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On the Interaction of Elementary Particles. I.

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§1. Introduction.

At the present stage of the quantum theory little is known about the nature of interaction between elementary particles. Heisenberg ~~introduced~~ ~~considered~~ the socalled "Platzwechsel"-interaction
~~assumed that~~ between the neutron and the proton, the interaction
consider the
by "Platzwechsel" ~~was~~ to be the most important.[1] Recently Fermi treated the problem of β-ray disintegration on the hypothesis of "neutrino"[2] to the nuclear structure,
According to this theory the neutron and the proton can interact by emitting and absorbing a neutrino and an electron. Unfortunately the interaction energy calculated on such assuption is much too small to account for the binding energies of neutrons and protons in the ~~and~~ nucleus.[3]

To remove this defect it seems natural to modify the theory of Heisenberg and Fermi in the following way. The transition of a heavy particle from neutron state to proton state is not always accompanied by the emission of light particles, i.e. a neutrino and an electron, but the energy liberated by the transition is

(1) W. Heisenberg, Zeits. f. Phys. **77**, 1 (1932); **78**, 156 (1932); **80**, 587 (1933). We shall denote them I, II and III.
(2) E. Fermi, ibid. **88**, 161 (1934).
(3) Ig. Tamm, Nature **133**, 981 (1934); D. Iwanenko, ibid. 981 (1934).

(2)

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~~is~~ taken up sometimes by another heavy particle, which in turn transfers from proton state into neutron state. If the probability of occurrence of the latter process is much larger than that of the former, the interaction between the neutron and the proton increases much more than in the case of Fermi, whereas the probability of emission of light particles is not affected essentially.

Now such interaction between the elementary particles can be described by means of a field of force, just as the interaction between the charged particles is described by the electromagnetic field. The above considerations show that the interaction of heavy particles with this field is much larger than that of light particles with it.

In the quantum theory this field should be accompanied by a new sort of quantum, just as the electromagnetic field is accompanied by the photon.

In this paper the possible natures of this field and the quantum ~~accompan~~ accompanying it will be discussed briefly and also their bearing on the nuclear stucture will be considered.

Besides such an exchange force and the ordinary electric and magnetic forces there may be other forces between the elementary particles, but we disregard the latter for the moment.

Fuller account will be made in the next paper.

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§2. Field describing the interaction.

In analogy to scalar potential ~~ϕ~~ of the electromagnetic field we introduce a function U(x,y,z,t) describing the field between the neutron and the proton. This function will satisfy an equation similar to the wave equation for the electromagnetic potential.

Now the equation

$$\left\{ \Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \right\} U = 0 \qquad (1)$$

has only static solution with central symmetry $\frac{1}{r}$, except the additive and multiplicative constants. The potentials of force between the neutron and the proton should, however, not be of Coulomb type, but decreas more rapidly with distance. It can be expressed for example by

$$+ \text{ or } - g^2 \frac{e^{-\lambda r}}{r} \qquad (2)$$

where g is a constant with the dimension of electric charge, i.e. $\text{cm.}^{\frac{3}{2}}\ \text{sec.}^{-1}\ \text{gr.}^{\frac{1}{2}}$, and λ with dimension cm.^{-1}

Since this function is a static solution with central symmetry of the wave equation

$$\left\{ \Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2 \right\} U = 0, \qquad (3)$$

let us assume (3) to be the correct equation for U in vacuum. In the presence of the heavy particles the U-field interact with them and causes the transition from neutron state to proton state.

Now, if we introduce the matrices

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$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and denote the neutron state and the proton state by $\tau_3 = 1$ and $\tau_3 = -1$ respectively,[4] the wave equation is given by

$$\left\{ \Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2 \right\} U = -4\pi g \tilde{\Psi} \frac{\tau_1 - i\tau_2}{2} \Psi, \qquad (4)$$

where Ψ denotes the wave function of the heavy particles and is a function of time, position, spin and also of ~~x,y,z,t and~~ τ_3, which takes the value +1 or −1. ~~eigenvalues ±1 of τ_3.~~

Next, corresponding to the inverse[5] transition from proton to neutron state, the conjugate complex function $\tilde{U}(x,y,z,t)$, satisfying the equation

$$\left\{ \Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2 \right\} \tilde{U} = -4\pi g \tilde{\Psi} \frac{\tau_1 + i\tau_2}{2} \Psi, \qquad (5)$$

is introduced.

Similar equation~~s~~ can ~~will~~ be considered ~~will hold~~ for the vector function~~s~~, which is the analogue of the ~~corresponding to~~ vector potential of the electromagnetic field, but we disregard them for the moment, since there's no correct relativistic theory for the heavy particles. Henceforth, simple non-relativistic wave equation neglecting spin will be used for the heavy particle, which is

$$\left\{ \frac{\hbar^2}{4}\left(\frac{1+\tau_3}{M_N} + \frac{1-\tau_3}{M_P} \right)\Delta + i\hbar \frac{\partial}{\partial t} - \frac{1+\tau_3}{2} M_N c^2 - \frac{1-\tau_3}{2} M_P c^2 \right.$$

$$\left. - g\left(\tilde{U} \frac{\tau_1 + i\tau_2}{2} + U \frac{\tau_1 - i\tau_2}{2} \right) \right\} \Psi = 0, \qquad (6)$$

(4) Heisenberg, loc. cit. I.

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where $\hbar$ is Planck's constant divided by 2π and M_N, M_P are the masses of neutron and proton respectively. The reason for taking the negative sign before g will be mentioned later.

The equation (6) corresponds to the Hamiltonian

$$H = \left(\frac{1+\tau_3}{4M_N} + \frac{1-\tau_3}{4M_P}\right)\mathbf{p}^2 + \frac{1+\tau_3}{2}M_N c^2 + \frac{1-\tau_3}{2}M_P c^2$$
$$+ g\left(\tilde{U}\,\frac{\tau_1 - i\tau_2}{2} + U\,\frac{\tau_1+i\tau_2}{2}\right), \qquad (7)$$

where $\mathbf{p}$ is the momentum of the particle. If we put $M_N c^2 + M_P c^2 = D$ and $M_N + M_P = 2M$, (7) becomes approximately

$$H = \frac{\mathbf{p}^2}{2M} + \frac{g}{2}\left\{\tilde{U}(\tau_1 - i\tau_2) + U(\tau_1 + i\tau_2)\right\} + \tau_3 D, \qquad (8)$$

where the constant term Mc^2 is omitted.

Now consider two heavy particles at points (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively and assume their relative velocity to be small. The fields at (x_1, y_1, z_1) due to the particle at (x_2, y_2, z_2) are, from (4) and (5),

$$\left.\begin{aligned} U(x_1, y_1, z_1) &= g\,\frac{e^{-\lambda r_{12}}}{r_{12}}\,\frac{(\tau_1^{(2)} - i\tau_2^{(2)})}{2} \\ \text{and}\quad \tilde{U}(x_1, y_1, z_1) &= g\,\frac{e^{-\lambda r_{12}}}{r_{12}}\,\frac{(\tau_1^{(2)} + i\tau_2^{(2)})}{2} \end{aligned}\right\} \qquad (9)$$

where $(\tau_1^{(1)}, \tau_2^{(1)}, \tau_3^{(1)})$ and $(\tau_1^{(2)}, \tau_2^{(2)}, \tau_3^{(2)})$ are the matrices relating to the first and the second particles respectively, and r_{12} is the distance between them.

Hence, the Hamiltonian for the system is given, in the absence of the external fields, by

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$$H = \frac{\mathbf{p}_1^2}{2M} + \frac{\mathbf{p}_2^2}{2M} + \frac{g^2}{4}\{(\tau_1^{(1)} - i\tau_2^{(1)})(\tau_1^{(2)} + i\tau_2^{(2)})$$

$$+ (\tau_1^{(1)} + i\tau_2^{(1)})(\tau_1^{(2)} - i\tau_2^{(2)})\}\frac{e^{-\lambda r_{12}}}{r_{12}} + (\tau_3^{(1)} + \tau_3^{(2)})D$$

$$= \frac{\mathbf{p}_1^2}{2M} + \frac{\mathbf{p}_2^2}{2M} + \frac{g^2}{2}(\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)})\frac{e^{-\lambda r_{12}}}{r_{12}}$$

$$+ (\tau_3^{(1)} + \tau_3^{(2)})D, \qquad (10)$$

where $\mathbf{p}_1, \mathbf{p}_2$ are the momenta of the particles.

This Hamiltonian is equivalent to Heisenberg's Hamiltonian (1~~7~~),[5] if we take for~~e~~ "Platzwechsel~~/~~integral"

$$J(r) = -g^2 \frac{e^{-\lambda r}}{r}, \qquad (11)$$

with the exception that the interaction between the neutrons and the electrostatic repulsion between the protons are not taken into account in our case. Heisenberg took the sign of J(r) positive, so that the spin of the lowest energy state of H^2 was 0, whereas in our case, owing to the negative sign before g^2, the lowest energy state has the spin 1, which is required from the experiment.

Two constants g, λ ~~/~~ appearing in the above equations should be determined by comparing with experiment. For example, using the Hamiltonian (10~~/~~) for two heavy particles, we can calculate the mass defect of H^2 and the probability of scattering of a neutron

(5) Heisenberg, I.

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~~and~~ by a proton, provided that the relative velocity is small compared with the light velocity.(6)

Rough estimation shows that the calculated values agree with the experimental results, if we take λ between $10^{12}\,\mathrm{cm}^{-1}$ and $10^{13}\,\mathrm{cm}^{-1}$, and g a few times of the elementary charge e, although no direct relation between them (g and e) was suggested in the above considerations.

§3. Nature of the quanta accompanying the field.

The U-field above considered should be quantized according to the general method of the quantum theory. Since the neutron and the proton both obey Fermi's statistics, the quanta accompanying the U-field should obey Bose's statistics and the quantization can be carried on the similar line with that of the electromagnetic field.

The law of conservation of the electric charge demands that the quantum should have the charge +e or −e. The field quantity U corresponds to the operator which increases the number of ~~positively~~ negatively charged quanta by one and ~~decreases~~ decreases the number of ~~the~~ positively charged particles by one. $\tilde{U}$, which is the complex conjugate of U and does not commute with it, corresponds to the inverse operator.

Next, denoting

$$p_x = -i\hbar\frac{\partial}{\partial x}, \quad \text{etc.}, \quad W = i\hbar\frac{\partial}{\partial t},$$

$$m_U c = \lambda\hbar,$$

(6) These calculations were made previously, according to the theory of Heisenberg, by Mr. Tomonaga, to whom the writer owes much. A little modification is necessary in our case. Detailed accounts will be made in the next paper.

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the wave equation for U in free space can be written in the form

$$\left\{ p_x^2 + p_y^2 + p_z^2 - \frac{W^2}{c^2} + m_U c^2 \right\} U = 0, \qquad (12)$$

so that the quantum accompanying the field has the proper mass $m_U = \frac{\lambda \hbar}{c}$. Assuming $\lambda = 5 \times 10^{12}\,\mathrm{cm}^{-1}$, we obtain for m_U a value 2×10^2 times as large as electron mass. As such a quantum with large mass and positive or negative charge has never been found by the experiment, the above theory seems to be on a wrong line. We can show, however, that, in the ordinary nuclear transformation, such a quantum can not be emitted into outer space.

Let us consider, for example, the transition from a neutron state of energy W_N to ~~astate~~ a proton state of energy W_P, both including the proper energies. These states can be expressed by the wave functions

$$\Psi_N(x,y,z,t,1) = u(x,y,z)\, e^{-iW_N t/\hbar}, \quad \Psi_N(x,y,z,t,-1) = 0$$

and

$$\Psi_P(x,y,z,t,1) = 0, \quad \Psi_P(x,y,z,t,-1) = v(x,y,z)\, e^{-iW_P t/\hbar},$$

so that, on the right hand side of the equation (4), the term

$$-4\pi g\, \tilde{v} u\, e^{-\frac{it}{\hbar}(W_N - W_P)}$$

appears.

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Putting $U = U'(x,y,z)e^{-i\omega t}$, we have from (4)

$$\left\{ \Delta - \left(\lambda^2 - \frac{\omega^2}{c^2}\right) \right\} U' = -4\pi g \tilde{v} u, \qquad (13)$$

where $\omega = \frac{W_N - W_P}{\hbar}$. Integrating this, we obtain a solution

$$U'(\mathbf{r}) = g \iiint \frac{e^{-\mu|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \tilde{v}(\mathbf{r}') u(\mathbf{r}') dv', \qquad (14)$$

where $\mu = \sqrt{\lambda^2 - \frac{\omega^2}{c^2}}$.

If $\lambda > \frac{\omega}{c}$ or $m_U c^2 > W_N - W_P$, μ is real and the function $J(r)$ of Heisenberg has the form $-g^2 \frac{e^{-\mu r}}{r}$, in which μ, however, depends on $W_N - W_P$ and becomes smaller and smaller as the latter becomes nearer and nearer to $m_U c^2$. This means that the range of interaction between a neutron and a proton increases as $W_N - W_P$ increases.

Now the scattering (elastic or inelastic) of a neutron by a nucleus can be considered as the result of the following double process: the neutron falls into a proton level in the nucleus and a proton in it jumps to a neutron state of positive kinetic energy, the total energy being conserved throughout the process. The above argument, then, shows that the probability of scattering may in some case increase with the velocity of the neutron.

According to the experiment of Bonner,[7] the collision cross section of the neutron ~~decreases~~ increases, in fact, with the velocity in case of lead, whereas it decreases in case of carbon and hydrogen, the rate of decrease being slower in the former than in the latter. The

(7) T.W. Bonner, Phys. Rev. **45**, 606 (1934).

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origin of this effect is not clear, but the above considerations do not, at least, contradict with it. For, if the binding energy of the proton in the nucleus becomes comparable to $m_U c^2$, the range of interaction of the neutron with it will increase considerably with the velocity of the neutron, so that the cross section will decrease slower in such case than in case of hydrogen, i.e. free proton. Now the binding energy of the proton in C^{12} is estimated from the difference of masses of C^{12} and B^{11}, which is

$$12.0036 - 11.0110 = 0.9926.$$

This corresponds to a ~~b~~ binding energy 0,0152 in mass unit, being thirty times as large as the electron mass. Thus in case of carbon we can expect a sort of Ramsauer the effect observed by Bonner.(8) The arguments are only tentative and other explanations, of course, are not excluded.

Next if $\lambda < \frac{\omega}{c}$ or $m_U c^2 < W_N - W_P$, μ becomes pure imaginary and U expresses a ~~s~~spherical undamped wave, which means that a quantum with energy greater than $m_U c^2$ can be emtted in outer space by the transition of the heavy particle from neutron state to proton state, provided that $W_N - W_P > m_U c^2$.

The velocity of U-wave is greater than the light velocity c, but the group velocity is smaller it, as in the case of the electron wave.

The reason why such massive quanta ~~are no~~, if they exist, are not yet discovered may be that the mass m_U is so large that the condition $W_N - W_P > m_U c^2$ is not fulfilled in ordinary nuclear transformation.

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(8) We can ~~also~~ expect the essential difference in angular distributions of scattered neutrons in two cases. ~~In the~~ case of hydrogen, neutrons should ~~have~~ be ~~scattered~~ maximum in the direction ~~at angle~~ right angle to the ~~direction~~ initial direction of scattering, whereas in case of ~~carbon~~ heavier nucleus ~~the distri~~ angular distribution should almost ~~be~~ ~~spherically symmetric~~ slightly small angle forward scattering will be larger ~~in forward~~ than the large angle scattering.

~~These seems to~~ not to ~~contra~~ agree ~~with the experimental~~ various results of predictions.

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§4. Theory of β-disintegration.

Hitherto we have considered only the interaction of U-quanta with heavy particles. Now, according to our theory, the quantum, emitted when a heavy particle jumps from a neutron state to a proton state, can be absorbed by a light parical which jumps consequently from a neutrino state of negative energy to an electron state of positive energy. Thus an anti-neutrino and an electron are emitted from the nucleus. Such intervention of a massive quantum does not alter essentially the probability of β-disintegration, which has been calculated on the hypothesis of direct coupling of a heavy particle and a light particle, just as, in the theory of internal conversion of γ-ray, the intervention of the photon does not affect the final result.[8] Our theory, therefore, does not differ essentially from Fermi's thory.

Fermi considered that an electron and a neutrino are emitted simultaneously from the radioactive nucleus , but this assumption is formally equivalent to assume that a light particle jumps from a neutrino state of negative energy to an electron state of positive energy.

For, if the eigenfunctions of the electron and the neutrino be ψ_k, φ_k respectively, where k=1,2,3,4, a term of the form

$$-4\pi g' \sum_{k=1}^{4} \tilde{\psi}_k \varphi_k \qquad (15)$$

(8) H.A.Taylor and N.F.Mott, Proc.Roy.Soc. A, **138**, 665 (1932).

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should be added on the right hand side of the equation (4) for U, where g' is a new constant with the same dimension as g.

Now the eigenfunctions of the neutrino state with energy and momentum just opposite to those of the state φ_k is given by

$$\varphi'_k = -\delta_{kl}\tilde{\varphi}_l ,$$

~~where~~ and conversely

$$\varphi_k = \delta_{kl}\tilde{\varphi}'_l ,$$

where

$$\delta = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} ,$$

so that (15) becomes

$$-4\pi g' \sum_{k,l=1}^{4} \tilde{\psi}_k \delta_{kl} \tilde{\varphi}'_l . \qquad (16)$$

From equations (13) and (15), we obtain for the matrix element of the interaction energy of the heavy particle and the light particle an expression

$$gg' \int\!\!\int \tilde{v}(\mathbf{r}_1) u(\mathbf{r}_1) \sum_k \tilde{\psi}_k(\mathbf{r}_2) \varphi_k(\mathbf{r}_2) \frac{e^{-\lambda r_{12}}}{r_{12}} dv_1 dv_2 , \qquad (17)$$

corresponding to the following double process: a heavy particle in the neutron state with the eigenfunction $u(\mathbf{r})$ falls into the proton state with the eigenfunction $v(\mathbf{r})$ and simultaneously a light particle in the neutrino state $\varphi_k(\mathbf{r})$ of negative energy to the electron state ~~of~~ $\psi_k(\mathbf{r})$ of positive energy. In (17) λ is taken instead of μ, since the difference of energies of the neutron state and the proton state, which is equal to the upper limit of the ~~energy spectrum of -rays added by the proper masses of the electron and the neutrino~~,

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energy spectrum of β-rays added by the proper energies of the electron and the neutrino, is always small compared with $m_\nu c^2$.

As λ is much larger than the wave numbers of the~~ø~~ electron state and the neutrino state, the function $\frac{e^{-\lambda r_{12}}}{r_{12}}$ can be regarded approximately as a δ-function multiplied by $\frac{4\pi}{\lambda^2}$, for the integrations with respect to x_2, y_2, z_2. The factor $\frac{4\pi}{\lambda^2}$ is determined from

$$\iiint \frac{e^{-\lambda r_{12}}}{r_{12}} dv_2 = \frac{4\pi}{\lambda^2}.$$

Hence (17) becomes

$$\frac{4\pi g g'}{\lambda^2} \iiint \tilde{v}(\mathbf{r}) u(\mathbf{r}) \sum_{k=1}^{4} \tilde{\psi}_k(\mathbf{r}) \varphi_k(\mathbf{r}) dv \qquad (18)$$

or by (16)

$$\frac{4\pi g g'}{\lambda^2} \iiint \tilde{v}(\mathbf{r}) u(\mathbf{r}) \sum_{k,l} \tilde{\psi}_k(\mathbf{r}) \delta_{kl} \tilde{\varphi}'_l(\mathbf{r}) dv, \qquad (19)$$

which is the same as the expression (21) of Fermi, corresponding to the emission of a neutrino and an electron of positive energy states $\varphi_k^{(+)}(\mathbf{r})$ and $\psi_k(\mathbf{r})$, except that the factor $\frac{4\pi g g'}{\lambda^2}$ is substituted for Fermi's g.

Thus the result is the same as that of ~~Feri~~ Fermi's ~~f~~ theory, in this approximation, if we take

$$\frac{4\pi g g'}{\lambda^2} = 4 \times 10^{-50} \text{ cm}^3 \text{ erg},$$

from which the constant g' can be determined. Taking, for example, $\lambda = 5 \times 10^{12}$ and $g = \cancel{8}\,2 \times 10^{-9}$, we~~k~~ have $g' \cong 4 \times 10^{-17}$, which is about 10^{-8} times as small as g.

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This means that the interaction between ~~a~~ the neutrino and the electron is much smaller than that between the neutron and the proton, so that the neutrino will be far more penetrating than the neutron and consequently more difficult to observe. The difference of g and g' may be due to the difference of massses of heavy and light particles.

§5. Summary.

The interaction of elementary particles are described by considering a hypothetical quantum which has the elementary charge and the proper mass and obeys Bose's statistics. The interaction of such a quantum with the heavy particle should be far greater than with the light particle to account for the large interaction of the neutron and the proton and ~~also~~ the small probability of β-disintegration.

Such quanta, if they exist and come close to the matter, will sometimes be absorbed, giving their charge and energy to it. If, then, the quanta with negative charge come in excess, the matter will be charged to a negative potential.

These arguments, of course, of merely speculative character, agree with the view that the high speed positive particles in cosmic ray are generated by the electrostatic field of the earth, which ~~are~~ is charged to a negative potential.(9)

(9) L.G.H.Huxley, Nature **134**, 418, 571 (1934); Johnson, Phys.Rev. **45**, 569 (1934).

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The massive quanta may also have some bearing on the shower produced by cosmic ray.

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