Collision of N and P (1)

$$\frac{d^2\psi(r)}{dr^2}+\frac{2}{r}\frac{d\psi(r)}{dr}+\left\{\frac{M_pE}{\hbar^2}\mp\frac{M_pg^2e^{-\lambda r}}{\hbar^2 r}-\frac{l(l+1)}{r^2}\right\}\psi(r)=0$$

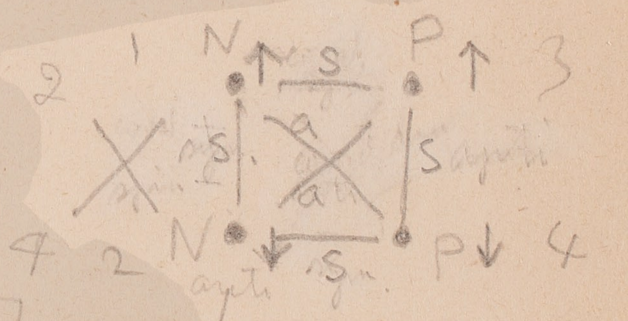
$$\lambda r = x \qquad \frac{\psi}{r}=y$$

$$\frac{d^2y}{dx^2}+\left(A\mp B\frac{e^{-x}}{x}-\frac{C}{x^2}\right)y=0$$

$$A=\frac{M_pE}{\hbar^2\lambda^2} \qquad B=\frac{M_pg^2}{\hbar^2\lambda}$$

$$C=l(l+1)$$

− sym.
+ anti.

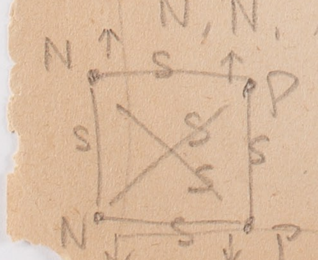


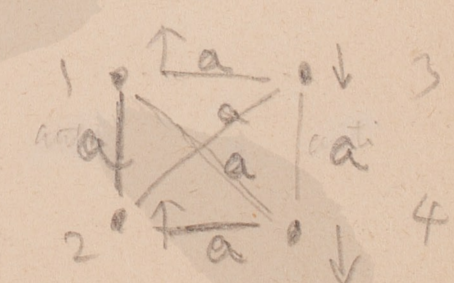
(12)

N, P の交換に対し
symmetric とすると

τ		spin	coord	
sym	$+J$	sym	sym	!!
anti	$-J$	anti	anti	
		sym	anti	
		anti	sym	

	τ	spin	coord	
P, P,	sym	sym	anti	
N, N,	sym	anti	sym	!!
		sym	anti	
		anti	sym	!!





(−) 2 3
1 4

(+) 2 1
3 4

(−) 3 1
2 4

$$H = \frac{1}{2}(1+\tau_3^{(1)})H_N(1) + \frac{1}{2}(1-\tau_3^{(1)})H_P(1)$$

$$+ \frac{1}{2}(1+\tau_3^{(2)})H_N(2) + \frac{1}{2}(1-\tau_3^{(2)})H_P(2)$$

$$+ \frac{1}{2}(\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)})J(r_{12}) + \ldots$$

$$\frac{1}{2}\left\{\begin{pmatrix}0&1\\1&0\end{pmatrix}^{(1)}\begin{pmatrix}0&1\\1&0\end{pmatrix}^{(2)} + \begin{pmatrix}0&-i\\i&0\end{pmatrix}^{(1)}\begin{pmatrix}0&-i\\i&0\end{pmatrix}^{(2)}\right\}$$

$$\begin{pmatrix}1\\0\end{pmatrix}^{(1)}\begin{pmatrix}0\\1\end{pmatrix}^{(2)} = \frac{1}{2}\left\{\begin{pmatrix}0\\1\end{pmatrix}^{(1)}\begin{pmatrix}1\\0\end{pmatrix}^{(2)} + \begin{pmatrix}0\\i\end{pmatrix}^{(1)}\begin{pmatrix}-i\\0\end{pmatrix}^{(2)}\right\}$$

$$= \begin{pmatrix}0\\1\end{pmatrix}^{(1)}\begin{pmatrix}1\\0\end{pmatrix}^{(2)} \qquad \text{etc}$$

$$\frac{1}{2}\{\quad\}\begin{pmatrix}1\\0\end{pmatrix}^{(1)}\begin{pmatrix}1\\0\end{pmatrix}^{(2)} = \frac{1}{2}\left\{\begin{pmatrix}0\\1\end{pmatrix}^{(1)}\begin{pmatrix}0\\1\end{pmatrix}^{(2)} - \begin{pmatrix}0\\i\end{pmatrix}^{(1)}\begin{pmatrix}0\\i\end{pmatrix}^{(2)}\right\}$$

$$= 0 \qquad \text{etc}$$

$$\psi(q_1, q_2, \tau_3'^{(1)}, \tau_3'^{(2)})$$

$$\begin{pmatrix}\psi(q_1,q_2,1,1) & \psi(q_1,q_2,1,-1)\\ \psi(q_1,q_2,-1,1) & \psi(q_1,q_2,-1,-1)\end{pmatrix}$$

$$\psi(q_1, q_2, \tau_3'^{(1)}, \tau_3'^{(2)}) =$$

$$\eta_0 = \frac{g^2 M_p}{4\hbar^2 k} \log\left(\frac{\lambda^2+4k^2}{\lambda^2+\cancel{4k^2}}\right)$$

$$(1-bk^2)^2+4k^2$$

$$k = \frac{M_p v_N}{2\hbar} = \frac{M_p' \cdot 10^{-24} \cdot 10^{9} v_N'}{2 \cdot 10^{-27}} = \frac{M_p' v_N'}{2 \cdot 10^{12}} (1-bk^2)^2$$

$$\eta_0 = \frac{g^2}{2\hbar \cdot v_N} \log\left(\frac{\lambda^2+4k^2}{\lambda^2+\cancel{4k^2}}\right) \qquad \lambda = 1-bk^2$$

$$\frac{\left(\frac{2k}{\lambda}\right)^2}{k} \qquad \frac{4k}{\lambda^2}$$

$$Q = \frac{1}{2} \cdot \frac{4\pi}{k^2} \cdot \left(\frac{1}{3}\right) \cdot \eta_0^2$$

$$= \frac{1}{2} \cdot \frac{\pi g^4 M_p^2}{4\hbar^4 k^4} \cdot \log\left(\frac{\lambda^2+4k^2}{\lambda^2+\cancel{4k^2}}\right)$$

$$\frac{g^2 M_p}{\hbar^2 \cdot 10^{48}} = \frac{g'^2 M_p' \cdot 10^{-20} \cdot 10^{-24}}{10^{-54} \cdot 10^{24}\, k'^2}$$

$$= g'^2 M_p' \cdot 10^{-14} / k'^2$$

$$M_p' = 1.66$$

$\lambda \gg k$

$$Q = g'^4 M_p'^2 \cdot 10^{-28} \qquad k \gg \lambda$$

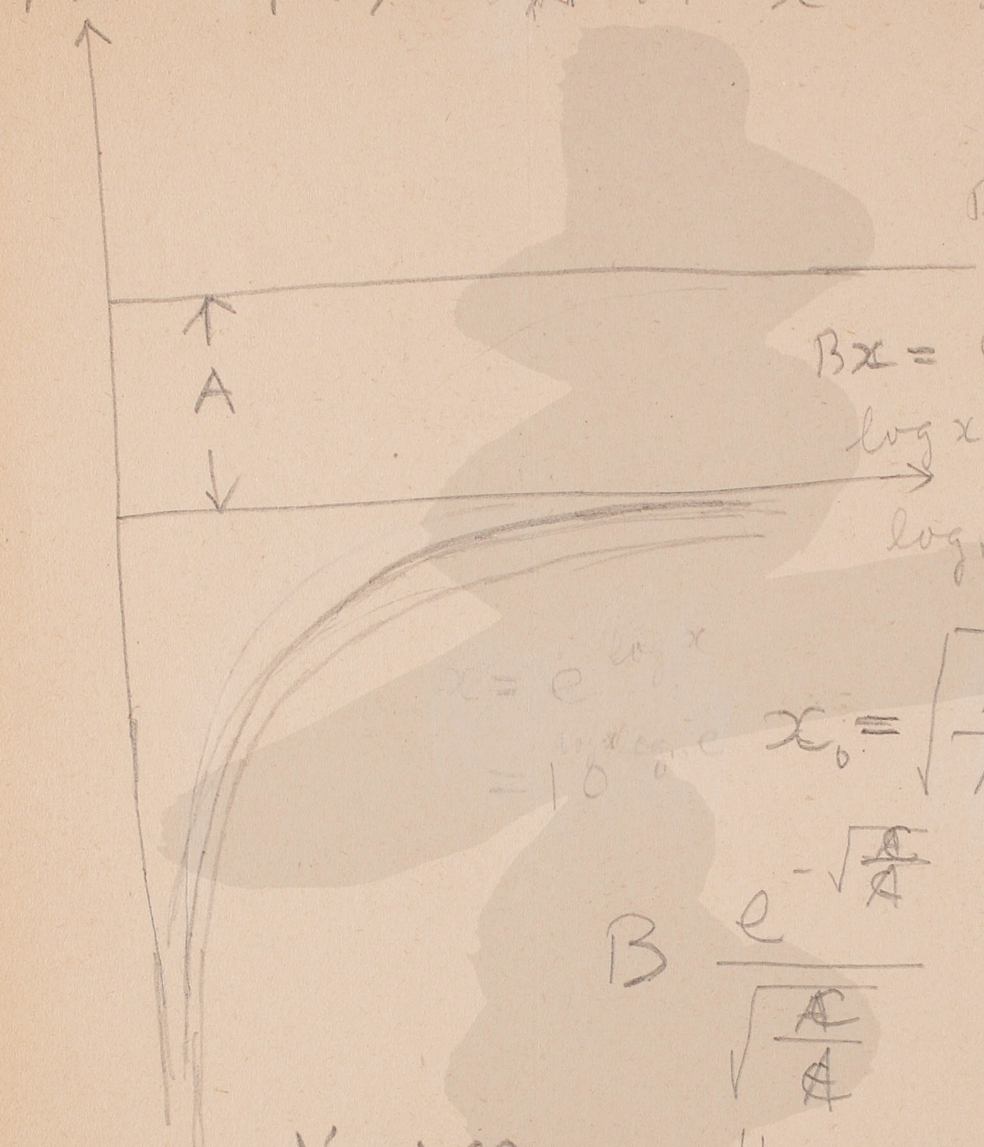
$$\lambda^2 = \lambda_0^2 - bk^2$$

$$\times \frac{4}{\lambda'^2 k'^2}$$

$$\lim_{x\to 0} \frac{\log\left(1+\left(\frac{2k}{\lambda}\right)^2\right)}{x} = \frac{\frac{2x}{1+x^2}}{1} = 0$$

$$F(r) = \frac{M_p E}{\hbar^2} \mp \frac{M_p g^2 e^{-\lambda r}}{\hbar^2 \, r} - \frac{l(l+1)}{r}$$

$$F(x) = A \mp B\frac{e^{-x}}{x} - \frac{C}{x^2}$$



$$B\frac{e^{-x}}{x} = \frac{C}{x^2}$$

$$Bx = C e^{-x}$$

$$\log x = -\frac{C}{B} \cdot x$$

$$\log_{10} x \cdot \log_{10} e = -\frac{C}{B} x$$

$$x_0 = \sqrt{\frac{C}{A}}$$

$$B \frac{e^{-\sqrt{\frac{C}{A}}}}{\sqrt{\frac{C}{A}}}$$

$l = 1$: $\lambda' = 1.67$ $\quad E = 2.1 \times 10^6 \, eV.$

$C = 2$

$A = 2$ $\quad x_1 = 1. \; 4.67.$ $\quad \frac{e^{-1}}{1} \cong 2.$

$r_p = \frac{1}{1.67 \times 10^{12}}$

$l = 2$: $C = 6.$ $\quad \frac{e^{-1.73\cdots}}{1.73\cdots} \cong \frac{1}{2}$ $\quad 4.67$

$A = 2$ $\quad x_2 = \frac{1}{1.73}$

$r_\ell < r_1$

$r_\ell > r_2$

$$r_\ell = \frac{\lambda}{\lambda\sqrt{A}} = \frac{1.67 \times 10^{-12}}{1.414 \times 1.67} \cong 0.4 \times 10^{-12}$$

$$H\,\psi(q_1, q_2, \tau_3, \tau_3) = E\,\psi.$$

$$\{H_N(1)+H_N(2)\}\psi(1,1) = E\,\psi(1,1)$$

$$\{H_N(1)+H_P(2)\}\psi(1,-1) + J(r_{12})\,\psi(-1,1) = E\,\psi(1,-1)$$

$$\{H_P(1)+H_N(2)\}\psi(-1,1) + J(r_{12})\,\psi(1,-1) = E\,\psi(-1,1)$$

$$\{H_P(1)+H_P(2)+\cdots\}\psi(-1,-1) = E\,\psi(-1,-1)$$

$$\psi(1,1) = \psi(-1,-1) = 0.$$

$$\begin{pmatrix} 0 \\ \psi_1(q_1,q_2) \\ \psi_2(q_2,q_1) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \psi(q_1,q_2) \\ \psi(q_1,q_2) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \psi'(q_1,q_2) \\ -\psi'(q_1,q_2) \\ 0 \end{pmatrix}$$

$1 \rightleftarrows 2$

$$\begin{pmatrix} 0 \\ \psi_2(q_2,q_1) \\ \psi_1(q_1,q_2) \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ \psi(q_2,q_1) \\ \psi(q_2,q_1) \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ -\psi'(q_2,q_1) \\ \psi'(q_2,q_1) \\ 0 \end{pmatrix}$$

$B\frac{e^{-x}}{x} \ll \frac{C}{x^2}$ for $(Ax)^2 \sim \sqrt{C}$

$B\,x \ll C\,e^{x}$

$\frac{B}{A} \ll \sqrt{C}\,e^{\frac{\sqrt{C}}{A}}$

$l=1$, $C=2$

$E=2$ milivolt $\lambda = 4.67 \times 10^{12}$ cm

$A=2$

$B=4.67$

$\lambda = 3.33 \times 10^{12}$ cm

$A=0.5$

$B=3.23$

$6.46 \ll 1.414 \cdot e^{2.828}$

0.44994

$\cancel{2}$3 828

52

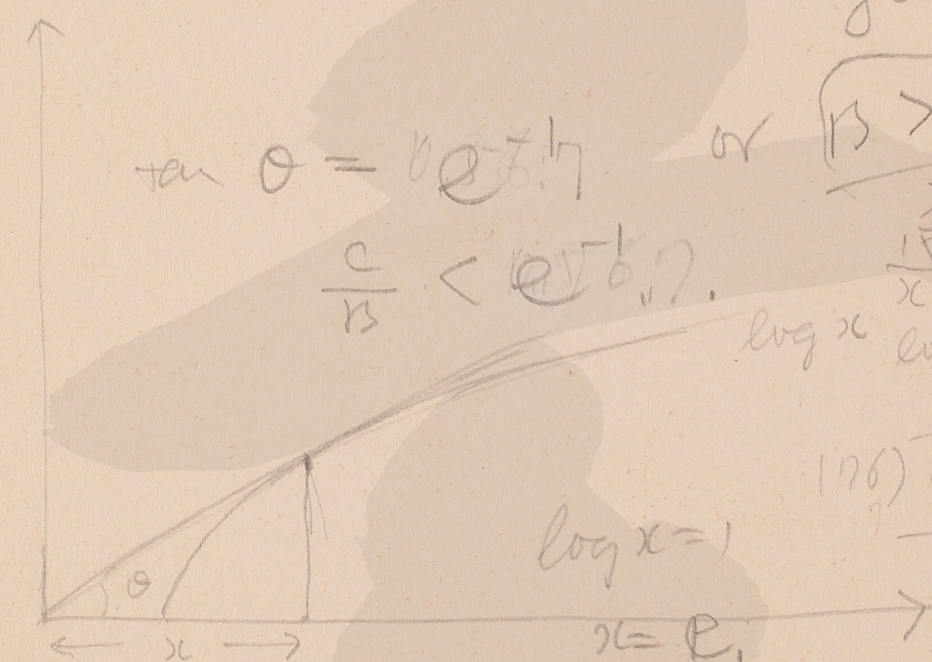
$F(x) = A \mp B\dfrac{e^{-x}}{x} - \dfrac{e^{-x}}{x^2}$ $\quad C x^x$

log"

$B\dfrac{e^{-x}}{x} = C/x^2$ $\qquad Bx < Ce^x$

$\log x = \dfrac{C}{B}x.$

$\log_{10} x = \dfrac{C}{B}x / \log_{10} e.$



$\tan\theta = e^{?}$ or $\boxed{B > e \cdot C}$

$\dfrac{C}{B} < e^{?}$

$\tan\theta = \dfrac{1}{x} = \log x / x.$

$x = 1.8$ $\quad \log x = 0.5878$ $\quad \dfrac{1}{x} = 0.555$

$x = 1.78$ $\quad \log x = 0.5766$ $\quad \dfrac{1}{x} = 0.562$

$x = 1.77$ $\quad \log x = 0.5710$ $\quad \dfrac{1}{x} = 0.565$

$x = 1.76$

$\dfrac{1}{x} = 0.568$

$\log x = 0.5653$

0.568

176) 10000
880
1200
1056
144

1440
1408

0.565
177) 1000000
885
1150
1062
880

0.555
18) 100
90
100

0.562
178) 1000
890
1100
1068
320

$$\{H_N(1)+H_p(2)\}\psi'(q_1 q_2)+J(r_{12})\psi'(q_1 q_2)$$
$$=E\psi'(q_1 q_2)$$

$\psi'(x_1 x_2\ s_1 s_2)$

$$\begin{pmatrix} \psi'(x_1 x_2, 1\ 1) & \psi'(x_1 x_2, 1, -1) \\ \psi'(x_1 x_2, -1, 1) & \psi'(x_1 x_2, -1, -1) \end{pmatrix}$$

$\frac{d\psi}{dr}$

$\psi(q_1 q_2, \tau_3^{(1)} \tau_3^{(2)})$

$$= \psi(q_1 q_2)\cdot\{\alpha(\tau_3^{(1)})\beta(\tau_3^{(2)})\pm\beta(\tau_3^{(1)})\alpha(\tau_3^{(2)})\}$$

$$\{H_N(1)+H_p(2)\pm J(r_{12})\}\psi(q_1 q_2)$$
$$=E\psi(q_1 q_2)$$

$$H_N(1)=\frac{p_1^2}{2M}-D \qquad H_p(2)=\frac{p_2^2}{2M}.$$

$\psi(q_1 q_2)=\psi_s(r_1\ r_2)\cdot$(antisym. of spin) (sym.)

or $=\psi_a(r_1\ r_2)$ (symm. of spin) anti

$\mu=\frac{M}{2}$, $\quad r_1-r_2=r$,

$$\{\Delta \pm \frac{M_p}{\hbar^2}(E\mp J(r))\}\psi(r)=0$$

$\mp$ sym. anti. | spin coord

N, Pの交換に対しanti-sym. とすると

τ

			spin	coord	
sym.	J	<	sym	anti	l: odd
			anti	sym	l: even
anti.	−J	<	sym	sym	
			anti	anti	

total cross section (~~for~~ relative coord について)

$$Q = \frac{1}{2}\frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\, g_l \sin^2\eta_l$$

$$k^2 = \frac{M_p E}{\hbar^2}$$

$$\vec{V} = \frac{\vec{v_N}}{2}$$

$$E = \frac{M_p}{2\cdot 2}\, v^2$$

$$v = \frac{v_N}{2}$$

$$k = \frac{M_p \cdot v_N}{2\cdot\hbar}$$

τ		spin	coord	l	g_l	
sym	; J	sym	anti	odd	3	$= 2-(-1)^l$,
		anti	sym	even	1	
anti	; $-J$	sym	sym	~~odd~~ even	3	$= 2+(-1)^l$,
		anti	anti	odd	1	

sym $\quad g^l = 2-(-1)^l$,

$$F(x) = A + B\frac{e^{-x}}{x} - \frac{C}{x^2} = 0,$$

$$\frac{dF}{dx} = \frac{Be^{-x}}{x^2} + \frac{Be^{-x}}{x} - \frac{C}{x^2} = 0,$$

$$B(1+x)e^{-x} - Cx = 0,$$

$F(x) = 0$: $\quad Bxe^{-x} = Ax^2 - C$,

$\cancel{k \gg \lambda}$

$\eta_0 \ll 1$

$$\eta_0 = -\frac{\pi}{2}\frac{M\,[illegible]}{\hbar^2}\cdot\int_0^\infty V(r)\left(J_{\frac{1}{2}}(kr)\right)^2 r\,dr$$

$$J_{\frac{1}{2}}(kr) = \sqrt{\frac{2}{\pi \cdot kr}}\cdot \sin kr$$

$$\eta_0 = +\frac{g^2 M_p}{\hbar^2}\int_0^\infty \frac{e^{-\lambda r}}{kr}\cdot \sin^2 kr\,dr$$

$$kr = y$$

$$\eta_0 = \frac{g^2 M_p}{\hbar^2 k}\int_0^\infty \frac{e^{-\mu y}}{y}\sin^2 y\cdot dy$$

$$\mu = \frac{\lambda}{k}$$

$$\sin^2 y = \frac{1}{2}(1 - 2\cos 2y)$$

$$2y = z$$

$$\eta_0 = \frac{g^2 M_p}{2\hbar^2 k}\int_0^\infty \frac{e^{-xz}}{z}(1-\cos z)\,dz$$

$$x = \frac{\mu}{2} = \frac{\lambda}{2k}$$

$$\int_0^\infty e^{-xz}(1-\cos z)\,dz = \frac{1}{x} - \frac{x}{x^2+1}$$

$$\int_x^\infty \int_0^\infty e^{-xz}(1-\cos z)\,dz$$

$$= \log\frac{(x^2+1)^{\frac{1}{2}}}{x} = \frac{1}{2}\log\left(\frac{\lambda^2+4k^2}{\lambda^2}\right)$$