Mass ... (1)

$$\frac{d^2\psi}{dr^2} + \frac{2}{r}\frac{d\psi}{dr} + \frac{2m}{\hbar^2}\left(E + \frac{g^2 e^{-\lambda r}}{r}\right)\psi = 0.$$

$$\frac{a e^{-br}}{br}$$

$\lambda r = x$: $\frac{\psi}{r} = y$

$$\frac{d^2y}{dx^2} + \left(A + B\frac{e^{-x}}{x}\right)y = 0$$

$a^2 = g^2\lambda$
$b = \lambda$

$\frac{E}{g^2\lambda}.\ \frac{g^2}{\lambda}$ $\quad A = \frac{2mE}{\hbar^2\lambda^2}$ $\quad B = \frac{2mg^2}{\hbar^2\lambda}$

$\frac{A}{B}.$

$$m \cong \frac{M_p}{2} \cong 1.66 \times 10^{-24}/2 = 0.83 \times 10^{-24}.$$

$$\hbar \cong 1.042 \times 10^{-27}$$

$$E^{erg} = 1.6 \times 10^{-6} \cdot E^{\text{millivolt}}$$

$$\lambda^{cm} = 10^{12}\lambda' \ (10^{-12}\ cm)$$

$$\frac{2m}{\hbar^2} = \frac{1.66 \times 10^{-24}}{1.08 \times 10^{-54}} = 1.54 \times 10^{30}$$

$$A = \frac{2.64 \times \cancel{X} \cdot E^{\text{millvolt}}}{X^2}$$

$$\frac{mc}{\hbar} = \frac{12.7}{1.04} \cdot 10^{10}$$

$$g = g' e = g' \, 4.77 \times 10^{-10}$$

$$B = 1.54 \cdot 10^{30} \times (4.77)^2 \times 10^{-20} \times 10^{-12} \cdot \frac{g'^2}{\lambda'}$$

$$\frac{e^2}{mc^2} = \frac{(4.77)^2}{9.81} \cdot 10^{-12}$$

$$= 0.35 \times \frac{g'^2}{\lambda'}$$

1.54	4.77
1.6.6	4.77
924	3339
154	3339
24.64	1908
	22.7529

22.76
1.54
9104
11380
2276
35.0504

$$* \quad C^2\int_0^\infty x^2 e^{-2ax}\,dx$$

$$= C^2\int_0^\infty \frac{2x\cdot e^{-2ax}}{2a}\,dx = C^2\int_0^\infty \frac{e^{-2ax}}{2a^2}\,dx$$

$$= \frac{C^2}{4a^3} = 1$$

$$2a\int_0^\infty x e^{-2ax}\,dx = \int_0^\infty e^{-2ax}\,dx = \frac{1}{2a}$$

$\frac{a}{B} = b.$

$a \ll 1.$ $\cancel{2a \cdot 1} = 6a^2 B$

$+4a+4a^2$

$\frac{(2b+1/B)^3}{2b(2b+3/B)} = 1.$

$\frac{A}{B} = b^2 B - \frac{4b^3 B}{(2b+1/B)^2}$

$B \gg 1:$ $a \cong \frac{3}{4}$

$3(2a+1) - 4a = 0$

$2a+3=0$

$2a(2a+3)(2+\delta) = (2a+1)^3$

$12a + 8a^2 + 6a\delta = 1.$

$(2a+1)^3 = B \cdot 2a(2a+3)$

$a \cong \frac{B}{2}$

$A \cong \frac{B^2}{4} - \frac{4B^2}{B^2 2}$

$- \frac{B}{4}$

$\frac{A}{B} \cong \frac{B}{4}$

$b^3 + b^2 + \frac{b}{3} + \frac{1}{27}$

$-b^2 - \frac{2b}{3} - \frac{1}{9}$

$-A'b - \frac{A'}{3}$

$-3A'$

$9 \cdot 9 \cdot 9$

$\frac{1}{27}$

(ii) $y^3 + py \pm q = 0$ $\quad p > 0$

$y = \mp\sqrt{\frac{4p}{3}} \sinh \theta, \quad \sinh 3\theta = \sqrt{\frac{27q^2}{4p^3}}$

$p = A' - \frac{1}{3}$ $\qquad \frac{4p}{3} = \frac{4}{3}\left(A' - \frac{1}{3}\right)$

$q = \frac{2}{3}\left(5A' - \frac{1}{9}\right)$ $\qquad \sqrt{\frac{27q^2}{4p^3}} = \sqrt{\frac{3 \cdot \left(5A' - \frac{1}{9}\right)^2}{\left(A' - \frac{1}{3}\right)^3}}$

$\lambda = 0$;　　$\psi = e^{-\frac{r}{a}}$

$$\frac{1}{a^2} + \frac{2}{ra} + \frac{2m}{\hbar^2}\left(E + \frac{g^2}{r}\right) = 0.$$

$$\frac{1}{a^2} = -\frac{2mE}{\hbar^2} = \left(\frac{m g^2}{\hbar^2}\right)^2$$

$$2.64 \times 10^{24}\, E^{\text{mill volt}} = \left(0.175 \times 10^{12} g'^2\right)^2$$

$$g'^2 \cong 10 \cdot E^{\text{mill volt}}$$

$$A = \min \int_0^\infty \left\{\left(\frac{dy}{dx}\right)^2 - B\frac{e^{-x}}{x}y^2\right\}dx$$

$$y = C x e^{-ax}, \qquad C = 2\sqrt{a^{\frac{3}{2}}}$$

$$\frac{dy}{dx} = C(1-ax)e^{-ax}$$

$$A = \text{min of } 4a^3 \int_0^\infty \left\{(1-ax)^2 e^{-2ax} - Bxe^{-(2a+1)x}\right\}dx$$

$$= \text{min of } 4a^3\left\{\frac{1}{2a} - \frac{1}{2a} + \frac{1}{4a} - \frac{B}{(2a+1)^2}\right\}$$

$$= \min\left\{a^2 - \frac{4a^3 B}{(2a+1)^2}\right\}$$

$$-A' = m\left\{a'^2 - \frac{2a'^3 B}{(a'+1)^2}\right\}$$

$B = 2$

$$-A' = \frac{a'^2(a'-1)^2}{(a'+1)^2}$$

$\frac{\partial A}{\partial a} = 0$;

$\alpha = \frac{1}{6B}$

$$2a - \frac{12a^2B}{(2a+1)^2} + \frac{16a^3B}{(2a+1)^3} = 2a\left[1 + \frac{B\cdot 2a^2}{(2a+1)^3}\{B(2a+1) - 4a\}\right] = 0$$

$$B = \frac{(a^2-A)(2a+1)^2}{4a^3} = \frac{(a'^2+A')(a'+1)^2}{2a'^3}$$

$$= \frac{(2a+1)^3}{2a(2a+3)} = \frac{(a'+1)^3}{a'(a'+3)}$$

a'^2

$$(a^2-A)(2a+3) = 2a^2(2a+1)$$

or $2a = a'$ $4A = -A'$ $\frac{A}{B} = \frac{a'^3(a'-1)}{(a'+1)^3}$

$$(a'^2+A')(a'+3) = 2a'^2(a'+1)$$

$A' = -a'^2$

$$a'^3 - a'^2 + A'a' + 3A' = 0$$

$$+ \frac{2a'^2(a'+1)}{a'+3}$$

$$a' = b + \frac{1}{3}$$

$$B = \frac{(a'+1)^3}{a'(a'+3)} = \frac{2a'^2(a'-1)}{a'+3}$$

$$b^3 - \left(A' + \frac{1}{3}\right)b + \left(\frac{10}{3}A' - \frac{2}{27}\right) = 0$$

$$\left(-\frac{2}{3}\left(5A' + \frac{1}{9}\right)\right)$$

$$D = -27\left\{\frac{2}{3}\left(5A' + \frac{1}{9}\right)\right\}^2 + 4\left(A' + \frac{1}{3}\right)^3$$

$$= -\frac{4}{27}\left\{(45A'+1)^2 - (3A'+1)^3\right\}$$

$$= \frac{4}{27}\left\{(3A'+1)^3 - (45A'+1)^2\right\} \quad (3A'-1)^3$$

2,345

1,345
4,345

0,3701
1,1103
0,1287
0,6380
0,7667

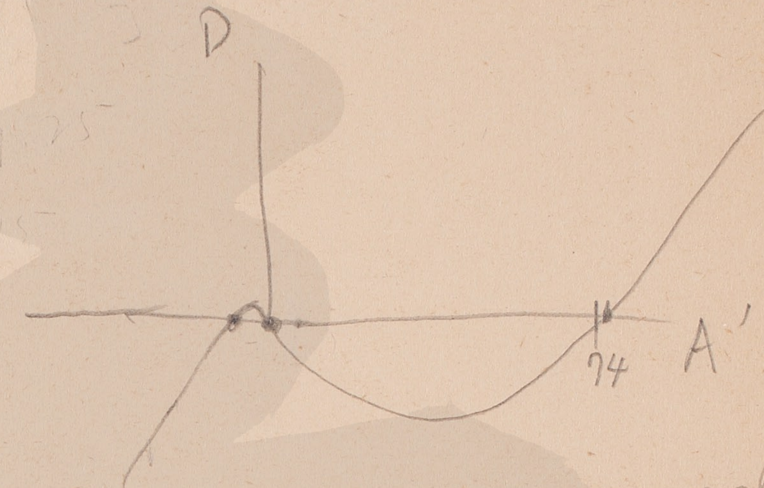
1,1103
0,7667
0,3436 − 2,21

37
3.7
259
111
1369
3
1372

$$= \frac{4}{27} A' \{ 3^3 A'^2 - 27 \cdot 74 A' - 9^2 \}$$

$$= 4 \cdot A' \{ A'^2 - 74 A' - 3 \}$$

$D \geq 0$: $A' = 0$ or $37 \pm \sqrt{(37)^2 + 3}$



$D < 0$:

$$b = \sqrt{\frac{4}{3}\left(A' + \frac{1}{3}\right)} \cosh \theta = \overbrace{\frac{2}{3}\sqrt{3A'+1}}^{\cosh\theta}$$

$$\cosh 3\theta = \sqrt{\frac{27 \{ \frac{2}{3} (5A' + \frac{1}{9}) \}^2}{4 \left(A' + \frac{1}{3}\right)^3}}$$

$$= \sqrt{\frac{(45A' + 1)^2}{(3A' + 1)^3}}$$

$$a' = \frac{1}{3} \{ 2 \sqrt{3A'+1} \cosh \theta + 1 \}$$

$$B = \frac{(a'+1)^3}{a'(a'+3)}$$

$$A' = -\frac{8mE}{\hbar^2\lambda^2} = 10.56 \times \frac{E^{\text{mill. volt}}}{\lambda'^2}$$

$2.64 \times 4 = 10.56$

$74 \overline{)\,10.56\,} = 0.143$
74
316
296

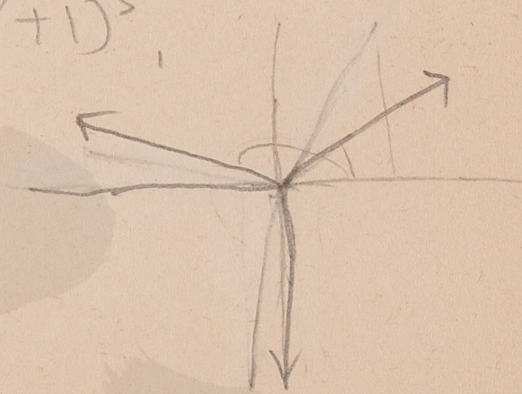
$A' = 74$; $\lambda'^2 = 0.143 \cdot E^{\text{mill. volt}}$

$D > 0.$ $\quad b = \sqrt{\frac{4}{3}\left(A' + \frac{1}{3}\right)} \cdot \cos\theta$.

″ $\quad \cos\left(\theta + \frac{2}{3}\pi\right)$,

″ $\quad \cos\left(\theta + \frac{4}{3}\pi\right)$

$$\cos 3\theta = \sqrt{\frac{(45A'+1)^2}{(3A'+1)^3}} ,$$



A'	$\frac{(4.5A'+1)}{(3A'+1)^{3/2}}$	3θ	$\cosh\theta$	a'	$B.$
0	1	0	1	1	2
1	5.75	2.435	1.346	2.128	2.80
2	4.913	2.274	1.302	2.634	3.23
5 (4)	3.531	1.934	1.245	3.573	4.07
8 (5)	2.888	1.722	1.169	4.230	4.67
16 (7)	2.102	1.373	1.107	5.399	5.78
21 33 (8) (10)	1.486	0.950	1.050	7.333	7.64
40 (11)	1.352	0.816	1.037	7.938	8.22
56 (13)	1.147	0.536	1.016	9.139	9.40

~~9~~6

$A' \cong 74$　　$a' \cong 10$　　$B \cong 10.3$

70

$$-A = \frac{-2mE}{\hbar^2\lambda^2} = \frac{2.8[illegible] \times E}{\lambda'^2} \qquad B = 0.35 \times \frac{g'^2}{\lambda'}$$

A'	$-A$	λ'^2/E	B
0	0	∞	2
1	0,25	10,56	2,80
2	0,5	5,28	3,23
5	1,25	2,11	4,07
8	2.	1,32	4,67
16	4.	0,66	5,78
33	8,25	0,32	7,64
40	10,	0,264	8,22
56	14.	0,189	9,40
74	18,5		~10.3

$$A' = \frac{10{,}56 \times E}{\lambda'^2}$$

Chadwick (Nature 134, 237, 1934)

$2{,}1 \times 10^6$ eV

E = 2,1.

~~E = 1,45~~

	λ'^2
1	15,312
2	7,656
5	3,062
8	1,914
16	0,957
~~33~~	
40	0,384
56	0,273

$g' = B^{\frac12}\, 0{,}35^{-\frac12} \cdot \lambda'^{\frac12}$

11,1

$E = 2.1$

	λ'^2	$\log \lambda'^2$	λ'	$g' = 0.35^{-\frac{1}{2}} \lambda'^{\frac{1}{2}} B^{\frac{1}{2}}$
1	22.176	1.34588	4.71	6.14
2	11.088	1.04485	3.33	5.54
5	4.435	0.64689	2.11	4.95
8	2.772	0.44279	1.67	4.71
16	1.386	0.14176	1.18	~~4.61~~
33	0.672	$\bar{1}.82737$	0.820	
40	0.554	$\bar{1}.74351$	0.744	
56	0.396	$\bar{1}.59770$	0.629	4.11