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DATE

NO. 1

On the Interaction of Elementary Particles. I.

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(Read Nov. 17, 1934)

§ 1. Introduction

At the present stage of the quantum theory little is known about the nature of interaction between elementary particles. Heisenberg assumed that between the neutron and the proton the interaction by "Platzwechsel" was most important.[1] Recently Fermi treated the problem of β-ray disintegration on the hypothesis of the "neutrino".[2] According to this theory the neutron and the proton can interact by emitting and absorbing a neutrino and an electron. Unfortunately the energy calculated on such assumption is much too small to account for the binding energies between a neutron and a proton ~~to~~ in the nucleus.[3]

(1) W. Heisenberg, Zeits. f. Phys. **77**, 1 (1932); **78**, 156 (1932); **80**, 587 (1933). We shall denote them I, II and III.
(2) E. Fermi, ibid. **88**, 161 (1934).
(3) Ig. Tamm, Nature **133**, 981 (1934); D. Iwanenko, ibid. 981 (1934).

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To remove this defect it seems natural to modify the theory of Heisenberg and Fermi in the following way.

The transition of a heavy particle from neutron state to proton state is not always accompanied by the emission of light particles, i.e. a neutrino and an electron, but the energy liberated by the transition is taken up sometimes by another heavy particle, which consequently transfers from proton state into neutron state. If the probability of the latter process is much larger than that of the former, the interaction between the neutron and the proton increases much more than that in the case of Fermi, whereas the probability of emission of light particles is not affected essentially.

Now such interaction between the elementary particles can be described by means of a field of force just as the interaction between charged particles are described by electromagnetic field. The above considerations show that the interaction of heavy particles with this field is much larger than that of light particles with it.

In the quantum theory this field will be accompanied by a new sort of quanta, just as electromagnetic field is accompanied by photons.

In this paper the nature of this field and the quanta accompanying it will be discussed briefly and also their bearing on the nuclear structure, β-ray disintegration etc. will be considered.

Besides such a force and the ordinary electromagnetic force other forces may exist between elementary particles, but we neglect them for the moment.

Fuller account will be made in the next paper.

§ 2. Field describing the interaction of elementary particles.

In analogy to scalar and vector potentials of the electromagnetic field we introduce a scalar function $U(x,y,z,t)$ and a vector function $\mathfrak{B}(x,y,z,t)$ describing the field of force between the neutron and the proton. These functions will satisfy equations similar to the wave equations for the electromagnetic potentials.

Now the equation

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right\} U = 0 \qquad (1)$$

has only static solution with central symmetry

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except the additive and multiplicative constants $\frac{1}{r}$.
The potential of force between the neutron and the proton, however, ~~is reg recog~~ should not be of Coulomb type, but ~~to~~ decrease more rapidly with distance. It can be expressed for example by

$$+ \text{ or } - \quad g^2 \frac{e^{-\lambda r}}{r}, \qquad (2)$$

where g is a constant with the dimension of electric charge, i.e. $cm.^{\frac{3}{2}}\ sec.^{-1}\ gr.^{\frac{1}{2}}$, and λ with dimension cm^{-1}.
Since this function is a static solution with central symmetry of the wave equation

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right\} U = 0, \qquad (3)$$

let us assume (3) to be the correct equation for U in vacuum. In the presence of the heavy particles the U-field interact with them and causes the transition from neutron state to proton state.
Now, if we introduce the matrices[4]

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and denote the neutron state and the proton state by $\tau_3 = 1$ and $\tau_3 = -1$ respectively, the wave

(4) Heisenberg, loc. cit. I.

equation is given by

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right\} U = -4\pi g \tilde{\Psi} \frac{\tau_1 + i\tau_2}{2} \Psi, \qquad (4)$$

where Ψ denotes the wave function of the heavy particle and is a function of x, y, z, t and τ_3', which takes eigenvalues ± 1 of τ_3.

Next, corresponding to the inverse transition from proton to neutron state, the complex conjugate potential $\tilde{U}(x, y, z, t)$, satisfying the equation

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right\} \tilde{U} = -4\pi g \tilde{\Psi} \frac{\tau_1 - i\tau_2}{2} \Psi, \qquad (5)$$

is introduced.

Similar equations will hold for the vector potentials, but we disregard it for the moment, since there's no correct relativistic theory for the heavy particle. Henceforth, simple non-relativistic ~~Schrödinger~~ equation will be used for the heavy particle, which is

$$\left\{\frac{\hbar^2}{4}\left(\frac{1+\tau_3}{M_N} + \frac{1-\tau_3}{M_P}\right)\Delta + i\hbar\frac{\partial}{\partial t} - \frac{1+\tau_3}{2}M_N c^2 - \frac{1-\tau_3}{2}M_P c^2 - g\left[U\frac{(\tau_1 - i\tau_2)}{2} + \tilde{U}\frac{(\tau_1 + i\tau_2)}{2}\right]\right\}\Psi = 0, \qquad (6)$$

where $\hbar$ is ~~the~~ Planck's constant divided by 2π and M_N, M_P are the masses of neutron and proton

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respectively. The reason for taking − sign before g will be mentioned later.

The equation (6) corresponds to the Hamiltonian

$$H = \left(\frac{1+\tau_3}{4M_N} + \frac{1-\tau_3}{4M_P}\right)p^2 + \frac{1+\tau_3}{2}M_N c^2 + \frac{1-\tau_3}{2}M_P c^2$$
$$+ \frac{g}{2}\{U(\tau_1 - i\tau_2) + \tilde{U}(\tau_1 + i\tau_2)\}, \qquad (7)$$

where p is the momentum of the heavy particle. If we put $M_N - M_P = \frac{D}{c^2}$ and $M_N + M_P = 2M$, (7) becomes approximately

$$H = \frac{p^2}{2M} + \frac{g}{2}\{U(\tau_1 - i\tau_2) + \tilde{U}(\tau_1 + i\tau_2)\}$$
$$+ \tau_3 D \qquad (8)$$

where the constant term Mc^2 is omitted.

Now, consider two heavy particles at points (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively and assume their ~~relatuvel~~ relative velocity to be small. The field at (x_1, y_1, z_1) due to the particle at (x_2, y_2, z_2) are, from (4) and (5),

$$\left.\begin{aligned} U(x_1, y_1, z_1) &= g\frac{e^{-\lambda r_{12}}}{r_{12}}\frac{(\tau_1^{(2)} + i\tau_2^{(2)})}{2} \\ \text{and} \quad \tilde{U}(x_1, y_1, z_1) &= g\frac{e^{-\lambda r_{12}}}{r_{12}}\frac{(\tau_1^{(2)} - i\tau_2^{(2)})}{2}, \end{aligned}\right\} \qquad (9)$$

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where $(\tau_1^{(1)}, \tau_2^{(1)}, \tau_3^{(1)})$ and $(\tau_1^{(2)}, \tau_2^{(2)}, \tau_3^{(2)})$ are the matrices relating to the first and the second particles respectively and r_{12} is the distance between them. Hence, the Hamiltonian for the system is given, in the absence of the external fields, by

$$H = \frac{p_1^2}{2M} + \frac{p_2^2}{2M} + \frac{g^2}{4}\{(\tau_1^{(1)} - i\tau_2^{(1)})(\tau_1^{(2)} + i\tau_2^{(2)})$$
$$+ (\tau_1^{(1)} + i\tau_2^{(1)})(\tau_1^{(2)} - i\tau_2^{(2)})\}\frac{e^{-\lambda r_{12}}}{r_{12}} + (\tau_3^{(1)} + \tau_3^{(2)})D$$
$$= \frac{p_1^2}{2M} + \frac{p_2^2}{2M} + \frac{g^2}{2}(\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)})\frac{e^{-\lambda r_{12}}}{r_{12}}$$
$$+ (\tau_3^{(1)} + \tau_3^{(2)})D, \qquad (10)$$

where p_1, p_2 are the momenta for the particles.
This Hamiltonian is equivalent to Heisenberg's Hamiltonian (1)[5], if we take for "Platzwechselintegral"

$$J(r) = -g^2\frac{e^{-\lambda r}}{r}, \qquad (11)$$

with the exception that the interaction between neutrons and the electrostatic repulsion between protons are not taken into account in our case.
Heisenberg took the sign of $J(r)$ positive, so that the spin of the lowest energy state of H^2 was zero,

(5) Heisenberg, loc. cit. I.

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whereas in our case, owing to the negative sign before g^2, the lowest energy state has the spin 1, which is required from the experiment.

Two constants g, λ appearing in the above equations should be determined by comparing with experiment. For example, using the Hamiltonian (10) for two heavy particles, we can calculate the mass defect of H^2 and the probability of scattering of a neutron by a proton, provided that the relative velocity is small compared with the light velocity[6].

Rough estimation shows that the calculated values agree with the experimental results if we take λ between 10^{12} cm^{-1} and 10^{13} cm^{-1} and g ~~few multi~~ several times of the elementary charge e. Thus the ~~numb~~ numerical values of g and e are of the same order, although no direct relations between them were suggested in the above considerations.

§ 3. Nature of the quanta accompanying the field.

(6) Mr. Tomonaga has previously made these calculations according to the theory of Heisenberg. A little modification is necessary in our case. Detailed accounts will be made in the next paper.