wave eq. solution

$\{\Delta - \lambda^2\} U = 0.$

$U(r)$: $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dU}{dr}\right) - \lambda^2 U = 0.$

$U = \frac{Ce^{-\lambda r}}{r}$. $\frac{dU}{dr} = \frac{-\lambda C e^{-\lambda r}}{r} - \frac{Ce^{-\lambda r}}{r^2}$

$\frac{d}{dr}\left(r^2\frac{dU}{dr}\right) = -\lambda C \frac{d}{dr}\left(re^{-\lambda r}\right) + C\lambda e^{-\lambda r}$

$= -C\lambda e^{-\lambda r} + C\lambda^2 r e^{-\lambda r} + C\lambda e^{-\lambda r}$

potential をもとめるに origin をうつして.

$\{\Delta - \lambda^2\} U(r) = -4\pi \rho'(r)$

とすると

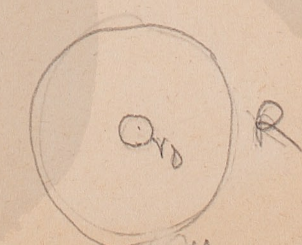
$\iiint \{U\{\Delta V\} - V\{\Delta U\}\} dv$

$\{-\lambda^2 V\}$ $\{-\lambda^2 U\}$

$= \iiint \mathrm{div}\{U\,\mathrm{grad}\,V - V\,\mathrm{grad}\,U\} dv$

$= \iint \{U\,\mathrm{grad}\,V - V\,\mathrm{grad}\,U\} df$

$V = \frac{e^{-\lambda r}}{r}$



$\sin\theta\, r^2 d\theta d\varphi$

$4\pi \iiint \frac{\rho' e^{-\lambda r}}{r} dv = 4\pi \iint_{r=R} U\left(\frac{\lambda e^{-\lambda r}}{r} + \frac{e^{-\lambda r}}{r^2}\right) - \frac{e^{-\lambda r}}{r}\frac{\partial U}{\partial r}$

$$+\iint_{r=r_0}\left\{U\left(\frac{\lambda e^{-\lambda r}}{r}+\frac{e^{-\lambda r}}{r^2}\right)+\frac{e^{-\lambda r}}{r}\frac{\partial U}{\partial r}\right\} r^2\sin\theta\, d\theta\, d\varphi$$

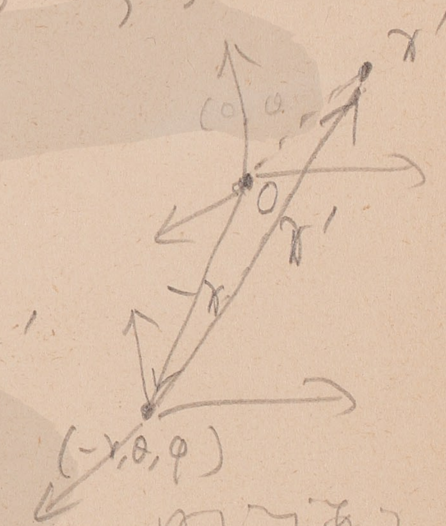
$$=\begin{cases} U,\ \dfrac{\partial U}{\partial r}:\ \text{finite for } r=0 \\ U,\ \dfrac{\partial U}{\partial r}:\ \text{finite for } r=\infty \end{cases}$$

ならば。

$$U_{r=0}=\iiint\frac{\rho e^{-\lambda r}}{r}\,dv$$

origin を $r(-r,\theta,\varphi)$ にうつせば。

$$U(r,\theta,\varphi)$$



$$=\iiint\frac{\rho(r')e^{-\lambda|r-r'|}}{|r-r'|}\,dv'$$

(勿論、新の ρ と旧の ρ とは意味はちがっている。functional form は異なる)

$$\rho(r'-r)\Rightarrow\rho(r')$$

$$(\Delta-\lambda^2)U=0$$

$$\frac{3}{2}r^{\frac{1}{2}}\frac{dV}{dr}$$

$$U=U(r)P_{l,m}^{m}(\theta,\varphi)$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dU}{dr}\right)-\left(\lambda^2+\frac{l(l+1)}{r^2}\right)U=0,$$

$$U(r)=\frac{V(r)}{r^{\frac{1}{2}}} \qquad r^2\frac{dU}{dr}=r^{3/2}\frac{dV}{dr}-\frac{1}{2}r^{1/2}V$$

$$\frac{1}{r^2}\frac{d^2}{dr}\left(r^2\frac{dU}{dr}\right)=\frac{1}{r^{1/2}}\cdot\frac{1}{r^{3/2}}\left\{r^{3/2}\frac{d^2V}{dr^2}+r^{1/2}\frac{dV}{dr}+\frac{1}{4}r^{-1/2}V\right.$$

$$\frac{d^2V}{dr^2}+\frac{1}{r}\frac{dV}{dr}-\left(\lambda^2+\frac{(l+\frac{1}{2})^2}{r^2}\right)V=0$$

$$x=i\lambda r$$

$$\frac{d^2V}{dx^2}+\frac{1}{x}\frac{dV}{dx}+\left\{1-\frac{(l+\frac{1}{2})^2}{x^2}\right\}V=0$$

$$V=H^{(1)}_{l+\frac{1}{2}}(x)$$

$$\cong\sqrt{\frac{2}{\pi x}}\cdot e^{i(x-\frac{2\nu+1}{4}\pi)} \quad \text{for } |x|\gg1.$$

$$V=\frac{H^{(1)}_{l+\frac{1}{2}}(x)+\overline{H^{(1)}_{l+\frac{1}{2}}(x)}}{2}$$

$$\text{or}=\frac{H^{(1)}_{l+\frac{1}{2}}(x)-\overline{H^{(1)}_{l+\frac{1}{2}}(x)}}{2i}$$

$$H_{\nu}^{(l)} =$$

$$U_l(r) = C_l \cdot r^{l+1} \cdot \frac{d^l}{d(r^2)^l} \frac{e^{-\lambda r}}{r}. \qquad \frac{d}{dr} = 2r \frac{d}{d(r^2)} = \frac{d}{}$$

$$\frac{dU_l}{dr} = 2 r^l \frac{d}{d(r^2)} \frac{U_{l-1}}{r^{l-1}} = \frac{dU_{l-1}}{dr} - (l-1)\frac{U_{l-1}}{r}$$

$$= l\, r^{l-1} \cdot \frac{d}{d(r^2)} \cdot \frac{U_{l-1}}{r^{l-1}} + 2 r^{l+1} \frac{d}{d(r^2)} \frac{1}{r^2} \frac{dU_{l-1}}{dr} \}$$

$$- (l-1) r^l \frac{d}{d(r^2)} \cdot \frac{U_{l-1}}{r^l}$$

$$r^2 \frac{d}{dr}\{ \quad \} = \{\lambda^2 r^2 + l(l-1)\} U_{l-1}$$

$$r^2 \frac{d}{dr}\left\{ \frac{dU_{l-1}}{dr} - (l-1)\frac{U_{l-1}}{r} \right\} = \frac{d}{dr}\left\{ r^2 \frac{dU_{l-1}}{dr} \right\}$$

$$-(l+1)$$

$$\frac{2}{r} \cdot r^2$$

$$- 2r \frac{dU_{l-1}}{dr} - (l-1) r \frac{dU_{l-1}}{dr} + (l-1) U_{l-1}$$

$$\frac{1}{r^2} \frac{d}{dr}\left\{ r^2 \frac{d}{dr}\{ \quad \}\right\} = \left\{ \lambda^2 + \frac{l(l-1)}{r^2} \right\} \frac{dU_{l-1}}{dr}$$

$$-\frac{(l+1)}{r}\left\{ \lambda^2 + \frac{l(l-1)}{r^2} \right\}$$

$$+ \frac{2\lambda^2}{r} U_{l-1} - \frac{l+1}{r^3} \frac{d}{dr}\left(r^2 \frac{dU_{l-1}}{dr} \right) + \frac{l+1}{r^2} \frac{dU_{l-1}}{dr}$$

$$+ \frac{(l-1)}{r^2} \frac{dU_{l-1}}{dr} = \left\{ \lambda^2 + \frac{l(l+1)}{r^2} \right\} \frac{dU_{l-1}}{dr}$$

$$- (l-1) \frac{U_{l-1}}{r} \}$$

$$u_\ell(r) = C_\ell r^\ell \left(\frac{d}{r\,dr}\right)^\ell \cdot \frac{e^{-\lambda r}}{r}$$

$$= \frac{e^{-\lambda r}}{r}\left(\frac{a_0}{r^{\ell+1}} + \frac{a_1}{r^{\ell-1}} + \cdots\right)$$

$$= \frac{e^{-\lambda r}}{r^{\ell+1}}\left(a_0 + a_1 r + \cdots + a_\ell r^{\ell+1}\right)$$