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$$\frac{d^2 y}{dx^2} + \left(A + B \frac{e^{-x}}{x} - \frac{C}{x^2} \right) y = 0.$$

$$\frac{d^2 y}{dx^2} = -p(x)y$$

$$\frac{d^3 y}{dx^3} = -p(x) \frac{dy}{dx} - p'(x)y$$

$$\frac{d^3 y}{dx^3} = -p(x)$$

$$\frac{1}{x} = z \quad \frac{dy}{dx} = -\frac{1}{x^2} \frac{dy}{dz} = -z^{-2} \frac{dy}{dz}$$

$$\frac{d^2 y}{dx^2} = z^4 \frac{d^2 y}{dz^2} - \frac{2}{x^3} \frac{dy}{dz}$$

$$= z^4 \frac{d^2 y}{dz^2} - 2z^3 \frac{dy}{dz}$$

$$\frac{d^2 y}{dz^2} - \frac{2}{z} \frac{dy}{dz} + \frac{1}{z^2} \left(A + B \cdot e^{-\frac{1}{z}} \cdot z - C z^2 \right) y = 0$$

$$z = \frac{x}{x+a} \quad (1-z)x = az \quad x = \frac{az}{1-z} \quad x+a = \frac{a}{1-z}$$

$$\frac{dy}{dx} = \frac{1}{\left(\frac{a}{1-z} - \frac{az}{1-z}\right)^2} \frac{dy}{dz} = \frac{a}{(x+a)^2} \frac{dy}{dz} = \frac{(1-z)^2}{a} \frac{dy}{dz} \quad az + a - az$$

$$\frac{dy}{dx^2} = \frac{a^2}{(x+a)^4} \frac{d^2y}{dz^2} - \frac{2a}{(x+a)^3} \frac{dy}{dz} = \frac{(1-z)^4}{a^2} \frac{d^2y}{dz^2} - \frac{2(1-z)^3}{a} \frac{dy}{dz}$$

$$a=1, \quad \frac{dy}{dz^2} - \frac{2}{(1-z)} \frac{dy}{dz} + \frac{1}{(1-z)^2} \left(A + B \frac{1-z}{z} e^{-\frac{z}{1-z}} \frac{(1-z)^2}{z^2} \right) y = 0$$

$$z = \frac{x+1}{x+1} \quad \frac{dy}{dx} = \frac{-1}{(x+1)^2} \frac{dy}{dz} = -z^2 \frac{dy}{dz}$$

$$x+1 = \frac{1}{z} \quad \frac{dy}{dx^2} = \frac{1}{(x+1)^4} \frac{d^2y}{dz^2} - \frac{2}{(x+1)^3} \frac{dy}{dz} = z^4 \frac{d^2y}{dz^2} - 2z^3 \frac{dy}{dz}$$

$$x = \frac{1-z}{z} \quad \frac{dy}{dx^2} - \frac{2}{z} \frac{dy}{dz} + \frac{1}{z^4} \left(A + B \frac{z}{1-z} e^{-\frac{1-z}{z}} - C \frac{z^2}{(1-z)^2} \right) y = 0$$

$$\frac{dy}{dx} = u, \quad \frac{du}{dz} = \frac{2}{z} u - \frac{1}{z^4} \left(\dots \right) y$$