

On the Interaction
of Elementary Particles. ~~II~~.

II. ~~Generalization of Formulation Scheme~~
Generalization of the Mathematical Scheme

By Hideki Yukawa

(Read ^{Nov. 28} sept, 1954)

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Elementary Particles. II,

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Abstract

(1)

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§ 1. Introduction

In the paper¹⁾ published early in 1935, the present author introduced a new field, which was responsible both for the short range force between the neutron and the proton and for the β -disintegration. It was shown that this field was ~~shown~~ to be accompanied by quanta with the elementary charge $+\epsilon$ or $-\epsilon$ and the mass about $\frac{1}{10}$ of that of the proton. Such quanta, which if it ever ~~was~~ were shown further that such quanta could not be emitted by ordinary nuclear reaction and might be present only in the cosmic ray. ~~At that time~~ There ~~was no~~ ^{had been} evidence, ^{however} in favour of the existence of such heavy quanta, so that further development of the theory ~~the author hesitated to develop the theory further.~~ ^{with late work} Recent researches of Anderson and Neddermeyer²⁾ and Stevenson³⁾ and ~~Nishina, Takeuchi, Ichimura and others~~ ^{Street} indicated the existence of in the cosmic ray of particles which can be identified neither with the electrons nor ~~the~~ ^{with} protons, ^{as long as} if we accept the present quantum theory. Thus the existence of the charged particles with intermediate mass predicted by the theory became very probable, and it seems, so that and Thus the existence of the charged particles with intermediate mass became very probable⁴⁾ which can naturally be identified with heavy quanta above considered. ^{and it seems reasonable to identify them}

5) Yukawa, Proc. Phys.-Math. Soc. Japan 19, 712, 1937.

Hereafter this paper will be referred to as I.

- 1) Yukawa, Proc. Phys.-Math. Soc. Japan 17, 48, 1935.
- 2) Anderson and Neddermeyer, Phys. Rev. 50, 273, 1936; 51, 884, 1937.
- 3) Street and Stevenson, *ibid.* 51, 1005, 1937.
- 4) According to the preliminary result obtained by Nishina and his coworker, the mass of the new particles lies between $\frac{1}{8}$ and $\frac{1}{7}$ the protonic mass, which is in fair agreement with the theoretical ^{prediction} about $\frac{1}{10}$ expectation.

⑤

Our aim is to deal with elucidate the phenomena
concerning the clear structure, β -disintegration and the concrete
in the light of the new field, and instead and
phenomena theory to give a
from a unified point which is the common
background

In this paper, we begin with the formulation
of the mathematical scheme the general scheme, describing
the relation of the concrete problems being
deal to deal with

1) Yukawa, Proc. Phys.-Math. Soc. Japan 14, 115, 1947.
Therefore this paper will be referred to as I.

2) Yukawa, Proc. Phys.-Math. Soc. Japan 17, 48, 1952.
3) Anderson and Heisenberg, Phys. Rev. 50, 215, 1942; 51, 884, 1943.

4) Heisenberg and Pauli, Z. Phys. 59, 170, 1951.

5) According to the preliminary result obtained by Yukawa and his
collaborer, the mass of the new particles is less than $\frac{1}{2}$ of the proton
mass, which is in fair agreement with the theoretical prediction
to be expected.

Thus the most serious drawback in my theory seems to be removed.

Similar conclusion was reached by Oppenheimer and Serber⁶⁾ and by Stückelberg⁷⁾. It should be noticed, however, that there are still many points in the theory to be corrected or completed by further development of the theory.

Firstly, ~~only the exchange force of Heisenberg type between ~~charge~~ unlike nuclear particles was deduced from the scalar potential in I.~~

We have to consider the interaction of the heavy quanta with the electromagnetic field in detail. ~~of the ~~vector~~ potentials~~ The mathematical formulation will be the same with that given by ^{Pauli and Weisskopf⁸⁾} account,

Pauli and Weisskopf⁸⁾, except ~~that the mass larger mass and~~ the presence of source and sink ^{due to the presence of the transition} of the heavy or light particles. (§ 2) Various processes,

Secondly, only exchange force of Heisenberg type between ~~the neutron and the proton~~ unlike heavy particles was deduced in I. Like pair forces between like particles can be obtained by considering higher order processes as in the usual neutrino-electron field theory, whereas the exchange force of Majorana type between unlike particles can be included, ~~also~~ only if ~~tensor field potentials is~~ introduced instead of ~~for~~ those of four vector of tensor ~~by~~ character.

(§ 4) ~~Recent experimental~~ It is argued recently ~~that~~ by many authors⁹⁾ that ~~the~~ experimental results on the scattering and ~~the~~ binding of nuclear particles can be explained by assuming ~~the same~~ forces between ~~like~~ particles ~~as well as as~~ ~~there~~ between unlike particles ~~and~~ that ^{further}

9)

6) Oppenheimer and Serber, Phys. Rev. 51, 1113, 1937.

7) Stückelberg, Phys. Rev. 52, , 1937.

8) Pauli and Weisskopf, Helv. Phys. 7, 709, 1934.

+ which ^{can be raised} ~~are given~~ ^{mentioned} ~~are~~ by the passage of heavy quanta through matter, will be ~~discussed~~ and their bearing on ~~the phenomena~~ cosmic ray phenomena ~~enumerated~~ will be discussed. (§ 3).

a certain combination of ^{four} types of forces ^{can be selected,} ~~is necessary,~~
~~so as to~~ be consistent with ~~all the~~ the experiment ~~as the~~
whole. ~~That~~ Unfortunately, ~~we can not reproduce these~~
consequences ~~on our theory, unless~~ further assumptions are
~~can not be reproduced strictly,~~

made, as pointed out by Oppenheimer ⁵⁾ and Serber ⁶⁾. ~~It should~~
be kept in mind, however, that ~~the present~~ ^{existing} experimental data

~~is~~ are not sufficient for ~~the unique determining~~
the existing experimental data, ~~are not sufficient~~ ^{are not sufficient}
for determining the nuclear forces uniquely, ~~so that~~

These finer detail ~~points~~ (This is not a serious defect
of our theory, ~~since however, since~~ Nevertheless)

Thirdly, ~~the magnetic anomalous~~ magnetic moments of the
~~proton~~ ^{theoretical deduction} will be an important
problem in our theory, as long as ~~the~~ wave equations of
Dirac type ~~is~~ are assumed for the heavy particle.
Detailed discussion of this problem will ~~not~~ be made ~~in this~~
by Sakata ¹⁰⁾ ~~in other place~~

Fourthly, ~~the original~~ the theory of β -decay given in I
corresponds to was equivalent to that of Fermi ¹¹⁾, we can
~~no~~ easily modify it into the form, which is equivalent to
that of Konopinski and Uhlenbeck ¹²⁾ corresponding to
Recent experimental results seems to show that, however, that
neither Fermi's ~~original~~ nor K.-U. Fermi nor of K.-U.
is ~~sufficient~~ for the complete explanation ^{the formula of} of the experiments
results, so that v

except ~~that~~ ~~when the energy of~~ ~~disintegration is very~~
large.

- 10) Sakata, Proc. Phys.-Math. Soc. Japan 19, impress.
- 11) Fermi, Zeits. f. Phys. 88, 161, 1934.
- 12) Konopinski and Uhlenbeck, Phys. Rev. 48, , 1935.

so that we should like to ~~defend~~ ^{put off} further discussions of this problem.

§ 2. Quantization of U-field

The field ~~is~~ ^{in vacuum} can be derived from the Lagrangian $L_U = \iiint L_U dv$, where L_U is derived from the wave equations for (3) and (4) in I for U-field and its conjugate complex (2).

$$L_U = \frac{1}{4\pi} \left\{ \frac{1}{c^2} \frac{\partial \tilde{U}}{\partial t} \frac{\partial U}{\partial t} - \text{grad } \tilde{U} \cdot \text{grad } U - \lambda^2 \tilde{U} U \right\}$$

where the ~~variables~~ ^{variables} canonically conjugate operator canonically conjugate variables to U and \tilde{U} are

$$U^\dagger = \frac{\delta L_U}{\delta \frac{\partial U}{\partial t}} = \frac{1}{4\pi c^2} \frac{\partial \tilde{U}}{\partial t}$$

$$\tilde{U}^\dagger = \frac{1}{4\pi c^2} \frac{\partial U}{\partial t} \quad \text{commutation}$$

respectively, which satisfy the relations of

$$U^\dagger(\vec{r}, t) U(\vec{r}', t) - U(\vec{r}, t) U^\dagger(\vec{r}', t) = -i\hbar \delta(\vec{r}, \vec{r}') \quad (a)$$

$$\tilde{U}^\dagger(\vec{r}, t) \tilde{U}(\vec{r}', t) - \tilde{U}(\vec{r}, t) \tilde{U}^\dagger(\vec{r}', t) = -i\hbar \delta(\vec{r}, \vec{r}')$$

$$U(\vec{r}, t) U(\vec{r}', t) - U(\vec{r}', t) U(\vec{r}, t) = 0$$

The Hamiltonian for the U-field becomes thus $\tilde{U}^\dagger(\vec{r}, t) \tilde{U}^\dagger(\vec{r}', t) - U^\dagger(\vec{r}, t) U^\dagger(\vec{r}', t) = 0$

$$\tilde{H}_U = \iiint H_U dv, \quad (b)$$

where

$$H_U = \frac{\partial U}{\partial t} U^\dagger + \tilde{U}^\dagger \frac{\partial \tilde{U}}{\partial t} - L_U = 4\pi c^2 \tilde{U}^\dagger U^\dagger + \frac{1}{4\pi} \text{grad } \tilde{U} \cdot \text{grad } U + \frac{\lambda^2}{4\pi} \tilde{U} U \quad (5)$$

In the presence of the heavy particle, the Hamiltonian

for the total system is given by,

$$\bar{H}_M = \int \int \int \Psi \left[\frac{\vec{p}^2}{2M} - \frac{g}{2} \{ \tilde{U}(\tau_1 - i\tau_2) + U(\tau_1 + i\tau_2) \} + \frac{D}{2} \tau_3 \right] \Psi d\tau \quad (7)$$

$$\bar{H} = \bar{H}_U + \bar{H}_M, \quad (6)$$

where M, D, \vec{p} and (τ_1, τ_2, τ_3) are the mass, momentum and isotopic spin generators of the heavy particle and D is the difference of mass of the neutron and the proton. $\vec{p} = -i\hbar \text{grad}$ of the neutron and the proton.

$$\begin{aligned} \Psi(\vec{r}, t) \Psi(\vec{r}', t) + \Psi(\vec{r}, t) \tilde{\Psi}(\vec{r}', t) &= \delta_{\tau_3 \tau_3'} \delta(\vec{r}, \vec{r}') \\ \Psi(\vec{r}, t) \Psi(\vec{r}', t) + \Psi(\vec{r}', t) \Psi(\vec{r}, t) &= 0 \\ \tilde{\Psi}(\dots) \tilde{\Psi}(\dots) + \tilde{\Psi}(\dots) \tilde{\Psi}(\dots) &= 0 \end{aligned} \quad (8)$$

By using (7) and the commutation relations (3) and (8), the theory can be developed on the line similar to that of the ordinary quantum electrodynamics. We can, for example, deduce the wave equations (4) (5) in I for the U-field in the presence of the heavy particle and the wave eq further the wave equations for the heavy particle in the presence of the U-field, which are identical with (6) in I, except for the ~~sign~~ positive sign before the term containing g in ~~stead~~ place of the negative sign. This results further in the change of sign of the ~~exchanged~~ ^{potential} energy between the neutron and the proton. Namely, the ~~interaction~~ ^{interaction} energy of two heavy ~~(I)~~ particles 1 and 2 at a distance r

(1)

becomes

$$-\frac{g^2}{2} (\tau_1^{(1)} \tau_1^{(2)} + \tau_2^{(1)} \tau_2^{(2)}) \frac{e^{-\lambda r}}{r}$$

instead of

$$+\frac{g^2}{2} (\tau_1^{(1)} \tau_1^{(2)} + \tau_2^{(1)} \tau_2^{(2)}) \frac{e^{-\lambda r}}{r}$$

as given in I, so that the neutron and the proton attracts with each other when their ordinary spins are anti-parallel in contradiction with the fact that H^+ has spin 1 in the normal state. (18) ~~this~~ difficulty of similar kind ^{will} ~~appears~~ ^{later} also, when the Majorana type of force is taken into account. ~~this~~ This can be

We can ~~perhaps~~ get rid of this diff. if we consider the vector potentials, ~~In~~ ^{In} this case, the Hamiltonian ~~for the scalar potentials~~ ^{involving} ~~U, \tilde{U} changes sign and can be written~~ ^{becomes} in the form

$$\bar{H}_0 = \iiint H_0 dv \quad (9)$$

$$H_0 = -4\pi c^2 \tilde{U}^t U^t - \frac{1}{4\pi} \text{grad } \tilde{U} \text{ grad } U - \frac{\lambda^2}{4\pi} \tilde{U} U \quad (10)$$

$$+ 4\pi c^2 \sum_i \tilde{B}_i^t B_i^t + \frac{1}{4\pi} \sum_i \text{grad } B_i \text{ grad } B_i + \frac{\lambda^2}{4\pi} \sum_i \tilde{B}_i B_i$$

if we ~~perpet~~ ^{persist} in the ~~parallel~~ ^{analogy} between the heavy and the light quanta. ~~On the other hand~~ The Hamiltonian ~~for the heavy particles~~ ^{becomes}

$$\bar{H}_M = \iiint \Phi \left\{ \frac{1}{2m} \left[\vec{p} + \frac{g}{2} (\tau_1 - i\tau_2) \vec{B} \right] + (\tau_1 + i\tau_2) \vec{B} \right\}^2$$

$$+ \frac{g}{2} \{ \tilde{U} (\tau_1 - i\tau_2) + U (\tau_1 + i\tau_2) \} \Phi dv, \quad (11)$$

The Hamiltonian for the total system being again

$$\bar{H} = \bar{H}_0 + \bar{H}_M. \quad (12)$$

§ 3. Interaction of U-field with the Electro-Magnetic Field.

So far we have altogether neglected the interaction of U-field with the electromagnetic field, which ~~was~~ appears ~~never~~ always, when the proton ~~should~~ be appears, for example, in the presence of the ~~it~~ should be considered proton, but the inclusion of ~~the~~ it can be included in the usual manner, as follows. Namely ~~grad U in (10) should~~ be replaced by ~~grad U and grad U~~ etc

and ~~(grad + \frac{ie}{\hbar c} \vec{A}) U~~
~~(grad - \frac{ie}{\hbar c} \vec{A}) \tilde{U}~~

respectively and the term of the form
 Namely, we start from the Lagrangian for the U-field

$$L_U = \int \int \int L_U dV, \tag{13}$$

where

$$L_U = \frac{1}{4\pi} \left\{ \frac{1}{c^2} \left(\frac{\partial \tilde{U}}{\partial t} - \frac{ie}{\hbar c} V \tilde{U} \right) \left(\frac{\partial U}{\partial t} + \frac{ie}{\hbar c} V U \right) \right. \tag{14}$$

$$+ \left(\text{grad} \tilde{U} + \frac{ie}{\hbar c} \vec{A} \tilde{U} \right) \left(\text{grad} U - \frac{ie}{\hbar c} \vec{A} U \right) + \lambda^2 \tilde{U} U \left. \right\}$$

$$+ \frac{1}{4\pi} \sum_{i,j} \left\{ \frac{1}{c^2} \left(\frac{\partial \tilde{B}_i}{\partial t} - \frac{ie}{\hbar c} V \tilde{B}_i \right) \left(\frac{\partial B_i}{\partial t} + \frac{ie}{\hbar c} V B_i \right) \right.$$

$$- \left(\text{grad} \tilde{B}_i + \frac{ie}{\hbar c} \vec{A} \tilde{B}_i \right) \left(\text{grad} B_i - \frac{ie}{\hbar c} \vec{A} B_i \right) \left. \right\}$$

field

and V, \vec{A} are the scalar and the vector potentials of the electro-magnetic field
 The canonical conjugate variables are the observables canonical

conjugate to U and \tilde{U} becomes

$$\tilde{U}^\dagger = \frac{1}{4\pi c^2} \left(\frac{\partial U}{\partial t} - \frac{ie}{\hbar c} V U \right) \quad U^\dagger = -\frac{1}{4\pi c^2} \left(\frac{\partial \tilde{U}}{\partial t} + \frac{ie}{\hbar c} V \tilde{U} \right)$$

$$\tilde{\beta}_i^\dagger = \frac{1}{4\pi c} \left(\frac{\partial \tilde{\beta}_i}{\partial t} - \epsilon \frac{ie}{\hbar} V \tilde{\beta}_i \right) \quad \tilde{\beta}_i^\dagger = \frac{1}{4\pi c} \left(\frac{\partial \beta_i}{\partial t} + \epsilon \frac{ie}{\hbar} V \beta_i \right)$$

respectively, so that the Hamiltonian takes the form

$$\bar{H}_U = \int H_U dv, \quad (16)$$

where

$$\begin{aligned} H_U = & -4\pi c^2 \tilde{U}^\dagger U^\dagger - \frac{\epsilon ie}{\hbar} (U^\dagger V U - \tilde{U}^\dagger V \tilde{U}) \\ & - \frac{1}{4\pi} \left\{ (\text{grad } \tilde{U} + \frac{\epsilon ie}{\hbar c} \vec{A} \tilde{U}) (\text{grad } U - \frac{\epsilon ie}{\hbar c} \vec{A} U) \right. \\ & \left. - \lambda^2 \tilde{U} U \right\} \\ & + 4\pi c^2 \sum_i \left\{ \tilde{\beta}_i^\dagger \beta_i^\dagger + \frac{\epsilon ie}{\hbar} (\beta_i^\dagger V \beta_i + \tilde{\beta}_i^\dagger V \tilde{\beta}_i) \right\} \\ & + \frac{1}{4\pi} \left\{ (\text{grad } \tilde{\beta}_i + \frac{\epsilon ie}{\hbar c} \vec{A} \tilde{\beta}_i) (\text{grad } \beta_i - \frac{\epsilon ie}{\hbar c} \vec{A} \beta_i) \right. \\ & \left. + \lambda^2 \tilde{\beta}_i \beta_i \right\}. \end{aligned} \quad (17)$$

On the other hand, ^{non-relativistic} these equations the Hamiltonian for the heavy particle in the presence of the electromagnetic field is given by

$$\begin{aligned} \bar{H}_M = & \int \Psi^\dagger \left\{ \frac{1}{2M} \left(\vec{p} - \frac{e}{c} \frac{1-i\tau_3}{2} \vec{A} \right)^2 + \frac{D}{2} \tau_3 \right. \\ & \left. + \frac{g}{2} (\tau_1 - i\tau_2) \vec{B} + \frac{g}{2} (\tau_1 + i\tau_2) \vec{B} \right\} \\ & - \frac{g}{2} \left[\tilde{U} (\tau_1 - i\tau_2) + U (\tau_1 + i\tau_2) \right] \Psi dv, \end{aligned} \quad (18)$$

so that the total Hamiltonian for the system containing the heavy particle, U-field and electro-magnetic field becomes

$$\bar{H} = \bar{H}_M + \bar{H}_U + \bar{H}_{RE}, \quad (19)$$

where

$$\bar{H}_E = \frac{1}{8\pi} \iint (\vec{E}^2 + \vec{H}^2) dv \quad (20)$$

Further, we should like to assume that U, \vec{B} and $\tilde{U}, \tilde{\vec{B}}$ should satisfy auxiliary conditions

$$\left. \begin{aligned} \frac{1}{c} \frac{\partial U}{\partial t} + \text{div } \vec{B} &= 0, \\ \frac{1}{c} \frac{\partial \tilde{U}}{\partial t} + \text{div } \tilde{\vec{B}} &= 0 \end{aligned} \right\} \quad (21)$$

which are intimately connected with the equation of continuity for the heavy of for the number

$$\frac{\partial}{\partial t} \left(\frac{1}{2M} \frac{\partial \Psi}{\partial t} \right) + \text{div} \left(\frac{1}{2M} \text{grad } \Psi \right) = 0$$

The reason which are necessary for deriving the interaction relativistically invariant forces between heavy particles, namely, the Hamiltonian in the absence of the electromagnetic field

$$H_0 = \iint H_0 dv$$

$$\text{with } H_0 = -4\pi c^2 \sum_i \psi_i^\dagger \psi_i + \frac{1}{4\pi} \text{grad } \tilde{U} \text{ grad } U + \lambda^2 \tilde{U} U$$

$$+ 4\pi c^2 \sum_i \vec{B}_i^\dagger \vec{B}_i + \frac{1}{4\pi} \text{grad } \tilde{\vec{B}}_i \text{ grad } \vec{B}_i + \lambda^2 \tilde{\vec{B}}_i \vec{B}_i$$

can be written in the form, by using (21),

$$H_0 = \frac{1}{4\pi c^2} \text{div } \vec{B} \text{ div } \tilde{\vec{B}} - \frac{1}{4\pi} \text{grad } \tilde{U} \text{ grad } U + \lambda^2 \tilde{U} U$$

§ 4. Deduction of static interaction between the neutron and the proton can be made in a manner similar that by of Fermi⁽¹⁾

(1) E. Fermi, Rev. Mod. Phys. 4, 131, 1932; W. Heitler, Quantum Theory of Radiation, p. 46, Oxford, 1936, p. 46.

{ Quanta
{ Quanta

E02130P12

- § Properties of Heavy Quanta
- ↳ Magnetic Moment of Heavy Particles
- § Interaction of Heavy Particles
- ~~Static~~
- § Passage of Heavy Quanta through Matter

第 110 回 物理談話會

9 月 30 日 木 曜 午後 4 時 30 分から理學部

大講義室に於て下記の講演を行います

伊藤 順吉 君

Note on Resonance in Transmutation of Light
Nuclei

(Kalckar, Oppenheimer and Serber, Phys. Rev. Aug 15, 1937)

岡谷 辰治 君

Viscosity に就て

尚以上に先立って午後 4 時 分から下記の
通り學生の輪講があります

山口 省太郎 君

Unidirectional Measurement of Cosmic Ray
Latitude Effect

(Johnson and Read, Phys. Rev. April 1, 1937)

留意、文化講義がありますので輪講の開始を 4 時
に致します。

10 月 9 日 27