On the Interaction of Elementary Particles II.

By Hideki Yukawa

(Read Nov. 28, 1936)

§1. ~~Introduction~~ Recapitulation and Revision of the Previous ~~Former~~ Paper

About two years ago, the ~~author~~ put forward the following idea to remove the difficulties of the so called "β-Hypothesis of N.F." which ~~remain~~ seems to remain as yet[2])
約二年前 筆者は所謂 "β- Hypothesis of Nuclear Forces" の困難を除くために次の如き考を出した。 Namely, the transition of a heavy particle from the neutron state to the proton state, is not always accompanied by the emission of light particles, i.e. a neutrino and an electron, but the energy thereby liberated is taken up sometimes by another heavy particle, which in turn will be transformed from the proton state ~~to the~~ into the neutron state. If the probability of occurrence of the latter process is much larger than that of the former, the interaction between the neutron and the proton will be much larger than in the case of Fermi, whereas the probability of emission of light particles is not affected essentially. Such an interaction was described by means of a field of force, just as the interaction between the charged particles was described by the electromagnetic field. The only difference was that ~~the potential of force in this case~~ the force in ~~this~~ our case ~~is~~ should not be of Coulomb type, but ~~decrease~~ should decrease more rapidly with distance. ~~The most simple~~ Hence, it was assumed that the force had a potential of the type

$$\text{const.} \times \frac{e^{-\lambda r}}{r} \qquad (1)$$

~~of the~~ which was a static solution with central symmetry of the wave equation

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right\} U = 0 \qquad (2)$$

2) An excellent ~~detailed~~ discussion of the subject will be found in Bethe and Bacher, Rev. Mod. Phys. 8, 201 ff, 1936.
1) Yukawa, Proc. Phys.-Math. Soc. Japan 17, 48, 1935.

$$\tau_1^{(3)}\tau_2^{(3)}\cdot\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2=-1$$

$$\begin{pmatrix}1&0\\0&-1\end{pmatrix}_1\begin{pmatrix}1&0\\0&-1\end{pmatrix}_2$$

$$\underbrace{\{1+(\sigma_1\sigma_2)\}}_{3}+\cancel{\tfrac{1}{2}}\,1 \qquad \cancel{\tfrac{3}{2}}\;3\;\cancel{=}\;1$$

$$\tau_1^{(1)}\tau_2^{(1)}+\tau_1^{(2)}\tau_2^{(2)}=\begin{pmatrix}0&1\\1&0\end{pmatrix}_1\begin{pmatrix}0&1\\1&0\end{pmatrix}_2+\begin{pmatrix}0&-i\\i&0\end{pmatrix}_1\begin{pmatrix}0&-i\\i&0\end{pmatrix}_2$$

$$\frac{(\;)^{(1)}}{2}\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2=\frac{1}{2}\left\{\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2+\begin{pmatrix}0\\i\end{pmatrix}_1\begin{pmatrix}-i\\0\end{pmatrix}_2\right\}$$

$$=\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2$$

$$\frac{(\;)^{(1)}+(\;)^{(2)}}{2}\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2=\cancel{\times}\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 \qquad \begin{matrix}2 & -2\\ -1 & -1\end{matrix}$$

$$(\sigma_1\sigma_2)\;\Psi_{\substack{sym\\anti}}(\sigma)=\cancel{\ }\,\Psi_{sym}(\sigma)\text{ or }-3\,\Psi_{anti}(\sigma)$$

$$-\frac{(\tau_1^{(1)}\tau_2^{(1)}+\tau_1^{(2)}\tau_2^{(2)})}{2}\cdot\Psi=\Psi_{sym}(r)\begin{cases}\Psi_{sym}(\sigma)\Psi_{anti}(\tau)=\cancel{\ }+1\cdots\\ \Psi_{anti}(\sigma)\Psi_{sym}(\tau)=+3\cdots\end{cases}$$

$$(\sigma_1\sigma_2) \qquad \cancel{1}+(\sigma_1\sigma_2)''$$

$$\Psi_{antisym}(r)\begin{cases}\Psi_{sym}(\sigma)\Psi_{anti}(\tau)=-\cancel{3}+1\cdots\\ \Psi_{anti}(\sigma)\Psi_{sym}(\tau)=-3\cdots\end{cases}$$

$(1-\beta^{(1)}\beta^{(2)}) + \beta^{(1)}\beta^{(2)}(1+\sigma^{(1)}\sigma^{(2)})$ $\quad \frac{\tau_1^{(2)}+i\tau_2^{(2)}}{2}\sigma^{(2)}\frac{\tau_1^{(1)}-i\tau_2^{(1)}}{2}\sigma^{(1)}$

$\overset{\circ}{H}$ $\quad M.$ $\quad + \frac{\tau_1^{(2)}-i\tau_2^{(2)}}{2}\sigma^{(2)}\frac{\tau_1^{(1)}+i\tau_2^{(1)}}{2}\sigma^{(1)}$ $(1+\alpha^{(1)}\alpha^{(2)})$ $\tfrac{1}{2}(\tau_1^{(1)}\tau_1^{(2)}+\tau_2^{(1)}\tau_2^{(2)})$

instead of const.× $\frac{1}{r}$, which ~~is the~~ was the corresponding solution of the ordinary D'Alembertian wave equation

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right\}U = 0. \tag{3}$$

~~The~~ In the presence of ~~the heav a neutron~~ the ~~neutron~~ heavy particle, ~~the equation~~ it was assumed that ~~the equation (2) was~~ to be replaced by an equation

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right\}U = -4\pi g\,\tilde{\Psi}\,\frac{\tau_1 - i\tau_2}{2}\,\Psi, \tag{4}$$

where Ψ denoted the wave function of the heavy particle depending on time, position and spin as well as ~~τ_3 which~~ τ_3, which ~~takes~~ ~~τ_3~~ took the value either 1 or −1 according as the particles was in the neutron or the proton state. τ's ~~are~~ were the matrices

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{5}$$

of Heisenberg[1] and g was a constant with the dimension of the electric charge e. Thus, ~~the right hand side of the~~ (4) expressed ~~corresponded to~~ the possibility of the transition of the heavy particle from a neutron to a proton state ~~by~~ due to the interaction with the U-field. Further, the conjugate complex equation i.e. eq. ?

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right\}\tilde{U} = -4\pi g\,\tilde{\Psi}\,\frac{\tau_1 + i\tau_2}{2}\,\Psi \tag{6}$$

was assumed, corresponding to the inverse transition from a proton to a neutron state.

From these equation, it was easy to ~~prove~~ deduce that the ~~two heavy particles~~ a neutron and a proton ~~interact~~ at a distance r is exerted by a exchange force ~~of~~ with the potential

[1] Heisenberg, Zeits. f. Phys. 77, 1, 1932.

$$J(r) = -g^2 \frac{e^{-\lambda r}}{r}, \qquad (7)$$

the Hamiltonian for two heavy particles becoming

$$H = \frac{\vec{p}_1^{\,2}}{2M} + \frac{\vec{p}_2^{\,2}}{2M} + \frac{g^2}{2}\left(\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}\right)\frac{e^{-\lambda r_{12}}}{r_{12}} + \left(\tau_3^{(1)} + \tau_3^{(2)}\right) D, \qquad (8)$$

where D is the difference of the ~~proton~~ proper energies of the neutron and the proton.; ~~Thus, the result~~ ~~Thus~~ the force thus deduced This Hamiltonian, ~~was the same with~~ being of the that of Heisenberg's type, ~~except that the interactions between likes particles was were not take neglected~~, was not satisfactory instead of Majorana type. ~~This point should will be corrected in this paper.~~

~~Next,~~ the interaction of the U-field with the light particle ~~In this paper, the~~ ~~the equations (4) and (6) will be reformed so as to involve the~~ ~~interact~~ the force of ~~Majorana type, also~~. Further, ~~If the~~ ~~Next, the~~ in order to obtain the force of Majorana type, one should ~~consider~~ vector functions $\vec{B}$, and $\vec{\tilde{B}}$ ~~shown~~ which satisfy the equations

$$\left\{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right\}\vec{B} = -4\pi g\, \tilde{\Psi}\frac{\tau_1 - i\tau_2}{2}\vec{\sigma}\Psi \qquad (9)$$

$$\left\{\quad ,, \quad\right\}\vec{\tilde{B}} = -4\pi g\, \tilde{\Psi}\frac{\tau_1 + i\tau_2}{2}\vec{\sigma}\Psi, \qquad (10)$$

are introduced, the interaction between ~~the next~~ two heavy particles ~~through~~ by means of ~~the scalar potential U and the vector potential respectively~~ the fields given by ~~becomes~~

The Hamiltonian of ~~the system~~ two heavy particles becomes

$$H = \frac{\vec{p}_1^{\,2}}{2M} + \frac{\vec{p}_2^{\,2}}{2M} + \frac{g^2}{2}\left(\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}\right)\left(1 + \vec{\sigma}_1\cdot\vec{\sigma}_2\right)\frac{e^{-\lambda r_{12}}}{r_{12}} + \left(\tau_3^{(1)} + \tau_3^{(2)}\right) D \qquad (11)$$

where $\vec{\sigma}$'s ~~are~~ denote the spins of the heavy particles. ~~Thus, the net~~ This expression shows that the ~~interaction~~ ~~force~~ interaction between

1) loc. cit.

( ) $\boxed{\alpha_i \alpha}$ scalar and ~~a~~ vector

by means of the fields $U, \vec{B}$,

~~Heavy~~ the neutron and the proton ~~are~~ is ~~that of~~ pure Majorana ~~force~~ type.

It ~~these~~ should ~~be~~ be noticed, however, that since U and $\vec{B}$ ~~does~~ not form a four vector ~~and~~, the small force of Heisenberg type can

(~~The relativistic invariance demand~~ necessitates a ~~scalar~~ four

The consideration of vector $U_0, \vec{U}$ U_i instead of a scalar U and a sixvector $\vec{B}, \vec{E}$ B_{ij} instead of a space vector $\vec{B}$, which satisfy the equations

$$\{\ \ ''\ \ \} U_i = -4\pi g \tilde{\Psi} \frac{\tau_1 - i\tau_2}{2} \beta \gamma_i \Psi \qquad (12)$$

$$\{\ \ ''\ \ \} B_{ij} = -4\pi g \tilde{\Psi} \frac{\tau_1 \mp i\tau_2}{2} \beta \alpha_i \alpha_j \gamma_i \gamma_j \Psi \qquad (13))$$

be expected from the relativistic extension of the above ~~above~~ formulation.

~~Next, the~~ Similarly, the interaction of the above ~~new~~ fields U and $\vec{B}$ ~~with~~ ~~can~~ be with the light particle ~~can~~ ~~should be ca~~ ~~was~~ ~~not~~ can be ~~assumed to~~ expressed by the equations

$$\{\ \ ''\ \ \} U = -4\pi g' \tilde{\Psi} \frac{\tau_1 - i\tau_2}{2} \beta \left(\frac{\partial}{\partial t} - \vec{\alpha} \frac{\partial}{\partial \vec{r}}\right) \Psi \qquad (12)$$

$$\{\ \ ''\ \ \} \vec{B} = -4\pi g' \tilde{\Psi} \frac{\tau_1 - i\tau_2}{2} \beta \vec{\sigma} (\ \ ''\ \ ) \Psi. \qquad (13)$$

~~from which~~ From ( ) ( ) (12) and (13) the interaction potential ~~between~~ ~~the light~~

(14)

between the light and the heavy particles, which is nothing but the form given by Gamow and Teller[3], which agrees ~~well~~ better than ever ~~with~~ the scheme of nuclear spins of the Thorium family.

3) Gamow and Teller, Phys. Rev. 49, 895, 1936.

§3. Interaction of the heavy and the light quanta

neutron, proton 又は neutrino electron の transition に伴って出来る field に伴ふ quantum は

$$m_U = \frac{\lambda\hbar}{c}$$

なる proper mass を有し、又 ±e の charge を有するものである。U は +e charge の quantum を一つへらし、−e charge の quantum を一つ増加する operator に対応する。$\tilde{U}$ はその逆。この quantum を light quantum に対して heavy quantum と名づけることにすると、両者の間には当然 interaction がある。

mass, charge を有し Bose の statistics に従ふ quantum に対する理論は Pauli と Weisskopf[1] が electron に対する scalar relativistic wave equation によって陽電子の問題を論じたのと全く同じ様に進めることが出来る。[†]
此の場合 charge density は

$$\rho = \frac{e}{i}\left\{\tilde{U}\left(i\hbar\frac{\partial U}{\partial t} + eA_0 U\right) - \left(i\hbar\frac{\partial \tilde{U}}{\partial t} - eA_0\tilde{U}\right)U\right\}$$

同様に current density は

$$\vec{I} = \frac{e}{i}\left\{\tilde{U}\left(-i\hbar\frac{\partial U}{\partial \vec{r}} + e\vec{A}U\right) - \left(-i\hbar\frac{\partial \tilde{U}}{\partial \vec{r}} - e\vec{A}\tilde{U}\right)U\right\}$$

である。U, $\tilde{U}$ に対する canonical conjugate は夫々

$$\left.\begin{aligned} V &= \frac{1}{i}\left(i\hbar\frac{\partial \tilde{U}}{\partial t} - eA_0\tilde{U}\right) \\ \tilde{V} &= \frac{1}{i}\left(i\hbar\frac{\partial U}{\partial t} + eA_0 U\right) \end{aligned}\right\}$$

である。従って、

$$\rho = +e\,\tilde{U}\tilde{V} - eVU$$

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† と U, $\tilde{U}$ に対する wave equations は particles が存在しない場合は

$$\left.\begin{aligned} &\left\{-\frac{1}{c^2}\left(i\hbar\frac{\partial}{\partial t} + eA_0\right)^2 + \left(i\hbar\frac{\partial}{\partial \vec{r}} + \frac{e}{c}\vec{A}\right)^2 + m_U^2c^2\right\}U = 0 \\ &\left\{-\frac{1}{c^2}\left(i\hbar\frac{\partial}{\partial t} - eA_0\right)^2 + \left(i\hbar\frac{\partial}{\partial \vec{r}} - \frac{e}{c}\vec{A}\right)^2 + m_U^2c^2\right\}\tilde{U} = 0 \end{aligned}\right\}$$

で与へられる。†

1) Helv. Phys. ~~Acta~~. 7, 709, 1934.

V's U's は次の如き V. R. を満足せねばならぬ

$$i[V(\vec{r}), U(\vec{r}')] = \delta(\vec{r}-\vec{r}') \qquad i[\tilde{V}(\vec{r}), \tilde{U}(\vec{r}')] = \delta(\vec{r}-\vec{r}')$$

~~但し、こゝまでの所では ±e の charge の quantum が separate されてゐない~~

従って　~~U=~~ U ある Raumstelle $\vec{r}_0$ に於て ρ は

$$\rho(\vec{r}) = N\,\delta(\vec{r}-\vec{r}_0)$$

なる Eigenwert を有する. 但し, $N = 0, \pm 1, \cdots$

更に

$$U = \frac{1}{\sqrt{V}}\sum q_{\vec{k}}\, e^{i\vec{k}\vec{r}} \qquad \tilde{U} = \frac{1}{\sqrt{V}}\sum \tilde{q}_{\vec{k}}\, e^{-i\vec{k}\vec{r}}$$

$$V = \frac{1}{\sqrt{V}}\sum p_{\vec{k}}\, e^{-i\vec{k}\vec{r}} \qquad \tilde{V} = \frac{1}{\sqrt{V}}\sum \tilde{p}_{\vec{k}}\, e^{+i\vec{k}\vec{r}}$$

なる expansion を行ふ†

†† $\tilde{a}_{\vec{k}} a_{\vec{k}}$ 　($\tilde{a}$ ~~...~~) ($\tilde{b}_{\vec{k}} b_{\vec{k}}$)

は夫々 charge が −e (又は +e) で, momentum $\hbar\vec{k}$ (又は $-\hbar\vec{k}$) なる heavy quantum の数をあらはす.

以下は省略して理論を考慮すると, その結果として期待されるのは

i) heavy quantum は electric or magnetic field に於て曲げられること. その曲り方は + 又 − で, その大いさは proton と electron の中間.

ii) 速い急速に於て atom を ionize する. その能力は ~~...~~ electron より proton に近い,

iii) $2m_0c^2 \cong 2\times 10^8$ eV 以上の ~~...~~ light quantum は nucleus と衝突して heavy quantum の pair を作る可能性が

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† ~~$a_{\vec{k}} = \frac{\sqrt{E_k}}{\sqrt{2}}(\ldots \tilde{c}_{\vec{k}} + d_k)$~~ 　$b_{\vec{k}} =$

$\tilde{a}_{\vec{k}} =$ 　$a_k = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{E_k}}\tilde{p}_k - i\sqrt{E_k}\,q_k\right)$ 　$\tilde{a}_k = \frac{1}{\sqrt{2}}\left(\frac{p_k}{\sqrt{E_k}} + i\sqrt{E_k}\,\tilde{q}_k\right)$

$b_k = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{E_k}}p_k - i\sqrt{E_k}\,\tilde{q}_k\right)$ 　$\tilde{b}_k = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{E_k}}\tilde{p}_k + i\sqrt{E_k}\,q_k\right)$ ††

とおくと

は[illegible]されるかり

ある。 ~~その~~ これと通常の electron pair を作る prob. ~~との数字は~~
~~簡単(以下詳しく述べる)~~

iv) nucleus に ~~捕獲されて~~ proton or neutron を飛出す probability は極めて大きいであろう。~~... energy ...~~ ~~nucleus~~
又 nucleus が ~~...~~ compton scattering をやるとか、inelastic scattering によって nucleus を excite する等の場合が ~~相当~~ に多く考へられる。
これ等の process によって出来た nucleus が radioactive で electron, neutrino, positron を出す ~~process~~ 場合も可能である。
~~nucleus~~ 一つの heavy quantum が次々に inelastic scattering をやって多数の ~~highly radi~~ radioactive (+or−) nucleus を作ることも可能であろう。 その際 quantum の charge の sign は途中で変はりうる。

v) 非常な high speed proton or neutron が nucleus と衝突して、(必ずしも capture によって除いて) heavy quantum を出す可能性が考へられる。 この時出てくる quantum の energy (kinetic) が小さければ ~~nucleus によって~~ field によっても曲げられるであろうし 又 ~~...~~ ionization も盛んであろうし、又 nucleus とも衝突し易く、結局 range の短いものになり、之が shower の不明になり終る。

かかる種々な性質を有する heavy quantum が実際あるならば Cosmic Ray の実験に現はれる筈であるが、high energy のものは proton 或は electron のいづれとも区別がつかぬであろう。~~...~~ low energy のものは proton と間違ふ恐れが充分ある。

Anderson and Neddermeyer (Phys. Rev. 50, 270, 1936) の実験 Fig. 12 (写真) に現はれている track 等は proton とすると range から energy が 1.5 MEV 以上になり、~~ener~~ 考へると $H\rho = 1.7\times10^5$ ・ $\rho = 20$ cm 以上となり、観測された他の曲率以上になるから否。 この他の shower と通常と異なることは注意すべきである。(positive なもの ばかりである)

$.9546$

$3)4.9712$

9546
$\times 8$
0.76368

e ν

$$\boxed{2\times10^{-13}\,\mathrm{cm}}$$

$$\lambda = \frac{\hbar}{m_\mu c} \doteqdot 5\times10^{12}$$

$$\gamma \sim \frac{W^2}{M} \sim \frac{W^{\frac{5}{2}}}{M}$$

$$\rho \sim \cancel{\quad} W^{\frac{1}{2}} \cdot M^{\frac{1}{2}} \qquad \gamma = \frac{\rho^3}{M^{\frac{5}{2}}}$$

M^3

$$\boxed{\gamma = \mathrm{const}\, M \cdot \rho^4 / M^3} \quad . M^{\frac{5}{2}} \quad 81$$

$$= \cancel{\mathrm{const}\, m_\mu}\ \rho_\mu^4$$

$$1 = \frac{m_\mu^3}{M^3} \cdot \frac{\rho^4}{\rho_\mu^4} \longrightarrow \frac{m_\mu^{\frac{5}{2}}}{M^{\frac{5}{2}}}\left(\frac{\rho^5}{\rho_\mu^5}\right)$$

$$\cong \left(\frac{\rho}{\rho_\mu}\right)^{\frac{8}{5}}$$

$$\frac{m_\mu}{M} = \left(\frac{\rho}{\rho_\mu}\right)^{\frac{4}{3}} \cong 4.\ 3 \quad = 5.8$$

$$\boxed{4 \sim 6}$$

i) Deflection of charged Quantum in Magnetic Field†

$$\frac{1}{\rho} = \frac{e\left|\left[\frac{v}{v}\, H\right]\right|}{\sqrt{E^2-(m_U c^2)^2}}$$

†. a) $E - m_U c^2 = W \ll m_U c^2$

$$\frac{1}{\rho} \cong \frac{e\left|\left[\frac{v}{v}\, H\right]\right|}{\sqrt{W}\cdot\sqrt{2m_U c^2}}$$

従って 同じ energy (kinetic) の proton との radius of curvature の比は

$$\frac{\rho_U}{\rho_P} = \sqrt{\frac{m_U}{m_P}}$$

$m_U = 2\times10^2\cdot m$ とすると．　$\frac{\rho_U}{\rho_P} \cong \frac{1}{3}$

b) $E \gg m_U c^2$.　$\frac{1}{\rho} = \frac{e\left|\left[\frac{v}{v}, H\right]\right|}{E}$

ii) Ionization

a) $W \ll m_U c^2$.

$$-\frac{dE}{dx} = \frac{4\pi Ne^4 z^2}{mv^2}\log\frac{2mv^2}{1.103 Ry}$$

$$= \frac{2\pi Ne^4 z^2 m_U}{m W}\log\frac{4mW}{m_U(1.103)Ry}$$

~~b) $W \gg m_U c^2$~~ 同じ energy の proton の Ionization とくらべると，

$$\frac{dE_U}{dE_P} \cong \frac{m_U}{m_P} \cong \frac{1}{9}$$

range は大体 9 倍になる．

$$\frac{m}{M_U}\frac{2E^2}{Ry\,mc^2}\quad \cancel{E} \cong \frac{2mE^2}{Ry(mc^2)}$$

b) $E \gg m_U c^2$.

$$-\frac{dE}{dx} = \frac{2\pi Ne^4 Z^2}{mc^2}\log\left(\frac{2mv^2}{Ry(1-v^2/c^2)}\right)^2$$

$$= \frac{4\pi Ne^4 Z^2}{mc^2}\log\left(\frac{m}{M_U}\frac{2E}{Ry\,mc^2}\right)$$

$$\frac{dE_U}{dE_P} \cong \cancel{\frac{\log m_P}{\log m_U}}\ 1$$

Range in air cm (Radiative lossを考へずに)

	10^4	10^5	10^6	10^7	10^8	10^9	10^{10}
Electron	0.2	11	360	$4.3\cdot10^3$	$3.3\cdot10^4$	$2.7\cdot10^5$	$2.3\cdot10^6$
Proton			1.3	100	$6\cdot10^3$	$2.3\cdot10^5$	$3.4\cdot10^6$
α-Particle			0.1	6	400	$2.5\cdot10^4$	$6\cdot10^5$

Energy loss eV/cm^2 (air)

ε	19.5	3.67	1.69	1.95	2.47	2.99	3.48
P			300	47	7.6	2.31	2.31
α			4100	680	105	18.8	9.6

$H\rho \ll 1.4\times10^5$.　proton energy $\ll 10^6 eV$　m_u-energy $< 9\times10^6 eV$

range (×5 cm observed)　range < 2 cm

~~飛跡のrangeを考へれば充分説明出来る.~~

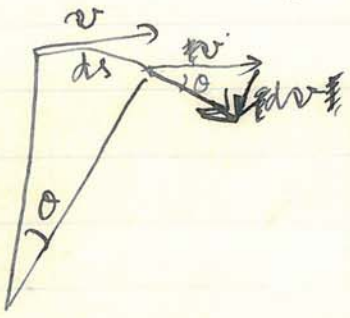
ionization以外のenergy lossを考へれば説明がつくであらう.

$H\rho = \frac{mc^2}{e}(\varepsilon^2-1)^{\frac{1}{2}}$

+

$$m\frac{d}{dt}\left(\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}\right) = -\frac{e}{c}[vH]$$

$$\cancel{mv\frac{d}{dt}} \quad mv\frac{d}{dt}\left(\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{m}{2}\frac{\frac{dv^2}{dt}}{\sqrt{1-\frac{v^2}{c^2}}} = 0.$$



$$\frac{|dv|}{v\,ds} = \frac{d\theta}{ds} = \frac{1}{\rho}$$

$$\text{or } \cancel{\frac{\left|\frac{dv}{dt}\right|}{v^3} = \frac{v^2}{\rho}} \qquad \left|\frac{dv}{dt}\right| = \frac{v^2}{\rho}$$

$$\frac{m}{\sqrt{1-\frac{v^2}{c^2}}}\frac{v^2}{\rho} = \frac{e}{c}|[vH]| \qquad H\rho = \frac{\sqrt{E^2-(mc^2)^2}}{e}$$

$$\frac{1}{\rho} = \frac{e\left|\left[\frac{v}{v}, H\right]\right|}{\sqrt{E^2-(mc^2)^2}}, \qquad E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \#.$$

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iii) Bremsstrahlung ~~の~~ nucleus との Electromagnetic interaction による。(short range force によるものは ~~[illegible]~~ 多少に異なる)
energy loss の cross section は $E \gg m_0c^2$ に於いて

$$\phi_{rad} \cong Z^2\left(\frac{e^2}{m_0c^2}\right)^2 \cdot \frac{e^2}{\hbar c}$$

$$\phi_{rad}^{(e)} : \phi_{rad}^{(m)} = 200^2 : 1 \qquad \text{for } E \gg m_0c^2,$$
$$\phi_{rad}^{(m)} : \phi_{rad}^{(p)} = 9^2 : 1 \qquad \text{for } E \gg M_pc^2.$$

~~b) Nucleus との 特有の interaction による。~~
~~[illegible]~~

iV) nucleus による Compton scattering
$E \gg m_0c^2$ ならば これは $\phi_{comp.} \cong \left(\frac{e^2}{m_0c^2}\right)^2 \frac{2m_0c^2}{E} \times \pi$ の order

で極めて小さい。

$E \cong m_U c^2 \cong m_U c^2$ の程度の所では計算は light ~~quantum~~ の とは大分異なるであらう。†

v) nucleus による Inelastic Scattering
+change of quantum or
~~nucleus~~ 中の neutron を → proton にかへ、この proton ~~neutron~~ が又 neutron state に落ちた結果、~~scatte~~ quantum は初めと異なる方向に inelastic scattering をする。其時 nucleus は γ-ray を出すこともあらうし、又 ~~其の~~ energy が充分あれば heavy quantum を出すかも知れぬし、又 β-ray or shower を出すかも知れない。

vi) nucleus による capture
この場合には ~~[illegible]~~ 一個或は二個の heavy particle が放出されるであらう。
capture の process は $E - m_U c^2$ が小さい時に割合に大きいであらう。

~~[illegible]~~
† ○ 真空中における U-quantum と他の particle との interaction

$$E_U + E = E_U' + E'$$
$$\vec{p}_U + \vec{p} = \vec{p}_U' + \vec{p}'$$
$$E_U^2 - \vec{p}_U^2 = m_U^2 \qquad E^2 - \vec{p}^2 = m^2$$
$$(E_U + E)^2 - (\vec{p}_U + \vec{p})^2 = (E_U' + E')^2 - (\vec{p}_U' + \vec{p}')^2$$
$$E_U E - \vec{p}_U \vec{p} = E_U' E' - \vec{p}_U' \vec{p}'$$

$E_U, E_U' > 0$ で且 particle の final state の energy $E' > 0$ の為には
$$E_U E - \vec{p}_U \vec{p} > 0$$
故 $E > 0$ でなければならぬ。之は通常の Compton scattering に correspond する場合で、~~[illegible]~~ 真空中の pair production は不可能である。‖ ~~又真空中で U-quantum が annihilate すること~~ ~~も不可能である。~~‖

2×10^9

5.4
5.4
21.6
270
29.16

2.7
7
18.9
54

さらに考へると又 U-quantum が安定でなくなって neutrino と electron が出来る如き process もまた不可能である。

~~法~~ ~~elec~~

しかし nucleus は positive charge をもってゐるから、(E−mc² が非常に小さく無き) negative charge をもった quantum の大半が nucleus に capture されるであらう。

~~かくの~~

Anderson の写真 Fig 12. の下へ出てゐる 4つの particle は proton ~~程~~ には ionization が大き過ぎるが positron とすると其の energy が

~~10^8~~ , 1×10^8 , 2.5×10^8 , 5×10^8 , $0.6\times10^8\,eV$

である。之を U-quantum と見なすと、

$$\sqrt{E^2-(m_U c^2)^2} = 〃 \quad \cdots$$

~~2.5~~ 6.25　0.36
1　1.36
　1.

となり、$m_U c^2 = 10^8\,eV$ とおけば、

$$E = 1.4\times10^8\,eV,\ 2.7\times10^8,\ 5\times10^8,\ 1.2\times10^8\,eV$$

となる。かゝる energy の quantum は ~~proton~~ proton よりは相当細い track を示す筈であらう。

628
8.6
3768
5024
54

4.77
18
38.16
477
85.86

Fig. 8. $\rho H = 1.8\times10^6$ gauss cm

$$\sqrt{E^2-(m_U c^2)^2} = \frac{1.8\times10^6}{4.77\times10^{-10}} = 8.5\times10^{-4}\,erg$$

$$= 0.6285\times10^{12}\times8.5\times10^{-4}\,eV$$

$$= 5.4\times10^8\,eV.$$

proton: $W \cong \frac{1}{2}\frac{(5.4\times10^8)^2}{Mc^2} = \frac{3\times10^{17}}{2\times10^9} \cong 1.5\times10^8\,eV$

U-quantum: $E \cong 5.\cdot\cdot\times10^8\,eV.$

∴ これは U-quantum ではないであらう。

Fig. 7. の方は U-quantum かも知れない。

vii) heavy particle にしろ、light particle にしろ、その kinetic energy が $m_U c^2$ 位になると U-field を mittel とする interaction の range が急に増す。 $m_U c^2$ よりずっと大きくなると Coulomb type の force に~~なるであろう~~。なってしまふ。但し、、

viii) $E \gg 10^8$ ~~以上~~ の energy の particle は U-quantum と electron との区別が難しい、、

---

vi) nucleus による capture

U-quantum の energy が $m_U c^2$ に比して大きいならば、(但し Mc^2 に比しては大きくない) nuclear particle の一つの wave function を

Photoel. Effect 同様

$$\psi_a = \frac{1}{\sqrt{\pi a^3}} e^{-r/a} \qquad \psi_b = e^{i(pr)/\hbar c}$$

$$\alpha = \frac{Z\mu}{137} = \frac{\hbar c Z}{a_0} = \frac{\hbar c}{a}$$

$$d\phi = \frac{2\pi}{\hbar c} |H'|^2 \rho_E \, d\Omega$$

$$H' = -\frac{e}{\mu} p_e \sqrt{\frac{\alpha^3}{\pi \hbar^3 c^3}} \sqrt{\frac{2\pi \hbar^2 c^2}{k}} \cdot \frac{8\pi\alpha \hbar^3 c^3}{(\alpha^2 + q^2)^2}$$

$$= -\frac{e}{\mu} p_e a^{-\frac{3}{2}} \sqrt{\frac{2\pi \hbar^2 c^2}{k}} \cdot \frac{8\pi}{a\left(\frac{1}{a^2} + \left(\frac{q}{\hbar c}\right)^2\right)^2}$$

$$\rho_E = \frac{\mu p}{(2\pi \hbar c)^3}$$

$$d\phi = \frac{32\pi e^2}{(\hbar c)^2} \frac{p\, p_e^2 a^{-5}}{\mu k \left(\frac{1}{a^2} + \left(\frac{q}{\hbar c}\right)^2\right)^4} d\Omega$$

$$a^2 + q^2 \cong 2\mu k(1-\beta\cos\theta),$$

$$\frac{1}{a^2} + \left(\frac{q}{\hbar c}\right)^2 \cong \frac{2\mu k}{(\hbar c)^2}(1-\beta\cos\theta)$$

$$d\phi = \frac{32\pi e^2}{(\hbar c)^2}\frac{(\hbar c)^8}{(2\mu k)^4 \mu k}\frac{\cancel{(1-\beta\cos\theta)^4}\, d\Omega}{(1-\beta\cos\theta)^4}\, p p_e^2\, a^{-\frac{7}{2}}$$

$$p^2 = 2k\mu \qquad \frac{10}{2} \qquad \frac{3}{2}$$

$$\phi \cong \frac{4\sqrt{2}\pi e^2}{\cancel{(\hbar c)^2}}\frac{(\hbar c)^6}{(\mu k)^{\frac{7}{2}} a^{\frac{5}{2}}}$$

nuclear capture

$$\phi \cong 4\sqrt{2}\pi \cdot g^2 \cdot \frac{(\hbar c)^6}{(Mc^2)^{\frac{7}{2}}(k)^{\frac{7}{2}} a^{\frac{5}{2}}}$$

$$= 4\sqrt{2}\pi \cdot \frac{g^2}{\hbar c} \cdot \frac{\hbar^7}{M^{\frac{7}{2}} k^{\frac{7}{2}} a^{\frac{5}{2}}}$$

$$k \cong \frac{m_u c^2}{\cancel{\hbar}}$$

$$= 4\sqrt{2}\pi \cdot \frac{g^2}{\hbar c} \cdot \frac{\left(\frac{\hbar}{Mc}\right)^{\frac{7}{2}}\left(\frac{\hbar}{m_u c}\right)^{\frac{7}{2}}}{a^{\frac{5}{2}}}$$

$$\cong \left(\frac{\hbar}{Mc}\right)^2 \sim \left(\frac{\hbar}{m_u c}\right)^2$$

$$\cong \left(\frac{3.85\times 10^{-11}}{1840}\right)^2 \sim \left(\frac{3.85\times 10^{-11}}{200}\right)^2$$

$$\cong 10^{-28} \sim 10^{-26}$$

$$\sum_{Pb} N \cong 3.32\times 10^{22} \qquad \sum N\phi \cong 3 \times 10^{-4} \sim 3\times 10^{-2}$$

$k \simeq \cancel{\ldots} \, m_0 c^2$

$$\phi \lesssim \frac{4\sqrt{2}\pi g^2}{\hbar c} \cdot \frac{\left(\frac{\hbar}{Mc}\right)^{\frac{7}{2}} \cdot \left(\frac{\hbar}{m_0 c}\right)^{-\frac{1}{2}}}{\left(\frac{\hbar}{m_0 c}\right)^{5}} \simeq \left(\frac{\hbar}{Mc}\right)^{\frac{7}{2}} / \left(\frac{\hbar}{m_0 c}\right)^{\frac{3}{2}}$$

$$\simeq 10^{-29}.$$

Pb. $N \cong 3.32 \times 10^{22}$

$$\Sigma N\phi \simeq \cancel{3 \times 10^{-7}} \; 3 \times 10^{-5} \; cm^{-1}.$$

故に high speed heavy quantum の nuclear capture ~~absorption の問題~~ は negligibly small である。

low velocity になつて来ると、

$$\phi \cong \frac{4\sqrt{2}\pi g^2}{\hbar c} \cdot \frac{(\hbar c)^6}{(Mc^2)^{\frac{7}{2}} k^{\frac{5}{2}} k_0 a^5}$$

$$\simeq \quad " \quad \frac{\left(\frac{\hbar}{Mc}\right)^{\frac{7}{2}}}{\left(\frac{\hbar}{m_0 c}\right)^{3/2}} \cdot \left(\frac{c}{v}\right)^{\frac{5}{2}}$$

$$\cong 10^{-29} \left(\frac{c}{v}\right)^{\frac{5}{2}}.$$

$\frac{v}{c} \cong \frac{1}{16}$ $\quad \frac{m_0 v^2}{2} = 10^8 \frac{1}{2 \times 256} \cong 2 \times 10^5 \, eV$

$$\phi \cong 10^{-29} \cancel{\times 10^{3}} \; \cancel{2000} \; 1000 = 10^{-26} \; cm^2$$

$$\Sigma N\phi \sim 3 \times 10^{24} \times 10^{-26} = 3 \times 10^{-2} \; cm^{-1}.$$

故に capture の cross section は 非常に遅くならぬ限り大して大きくない。(非常に)遅くなると、charge の方が問題になる。