6 (p^2 configuration) with Gaussian potentials and have shown a satisfactory agreement with experiment, at least concerning the magnetic moment of the ground state and the interval between the $^3D_1$ and $^3S_1$ (ground) levels. It should be noticed however that the $^1S_0$ level falls below the ground level, as far as the force constants should be determined by Pease and Feshbach[1].

With the force constants adjusted for Li^6 the energy levels are calculated for several other nuclei, *i.e.*, the nuclei with higher p^n configurations and some of odd-odd nuclei. For the evaluation of the matrix elements of the tensor interaction, the expressions given by Talmi[4] were used and the generalized Slater integrals were also evaluated by the full use of the Talmi's procedure[5]. Numerical calculations are now in progress.

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On the Pseudoscalar Interaction in the Theory of Beta-Decay and RaE (*Comment*)

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To explain the beta-ray spectrum of RaE Petshek and Marshak have considered the contribution of the pseudoscalar interaction. Considering the pseudoscalar interaction, however, if the nuclear matrix element $\int\beta\gamma_5$ is small as is shown by Ahrens and Feenberg, we should proceed to the approximation of the next order. We have deduced the expressions for the correction factors of the interactions (T, P) and (A, P) for the first forbidden transition $0\longleftrightarrow 0$ (yes), giveng careful consideration to the effect of the next order approximation mentioned above and that of the finite size of the nucleus. Taking as the parameters $\lambda=\lambda_T/(\lambda_p/2M)$ and $\Gamma/\rho=-2iM\int\beta\gamma_5/\int\boldsymbol{\sigma}\cdot\boldsymbol{r}$, we have applied the expressions for (T, P) to the beta-ray spectrum of RaE. We have been able to determine the domains for λ and Γ/ρ separately, although not independently.

On the Beta-spectrum of RaE (*Comment*)

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The β-spectrum of RaE has been investigated taking into acconnt the finite de Broglie wavelength effect pointed out by Rose and Perry (Phys. Rev. **90**, 479) and other corrections, and the results which differ from those of Petschek and Marshak (Phys. Rev. **85**, 698) have been obtained. Our results are as follows: in the cases of ST 1–0 (scalar+tensor, spin change $1\to 0$), VA 1–0 and TP 0–0, very strongly energy dependent correction factors can be easily obtained, and choosing the ratios among the nuclear matrix elements suitably, correction factors necessary for RaE can be obtained. In the cases of VT 1–0, A 0–0 and AP 0–0, it is difficult to get strongly energy dependent correction factors and they seem unlikely, though they can not decisively excluded on account of many other corrections. Other cases can not fit the experiments.

The effect of finite nuclear size seems unimportant to get a strongly energy dependent correction factor. If we use the approximation $\alpha Z \ll 1$, it can be easily shown that the effect of finite nuclear size has no effect to get a correction factor with any shape, as far as nuclear matrix elements are treated as adjustable parameters.

Recently it has become probable that the G-T type interaction is tensor. Therefore, according to the above results, if the spin of RaE is one, the β-decay of RaE will take place with ST 1–0, and if the spin of RsE is zero, with TP 0–0. In the latter case the pseudoscalar interaction is necessary. However, the pseudoscalar coupling constant can not be so large as was proposed by Ahrens, Feenberg and Primakoff (Phys. Rev. **87**, 663). Such a large coupling constant is inconsistent with the allowed shape of high energy β-spectra (He^6 and B^{12}). It has been obtained that the ratio of the pseudoscalar coupling constant to that of tensor must lie in the region

$$-55 < G_p/G_t < 19$$

From the above results and that of Ruderman (Phys. Rev. **89**, 1227), the interaction of π-mesons and nucleons will be pseudoscalar type, if the spin of RaE is zero.

β-γ Angular Correlation of Sb^{124} and β-ν Angular Correlation of P^{32} (*Comment*)

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1. A consistant explanation has been obtained for both the β-γ angular correlation and the β-ray spectrum of Sb^{124}. It is concluded that the transition scheme, ground state of Sb^{124}→first excited state of Te^{124}→ground state of Te^{124}, is $3_- \to 2_+ \to 0_+$, where the β-decay is explained by the first forbidden transition in ST type interaction of the Fermi theory and the γ-decay is an electric quadrupole radiation which was measured by Langer[1] and Metzger[2].

We introuduce real parameters x, y and z as follows:

$$\frac{G_S \mathfrak{M}(\beta \boldsymbol{r})}{G_T \mathfrak{M}(\beta \boldsymbol{\sigma} \times \boldsymbol{r})} = -ix, \quad \frac{\mathfrak{M}(\beta \boldsymbol{\alpha})}{\mathfrak{M}(\beta \boldsymbol{\sigma} \times \boldsymbol{r})} = \frac{\alpha Z}{2\rho} y \quad \text{and} \quad \frac{\mathfrak{M}(B_{ij}{}^{\beta})}{\mathfrak{M}(\beta \boldsymbol{\sigma} \times \boldsymbol{r})} = iz$$

A linear combination of four nuclear matrix elements is symbolized by $(x, y, z)_1$.

Recently, Langer *et al*[1] Reported that a linear Combination of $\mathfrak{M}(\beta \boldsymbol{r})$, $\mathfrak{M}(\beta \boldsymbol{\alpha})$ and $\mathfrak{M}(\beta \boldsymbol{\sigma} \times \boldsymbol{r})$, $(1, 1, 0)_1$, has a good correction factor for β-ray spectrum. Unfortunately, $(1, 1, 0)_1$ has not a good angular correlation. Whatever the ratios x and y may be, without $\mathfrak{M}(B_{ij}{}^{\beta})$ the calculated angular correlations are not so large in absolute value as experimental one.

In order to remedy this defect, there is a possible way, namely, to use the linear combination of $\mathfrak{M}(B_{ij}{}^{\beta})$ and $(1, 1, 0)_1$, *i. e.* $(1, 1, z)_1$. The interferences between them will cause an advantageous effect on the angular correlation and do not exist in the correction factor. This combination makes Kurie plot straight because each of $\mathfrak{M}(B_{ij}{}^{\beta})$ and $(1, 1, 0)_1$ has this property. The suitable ratio z is $-6 > z > -9$. Another example is $(1, 1.54, z)_1$ where $-4.1 > z > -5.7$.

In these two cases the corrected ft values for $\mathfrak{M}(B_{ij}{}^{\beta})$ are of order of 10^{10}. Considering the shell structure we can prove large value of z.

If β-decay is second forbidden[2], the decay scheme is $4_+ \to 2_+ \to 0_+$. Generally speaking, when we make the angular correlation large in the high energy region of the electron, the Kurie plot is not straight and *vice versa*. The situation becomes worse in ST type than in T type only[3] by mixing $\mathfrak{M}(R_{ij}{}^{\beta})$.

2. We can make $1/W$ term in allowed spectrum vanish by putting

$$G_A G_T = 0 \quad \text{if } G_i\text{'s are real,}$$
$$\text{or} \quad G_A/G_T = g \exp(i\pi/2) \quad \text{if } G_i\text{'s are complex,}$$

where g is real. In the latter case β-ν angular correlation is

$$\mathfrak{M}_{0TA} = 1 + \frac{p}{3W} f(g) \cos\theta, \quad \text{where} \quad f(g) = \frac{1 + (6\alpha Z/p) g - g^2}{1 + g^2}$$

The data on P^{32} show $f_{exp.}(g) \gtrsim 1 \ (W = 2.2\, mc^2)$[4], which brings $|g| \lesssim 0.2$.

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Some Remarks on the Theory of Beta-Disintegration (*Comment*)

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Recently, some interesting contributions have been made by several researchers in this university to the development in the theory of beta-disintegration, in which two directions of approach may be discerned.

1. Three of their works have been done from the viewpoint of elementary process.

Radiative correction

The mesonic correction to beta-decay matrix element was considered by T. Kotani *et al.*[1] and its radiative one by *T.* Nakano *et al.*[2] in the lowest order approximation. S. Kamefuchi[3] has further analysed the latter question in the higher order approximations. He reached the sufficient cnnclusion that the divergences coming from beta-vertex can be removed in appropriate way, and those arising from internal lines and electromagnetics can be renormalized in the same way as in quantum electrodynamics, so that, in general, all the divergences are completely removed by the method of renormalization. He has also obtained a necessary and sufficient condition for the compensation of divergent terms to be possible without use of renormalization.

Fermi type? Yukawa type? or *Tanikawa type?*

S. Tanaka noted in his interesting paper[4] that it may be desirable to look for the theory, which is first renormalizable and further able to give the various Fermi's beta-couplings. Such a possibility may be found in the early works of Y. Tanikawa[5], where the boson field was considered as the charged scalar or pseudoscalar, and as having a heavier mass than a nucleon.

The interesting conclusion reached by S. Tanaka is that, from the theory of Tanikawa type, $(S+T+P)$ combination of beta-interactions is obtained, assuming the beta-transition to be caused by the intermediation $S(S)$ and $PS(PS)$ boson fields.

Ambiguity in the phase of coupling constants.

J. Yukawa, H. Umezawa and M. Morita have indicated the ambiguity in the phase of Fermi coupling constants, taking into account the demand for invariance under time reversal, and reexamined the beta-spectrum and beta-neutrino angular correlation. There are two alternative ways of time reversal transformation; one is Pauli type, which gives no restriction to the phases in beta-coupling constants, and the other, Wigner type which restricts them. However, there is no reason why either of two transformations should be taken. Therefore, there still remains the ambiguity in coupling constants.

2. The last one has been investigated from the view point of nuclear structure.

Nuclear Matrix Elements

K. Izumo and J. Yukawa[6] were the first to use the approximate solution of Dirac wave equation for a nucleon moving in the oscillator potential in order to evaluate the beta-decay nuclear matrix elements on the basis of single particle model. They have compared their results with the experimental data and with Brysk's[7] results in the case of square well potential. Their conclusion is as follows: (*a*) for transition via α- or γ_5-type interaction, the results yield better agreement with the data, (*b*) for the first forbidden transition via r-type interaction, experimental results favor Brysk's results more than their results, and (*c*) the fluctuations of ft values in Δl-forbidden transition still remain an unexplained problem, and generally the problem of unfavored factor is that remained in the future.

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On the Problem of Convergence in the Non-Local Field Theories

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Yukawa's attempts to overcome the divergence difficulties of the usual field theories by introduction of the notion of non-local fields have given renewed interest to the theories of local fields in non-local interaction since the theory of non-local fields in many respects are equivalent to theories of the latter type. The most important question is now if such theories really can be made convergent, i.e. if all the matrix elements of any field quantity are well-defined quantities independently of any approximation treatment, and if this over-all convergence is reconcilable with other necessary requirements. For definiteness let us consider a theory of neutral scalar mesons in non-local interaction with nucleons of the type developed by C.

Bloch[1] and by P. Kristensen and C. Møller[2]. The field equations of this system are of the form

$$(\gamma_\mu \partial/\partial x_\mu' + M)\psi(x') = -g\int F(x', x'', x''')u(x'')\psi(x''')dx''dx''' \tag{1a}$$

$$(\Box'' - m^2)u(x'') = g\int F(x', x'', x''')\bar{\psi}(x')\psi(x''')dx'dx''' \tag{1b}$$

where $F(x', x'', x''')$ is a c-number form factor depending on three space-time points describing the non-local character of the interaction and ψ, $\bar{\psi}$ are the q-number field variables of the nucleons while u is the Hermitean field variable of the scalar neutral mesons. At first sight such a theory does not seem to be of the Hamiltonian type since the variables ψ, $\bar{\psi}$, u, $\dot{u}$ are not canonical variables as in the corresponding local theory. The method of quantization adopted in the papers quoted above was therefore closely analogous to the method of Yang and Feldman and Källén[3] based on an S-matrix treatment of quantum electrodynamics. However, in a recent paper Pauli[4] has shown that the formalism (1) is intrinsically Hamiltonian and that an energy-momentum vector

$$P_\mu = P_\mu^0 + P_\mu^{int} \tag{2}$$

can be defined which is constant in time in virtue of the field equations (1). P_μ^0 is the energy-momentum vector of free fields while P_μ^{int} is the interaction part involving space-time integrals containing the form factor.

Pauli could further show that canonical variables exist which are certain functionals of the field variable ψ, $\bar{\psi}$, u, $\dot{u}$ and which satisfy the usual canonical commutation relations. For any displacement invariant field quantity $F(x)$ we therefore have the usual equations

$$i\,\partial F(x)/\partial x_\mu = [P_\mu, F(x)]. \tag{3}$$

Now, there are certain properties which one would like this form factur theory to have:

1) *Relativistic invariance*. This condition requires that F must be a rotational invariant function of the four-dimensional distances $x'-x''$, $x'-x'''$, only. Consequently the Fourier expansion of $F(x', x'', x''')$ must have the form

$$F(x', x'', x''') = (2\pi)^{-8}\int G(l^1, l^3)\exp\{i[l^1x' + l^3x''' - (l^1+l^3)x'']\}dl^1, dl^3, \tag{4}$$

where $G(l^1, l^3)$ is an invariant function of the two four vectors l^1, l^3. Hence, G is a function of f. inst. $(l^1)^2$, $(l^3)^2$ and $(l^1\cdot l^3)$ only.

2) *Reality requirment*. This leads to the condition

$$F(x', x'', x''') = F^*(x''', x'', x') \tag{5}$$

or

$$G(l^1, l^3) = G^*(-l^3, -l^1). \tag{6}$$

3) *Correspondence*. For slowly varying fields, i.e. if the field functions only contain waves of wave length large compared with a certain constant λ of the dimensions of a length the form factor is effectively a δ-function like in the local theory.

4) The form factor depends on the constant λ in such a way that $F(\lambda; x', x'', x''')$ in the limit $\lambda \to 0$ goes over into the corresponding quantity of the local theory, i.e.

$$\lim_{\lambda\to 0} F(\lambda; x', x'', x''') = \delta(x'-x'')\delta(x'-x''').$$

5) *Causality in the large:* If Ω and Ω' are two clearly sparated domains in space time whose linear dimensions are large compared with λ then any signal transmitted from Ω to Ω' should take place with velocity smaller than c and further the absorption process should occur later than the emission process.

6) *Convergence*. All selfenergies, all transition probabilities, all matrix elements of dynamical variables are finite for reasonable finite values of λ.

Among these requirements 1), 2), 5) and 6) are necessary conditions for the theory. If on the other hand also 3) and 4) are satisfied the theory could be used as a consequent way of carrying through the renormalization program for the so-called renormalizable systems. In fact, for finite λ the renormalization constants would be well defined finite quantities and after the renormalization has been performed we could let $\lambda \to 0$ in the renormalized field equations.

As regards the question which of the conditions 1)—6) can be satisfied simultaneously a closer investigation shows that it is easy to find form factors which satisfy 1)—5) and which remove many of the most disturbing divergences of the local theory. However, it was shown first by C. Bloch[1] that in order to be sure to get convergence of all terms in a series expansion in powers of the coupling constant g it is necessary to assume that $G(l^1, l^3)$ vanishes whenever one of the vectors l^1, l^3, l^1+l^3 are space-like. This can also be shown quite generally without use of series expansions. Followihg a proof given by P. Kristensen along the line developed by Källén in his paper on the exact formulation of the renormalization method in quantum electrodynamics[5] this may be seen in the following way. Condition 6) requires that every matrix element of any field variable must be finite. Let us first consider the matrix element $<a|u(x)|b>$ of $u(x)$ connecting to states a ann b with the values p^a and p^b, respectively, for the energy-momentum of the whole system. From (3) we get for any field variable $F(x)$

$$i\frac{\partial}{\partial x_\mu}<a|F(x)|b> = (p^a - p^b)_\mu \cdot <a|F(x)|b>. \tag{7}$$

Hence

$$<a|F(x)|b> = e^{i(p^b - p^a)x}<a|F|b>, \tag{8}$$

where $<a|F|b>$ is the matrix element of the field variable $F = F(0)$ at the origin of the space-time system. Applying (8) to the field variable $u(x)$ and introducing into the equation (1b) we get

$$<a|u(x)|b> = -\frac{\mathscr{F}_{ab}(x)}{(p^b - p^a)^2 + m^2}, \tag{9}$$

where

$$\mathscr{F}_{ab}(\boldsymbol{x}) = g\int F(\boldsymbol{x}', \boldsymbol{x}, \boldsymbol{x}''')<a|\bar{\Psi}(\boldsymbol{x}')\Psi(\boldsymbol{x}''')|b>d\boldsymbol{x}'d\boldsymbol{x}''' \tag{10}$$

is the matrix element of the nucleon source on the right hand side of Eq (1*b*).

Applying (9) to $\bar{\psi}(\boldsymbol{x}')$ and $\psi(\boldsymbol{x}''')$ we get

$$<a|\bar{\psi}(x')\psi(x''')|b> = \int d\boldsymbol{p}e^{i(p-p^a)x'+i(p^b-p)x'''}\sum_s <a|\bar{\psi}|\boldsymbol{p}s><\boldsymbol{p}s|\psi|b> \tag{11}$$

where s denotes the set of variables which together with the total energy-momentum vector $\boldsymbol{p}$ determines the intermediate state in the product of $\bar{\psi}$ and ψ. Using (4) and (11) in (10) and (9) we get after integration over $\boldsymbol{x}'$, $\boldsymbol{x}'''$, $\boldsymbol{l}^1$ and $\boldsymbol{l}^3$

$$<a|u(\boldsymbol{x})|b> = -e^{i(p^b-p^a)x}g\int d\boldsymbol{p}G(\boldsymbol{p}^a-\boldsymbol{p}, \boldsymbol{p}-\boldsymbol{p}^b)\mathfrak{f}_{ab}{}^{(1)}(\boldsymbol{p}^a, \boldsymbol{p}, \boldsymbol{p}^b), \tag{12}$$

with

$$\mathfrak{f}_{ab}{}^{(1)}(\boldsymbol{p}^a, \boldsymbol{p}, \boldsymbol{p}^b) = \sum_s \frac{<a|\bar{\psi}|\boldsymbol{p}s><\boldsymbol{p}s|\psi|b>}{(\boldsymbol{p}^b-\boldsymbol{p}^a)^2+m^2}. \tag{13}$$

By the same procedure one finds from Eq. (1a)

$$<a|\psi(\boldsymbol{x})|b> = e^{i(p^b-p^a)x}\int G(\boldsymbol{p}^b-\boldsymbol{p}^a, \boldsymbol{p}-\boldsymbol{p}^b)\mathfrak{f}_{ab}{}^{(2)}(\boldsymbol{p}^a, \boldsymbol{p}, \boldsymbol{p}^b)d\boldsymbol{p} \tag{14}$$

with

$$\mathfrak{f}_{ab}{}^{(2)}(\boldsymbol{p}^a, \boldsymbol{p}, \boldsymbol{p}^b) = \sum_s \frac{<a|u|\boldsymbol{p}s><\boldsymbol{p}s|(i\gamma_\mu(\boldsymbol{p}^b-\boldsymbol{p}^a)_\mu - M)\psi|b>}{(\boldsymbol{p}^b-\boldsymbol{p}^a)^2+M^2}. \tag{15}$$

In the local theory where $G=1$ the integrals on the right hand sides of (12) and (14) are divergent even if $\mathfrak{f}_{ab}{}^{(1)}$, $\mathfrak{f}_{ab}{}^{(2)}$ are assumed to be finite. The question is now if a form factor G can be found which assures convergence. Let us first consider the integral in (14) which may be written

$$\int G(\boldsymbol{l}^1, \boldsymbol{l}^3)\mathfrak{f}_{ab}{}^{(2)}(\boldsymbol{p}^a, \boldsymbol{l}^3+\boldsymbol{p}^b, \boldsymbol{p}^b)d\boldsymbol{l}^3 \tag{16}$$

where we have put $\boldsymbol{l}^1=\boldsymbol{p}^b-\boldsymbol{p}^a$. Since, according to condition 1), $G(\boldsymbol{l}^1, \boldsymbol{l}^3)$ can be a function only of $(\boldsymbol{l}^1)^2$, $(\boldsymbol{l}^3)^2$ and $(\boldsymbol{l}^1\cdot\boldsymbol{l}^3)$, the best we can do to assure convergence of this integral is to assume that G vanishes for large positive and negative values of $(\boldsymbol{l}^1)^2$, $(\boldsymbol{l}^3)^2$ and $(\boldsymbol{l}^1\cdot\boldsymbol{l}^3)$. If $\boldsymbol{l}^1=\boldsymbol{p}^b-\boldsymbol{p}^a$ is time-like this assumption is easily seen to make the integral convergent provided of course that $\mathfrak{f}_{ab}{}^{(2)}$ is finite.

However, if $\boldsymbol{l}^1$ is space-like the degree of divergence will be somewhat reduced by at least *1* (*and at most 2*). In fact, if we calculate (16) in the frame of reference where $\boldsymbol{l}^1=(l^1{}_1, 0, 0, 0)$ we have $(\boldsymbol{l}^3)^2=(\vec{l}^3)^2-(l^3{}_0)^2$, $(\boldsymbol{l}^1\cdot\boldsymbol{l}^3)=l^1{}_1\cdot l^3{}_1$ and we see that the above assumption only limits the domain of integration for $l^3{}_1$ and for one of the remaining components of the vector $\boldsymbol{l}^3$ while the two others are unlimited. Thus the integral would be divergent unless $\mathfrak{f}_{ab}{}^{(2)}$ for some peculiar reason vanishes for large values of $l^3{}_0$ or of both $l^3{}_2$ and $l^3{}_3$. There is no reason to believe that this is the case for all states a and b. On the contrary it is known not to be the

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case if $\mathfrak{f}_{ab}{}^{(2)}$ is calculated by a method of series expansions in powers of g. On the other hand $\mathfrak{f}_{ab}{}^{(2)}$ in (15) is finite provided that the matrix elements of u and ψ are finite so that no other sources of divergence come into play than the lack of limitation in the region of integration.

To obtain a convergent expression for the matrix elements $<a|\psi(\boldsymbol{x})|b>$ it is therefore necessary to assume that $G(\boldsymbol{l}^1, \boldsymbol{l}^3)$ for $(\boldsymbol{l}^1)^2>0$. On account of condition 2) G must then also be zero for space-like $\boldsymbol{l}^3$. In a similar way as above one finds that in order to make $<a|u(\boldsymbol{x})|b>$ finite $G(\boldsymbol{l}^1, \boldsymbol{l}^3)$ must vanish also if $\boldsymbol{l}^1+\boldsymbol{l}^3$ is space-like, in accordance with Bloch's result.

It is clear that a form factor satisfying Bloch's condition is not reconcilable with conditions 3) and 4) and probably also invalidates the causality condition 5). Anyhow such a theory would mean a radical change of the usual theory. For instance it is easily seen that the interaction between two nucleons would become zero to the second order in the coupling constants.

It follows from the preceding considerations that Bloch's condition is unavoidable if one wants to assure convergence in a theory with an invariant form factor. The only way to avoid this undesirable feature is to introduce non-invariant form factors. Now it is interesting to remark that the reformulation of the theory of non-local fields recently performed by H. Yukawa[5] leads to a covariant formalism which is equivalent with the introduction of non-invariant form factors. This theory satisfies the covergence requirement, but seems to have other unsatisfactory features.

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Appendix

Researches in Japan on the Theory of Elementary Particles

This is a list of unpublished papers, some of which will be published in the *Progress of Theoretical Physics* or other journals in the near future.

(I) Field Theory

a) Non-Local Theory and the Theory of the Structure of Interactions.

1) Y. Katayama (Kyoto Univ.): Hamilton Formalism in the Non-Local Interaction Theory.
2) C. Hayashi (Naniwa Univ.; Osaka): Hamilton Formalism and Bound States in the Non-Local Interaction Theory.
3) O. Hara, T. Marumori, Y. Oonuki and T. Shimodaira (Nagoya Univ.): On the Theory of the Structure of the Elementary Particles.
4) H. Goto (Gifu Tech. Univ.): Remarks on the Non-Local Field Theory.
5) Y. Ono (Hokkaido Univ.): On the Constants of Motion for the Case of Non-Localized Interactions.
6) T. Hiroshige and I. Inoue (Kyoto Univ.): S-matrix Theory for the Non-Local Field Theory.
7) Y. Murai (Saitama Univ.): On the Mass-spectrum of the Elementary Particles.
8) S. Kamefuchi and H. Umezawa (Nagoya Univ.): A Remark on the Connection between the Interactions of the Second Kind and the Non-Local Interactions.
9) Y. Takano (Yokohama Univ.): New Description of Field.
10) T. Shimose and C. Fujita (Ochanomizu Univ.; Tokyo): On the Theory of Quantization for Particle Dynamics in Non-Local Formalism.
11) H. Enatsu (Kyoto Univ.): On the Relativistic Eigenvalue Problem.
12) T. Nakano (Osaka City Univ.): Relativistic Field Theory of Rigid Sphere.
13) I. Fujiwara (Naniwa Univ.; Osaka): On the β-algebra of Quantized Field Operators.
14) S. Oneda and *H. Umezawa (Kanazawa Univ. and *Nagoya Univ.): On the Connection between the Commutation Relations and the Families of the Fermi Particles.
15) H. Umezawa, S. Kamefuchi and S. Tanaka (Nagoya Univ.): On the Invariance in the Quantum Field Theory.
16) H. Umezawa and Y. Takahashi (Nagoya Univ.): On the Quantization of the General Fields with General Interactions.
17) S. Ozaki, S. Nagata and R. Kitamura (Kyushu Univ.): Integral Quantum Mechanics.
18) R. Kitamura, S. Nagata and S. Ozaki (Kyushu Univ.): On the Relation between Wigner-Yang's Comutation Relation and Classical Equation of Motion.

b) Many Body Problem

1) N. Mugibayashi and *M. Namiki (Osaka Univ. and *Waseda Univ.): Perturbation Theory of Relativistic Two-Body Problems in an External Field.
2) K. Nishijima (Osaka City Univ.): Many-Body Problem in Quantum Field Theory.
3) H. Suura (Hiroshima Univ.): On a Treatment of Many-Body Problems in Quantum Field Theory.
4) S. Tanaka and H. Umezawa (Nagoya Univ.): On the Transition Matrix and the Green Function in the Quantum Field Theory.
5) S. Nakai (Osaka Univ.): On the Green Functions of Many Electron Problem in Quantum Electrodynamics.
6) S. Nakai (Osaka Univ.): Bound States and S-matrix in Quantum Field Theory.
7) H. Kita (Kyoto Univ.): Construction of Stationary State by Adiabatic Process and the Energy Shift.
8) H. Kita (Kyoto Univ.): On Solutions of Heisenberg's Equations of Motion.
9) Y. Munakata (Kyoto Univ.): A Non-Adiabatic Treatment of Relativistic Many Body Problem.
10) H. Tanaka (Kyoto Univ.): Construction of Stationary State by Adiabatic Process and the Energy Shift.

c) Convergence of S-Matrix

1) S. Hori (Kanazawa Univ.): On the Descending Power Expansion of the $S(\eta^*, \eta, J)$ and Green-Functions.
2) H. Tanaka and *C. Hayashi (Kyoto Univ. and *Naniwa Univ.): Stability of System of Coexistent Fields and Convergence of Transformation Functions.
3) S. Ozaki, S. Nagata and R. Kitamura (Kyushu Univ.): On the Sufficient Condition under which the Transformation Function is Obtained in a Closed Form.

(II) Mesons and Nuclear Forces

1) M. Otsuka (Osaka Univ.): On the Effect of High-Energy Mesons to the Self-Field of Nucleons in Pseudoscalar Meson Theory.
2) A. Kanazawa and M. Sugawara (Hokkaido Univ.): Pion-Nucleon Scatterings and Nucleon Isobars.
3) T. Hamada (Hokkaido Univ.): Effect of Nucleon Excited States upon Magnetic Moment Anomaly.
4) N. Nakabayashi, K. Hasegawa and I. Yamamura (Tohoku Univ.): Fourth Order Calculation of Pion-Nucleon Scattering.
5) I. Sato and Y. Ichikawa (Tohoku Univ.): Classical Treatment of Meson Corrections to Scattering of γ-Ray by Nucleons.

6) I. Sato and K. Itabashi (Tohoku Univ.): Meson Correction to Magnetic Moment of Deuteron.
7) N. Fukuda, M. Tanaka and T. Yamanouchi (Tokyo Educ. Univ.): Phase-Shift Analysis of Pion-Nucleon Scattering.

(III) Nuclear Physics

a) Nuclear Reactions

1) Y. Fujimoto, *S. Hayakawa and *K. Nishijima (Kyoto Univ. and *Osaka City Univ.): Theory of Rearrangement Collisions. I. Fundamental Formalism.
2) M. Kawaguchi and *K. Nishijima (Res. Inst. Fund.; Kyoto and *Osaka City Univ.): Theory of Rearrangement Collisions. II. Exchange Effect and Application to *n-d* Scattering.
3) S. Hayakawa (Osaka City Univ.): Theory of Rearrangement Collisions. III. Derivation of Dispersion Formula of Nuclear Reactions.
4) Y. Fujimoto and *Y. Ichikawa (Kyoto Univ. and *Tohoku Univ.): Theory of *d-p* Reactions.
5) Y. Fujimoto and *Y. Ichikawa (Kyoto Univ. and *Tohoku Univ.): On Oppenheimer Phillips Process.
6) Y. Nakano (Hokkaido Gakugei Univ.): On the Pick-Up Reactions $Be^9(d,t)Be^8$ and $Be^9(p,d)Be^8$.
7) S. Yoshida (Res. Inst. Fund.; Kyoto): Deuteron Stripping Reaction.
8) S. Hayakawa and *K. Kikuchi (Osaka City Univ. and *Osaka Univ.): Excitation of γ-Rays due to the Collision of Fast Neutrons.
9) O. Kawaguchi and T. Nishiyama (Kyoto Univ.): Note on Excitation Curve of Direct Photo-Nuclear Reaction.
10) S. Hayakawa and *K. Ito (Osaka City Univ. and *Kyoto Univ.): Z-Dependence of Capture Life Time of Negative Mesons.
11) T. Eguchi and M. Ohta (Kyushu Univ.): Capture of Mu-Mesons by Atomic Nuclei.
12) A. Sugie and *S. Yoshida (Tokyo Univ. and *Res. Inst. Fund.; Kyoto): Angular Distribution of the Deuteron Photo Disintegration at Moderate Energies.
13) A. Sugie and *T. Tamura (Tokyo Univ. and *Tokyo Educ. Univ.): On the Doublet Splitting of He^5.

b) Nuclear Structure, β-Decay and Isomeric Iransition.

1) M. Umezawa, (Tokyo Univ.): A Note on the *j-j* Coupling Shell Model.
2) H. Horie, T. Ishizu, S. Obi, M. Sato and S. Yanagawa (Tokyo Univ.): Effects of the Tensor Force on the Energy Levels of Light Nuclei.
3) H. Takebe (Tokyo Univ.): On the Pseudoscalar Interaction in the Beta-Decay and Ra-E Beta-Decay Spectrum.

4) M. Yamada (Tokyo Univ.): Correction to the Beta-Decay Nuclear Matrix Element.
5) M. Yamada (Tokyo Univ.): Theoretical Reinvestigation of Beta-Spectrum of Ra-E.
6) M. Yamada (Tokyo Univ.): Note on the Finite Nuclear Size Effect in Beta-Decay.
7) J. Fujita and M. Yamada (Tokyo Univ.): On the Upper Bound of the Pseudoscalar Coupling Constant.
8) M. Morita (Tokyo Univ.): Interference Terms of Beta-Ray Angular Correlations.
9) M. Morita and M. Yamada (Tokyo Univ.): Theoretical Reinvestigation of the β-γ Angular Correlation of Sb^{124}.
10) M. Morita (Tokyo Univ.): Note on the Coexistence of the Tensor and the Axialvector Interaction in Beta-Decay.
11) M. Morita, J. Fujita and M. Yamada (Tokyo Univ.): The Beta-Ray Spectra of Fe^{59}, Rb^{87}, Tc^{99} and Cs^{137} and the coupling Constants of the Scalar and the Tensor Interactions.
12) H. Horie and *T. Tamura (Tokyo Univ. and *Tokyo Educ. Univ.): Effect of Spin-Orbit Interaction to the Higher-Pole Isomeric Transition of Nuclei.
13) S. Suekane (Kyoto Univ.): Effect of Collective Properties on Beta-Decay.
14) O. Miyatake (Inst. of Polytechnics, Osaka City Univ.): A Non-Uniform Nuclear Model Obtained by Introducing the Idea of Magic Number and Its Applications to Fission, Nuclear Temperature, Fluorescent Gamma-Ray Intensity, Photonuclear Dipole Vibration, and Inelastic Scattering Cross Section of Neutrons, etc.
15) T. Sasakawa (Kyoto Univ.): Theory of Fission.

c) **Miscellaneous**

1) J. K. Knipp, T. Eguchi, M. Ohta and S. Nagata (Kyushu Univ.): Ionization of Gas by Electrons.

Bhabha　　Sept. 19, 1953

$V_1^0 \to p + \pi^- + Q$

$Q = 37 \pm 2$ MeV

Mass $= 2190 \pm 4\, m$

$\tau = 3.3^{+0.9}_{-0.5} \times 10^{-10}$ sec.

$V_4^0 \to \pi^+ + \pi^- + Q$

$Q = 214$

Mass $= 971 \pm 2\, m$

$\tau = 0.9^{+1.0}_{-0.3} \times 10^{-10}$ sec.

$\tau^\pm \to 2\pi^\pm + \pi^\mp + Q$

$Q = 74 \pm$ MeV

Mass $= 970\ (\pm 2)\, m$

$\chi^\pm \to \pi^\pm + \pi^0$

If $m_\chi = m_\tau$, then $E_\pi = 110$ MeV.

Bristol has 5 good cases of $E \sim 109$ MeV

$\kappa \to \mu + 2\nu$　　$(\mu + \nu + \gamma\ ?)$

Mass $= 960 \pm 28\, m$

$(p\beta)_{max} = 206$ MeV/c

S particle mass $\simeq 1200^{+220}_{-200}\, m$

γ ray $\sim \frac{4}{22}$

not π^0

$H^{\pm} \to N + \pi^{\pm} + Q$

Levi Setti $Q \sim 135 \pm 24$

120 ± 17

Bombay 135 ± 35

$H^{\pm} \to P + \pi^{0}$

$Q \fallingdotseq 115 \pm 3$

P

S

Λ

Cascade decay

4 cases primary negative

Marshak ; Brookhaven ; Yang

2.2 BeV. hydrogen diff. cloud chamber

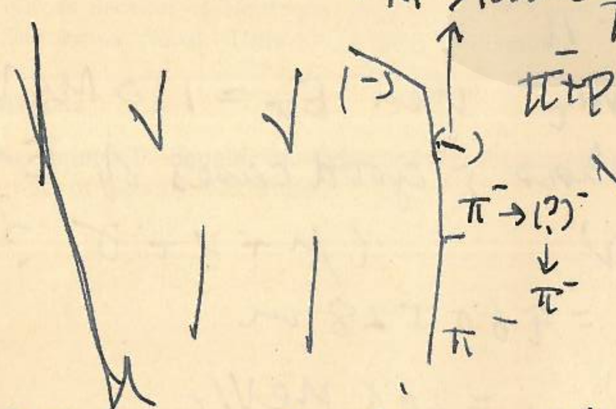
V_1^{0} in N-P collisions V_1^{0} in π^{-}-P collisions

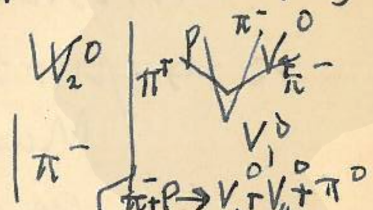
(Fowler, Shutt, Thorndike and Witemore)

$m > 1000\,m_e$ π^{-} 1.5 BeV

$\pi^{-} + P \to V_1^{0} +$ neutral

(1350 ± 70)





$(\pi^{-} + P \to V_1^{0} + V_4^{0} + \pi^{0})$

multiple production $N + P \to P + \pi^{+} + \pi^{-} + N$

$\to P + P + \pi^{-} + \pi^{0}$

26 $\pi^{+} + \pi^{-}$

≤ 7 $\pi^{-} + \pi^{0}$

≤ 7 π^{-}

Sept. 22　2.00 ~ 3.30
Informal discussions on V-particles
Marshak, Hayakawa

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