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On the Interaction of Elementary Particles. II.

By Hideki Yukawa and Shoichi Sakata.

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§1. Introduction and Summary.

In the first part of this paper,[1] one of the present authors introduced a new field, which was responsible both for the short range force between the neutron and the proton and for the β-disintegration. This field turned out to be accompanied by quanta each with the elementary charge either +e or -e, the mass about 1/10 of that of the proton and 0 or integer spin, obeying Bose statistics. It was shown further that such quanta, if they ever existed, could not be emitted by ordinary nuclear reactions, but might be present in the cosmic ray. There had been no evidence in favour of the last point, until recent researches of Anderson and Neddermeyer,[2] Street and Stevenson,[3] and Nishina, Takeuchi and Ichimiya[4] indicated the existence in the cosmic ray of particles, which could ~~can~~ be identified neither with the electrons nor with the protons, as long as we accepted the present theory of energy dissipation of high speed particles. Thus it seems likely that they are the ~~to be reasonable to identify~~ these particles with heavy quanta above considered.[5] Similar conclusions were

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1) Yukawa, Proc. Phys.-Math. Soc. Japan 17, 48, 1935. ~~Here~~ Hereafter, this paper will be referred to as I.
2) Anderson and Neddermeyer, Phys. Rev. 50, 273, 1936; 51, 884, 1937.
3) Street and Stevenson, ibid. 51, 1005, 1937.
4) According to the preliminary result of Nishina and others, the mass of the new particle is about 1/10 the protonic mass in fair agreement with the theoretical expectation.
5) Yukawa, Proc. Phys.-Math. Soc. Japan 19, 712, 1937.

reached by Oppenheimer and Serber[6] and by Stueckelberg.[7]

In this way, the most serious drawback in our theory is likely to have disappeared, although there are still many points to be completed or improved in course of development of the theory, which will be made in this and subsequent papers. We hope that the whole phenomena of the nuclear transformation, β-disintegration and cosmic ray will be elucidated in the light of the theory of the new field. In this paper, we begin with the problem of quantization of the scalar field, which can be solved in a manner similar to that of Pauli and Weisskopf.[8] The interaction between the heavy particle and the new field can be dealed with as that between the charged particle and the electromagnetic field in the quantum electrodynamics. (§2). The interaction between unlike particles, viz. the neutron and the proton, is derived as second order effect, confirming the result in I. (§3). Further, the interaction between like particles as higher order effect is discussed. (§4). Finally, the problem of passage of high energy heavy quanta through matter is considered in connection with cosmic ray phenomena. (§5). ~~The scalar theory~~ however ~~has several~~ is not sufficient for consistent interpretation ~~weak points.~~ (§6). The development of a more complete theory including the interaction of the heavy quanta with the electromagnetic field and with the light particles will be made in III etc, ~~It will~~ where ~~it will be shown the~~ the necessity of non-scalar field will be ~~shown~~ indicated.

6) Oppenheimer and Serber, Phys. Rev. 51, 1113, 1937.
7) Stueckelberg, ibid. 52, 41,1937. Stueckelberg arrived at the new field theory independently.
8) Pauli and Weisskopf, Helv. Phys. 7, 709, 1934.

§2. Quantization of the ~~New~~ Scalar Field.

The simplest conceivable form of the new field theory can be obtained by considering the potentials U and $\tilde{U}$ in I as the four dimensional scalars instead of time components of four vectors. Whether such a simplification is legitimate or not will be discussed later.

The wave equation (3) and its complex conjugate for U-field in vacuum can be derived from the Lagrangian

$$\bar{L}_U = \iiint L_U \, dv, \tag{1}$$

with

$$L_U = \frac{1}{4\pi}\left(\frac{1}{c^2}\frac{\partial \tilde{U}}{\partial t}\frac{\partial U}{\partial t} - \operatorname{grad}\tilde{U}\operatorname{grad}U - \kappa^2 \tilde{U} U\right) \tag{2}$$

where the constant κ is used in place of λ in I. The variables canonically conjugate to U and $\tilde{U}$ are

$$U^{\dagger} = \frac{\delta \bar{L}_U}{\delta \frac{\partial U}{\partial t}} = \frac{1}{4\pi c^2}\frac{\partial \tilde{U}}{\partial t} \quad \text{and} \quad \tilde{U}^{\dagger} = \frac{1}{4\pi c^2}\frac{\partial U}{\partial t}$$

respectively, satisfying the commutation relations

$$\left.\begin{aligned}
&U(\vec{r},t)\,U^{\dagger}(\vec{r}',t) - U^{\dagger}(\vec{r}',t)\,U(\vec{r},t) = i\hbar\,\delta(\vec{r},\vec{r}') \\
&\tilde{U}(\vec{r},t)\,\tilde{U}^{\dagger}(\vec{r}',t) - \tilde{U}^{\dagger}(\vec{r}',t)\,\tilde{U}(\vec{r},t) = i\hbar\,\delta(\vec{r},\vec{r}') \\
&U(\vec{r},t)\,\tilde{U}(\vec{r}',t) - \tilde{U}(\vec{r}',t)\,U(\vec{r},t) = 0 \\
&U^{\dagger}(\vec{r},t)\,\tilde{U}^{\dagger}(\vec{r}',t) - \tilde{U}^{\dagger}(\vec{r}',t)\,U^{\dagger}(\vec{r},t) = 0
\end{aligned}\right\} \tag{3}$$

The Hamiltonian for the U-field becomes thus

$$\bar{H}_U = \iiint H_U \, dv \tag{4}$$

with

$$H_U = \frac{\partial U}{\partial t} U^\dagger + \tilde{U}^\dagger \frac{\partial \tilde{U}}{\partial t} - L_U$$

$$= 4\pi c^2 \tilde{U}^\dagger U^\dagger + \frac{1}{4\pi} \operatorname{grad} \tilde{U} \operatorname{grad} U + \frac{\kappa^2}{4\pi} \tilde{U} U. \qquad (5)$$

In the presence of the heavy particle, the Hamiltonian for the total system is given by

$$\bar{H} = \bar{H}_M + \bar{H}_U + \bar{H}' \qquad (6)$$

with

$$\left.\begin{aligned} \bar{H}_M &= c \iiint \tilde{\Psi} \{ \vec{\alpha} \vec{p} + \beta (\tfrac{1+\tau_3}{2} M_N c + \tfrac{1-\tau_3}{2} M_P c) \} \Psi \, dv \\ \bar{H}' &= g \iiint \tilde{\Psi} (\tilde{U} Q^* + U Q) \beta \Psi \, dv \end{aligned}\right\} \qquad (7)$$

$$\vec{p} = -i\hbar \operatorname{grad}, \quad Q = \frac{\tau_1 + i\tau_2}{2}, \quad Q^* = \frac{\tau_1 - i\tau_2}{2}, \qquad (8)$$

where $(\vec{\alpha}, \beta)$ and (τ_1, τ_2, τ_3) are Dirac matrices and isotopic spin matrices respectively, assuming the wave equations for the heavy particle to be of Dirac's type. M_N and M_P are the masses of the neutron and the proton respectively. The wave functions Ψ and $\tilde{\Psi}$ have each eight components and satisfy the following commutation relations, for $i, j = 1, 2, \cdots 8$.

$$\left.\begin{aligned} \tilde{\Psi}^{(i)}(\vec{r},t)\Psi^{(j)}(\vec{r}',t) + \tilde{\Psi}^{(j)}(\vec{r}',t)\Psi^{(i)}(\vec{r},t) &= \delta_{ij}\,\delta(\vec{r},\vec{r}') \\ \Psi^{(i)}(\vec{r},t)\Psi^{(j)}(\vec{r}',t) + \Psi^{(j)}(\vec{r}',t)\Psi^{(i)}(\vec{r},t) &= 0 \\ \tilde{\Psi}^{(i)}(\vec{r},t)\tilde{\Psi}^{(j)}(\vec{r}',t) + \tilde{\Psi}^{(j)}(\vec{r}',t)\tilde{\Psi}^{(i)}(\vec{r},t) &= 0 \end{aligned}\right\} \qquad (9)$$

consider the whole system is ~~enclosed in the unit cube~~ enclosed in a large volume, an unit cube for instance, ~~a unit cube~~ ~~the U-field~~ ~~enclosed in the volume V~~ and change. If we ~~change~~ ~~the~~ variables describing the U-field by the Fourier transformation[9)]

$$\left.\begin{aligned} \tilde{U} &= -i\hbar c \sqrt{\frac{2\pi}{\cancel{V}}} \sum_k \frac{1}{\sqrt{E_k}} (a_k^* - b_k) e^{-i(\vec{k}\vec{r})} \\ U &= -i\hbar c \sqrt{\frac{2\pi}{\cancel{V}}} \sum_k \frac{1}{\sqrt{E_k}} (-a_k + b_k^*) e^{i(\vec{k}\vec{r})} \end{aligned}\right\} \qquad (10)$$

9) The suffix k ~~is the abbreviation of~~ stand for the vector $\vec{k}$, the components of which takes 0 or integer values. The volume integral, ~~[illegible]~~, ~~mean~~ should be performed ~~always~~ throughout the unit cube.

* Alternatively, we can ~~when there are~~ when n heavy particles are present,

Next, the variables describing ~~the field of heavy particles~~ can be ~~written~~ changed by

they can be described by the variables $\vec{x}_i, \vec{p}_i, \alpha_i, \beta_i$ ~~($i=1,2,\dots n$)~~ α ($i=1,2,\dots n$) instead of $\Psi, \tilde{\Psi}$, and the Hamiltonian ~~for the becomes~~ takes the alternative form

$$\bar{H}_m = c\sum_{i=1}^{n}\{(\vec{\alpha}_i \vec{p}_i) + \beta_i(\frac{1+\tau_3^{(i)}}{2}M_N c^2 + \frac{1-\tau_3^{(i)}}{2}M_P c^2)\}.$$

~~acco~~ The ~~H term~~ interaction term in the Hamiltonian (1) ~~tat~~ becomes, ~~at the same time~~ accordingly

$$\bar{H}' = g\sum_i\{Q_i^* \tilde{U}(\vec{x}_i) + Q_i U(\vec{x}_i)\}\beta_i, \quad (\ )$$

which ~~can~~ be considered as perturbation in actual calculations.

$\bar{H}'$ can involves the ~~operators changing the number the heavy particle from the neutron~~ the product of Q_i^* or Q_i causing the transition of ~~a~~ i-th heavy particle between the neutron and the proton states ~~as well as~~ with their simultaneous emission or absorption of ~~that a~~ heavy quantum, ~~the~~ $U(\vec{x}_i), \tilde{U}(\vec{x}_i)$.

with $E_k = \hbar c\sqrt{k^2+\kappa^2}$,

the Hamiltonian takes the form

$$\bar{H}_U = (a_k^* a_k + b_k^* b_k + 1)E_k, \tag{11}$$

where a_k, a_k^*, b_k and b_k^* satisfy the commutation relations

$$a_k a_l^* - a_l^* a_k = \delta_{kl}, \qquad b_k b_l^* - b_l^* b_k = \delta_{kl}, \tag{12}$$

any other two commuting with each other. The variable~~s~~ $N_k^+ = a_k^* a_k$ with eigenvalues 0, 1, 2, ... denotes the number of heavy quanta with the charge +e, the momentum $\hbar k$ and the energy E_k, whereas $N_k^- = b_k^* b_k$ denotes the number of those with the charge −e, the momentum $\hbar k$ and the energy E_k.

Similarly, the variables describing the heavy particles can be changed by expanding Ψ, $\tilde{\Psi}$ into series

$$\Psi = \sum_n c_n u_n + \sum_p d_p v_p, \qquad \tilde{\Psi} = \sum_n c_n^* \tilde{u}_n + \sum_p d_p^* \tilde{v}_p \tag{13}$$

where u_n is the unquantized wave function with four non-zero components for the neutron state n and v_p is that for the proton state p (n,p = 1, 2, ...). The new variables satisfy the commutation relations

$$c_n c_m^* + c_m^* c_n = \delta_{nm}, \qquad d_p d_q^* + d_q^* d_p = \delta_{pq}, \tag{14}$$

~~or~~ any other two anti-commuting with each other. Thus, we obtain

$$\bar{H}_M = \sum_n N_n W_n + \sum_p N_p W_p, \tag{15}$$

where $N_n = c_n^* c_n$ is the number of neutrons in the state n with the energy W_n and $N_p = d_p^* d_p$ is that of protons in ~~the~~ the state p with the energy W_p.*

The term

$$\bar{H}' = -i\kappa c g\sqrt{2\pi}\sum_{k,n,p}\iiint \tag{16}$$

denoting the interaction between the heavy particle and the U-field involves the operaors, which change the number of particles and quanta, and can be considered as perturbation inactual calculations.

$$\overline{H}' = \cancel{g\sum_i} -i\pi g c \sum_i \sum_k \sqrt{\frac{2\pi}{E_k}} \{ (a_k^* - b_k^*) e^{-i(\vec{k}\vec{r})} Q_i^* \beta_i$$

$$\cancel{+\sqrt{\frac{2\pi}{E_k}}} - (a_k^* - b_k^*) e^{+i(\vec{k}\vec{r})} Q_i \} \beta_i$$

§3. Deduction of the Force between Unlike Particles. ~~the Neutron and the Proton.~~

When a heavy particle 1 is in a neutron state m and 2 in a proton state p ~~n~~ in the absence of the heavy quanta initially, the energy for the unperturbed system becomes $(W_n + W_p)$, if we omit the zero point energy $\sum_k E_k$ for the U-field. The perturbation energy with the general form

$$\bar{H}' = g \sum_{\cancel{k,p,n,q,m}} \sum_{i=1,2} \{ Q_i^* \tilde{U}(\vec{x}_i) + Q_i U(\vec{x}_i) \} \rho_i \qquad (13)$$

indicates the possibility of the transition of 1 from the neutron to the proton state with the simultaneous emossion of a ~~positive~~ charge and conversely that of 2 from the proton to the neutron state with the simultaneous emission of a quantum of positive charge, the suffices 1 and 2 in (13) denoting ~~the~~ corresponding quantities for the particles 1 and 2 respectively.

The quantum thus emitted with ~~the~~ positive or negative can be absorbed by 2 or 1, which in turn changes into the neutron or the proton. In this way, the state of the system, in which the particle 1 is in the proton state q and 2 in the neutron state m, is linked together with the initial state ~~by~~ through the virtual intermediate states, which can not be realize owing second order process above considered. The corresponding ~~second order~~ matrix element of the second order perturbation energy come out to be

~~with~~ have a large energy and can not

each of ~~these~~ has the energy larger than that of the initial state due to the presence of a heavy quantum and can not be ~~so~~ far as $(W_N + W_P - M_N c^2 - M_P c^2) \ll M_U c^2$

$$H^{(2)mq}_{np} = -4\pi g^2 \hbar^2 c^2 \iint \sum_k \frac{e^{i\vec{k}\vec{r}}}{E_k^2} \{ \tilde{v}_q(\vec{r}_1)\beta_1 u_n(\vec{r}_1) \tilde{u}_m(\vec{r}_2)\beta_2 v_p(\vec{r}_2) \, dv_1 dv_2 \qquad (14)$$

~~By~~ Replacing the summation $\sum_k$ by the integration in the limit of the infinite ~~large~~ volume of the enclosure, we obtain

$$\sum_k \frac{e^{i\vec{k}\vec{r}}}{E_k^2} = \frac{1}{(2\pi)^3(\hbar c)^2}\int \frac{e^{i\vec{k}\vec{r}}}{k^2+\varkappa^2}\,d\vec{k}$$

$$= \frac{1}{4\pi(\hbar c)^2}\,\frac{e^{-\varkappa r}}{r}.$$

where $\vec{r} = \vec{r}_2 - \vec{r}_1$, so that (14) reduces to

$$H_{np}^{mq} = -g^2\int\!\!\int \frac{e^{-\varkappa r}}{r}\tilde{v}_q(\vec{r}_1)\rho_1 u_m(\vec{r}_1)\cdot\tilde{u}_m(\vec{r}_2)\rho_2 v_p(\vec{r}_2)\,dv_1 dv_2 \tag{15}$$

This expression shows that the interaction between the neutron and the proton ~~is the~~ can be described by the exchange force of Heisenberg Type with the potential

$$V_{PN} = J(r)\,P_{12}^H\,\rho_1\rho_2\,, \tag{16}$$

where
$$J(r) = g^2 e^{-\varkappa r}/r \tag{17}$$

and P_{12}^H is Heisenberg's exchange operator. In non-relativistic approximation, ρ_1 and ρ_2 are roughly equal to 1, so that (16) becomes the same with the result obtained in I, except for the change of sign. In order to obtain a result with the same sign as that of I, it will be necessary to change the sign of the energy H_J ~~of the U-field,~~ for the U-field which will obviously lead to serious difficulty of negative energy. This difficulty will be removed only by introducing non-scalar field, as will be shown later on.

§4. Deduction of the Force between Like Particles.

~~If we consider~~ ~~calculate~~ ~~the perturbation energy~~ Next, the interaction between like particles can be deduced as fourth order process due to the interaction of the heavy particles with the U-field. For example, if there are two neutrons initially in the absence of heavy quanta, they can interact through ~~following~~ three intermediate stages, as shown in the ~~following~~ scheme:

$$\begin{array}{ccccccccc} & & & & N'_1+N_2+U_++U_- & & & & \\ & \nearrow & P_1+N_2+U_- & & & \searrow & N'_1+P_2+U_- & \searrow & \\ N_1+N_2 & & & \rightrightarrows & P_1+P_2+U_-+U'_- & \rightrightarrows & & & N'_1+N'_2, \\ & \searrow & N_1+P_2+U'_- & & & \nearrow & N_1+N'_2+U'_- & \nearrow & \\ & & & & N_1+N'_2+U_++U'_- & & & & \end{array} \qquad (18)$$

where N_1 or N_2 denote the first or second particle in the neutron state, ~~and~~ P_1 or P_2 that in the proton state and U_+ or U_- the quantum with positive or negative charge, different states being distinguished by undashed and dashed letters. If we denote the initial states of neutrons by ~~u_n~~ u^n and u^m and the final states by n' and m' respectively, the corresponding matrix element of of the perturbation energy becomes

$$H^{(4)\,n'm'}_{\;nm} = \int\!\cdot\!\cdot\!\int K(\vec{r})\,\tilde{u}_{n'}(\vec{r}_1)\,u_n(\vec{r}_1)\,\tilde{u}_{m'}(\vec{r}_2)\,u_m(\vec{r}_2)\,dv_1 dv_2 \qquad (19)$$

if we neglect the recoil of the heavy neutron
where

$$K(\vec{r}) = -(2\pi)^2 (g\hbar c)^4 \left[4\sum_{k,l} \frac{e^{i(\vec{l}-\vec{k})\vec{r}}}{E_k E_l^3 (E_k+E_l)} + 2\sum_{k,l} \frac{e^{i(\vec{l}-\vec{k})\vec{r}}}{E_k^2 E_l^2 (E_k+E_l)} \right] \qquad (20)$$

the summations in (20) can be replaced by integrations and it reduces after some calculations to

$$K(\vec{r}) = \frac{g^4/\hbar c}{2\pi (\hbar c)^5 r^2} \int_0^\infty \frac{k \sin kr \cos kr\, dk}{\sqrt{(k^2+\kappa^2)^3}} \times (g\hbar c)^4$$

$$= \frac{g^4\, i\, H_0^{(1)}(2i\kappa r)}{\cancel{4\pi^2 (\hbar c)^5}\ \hbar c\; r} \qquad (21)$$

where $H_0^{(1)}$ is the Hankel function of zero order. This gives a ~~For Thus, the~~ short range attractive force between two neutrons, the range in these cases is ~~being~~ only half of that between the neutron and the proton, as K has an asymptotic form

and exactly the same result can be obtained for the case of two protons, in a similar way

$$K(r) \cong \frac{g^4}{\hbar c} \frac{e^{-2\kappa r}}{\sqrt{\pi \kappa r^3}} \left\{ 1 + O\left(\frac{1}{\kappa r}\right) \right\} \tag{22}$$

for large r.

Relative magnitude of the forces between like and unlike particles is given by ~~is~~

$$\frac{|K(r)|}{|J(r)|} = \frac{g^2}{\hbar c} e^{\kappa r} i H_0^{(1)}(2i\kappa r), \tag{23}$$

which varies with distance as follows[10] if we omit the constant $g^2/\hbar c$.

κr	0.05	0.1	0.25	0.5	1.0	1:5
$e^{\kappa r} i H_0^{(1)}(2i\kappa r)$	1.62	1.23	0.76	0.44	0.2	0.1

If we take $\kappa = 5\text{x}10^{12}\text{cm}^{-1}$, as in I $g^2/\hbar c$ becomes about 1/10, so that the like particle force between thus obtained ~~seems tobe too~~ is smaller by a factor 10 than ~~compared~~ with the unlike particle force. The ratio (23) becomes larger, if we take the rnge of forces κ smaller, so that it is not certain whether or not we have to introduce further neutral heavy quanta, in order to account for the approximate equality assumed in the current theory. If such neutral quanta ever exist, it ~~for~~ can not be detected easily, but may be ~~useful~~ ~~for the elucidation of~~ indirectory responsible for the neutro-electric effect discovered recently by Kikuchi, Husimi and Aoki,[11] just as the charged quantum was considered to be responsible for β-disintegration. Such speculations on ~~Discussion of~~ these subjects, of course ~~however~~, is premature at present.

10) Jahke-Emde, Tables of Functions ~~Funktionentafeln~~, p.286, 2nd Ed.

11) Kikuchi, Husimi and Aoki, Proc. Phys.-Mahth. Soc. Japan 18, 727, 1936. and subsequent papers. Kikuchi and Aoki, ibid. 19, 734, 1937. Takeda, ibid. 19, 835, 1937.

§5. Collision of High Speed Heavy Quanta with Atomic Nuclei.

If we assume that the hard component of the cosmic ray consists mainly of high energy heavy quanta above considered, the problem of collision of them with atoms or molecules becomes of primary importance. Since each quantum has either positive or negative charge, it will be scattered by the orbital electrons and the atomic nuclei just as the electron and the proton, resulting ~~in the elastic~~ eventually in excitation or ionization of the atoms or molecules. The energy loss by radiation is far smaller than that by ionization, as the former is proportional to the square of the scecific charge of the heavy quantum, which is about 200^{-2} of that of the electron.

Next, we consider the processes due to the non-electric interaction of the heavy quantum ~~particle~~ with the heavy particle~~s~~, while we can neglect altogether those due to the small non-electric interaction with the light particles. Among various processes, by which any number of, ~~one~~ or heavy particles, heavy quanta or photons are ~~can be~~ emitted after collision with the nucleus, two simplest cases ~~is~~ will be considered in detail. The first is ~~the~~ [illegible], e.g. elastic scattering of the heavy quantum of very high ~~kinetic~~ energy E_0 by the heavy free neutron, initially at rest. The cross section for this process can be calculated in a manner smilar to that for Compton scattering of the light quantum by the free electron~~s~~. The differential cross section ~~for~~ that the heavy quantum is scattered into the solid angle $d\Omega$ in the direction θ, φ is given by

$$d\sigma = \frac{g^4}{4M^2c^4}\cancel{\left(\frac{g}{\sqrt{4\pi}\hbar c}\right)^4}\frac{E}{\ } \quad \cancel{\frac{1}{64\pi^2 M^2c^4}\left(\frac{g}{\hbar c}\right)^4} \quad \frac{1}{4}\left(\frac{g^2}{Mc^2}\right)^2 \cdot \left(\frac{E}{E_0}\right)^2 \left\{ \left(1 + \frac{2Mc^2}{E_0}\right)^2 + \frac{E}{E_0} \right\} d\Omega \tag{24}$$

$$\frac{g^2}{\hbar c} = \frac{1}{10}$$

$$\frac{g^2}{Mc^2} = \frac{g^2}{\hbar c} \cdot \frac{\hbar}{Mc}$$

$$= \frac{e^2}{mc^2} \cdot \left(\frac{m}{M}\right) \cdot \left(\frac{g^2}{e^2}\right)$$

$$\frac{m}{M} = \frac{1}{1840}$$

$$\frac{g^2}{e^2} = \frac{13.7}{\cancel{70}}$$

$$\frac{2.8 \times 10^{-13} \times 13.7}{1.84 \times 10^3} = 2.1 \times 10^{-15}$$

13.7
2.8
1096
274
1.84) 38.36 (2.1
368
156
104
52

*when we ~~consider~~ assume the whole ~~mass of~~ its mass ~~to be originated~~ energy of ~~the~~ field surrounding it, just as ~~the mass of the electron is due~~ $\frac{e^2}{mc^2}$ is called ... $\frac{e^2}{mc^2}$, when

for E_0 (and) or E large compared with Mc^2, ~~where~~ where E is the energy of the ~~scattered~~ heavy quantum after scattering and M is the mass of the ~~neutron.~~ heavy particle. Integrating (24) with respect to angles, we have for the total cross section

$$\sigma_\phi = \pi\left(\frac{g^2}{Mc^2}\right)^2\left[\frac{(2+\gamma)^2}{\gamma}\frac{1}{1+2\gamma}+\frac{1+\gamma}{(1+2\gamma)^2}\right]; \quad (25)$$

where $\gamma = E_0/Mc^2$. ~~In (24), (25)~~ The length $\frac{g^2}{Mc^2}$, which appears in (24) and can be called the "radius of the heavy particle",* has a value about 2×10^{-15} cm ~~The factor before~~ the bracket on the in(25) is about $2\times10^{-29}\,cm^2$, which is so small that ~~so that the probability~~ the ~~cross section~~ contribution of this process to the absorption ~~for high~~ of the high energy quantum ~~enrgey~~ is negligible compared with that due to the electric interaction ~~be- tween the~~ of it with matter. Thus, the ~~factor before (25)~~ the bracket in (25) is about, so that the The cross section increases as E_0 decreases, but is ~~still~~ always negligible compared with those for other processes.

~~Ten~~ The second process is ~~the emission of a heavy particle, e.g. a neutron, when the heavy quantum is absorbed by the nucleus~~ the absortion of the heavy quantum with the negative charge ~~with the negative charge, for instance,~~ by the nucleus with subsequent emission of a ~~neutron a~~ heavy particle, e.g. a for instance, neutron. ~~The~~ The calculation of the cross section can be ~~calculated~~ made ~~in a man~~ in a manner smilar to that ~~for~~ for the photoelectric effect and comes out to be

$$\sigma = 64\pi\left(\frac{\hbar}{Mc}\right)\left(\frac{g^2}{Mc}\right)\left(\frac{c}{v}\right)\left(\frac{Mvc^2}{E_0}\right)^2\left(\frac{I}{E_0}\right)^{\frac{3}{2}}\left(1-\frac{I}{E_0}\right)^{\frac{3}{2}}\left(1-\frac{4I}{E_0}\right)^2 \quad (26)$$

if we assume the kinetic energy E_0 of the heavy quantum to be small compared with Mc^2, where v is the velocity of the heavy quantum and I is the energy necessary for taking the neutron out of the nucleus.

Now, the factor $64\pi\left(\frac{\hbar}{Mc}\right)\left(\frac{g^2}{Mc}\right)$ ~~before the bracket~~ has a value ~~about~~ 0.75 $10^{-26}\,cm^2$, ~~so t~~ and the remaining factors increases rapidly with as E_0 decreases, so that the cross section becomes fairly large for small E_0. The coefficient of absorption τ of the heavy quantum due to this process alone is given approximately by $\tau = NZ\sigma$, where

~~those~~ of the interaction of the heavy quanta with the light particles and that with the el.-mg field,

Z is the atomic number and N ~~is~~ the number of atoms per unit volume of the absorber. ~~Thus, we the a~~ The absorption coefficient τ in Pb, for example, is 0.02 cm^{-1} for E_0 of the order of 10^8eV and 4 cm^{-1} for E_0 about 10^7eV. For still smaller values of E_0, τ is so large that the heavy quanta slowed down by ionization will soon be absorbed into matter. The comparison of these results with the experiment is difficult at present, but they ~~do not~~ are probably ~~at least~~ not in ~~seem, to~~ contradict(ion) with each other.

~~In this paper, the problems of~~ β-disintegration, ~~the spin and the magnetic moment~~ the interaction of the heavy quanta with the electromagnetic field

There are many problems which are not dealt with ~~in~~ this paper, such as the another problem of ~~which~~ the latter will be important in relation to the spin and the magnetic moment of the heavy quanta, which will in turn have a close connection with the anomalous magnetic moment of the heavy particle. All these subjects will be considered successively in subsequent papers.

The former is responsible for the β-disintegration as shown in I,

Physics Department,
Osaka Imperial University,
Osaka, Japan.

In conclusion the authors wish to express heartfelt thanks to Dr. Y. Nishina and Prof. S. Kikuchi for their continued interest and valuable discussions.

Note added in Proof.

i) Recent experiment result of Street and Stevenson
(   )

ii) Landau and Rumer (   )

iii) Kobayashi