~~Interaction of Higher Orders~~

~~between the various sorts of phenomena, which~~
~~qualitative ... so as to ...~~

○ Quantization の問題

§2. Quantization of the U-field in the absence of the electromagnetic field

Lagrangian $\bar{L}_U = \iiint L_U \, dv$,

where $L_U = \frac{1}{4\pi}\left\{\frac{1}{c^2}\frac{\partial \tilde{U}}{\partial t}\frac{\partial U}{\partial t} - \mathrm{grad}\,\tilde{U}\,\mathrm{grad}\,U - \lambda^2 \tilde{U} U\right\}$.

Canonical Conjugate to U, $\tilde{U}$

$$\left.\begin{aligned} U^\dagger &= \frac{\delta \bar{L}_U}{\delta \frac{\partial U}{\partial t}} = \frac{1}{4\pi c^2}\frac{\partial \tilde{U}}{\partial t} \\ \tilde{U}^\dagger &= \frac{1}{4\pi c^2}\frac{\partial U}{\partial t}. \end{aligned}\right\}$$

Hence, $L_U = 4\pi c^2 \tilde{U}^\dagger U^\dagger - \frac{1}{4\pi}\mathrm{grad}\,\tilde{U}\,\mathrm{grad}\,U - \frac{\lambda^2}{4\pi}\tilde{U} U$.

Vertauschungsrel.

$$U^\dagger(\vec{r},t)U(\vec{r}',t) - U(\vec{r}',t)U^\dagger(\vec{r},t) = -i\hbar\,\delta(\vec{r}\vec{r}')$$

$$\tilde{U}^\dagger(\vec{r},t)\tilde{U}(\vec{r}',t) - \tilde{U}(\vec{r}',t)\tilde{U}^\dagger(\vec{r},t) = -i\hbar\,\delta(\vec{r}\vec{r}')$$

$$\bar{H}_U = \iiint H_U \, dv$$

$$H_U = \frac{\partial U}{\partial t}U^\dagger + \tilde{U}^\dagger\frac{\partial \tilde{U}}{\partial t} - L_U$$

$$= 4\pi c^2 \tilde{U}^\dagger U^\dagger + \frac{1}{4\pi}\mathrm{grad}\,\tilde{U}\,\mathrm{grad}\,U + \frac{\lambda^2}{4\pi}\tilde{U} U$$

$$\lambda = \frac{m_U c}{\hbar}$$

$$\cancel{U = \left(\frac{4\pi}{m_U}\right)^{\frac{1}{2}}\hbar\, u \;,\; \tilde{U} = \left(\frac{4\pi}{m_U}\right)^{\frac{1}{2}}\hbar\, \tilde{u}}$$

$$\cancel{U^\dagger = \frac{1}{4\pi c^2}}$$

$$\bar{H}_U =$$

$$\overline{H}_M = \iiint \tilde{\Psi} \Big[\frac{p^2}{2M} - \frac{g}{2}\{ \tilde{U}(\tau_1 - i\tau_2) + U(\tau_1 + i\tau_2)\}$$
$$+ \frac{D}{2}\tau_3 \Big] \Psi \, dv$$

(但し、(I) とは Ψ の第3の項の消去が省かれたから, 従って Heisenberg の original ~~Ar~~Arbeit より. $J(r) = g^2 \frac{e^{-\lambda r}}{r}$ とする)

~~Heavy~~ Particle
Current Density.

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{1}{2M}(-i\hbar \, \mathrm{grad})^2 \Psi - \frac{g}{2}\{ \tilde{U}(\tau_1 - i\tau_2) + U(\tau_1 + i\tau_2)\}\Psi$$

$$-i\hbar \frac{\partial \tilde{\Psi}}{\partial t} = \frac{1}{2M}(i\hbar \, \mathrm{grad})^2 \tilde{\Psi} - \frac{g}{2}\tilde{\Psi}\{ \tilde{U}(\tau_1 - i\tau_2) + U(\tau_1 + i\tau_2)\}$$

$$i\hbar \frac{\partial}{\partial t}(\tilde{\Psi}\Psi) = \frac{(i\hbar)^2}{2M} \overset{\mathrm{div}}{\cancel{\mathrm{grad}}} \{ \tilde{\Psi} \, \mathrm{grad}\, \Psi - \Psi \, \mathrm{grad}\, \tilde{\Psi} \}$$

Charge-Current Density

$\cancel{i\hbar \frac{1-\tau_3}{2}}$

$$i\hbar \frac{\partial}{\partial t}\left(\tilde{\Psi} \frac{1-\tau_3}{2} \Psi\right) = \frac{(i\hbar)^2}{2M} \mathrm{div} \left(\tilde{\Psi} \frac{1-\tau_3}{2} \mathrm{grad}\, \Psi - \cancel{\Psi \frac{1-\tau_3}{2} \mathrm{grad}\, \tilde{\Psi}} \right)$$
$$- \mathrm{grad}\, \tilde{\Psi} \cdot \frac{1-\tau_3}{2} \Psi$$

$$+ \frac{g}{2} \quad \tilde{\Psi}\Big\{ \tilde{U}\Big\{ \frac{\tau_3}{2}(\tau_1 - i\tau_2) - (\tau_1 - i\tau_2)\frac{\tau_3}{2} \Big\}$$
$$+ U \Big\{ \frac{\tau_3}{2}(\tau_1 + i\tau_2) - (\tau_1 + i\tau_2)\frac{\tau_3}{2} \Big\} \Big\} \Psi$$

$$= \quad \parallel \qquad i\tau_2 + i \cdot i\tau_1$$
$$i\tau_2 - \tau_1$$

$$- \frac{g}{2} \tilde{\Psi}\{ \tilde{U}(\tau_1 - i\tau_2) + U(\tau_1 + i\tau_2)\}$$

$\cancel{\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}}$

$$\tilde{U} \times \quad \frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} = (\Delta - \lambda^2) U + 4\pi g \tilde{\Psi} \frac{\tau_1 - i\tau_2}{2} \Psi$$

$$-\lambda U \quad \frac{1}{c^2}\frac{\partial^2 \tilde{U}}{\partial t^2} = (\Delta - \lambda^2) \tilde{U} + 4\pi g \tilde{\Psi} \frac{\tau_1 + i\tau_2}{2} \Psi$$

$$\frac{1}{c^2}\frac{\partial}{\partial t}\left(\tilde{U}\frac{\partial U}{\partial t} - \frac{\partial \tilde{U}}{\partial t}U\right) = \mathrm{div}\left(\tilde{U}\,\mathrm{grad}\,U - \mathrm{grad}\,\tilde{U}\cdot U\right)$$

$$+\ 4\pi g\,\tilde{\Psi}\left(\tilde{U}\frac{\tau_1 - i\tau_2}{2} + U\frac{\tau_1 + i\tau_2}{2}\right)\Psi$$

$$i\hbar\frac{\partial}{\partial t}\left\{\tilde{\Psi}\frac{1-\tau_3}{2}\Psi + \frac{1}{4\pi(\hbar i)c^2}\left(\tilde{U}\frac{\partial U}{\partial t} - \frac{\partial \tilde{U}}{\partial t}U\right)\right\}$$

$$= \mathrm{div}\left(\frac{(i\hbar)^2}{2M}\left(\tilde{\Psi}\frac{1-\tau_3}{2}\mathrm{grad}\,\Psi - \mathrm{grad}\,\tilde{\Psi}\frac{1-\tau_3}{2}\Psi\right)\right.$$

$$\left. + \frac{1}{4\pi}\left(\tilde{U}\,\mathrm{grad}\,U - \mathrm{grad}\,\tilde{U}\,U\right)\right)$$

∴ charge Density is

$$\rho = \frac{e}{4\pi\hbar i c^2}\left(\tilde{U}\frac{\partial U}{\partial t} - \frac{\partial \tilde{U}}{\partial t}U\right) = \frac{e}{\hbar i}\left(\tilde{U}\tilde{U}^\dagger - U^\dagger U\right)$$

$$\frac{U^\dagger}{4\pi i\hbar} = u^\dagger$$

$$\frac{\tilde{U}^\dagger}{i\hbar} = \tilde{u}^\dagger \qquad = e\left(\tilde{U}\tilde{u}^\dagger - u^\dagger U\right)$$

$$u^\dagger U' - U' u^\dagger = -\delta$$

$$\tilde{u}^\dagger \tilde{U}' - \tilde{U}' \tilde{u}^\dagger = \delta$$

~~∴ Uの方は neg charge の quantum~~
~~Ũの方は pos. charge の quantum~~ を表はす.

$$U = c(\varphi_1 + \tilde{\varphi}_2) \qquad U = ic'(\tilde{\varphi}_1 - \varphi_2)$$

$$\tilde{U}^\dagger = c(\tilde{\varphi}_1 + \varphi_2) \qquad \tilde{U} = -ic'(\varphi_1 - \tilde{\varphi}_2)$$

$$U^\dagger U' - U' U^\dagger = -i\hbar\delta$$

$$\tilde{U}^\dagger \tilde{U}' - \tilde{U}'\tilde{U}^\dagger = i\hbar\delta$$

$$\cancel{(\varphi_1+\tilde{\varphi}_2)\ i c'(\tilde{\varphi}_1'-\varphi_2) =}$$

$$\varphi_1 = \frac{U^\dagger}{2c} + \frac{i}{2c'}\tilde{U} \qquad \tilde{\varphi}_1 = \frac{\tilde{U}^\dagger}{2c} + \frac{i}{2c'} U$$

$$\varphi_2 = \frac{\tilde{U}^\dagger}{2c} + \frac{i}{2c'} U \qquad \tilde{\varphi}_2 = \frac{U^\dagger}{2c} - \frac{i}{2c'}\tilde{U}$$

$$\tilde{\varphi}_1 \varphi_1' - \varphi_1' \tilde{\varphi}_1 = \frac{i}{4cc'}(\tilde{U}^\dagger \tilde{U}' - \tilde{U}'\tilde{U}^\dagger)$$

$$- \frac{i}{4cc'}(U U^{\dagger\prime} - U^{\dagger\prime} U)$$

$$= + \frac{\hbar\,\delta}{4cc'} + \frac{\hbar\,\delta}{4cc'} = \frac{\hbar}{2cc'}\delta = -\delta,$$

$$c' = -\frac{\hbar}{2c}$$

$$U^\dagger = c_0(\varphi_1 + \tilde{\varphi}_2) \qquad U = -\frac{i\hbar}{2c_0}(\tilde{\varphi}_1 - \varphi_2)$$

$$\tilde{U}^\dagger = c_0(\tilde{\varphi}_1 + \varphi_2) \qquad \tilde{U} = +\frac{i\hbar}{2c_0}(\varphi_1 - \tilde{\varphi}_2)$$

$$\rho = e\{(\varphi_1 - \tilde{\varphi}_2)(\tilde{\varphi}_1 + \varphi_2) + (\varphi_1 + \tilde{\varphi}_2)(\tilde{\varphi}_1 - \varphi_2)\}$$

$$= e\{\varphi_1\tilde{\varphi}_1 - \tilde{\varphi}_2\varphi_2\}$$

δ-function から出る const. term を除けば

$$\rho = e\{\tilde{\varphi}_1\varphi_1 - \tilde{\varphi}_2\varphi_2\}$$

故 $\varphi_1, \tilde{\varphi}_1$ は positive charge の quantum
$\varphi_2, \tilde{\varphi}_2$ は negative 〃 〃 を表はす.

$$H_U = 4\pi c^2 c_0^2 (\tilde{\varphi}_1 + \varphi_2)(\varphi_1 + \tilde{\varphi}_2)$$

$$+ \frac{\hbar^2}{4\pi e \; 4c_0^2} \, \mathrm{grad}(\varphi_1 - \tilde{\varphi}_2) \, \mathrm{grad}(\tilde{\varphi}_1 - \varphi_2)$$

$$+ \frac{\lambda^2 \hbar^2}{4\pi \cdot 4c_0^2} (\varphi_1 - \tilde{\varphi}_2)(\tilde{\varphi}_1 - \varphi_2)$$

$$\varphi_1 = \sum a_1(\vec{p}) \exp \frac{i\vec{p}\vec{r}}{\hbar}$$

$$\varphi_2 = \sum a_2(\vec{p}) \exp \frac{i\vec{p}\vec{r}}{\hbar}$$

$$\bar{H}_U = \sum \left(4\pi c^2 c_0^2 + \frac{\hbar^2}{4\pi \cdot 4c_0^2} \cdot p^2 + \frac{\lambda^2 \hbar^2}{4\pi 4c_0^2} \right) a_1^{\dagger}(\vec{p}) \, a_1(\vec{p})$$

$+ \cdots$