Collision of [illegible] Protons (2)

$$\frac{d^2\psi}{dr^2}+\frac{2}{r}\frac{d\psi}{dr}+\left\{\frac{M_pE}{\hbar^2}\mp\frac{M_pg^2e^{-\lambda r}}{\hbar^2 r}-\frac{l(l+1)}{r^2}\right\}\psi=0$$

$\lambda r = x$ , $r\psi = y$

$$\frac{d^2y}{dx^2}+\left(A\mp B\frac{e^{-x}}{x}-\frac{C}{x^2}\right)y=0$$

$$A=\frac{M_pE}{\hbar^2\lambda^2}\ ,\quad B=\frac{M_pg^2}{\hbar^2\lambda}$$

$$C=l(l+1)$$



Differential Cross Section (Relative Coord)

$$Q(\theta,\varphi)\,d\omega=|f(\theta)|^2\sin\theta\, d\theta\, d\varphi$$

$$f(\theta)=\frac{1}{2ik}\sum_{n=0}^{\infty}(2l+1)(e^{2i\eta_l}-1)\,g_l\,P_l(\cos\theta)$$

| $\tau$ | | spin | coord | $l$ | $g_l$ | |
|---|---|---|---|---|---|---|
| sym | $J$ | sym | anti | odd | 3 | $> 2-(-1)^l$ |
| | | anti | sym | even | 1 | |
| anti | $-J$ | sym | sym | even | 3 | $> 2+(-1)^l$ |
| | | anti | anti | odd | 1 | |

$\tau$ anti : $g_l = 2+(-1)^l$

$$\psi\sim e^{ikz}+r^{-1}e^{ikr}f(\theta)$$

~~$\frac{3}{4}$~~ ~~$\frac{1}{2}$~~ ~~$\frac{3}{4}$~~ 2

Total Cross Section (Rel. Coord.)

1

$$Q=\frac{4\pi}{k^2}\sum_{n=0}^{\infty}(2l+1)\,g_l\sin^2\eta_l$$

$$k^2=\frac{M_pE}{\hbar^2}\ ,\quad E=\frac{M_p}{4}v_N^2\quad \therefore k=\frac{M_pv_N}{2\hbar}$$

$$\eta_l = \frac{\pi}{2}\frac{M}{\hbar^2}\int_0^\infty V(r)\left[J_{l+\frac{1}{2}}(kr)\right]^2 r\,dr$$

$$V(r) = -\frac{g^2 e^{-\lambda r}}{r}$$

$\lambda r = x$, $\kappa = k/\lambda$

$$\eta_l = \frac{\pi M_p g^2}{2\hbar^2\lambda}\int_0^\infty e^{-x} J^2_{l+\frac{1}{2}}(\kappa x)\,dx$$

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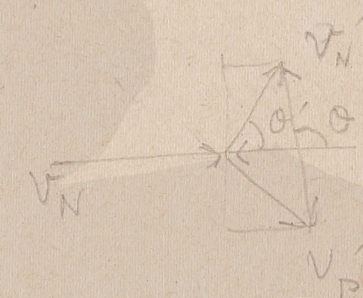
Differential Cross Section (Proton Fixed)

$\theta = 2\theta'$




$\tan\theta' = \frac{m_p \sin\theta}{m_p\cos\theta + m_N} \cong \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$

$2\cos^2\frac{\theta}{2} - 1$

$I(\theta'\varphi')\,d\omega'$

$= |f(2\theta')|\sin\theta\,d\theta\,d\varphi,$

$= |f(2\theta')|^2 \cdot 4\sin\theta'\cos\theta'\,d\theta'\,d\varphi'$

$v_N = v_N'\cos\theta' + v_p'\sin\theta'$

or $I(\theta',\varphi') = 4|f(2\theta')|^2\cos\theta'$ $\quad v_N'\sin\theta' = v_p'\cos\theta'$

$v_N =$

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$kr = y$, $\mu = \lambda/k$

$$\eta_l \doteq \frac{\pi M_p g^2}{2\hbar^2 k}\int_0^\infty e^{-\mu y} J^2_{l+\frac{1}{2}}(y)\,dy$$

$$J_{l+\frac{1}{2}}(y) = \sqrt{\frac{2}{\pi y}}\left\{P_l(y)\cos\left(y - \frac{l+1}{2}\pi\right) - Q_l(y)\sin\left(y - \frac{l+1}{2}\pi\right)\right\}$$

$$= \frac{(-1)^l}{(2^l)}\sqrt{\frac{2y}{\pi}}\frac{d^l}{d(y^2)^l}\left(\frac{\sin y}{y}\right)$$

$$\int_0^\infty e^{-\mu y} J_{\frac{1}{2}}(y)\,dy = \frac{2}{\pi}\int_0^\infty e^{-\mu y}\frac{\sin^2 y}{y}\,dy$$

$$= \frac{2}{\pi}\int_0^\infty e^{-\mu y}\frac{(1-\cos 2y)}{2y}\,dy$$

$\xi = \frac{\lambda}{2k}$

$$= \frac{1}{\pi}\int_0^\infty e^{-\xi y}\frac{1-\cos y}{y}\,dy$$

$$= \frac{1}{\pi}\int_\xi^\infty d\xi\int_0^\infty e^{-\xi y}(1-\cos y)\,dy$$

$$= \frac{1}{\pi}\int_\xi^\infty\left(\frac{1}{\xi}-\frac{\xi}{\xi^2+1}\right)d\xi$$

$$= \frac{1}{\pi}\log\frac{\sqrt{\xi^2+1}}{\xi} = \frac{1}{\pi}\log\sqrt{1+\frac{4k^2}{\lambda^2}}$$

$$= \frac{1}{2\pi}\log\left(1+\frac{4k^2}{\lambda^2}\right)$$

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$$\eta_0 = \frac{M_P g^2}{4\hbar^2 k}\log\left(1+\frac{4k^2}{\lambda^2}\right)$$

$$k = \frac{M_P v_N}{2\hbar} = \frac{1.66\times10^{-24}\times2\times3.14\times10^9}{2\times6.55\times10^{-27}}v_N'$$

$g = 10^{-10}g'$

$$= 0.8\times10^{12}v_N'$$

$$\frac{M_P g^2}{4\hbar^2 k} = \frac{1.66\times10^{-24}\times(3.14)^2\times10^{-20}\cdot g'^2}{4\cdot(6.55\times10^{-27})^2\cdot0.8\times10^{12}v_N'}$$

$$= 1.21\times10^{-3}\frac{g'^2}{v_N'}$$

$v_N = v_N' \times 10^9$

3.14
3.14
12.56
31.4
942
9.8586

6.55
6.55
3275
3275
3930
42.9

43
3.2
86
1.6129
188
1884
3.14
5.2124
5.240

$10^{-45}$
$10^{-40}$

| $\lambda'$ | | | |
|---|---|---|---|
| 4.77 | 4.774 | 6.14 | $g' = 29.3$ |
| 2.11 | | 4.95 | 23.6 |
| 1.18 | | 4.41 | 21.0 |

$v_N = 10^9$ cm/sec. $\quad \eta_0 = 1.21 \times 10^{-3} \cdot (4.77)^2 \cdot g'^2$

$\times \log\{1 + (\frac{1.6}{\lambda'})^2\}$

6.14 × 4.77: 4298, 4298, 2456 = 29.2878

4.95 × 4.77: 3465, 3465, 1980 = 23.6115

4.41 × 4.77: 3087, 3087, 1764 = 21.0357

$g'^2 = 858.$ $\quad \frac{M_p g^2}{4\pi^2 k} = 1.0$ $\quad (\frac{1.6}{\lambda'})^2 = 0.116$

557. $\quad$ 0.67 $\quad$ 0.634

441. $\quad$ 0.53 $\quad$ 1.84

4533 × 23: 13599, 9066 = 104259

858 × 1.21: 858, 1716, 858 = 1038.18

557 × 1.21: 557, 1114, 557 = 673.97

441 × 1.21: 441, 882, 441 = 533.61

$\log_{10} 1.116 = 0.0477$

0.634 = 0.2133

1.84 = 0.4533

$\log_e$ … = 0.100

… = 0.491

… = 1.043

$e^x = y$

$= 10^z$

$= e^{z \log 10}$

$x = z \times 2.3$

0.2123 × 1.6: 12738, 2123 = 0.33968

0.4975 × 1.6: 29850, 4975 = 0.79600

0.8475 × 1.6: 50850, 8475 = 1.35600

477 × 23: 1431, 954 = 10971

2133 × 23: 6399, 4266 = 49059

$\frac{d}{dk}\left[\frac{1}{k}\log\{1+(\frac{2k}{\lambda})^2\}\right] = -\frac{1}{k^2}\log(\ ) + \frac{4.2}{\cancel{\lambda^2}(1+(\frac{2k}{\lambda})^2)}$

$\log(1+\frac{2k}{\lambda}) = \frac{2(\frac{2k}{\lambda})^2}{1+(\frac{2k}{\lambda})^2}$

$\nu_N = 10^9$ cm/sec

$\lambda' = 4.71$ $\qquad \eta_0 = 0.114$
2.11 $\qquad 0.329$
1.18 $\qquad 0.553$
$\qquad 0.53$

~~104~~ ~~116~~ ~~104~~ ~~464~~ ~~116~~ ~~12064~~ $\qquad$ ~~0.67 2.11 67 61 122 12871~~

104 × 110 = 11.44 $\qquad$ 0.67 × 0.491 = 0.32897 $\qquad$ 1.043 × 0.53 = 0.55279

$$Q = \frac{4\pi}{k^2} \cdot 3 \cdot \eta_0^2 = \frac{12.56 \times 3}{8.64 \times 10^{24}} \eta_0^2$$

$\eta_0 = 0.114 \qquad = 0.59 \cdot \eta_0^2 \cdot 10^{-22}$

5.8.9
64) 37.70
320
570
512
580

0.114 × 0.114 = 0.012996 $\qquad$ 0.0128 × 0.60 = 0.00768

$\lambda' = 4.71$ : $Q = 0.97 \times 10^{-24}$ cm$^2$

$$\frac{d^2y}{dx^2}+\left(A+B\frac{e^{-x}}{x}-\frac{C}{x^2}\right)y=0$$

$$A=\frac{M_pE}{\hbar^2\lambda^2} \qquad B=\frac{M_pg^2}{\hbar^2\lambda}$$

$y=r\psi$

$x=\lambda r$

$$C=l(l+1)$$

$$\int\left\{\left(\frac{dy}{dx}\right)^2+B\frac{e^{-x}}{x}y^2+\cancel{\frac{C}{x^2}y^2}\right\}dx$$

$$y=\frac{\sin(\kappa x+\eta)}{x}$$

$$\int_0^X\left\{\kappa^2\cos^2(\kappa x+\eta)+B\frac{e^{-x}}{x}\sin^2(\kappa x+\eta)\right\}dx$$

$$=\frac{\kappa^2}{2}X+B\,\text{——}$$

$1+\cos 2(\kappa x+\eta)$ / $2$

$\xi=\frac{1}{2\kappa}$

$\kappa x=\frac{y}{2}$

$$\int_0^X\frac{e^{-x}}{x}\sin^2(\kappa x+\eta)\,dx=\int_0^{2\kappa X}\frac{e^{-\xi y}}{y}\sin^2\left(\frac{y}{2}+\eta\right)dy$$

$$=\int_0^{2\kappa X}\frac{e^{-\xi y}}{2y}\{1-\cos(y+2\eta)\}\,dy$$

$$=\int_0^{2\kappa X}\frac{e^{-\xi y}}{2y}(1-\cos y\cos 2\eta+\sin y\sin 2\eta)\,dy$$