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DATE Nov. 1, 1934

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# On the Interaction of Elementary Particles. I.

By Hideki Yukawa.

## § 1. Introduction

At the present stage of the quantum theory little is known about the nature of interaction between elementary particles. For example, the <sup>interaction</sup> force acting between a neutron and a proton ~~is not known~~ whether it is an ordinary attraction force or an "exchange interaction" first proposed by Heisenberg. Recently Fermi<sup>(1)</sup> has treated the problem of  $\beta$ -ray disintegration on the hypothesis of the existence of "neutrino". According to this theory a neutron and a proton can interact by emitting and absorbing a neutrino and an electron. Unfortunately the energy of interaction calculated on this such assumption<sup>(2)</sup> is much too small to account for the binding of neutrons and protons in the nucleus. To remove this deficit we <sup>may</sup> ~~cannot~~ <sup>have to</sup> modify the theory of Heisenberg or Fermi in the following way.

The transition of a heavy particle from a neutron state to a proton state is not always accompanied

<sup>(1)</sup> E. Fermi, Zeits. f. Phys. **88**, 161<sub>2</sub> (1934).

<sup>(2)</sup> J. J. Thomson, Nature, **133**, 981 (1934); D. Swanecko, *ibid.*, 981 (1934).

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by the emission of ~~or~~ the light particles, i.e. a neutron and an electron, but the energy liberated by the transition can be taken up by another heavy particle which changes transfers into a neutron state.

<sup>in photon state</sup> interaction <sup>between a neutron and a proton.</sup> increases will increase ~~much~~ the interaction <sup>interaction by</sup> of exchange of energy.

Now such direct interaction between the elementary particles can be described by <sup>the aid of</sup> a field of force just as the interaction between charged particles ~~are~~ are described by ~~electrons~~ <sup>the aid of</sup> the aid of electromagnetic field.

~~The~~ <sup>the</sup> author ~~first~~ <sup>with</sup> discusses briefly in this paper the nature of such a field of force and of the quantum, which accompanies ~~the field at~~ <sup>the field?</sup> just as the photon accompanies the electromagnetic field, when it is quantized. Also their bearing on the ~~theory~~ nuclear structure, and ~~of~~ <sup>of</sup>  $\gamma$ -ray disintegration will be discussed considered.

Fuller account will be made in the next paper. <sup>and the scattering of neutrons or neutinos</sup> ~~interaction~~ <sup>interaction</sup> the exchange interaction of elementary particles.

§ 2. Field describing the interaction of elementary particles. In analogy to electric scalar ~~and~~ <sup>and</sup> the vector potentials  $V, A$  of the electromagnetic field we introduce a scalar



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function  $U(x, y, z, t)$  and a vector function  $B(x, y, z, t)$  describing the field between a neutron and a proton. These functions will satisfy the wave equations similar to the wave equations for the electromagnetic potentials.

Now the equation  $\Delta U - \frac{1}{c^2} \frac{\partial^2 U}{\partial t^2} = 0$  has only static solution with central symmetry  $\frac{1}{r}$

except the additive and multiplicative constants. The force acting between a neutron and a proton, however, is recognized to not to be Coulomb type, but to decrease more rapidly with the distance, which it can be expressed for example by

$$g \frac{e^{-\lambda r}}{r} \quad (2)$$

where  $g$  is a constant corresponding to the electric charge in the case of electromagnetic field. Since this function is, as well known, the static solution with central symmetry of the wave equation

$$\Delta U - \frac{1}{c^2} \frac{\partial^2 U}{\partial t^2} - \lambda^2 U = 0, \quad (3)$$

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of  $U, U^\dagger$

The equations (5) and (4) can be deduced by the variation from a Lagrangian function

$$L = \int \left( \frac{\partial U}{\partial x} \frac{\partial U^\dagger}{\partial x} + \frac{\partial U}{\partial y} \frac{\partial U^\dagger}{\partial y} + \frac{\partial U}{\partial z} \frac{\partial U^\dagger}{\partial z} - U \frac{\partial U^\dagger}{\partial t} + \frac{\partial U}{\partial t} U^\dagger + \Phi U^\dagger + \tilde{\Phi} U \right) dV dt,$$

where by the variation of  $U, U^\dagger$ , where their values on the three dimensional boundary being fixed.

Next we consider the function  $U^\dagger$  complex conjugate to  $U$  which satisfies the equation

$$(\Delta - \frac{1}{c} \frac{\partial}{\partial t} - \lambda^2) U^\dagger = -g \tilde{\Phi} \Psi \quad (4)$$

This corresponds to the inverse transition from a proton state to a neutron state, similar equations will hold for the vector potential, but we do not consider it for the moment, because we do not know the correct relativistic wave equations for the proton and the neutron.

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Let us assume (2) to be the correct wave equation for  $\psi$  in vacuum, where  $\lambda$  is a constant with dimension  $\text{cm}^{-1}$ . In the presence of the heavy particles the  $\psi$ -field interact with them and causes the transition from the neutron state to the proton state, so that, if we denote the wave functions of neutron and proton by  $\psi(x, y, z, t)$  and  $\psi'(x, y, z, t)$  respectively, the right hand side of (2) can be substituted by the Hermitian term like wave equation becomes

$$\left\{ \Delta - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \lambda^2 \right\} \psi = -g \psi' \psi + 4\pi g \tilde{\psi} \psi \quad (4)$$

should be inserted in the right hand side of the equation (2). Next we consider the field  $\psi'$ , which is complex conjugate of  $\psi$ . This corresponds to the inverse transition from a proton state to a neutron state.

Let us satisfy the equation

$$\left\{ \Delta - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \lambda'^2 \right\} \psi' = -g \psi \psi' \quad (4')$$

Now the wave equations for heavy particles interacting with  $\psi$ -field, can be deduced from a Hamiltonian, function for the total system Lagrangian

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$$\Psi i\hbar \frac{\partial \Psi}{\partial t} + \tilde{\Psi} i\hbar \frac{\partial \tilde{\Psi}}{\partial t}$$

The kinetic and potential energies of the heavy particles are

$$\frac{\hbar^2}{2M_N} \left( \int \tilde{\Psi} \frac{\partial \Psi}{\partial x} \frac{\partial \Psi}{\partial x} + \frac{\partial \tilde{\Psi}}{\partial y} \frac{\partial \Psi}{\partial y} + \frac{\partial \tilde{\Psi}}{\partial z} \frac{\partial \Psi}{\partial z} \right) d\mathbf{r}$$

$$+ \frac{\hbar^2}{2M_P} \left( \int \tilde{\Psi} \frac{\partial \Psi}{\partial x} \frac{\partial \Psi}{\partial x} + \frac{\partial \tilde{\Psi}}{\partial y} \frac{\partial \Psi}{\partial y} + \frac{\partial \tilde{\Psi}}{\partial z} \frac{\partial \Psi}{\partial z} \right) d\mathbf{r}$$

and 
$$g \int \tilde{\Psi} U \Psi + \tilde{\Psi} U^\dagger \Psi d\mathbf{r}$$

so that the Lagrangian for function  $\Psi$  (the equation of motion can be derived from  $L_M$ ) would be for the heavy particle

$$L_M = \left( \int \tilde{\Psi} \frac{\hbar^2}{2M_N} \left( \frac{\partial \Psi}{\partial x} \frac{\partial \Psi}{\partial x} + \frac{\partial \tilde{\Psi}}{\partial y} \frac{\partial \Psi}{\partial y} + \frac{\partial \tilde{\Psi}}{\partial z} \frac{\partial \Psi}{\partial z} \right) + \frac{\hbar^2}{2M_P} \left( \frac{\partial \tilde{\Psi}}{\partial x} \frac{\partial \Psi}{\partial x} + \frac{\partial \tilde{\Psi}}{\partial y} \frac{\partial \Psi}{\partial y} + \frac{\partial \tilde{\Psi}}{\partial z} \frac{\partial \Psi}{\partial z} \right) - g \int \tilde{\Psi} U \Psi + \tilde{\Psi} U^\dagger \Psi \right) d\mathbf{r} dt$$

the variations of  $\Psi, \tilde{\Psi}, \dot{\Psi}, \dot{\tilde{\Psi}}$ , i.e.

$$\left( \frac{\hbar^2}{2M_N} \Delta + i\hbar \frac{\partial}{\partial t} \right) \Psi = -g U \Psi$$

$$\left( \frac{\hbar^2}{2M_P} \Delta + i\hbar \frac{\partial}{\partial t} \right) \tilde{\Psi} = -g U^\dagger \tilde{\Psi}$$

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Now the constants  $g$ ,  $\lambda$  appearing in the equations (4), (5) should be determined by comparing with experiment. For example, assuming the interaction between a neutron and a proton, (2) for the mass defect of  $H^2$  and the scattering of a neutron by a proton can be estimated <sup>probability of</sup>, which from which <sup>calculated and compared with experimental data.</sup> ~~it is estimated that~~  $\lambda$  should be a multiple lie between  $10^{12}$  cm and  $10^{17}$  cm and  $g$  should be a few multiple of elementary charge  $e$ .

§ 3. Quantum accompanying the  $U$ -field  
~~Now~~ In the quantum theory the  $U$ -field should be quantized according to the general principle,  
 the free space  $U$  denoting

$$p_{ri} = -i\hbar \frac{\partial}{\partial x_i}, \quad \text{etc} \quad W = i\hbar \frac{\partial}{\partial t}$$

<sup>wave</sup>  $m_U = \lambda \hbar$ , the equation for  $U$  in free space <sup>can be</sup> is written in the form

$$\{ p_{ri}^2 - W^2/c^2 + m_U^2 c^4 \} U = 0.$$

so that we see the quantum accompanying the  $U$ -field has a proper mass  $m_U = \frac{\lambda \hbar}{c}$ .

Assuming, for example,  $\lambda = 5 \times 10^{12}$  cm we obtain for  $m_U$  a value  $2 \times 10^{-2}$  electron mass  $m_e$ .

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Thus the proper mass of the quantum is very great and lie between the masses of the electron and the proton. If we consider now we consider a massive quantum the what case such a massive quantum will be emitted from the nucleus? The energies of If the energies of the neutrons state  $\Phi$  and the proton state  $\Psi$  be  $W_N, W_P$  respectively, we can write

$$\Phi = U(xyz) e^{-iW_N t/\hbar}$$

$$\Psi = \tilde{V}(xyz) e^{iW_P t/\hbar}$$

so that the hence the right hand side of the wave equation (4) becomes

$$-4\pi g \cdot \tilde{V} U e^{-i(W_N - W_P) t/\hbar}$$

so that putting  $\Psi = U'(xyz) e^{-i(W - W_P) t/\hbar}$ ,

we have

$$\Delta - (\lambda^2 - \frac{\omega^2}{c^2}) \Psi = -4\pi g \tilde{V} U,$$

where  $\omega = W - W_P/\hbar$ ,

Integrating this equation we obtain a solution

$$\Psi(x) = g \iiint \frac{e^{-\mu|x-x'|}}{|x-x'|} \tilde{V}(x') U(x') dx'$$

where  $\mu = \sqrt{\lambda^2 - \frac{\omega^2}{c^2}}$ .

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In case  $\lambda > \frac{w}{c}$  or  $m v c^2 > W_N - W_p$ ,  $M$  is real and the function  $J(r)$  of Heisenberg has the form  $e^{-\mu r}$ , but  $M$  is ~~not~~ depends on  $W_N - W_p$  and becomes smaller and smaller when the latter becomes nearer to  $m v c^2$ . This means the range of interaction between neutron and a proton extends ~~but~~ increases when  $W_N - W_p$  increases. (3) According to the experiment of Bonner (3) the effective cross section of ~~the~~ <sup>the</sup> neutron and ~~the~~ the nucleus ~~de-~~ increases with the velocity of the neutron in case of Pb lead and, whereas it decreases in case of carbon and hydrogen. Moreover the rate of decrease is slower in case of carbon than hydrogen. This effect is not clear ~~and~~ may be ~~the~~ <sup>the</sup> ~~superposition~~ <sup>the</sup> of inelastic scattering and the simple elastic scattering in case of the heavy nucleus. Above consideration show, however, a possible explanation of the effect, ~~for~~ that is, for, if the binding energy of the neutron to the nucleus becomes comparable to  $m v c^2$ , the interaction of the neutron and the proton will ~~change~~ <sup>increase</sup> considerably with the velocity of the neutron, so that the ~~the~~ cross section will decrease slower in ~~the~~ this case than in the case of hydrogen. Now the binding energy of the outer proton of  $C^{12}$  is determined estimated (3) W. Bonner, Phys. Rev. 45, 606 (1934).

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from the difference of masses of  $C^{11}$  and  $B^{11}$ , which is  
 $12.0036 - 11.0110 = 0.9926$ .

This corresponds to a binding-energy  
0.0152

in mass unit and ~~to~~ about thirty times of the electron mass, so that in case of carbon we can expect the effect observed by Posner. Of course, above argument is only tentative and the other explanations are not included.

What in case  $\lambda < \frac{h}{m_0 c}$  or  $m_0 c^2 < W_N - W_P$ ,  $\mu$  becomes imaginary and  $\psi$  expresses a spherical wave, so that ~~is~~ a quantum is emitted with energy greater than  $m_0 c^2$  is emitted by ~~the~~ transition of a neutron state to a proton state, provided ~~the~~ heavy particle from  $W_N - W_P > m_0 c^2$ . The phase velocity of  $\psi$ -wave is greater than the light velocity  $c$ , but the group velocity is smaller than  $c$ , as in the case of electron wave.

The mass  $m_0$  is, however, hundred times greater than electron mass, so that the condition  $W_N - W_P > m_0 c^2$  is never satisfied in ordinary case.

The quantum accompanying  $\psi$  ~~has~~ <sup>moreover</sup> should have a ~~positive~~ <sup>negative</sup> charge - $e$  and ~~also~~ <sup>follow</sup> the Posner's conservation of charge and ~~also~~ <sup>satisfy</sup>

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statistics, since both ~~the~~ neutrons and protons ~~are~~ <sup>obey</sup> Fermi statistics.

~~no~~ ~~the~~ quantization of W. field will be ~~reformed~~ <sup>but in the present</sup> analogous to electromagnetic field. ~~but in the present~~ paper we do not go farther, in this problem. The detailed account of the problem will be given in the

§ 4. Theory of  $\beta$ -disintegration  
According to our theory the ~~the~~ massive quantum emitted by a heavy particle when a heavy particle jumps from a neutron state to a proton state can be absorbed by a light particle which jumps from a neutrino state to an electron state <sup>of positive energy.</sup>

Thus an antineutrino and an electron are emitted from the nucleus. Such double processes intervention of a massive quantum does not alter ~~the transition~~ <sup>in general</sup> probability of  $\beta$ -disintegration, essentially, for which ~~has~~ has been calculated on the hypothesis of direct coupling of a heavy particle and a light particle, just as in the theory of internal conversion, the intervention of the ~~the~~ photon does not alter <sup>in general</sup> the coefficient of internal conversion. <sup>(\*)</sup>

(\*) Taylor and Motz, Proc. Roy. Soc. A. 138, 665 (1932).

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So that our theory differ does not differ essentially from Fermi's theory.

Fermi has assumed that an electron and a neutrino are emitted simultaneously from the radioactive nucleus, but <sup>his assumption is formally</sup> we ~~do~~ want ~~it~~ for the sake of simplicity in form we assume that a light particle equivalent to assume that a light particle jumps from a neutrino state of negative energy ~~into~~ a electron state of positive energy.

For if the eigenfunctions of the electron and the neutrino be  $\psi_k, \varphi_k$  respectively, where  $k=1, 2, \dots, 4$ , in the right hand side of the equation for  $W$  a term of the form

$$-4\pi g' \sum_{k=1}^4 \psi_k \varphi_k \quad (6)$$

will be added, where  $g'$  is a ~~constant~~ constant with the same dimension as  $g$ . Now ~~if~~ ~~we~~ be the eigenfunctions of the neutrino state with the same energy and momentum <sup>but</sup> opposite to the state that of the state of  $k$  is given by

$$\varphi'_k = \delta_{kl} \tilde{\varphi}_k$$

$$\delta = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

where

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so that (6) becomes

$$-4\pi g' \sum_{k=i}^4 \tilde{\psi}_k \delta_{kl} \psi_k \quad (17) \quad (A)$$

This expression of density is the same as that of Fermi except the conjugate complex and ~~the~~ absolute values are the same in both case. <sup>same as conjugate complex to</sup> The matrix element of

from (9) and (6) we obtain for the interaction energy of ~~the~~ the heavy particle and the light particle

$$gg' \iint \tilde{\psi}(x_1) u(x_1) \sum_k \tilde{\psi}_k(x_2) \psi_k(x_2) \times \frac{e^{-\lambda r_{12}}}{r_{12}} dv_1 dv_2, \quad (8)$$

from which we can calculate the probability of  $\delta$ -disintegration

Since  $\lambda$  is <sup>very large</sup> compared with the wave numbers of the electron and the neutrinos in ordinary case, the function for the integration with respect to  $x_1$  is

$$\frac{e^{-\lambda r_{12}}}{r_{12}} \int \frac{4\pi}{X} \delta_{kl} dv_2 = \frac{4\pi}{X^2}$$

can be regarded as a  $\delta$ -function multiplied by a factor

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so that (8) becomes

$$\frac{4\pi g g'}{\lambda^2} \iiint \tilde{v}(x) u(x) \sum_k \psi_k(x) \varphi_k(x) \cdot dv \quad (9)$$

which, considering the absolute square of the modulus of since this expression is the same as the equation (21) of Fermi's ~~that~~ except, if we substitute for Fermi's  $g$  the factor

$$\frac{4\pi g g'}{\lambda^2}$$

the our theory does not differ essentially from Fermi's theory. Comparing with experimental

data, Fermi has taken for his ~~value~~  $g$  a

value  $4 \cdot 10^{-10}$  cm<sup>2</sup> erg

we can determine  $g'$  ~~value~~  $3 \times 10^{-9}$  <sup>from which we take for</sup>  $g'$  becomes a

$$g' = \frac{\lambda^2}{4\pi g} \quad 4 \cdot 10^{-50}$$

$$= 3 \times 10^{-17}$$

which is  $10^8$  times smaller than  $g$ . This means that the interaction between a neutrino and a electron is much smaller than that between a neutrino

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and a proton, so that the neutrino <sup>will be</sup> far more penetrating than the neutron and more difficult to observe. The difference of  $g$  and  $g'$  will may be due to the difference of masses of heavy and light particles.

### § 5. Conclusion

In conclusion thus the interaction of elementary particles can be described by considering <sup>the</sup> quanta with elementary charge and proper mass, satisfying the Bose's statistics. The interaction of these quanta with heavy particles <sup>is</sup> far greater than that with light particles to account for the large interaction of neutron and a proton and the small probability of  $\beta$ -disintegration. ~~The~~ <sup>For larger than that of the electron</sup> ~~the~~ <sup>proton</sup> mass of these quanta should be surprisingly large to account for ~~the interaction~~ <sup>the</sup>.

Such quanta, if they come nearer to the nucleus, they will give their charge and energy to the nucleus, ~~the nucleus~~ <sup>the nucleus</sup> and the ~~charge~~ <sup>charge</sup> of the matter which changes so that if the quanta with negative charge come in excess, the matter will be charged up to negative potential and vice versa.

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~~But~~ these arguments merely of course, of merely speculative,  
agree with ~~the~~ view ~~recently~~ <sup>in</sup> that the cosmic high  
speed positive particles ~~emitting~~ the cosmic ray are  
generated by the earth's electrostatic field <sup>(5)</sup>  
of the earth, which are charged to a negative potential.

L.G.H.  
(5) <sup>(5)</sup> Tinsley, Nature, 134, 418, 571<sub>a</sub> (1934); Johnson, Phys. Rev.  
45, 569<sub>a</sub> (1934).