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On the Interaction of Elementary Particles. II.

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(Read Sept. 25, 1937.)

§ 1. Introduction and Summary.

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In the first part of this paper[1], one of the present authors introduced a new field, which was responsible for the short range force between the neutron and the proton as well as for the β-disintegration. This field turned out to be accompanied by quanta each with the elementary charge either $+e$ or $-e$, the mass about 1/10 of that of the proton and zero or integer spin, obeying the Bose statistics. It was shown further that such quanta, if they ever existed, could not be produced by ordinary nuclear reactions in the laboratory, but might be present in the cosmic ray as the primary or the secondary.

There had been no evidence whatever in favour of the last point, until recent researches of Anderson and Neddermeyer[2], Street and Stevenson[3], and Nishina, Takeuchi and Ichimiya[4] indicated the existence in the cosmic ray of particles, which could be identified neither with the electrons nor with the protons, as long as we accepted the present theory of energy dissipation of high speed particles. Thus it seems likely that these are the heavy quanta above considered[5]. Similar conclusions were reached by Oppenheimer and Serber[6] and by Stueckelberg[7]. If this is true, the only one serious drawback in our theory disappears, although there are still many points to be completed or improved in course of development of the theory, which will be made in this and subsequent papers. We hope that the whole phenomena of the nuclear transformation, β-disintegration and cosmic ray

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(1) Yukawa, Proc. Phys.-Math. Soc. Japan 17, 48, 1935. Hereafter, this paper will be referred to as I.

(2) Anderson and Neddermeyer, Phys. Rev. 50, 273; 51, 884, 1937. 1936　273, 1936;

(3) Street and Stevenson, ibid. 51, 1005, 1937.

(4) According to the preliminary result of Nishina and others, the mass of the new particle is about 1/10 the protonic mass in fair agreement with the theoretical expectation.

(5) Yukawa, Proc. Phys.-Math. Soc. Japan 19, 712, 1937.

(6) Oppenheimer and Serber, Phys, Rev. 51, 1113, 1937.

(7) Stueckelberg, ibid. 52, 41, 1937. Stueckelberg arrived at the new field theory independently.

will be explained in a unified way.

In this paper, we begin with the problem of quantization of the scaler field, which can be performed in a manner similar to that of Pauli and Weisskopf[8]. The interaction between the heavy particle and the new field can be dealed with as that between the charged particle and the electromagnetic field in quantum electrodynamics. (§ 2.) The interaction between unlike particles, viz. the neutron and the proton, is derived as second order effect, confirming the result in I. (§ 3.) Further, the interaction between like particles as fourth order effect is discussed. (§ 4.) Finally, the problem of passage of high energy heavy quanta through matter is considered and the cross sections for typical processes are calculated. (§ 5.)

The development of a more complete theory including the interaction of the heavy quanta with the electromagnetic field and with the light particles will be made in III etc., where the introduction of non-scaler field will be necessitated.

§ 2. Quantization of the Scalar Field.

The simplest conceivable form of the new field theory can be obtained by considering the potentials U and $\tilde{U}$ in I as four dimensianal scalars instead of time components of four vectors. Whether such a simplification is adequate or not will become clear later on.

The wave equation (3) in I and its complex conjugate for U-field in vacuum can be derived from the Lagrangian

$$\overline{L}_U=\iiint L_U dv, \tag{1}$$

with

$$L_U=\frac{1}{4\pi}\left(\frac{1}{c^2}\frac{\partial\tilde{U}}{\partial t}\frac{\partial U}{\partial t}-\operatorname{grad}\tilde{U}\operatorname{grad}U-\kappa^2\tilde{U}U\right), \tag{2}$$

where the constant κ is used in place of λ in I and is connected with the mass m_U of the heavy quantum by the relation $\kappa=\frac{m_U c}{\hbar}$. The variables canoncally conjugate to U and $\tilde{U}$ are

$$U^+=\frac{\delta\overline{L}_U}{\delta\frac{\partial U}{\partial t}}=\frac{1}{4\pi c^2}\frac{\partial\tilde{U}}{\partial t} \text{ and } \tilde{U}^+=\frac{1}{4\pi c^2}\frac{\partial U}{\partial t}$$

(8) Pauli and Weiskopf, Helv. Phys. 7, 709, 1934.

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respectively, satisfying the commutation relations

$$\left.\begin{aligned}
&U(\vec{r},t)U^{+}(\vec{r}',t)-U^{+}(\vec{r}',t)U(\vec{r},t)=i\hbar\delta(\vec{r},\vec{r}')\\
&\tilde{U}(\vec{r},t)\tilde{U}^{+}(\vec{r}',t)-\tilde{U}^{+}(\vec{r}',t)\tilde{U}(\vec{r},t)=i\hbar\delta(\vec{r},\vec{r}')\\
&U(\vec{r},t)\tilde{U}\,(\vec{r}',t)-\tilde{U}\,(\vec{r}',t)U(\vec{r},t)=0\\
&U(\vec{r},t)\tilde{U}^{+}(\vec{r}',t)-\tilde{U}^{+}(\vec{r}',t)U^{+}(\vec{r},t)=0.
\end{aligned}\right\}\qquad(3)$$

U^{+}

The Hamiltonian for the U-field becomes thus

$\overline{H}_U$

$$\overline{H}_U=\iiint H_U dv \qquad(4)$$

with
$$H_U=\frac{\partial U}{\partial t}U^{+}+\tilde{U}^{+}\frac{\partial\tilde{U}}{\partial t}-L_U$$
$$=4\pi c^2\tilde{U}^{+}U^{+}+\frac{1}{4\pi}\operatorname{grad}\tilde{U}\operatorname{grad}U+\frac{\kappa^2}{4\pi}\tilde{U}U. \qquad(5)$$

In the presence of the heavy particles, the Hamiltonian for the total system is given by

$$\overline{H}=\overline{H}_M+\overline{H}_U+\overline{H}' \qquad(6)$$

with
$$\overline{H}_M\equiv\iiint\tilde{\Psi}\left\{c\vec{\alpha}\vec{p}+\beta\left(\frac{1+\tau_3}{2}M_Nc^2+\frac{1-\tau_3}{2}M_Pc^2\right)\right\}\Psi dv \qquad(7)$$

$$\overline{H}'=g\iiint\tilde{\Psi}(\tilde{U}Q^*+UQ)\beta\Psi dv \qquad(8)$$

where $\vec{p}=-i\hbar\operatorname{grad},\quad Q=\frac{\tau_1+i\tau_2}{2},\ Q^*=\frac{\tau_1-i\tau_2}{2}.$

$(\vec{\alpha},\beta)$ and (τ_1,τ_2,τ_3) are Dirac matrices and isotopic spin matrices respectively for the heavy particle, assuming the wave equations to be of Dirac's type. M_N and M_P are the masses of the neutron and the proton respectively. The wave functions Ψ and $\tilde{\Psi}$ have each eight components and satisfy the commutation relations

$$\left.\begin{aligned}
&\Psi^{(i)}(\vec{r},t)\tilde{\Psi}^{(j)}(\vec{r}',t)+\tilde{\Psi}^{(j)}(\vec{r}',t)\Psi^{(i)}(\vec{r},t)=\delta_{ij}\delta(\vec{r},\vec{r}')\\
&\Psi^{(i)}(\vec{r},t)\Psi^{(j)}(\vec{r}',t)+\Psi^{(j)}(\vec{r}',t)\Psi^{(i)}(\vec{r},t)=0\\
&\vec{\Psi}^{(i)}(\vec{r},t)\tilde{\Psi}^{(j)}(\vec{r}',t)+\tilde{\Psi}^{(j)}(\vec{r}',t)\tilde{\Psi}^{(i)}(\vec{r},t)=0
\end{aligned}\right\}\qquad(9)$$

for $i,j=1,2,\ldots,8$.

Now, if we consider the whole system as enclosed in a large volume, a unit cube for example, we can change the variables describing the U-field by the Fourier transformation

$$\left.\begin{aligned} U &= i\hbar c\sum_k\sqrt{\frac{2\pi}{E_k}}\,(\cancel{a_k^*}\,a_k-b_k^*)e^{i(\vec{k}\,\vec{r})} \\ \tilde{U} &= -i\hbar c\sum_k\sqrt{\frac{2\pi}{E_k}}\,(a_k^*-b_k)e^{-i(\vec{k}\,\vec{r})} \end{aligned}\right\} \qquad (10)$$

where the suffix k stands for the vector $\vec{k}$, the components of which are each zero or integer multiplied by 2π, and $E_k=\hbar c\sqrt{k^2+\kappa^2}$. The new variables a_k, a_k^*, b_k and b_k^* satisfy the commutation relations

$$a_k a_l^* \cancel{-} a_l^* a_k=\delta_{kl},\quad b_k b_l^*-b_l b_k^*=\delta_{kl}, \qquad (11)$$

any other two commuting with each other. Thus, $N_k^+=a_k^*a_k$ has eigenvalues $0, 1, 2, \ldots$ and denotes the number of heavy quanta with the +e charge $\cancel{\pm}e$, the momentum $\hbar k$ and the energy E_k, whereas $N_k^-=b_k^*b_k$ denotes the number of those with the charge $-e$, the momentum $-\hbar k$ and the energy E_k, the Hamiltonian for the U-field (4) being reduced to the simple form

$$\bar{H}_U=\sum_k(N_k^++N_k^-+1)E_k. \qquad (12)$$

Next, when n heavy particles are present, the variables $\vec{r}_i$, $\vec{p}_i$, $(\vec{\alpha}_i, \beta_i)$ $(\tau_1^{(i)}, \tau_2^{(i)}, \tau_3^{(i)})$ denoting the coordinates, momenta, Dirac matrices and isotopic spin matrices for i-th particle with $i=1, 2, \ldots, n$ can be used instead of Ψ, $\tilde{\Psi}$, so that the Hamiltonian (7) taken an alternative form

$\tilde{\Psi}$

$$\bar{H}_M=\sum_i\left\{c\vec{\alpha}_i\,\vec{p}_i+\beta_i\left(\frac{1+\tau_3^{(i)}}{2}M_Nc^2+\frac{1-\tau_3^{(i)}}{2}M_Pc^2\right)\right\}. \qquad (13)$$

The interaction term (8) becomes accordingly

$$\bar{H}'=g\sum_i\{Q_i^*\tilde{U}(\vec{r}_i)+Q_iU(\vec{r}_i)\}\beta_i,$$

or

$$=-i\hbar gc\sum_i\sum_k\sqrt{\frac{2\pi}{E_k}}\{(a_k^*-b_k)e^{-i(\vec{k}\,\vec{r})}Q_i^*-(a_k-b_k^*)e^{i(\vec{k}\,\vec{r})}Q_i\}\beta_i \qquad (14)$$

where

$$Q_i=\frac{\tau_1^{(i)}+i\tau_2^{(i)}}{2},\qquad Q_i^*=\frac{\tau_1^{(i)}-i\tau_2^{(i)}}{2}.$$

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(14) involves the operators causing the transition of the heavy particle between the neutron and the proton states with the simultaneous emission or absorption of the heavy quantum and can be considered as perturbation in actual calculations.

§ 3. Deduction of the Force between Unlike Particles.

When a heavy particle 1 is in a neutron state u_n with the energy W_n and 2 in a proton state v_p with the energy W_p in the absence of the heavy quanta initially, the energy for the unperturbed system is $W_n + W_p$, if we omit the zero point energy $\sum_k E_k$ for the U-field. The perturbation given by (14) indicates the possibility of the transition of 1 from the neutron to the proton state with the simultaneous emission of a quantum of negative charge and conversely that of 2 from the proton to the neutron state with the simultaneous emission of a quantum of positive charge, although these transitions are energetically forbidden as long as $W_n - M_N c^2 + W_p - M_p c^2 \ll m_U c^2$. The quantum of negative or positive charge thus emitted virtually can be absorbed by 2 or 1, which in turn charges into the neutron or the proton, so that the state of the system, in which the particles 1 is in a proton state, q say, and 2 in a neutron state, m say, is linked together with the initial state through the intermediate states above considered. The corresponding matrix element of the second order perturbation energy comes out to be

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$$H^{(2)\,mq}_{\;\;np} = -4\pi g^2 \hbar^2 c^2 \int \cdots \int \sum_k \frac{e^{i\vec{k}\vec{r}}}{E_k^2} \tilde{v}_p(\vec{r}_1)\beta_1 u_n(\vec{r}_2)\, dv_1 \cdot \tilde{u}_n(\vec{r}_1)\beta_2 v_p(\vec{r}_2)\, dv_2 \quad (15)$$

where $\vec{r} = \vec{r}_2 - \vec{r}_1$. Replacing the summation $\sum_k$ by the integration in the limit of infinitely large enclosure we obtain

$$\sum_k \frac{e^{i\vec{k}\vec{r}}}{E_k^2} = \frac{1}{(2\pi\hbar c)^3} \int \frac{e^{i\vec{k}\vec{r}}}{k^2+\kappa^2}\, d\vec{k} = \frac{1}{4\pi(\hbar c)^3} \cdot \frac{e^{-\kappa r}}{r},$$

so that (15) reduces to

$$H^{(2)\,mq}_{\;\;np} = -g^2 \int \cdots \int \frac{e^{-\kappa r}}{r} \tilde{v}_q(\vec{r}_1)\beta_1 u_n(\vec{r}_1) \cdot \tilde{u}_m(\vec{r}_2)\beta_2 v_p(\vec{r}_2)\, dv_1 dv_2 \quad (16)$$

This expression shows that the interaction between the neutron and the proton can be described by the exchange force of Heisenberg type with the potential

$$V_{PN} = J(r) P_{12}^{H} \beta_1 \beta_2, \quad (17)$$

where

$$J(r) = g^2 e^{-\kappa r}/r \quad (18)$$

and P_{12}^{H} is Heisenberg's exchange operator. In non-relativistic approximation, β_1 and β_2 reduces to 1, so that (17) becomes the same with

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the result in I except the sign. In order to obtain a result exactly the same as that of I, we have to change the sign of ~~H'~~ H_U, which will obviously lead to serious ~~difficulty~~ difficulties of negative energy for the U-field. Whether or not this defect can be removed by introducing non-scalar field will be discussed in III etc.

The exchange force of Majorana type can be deduced by introducing terms involving the ~~sign~~ spin of the heavy particle in the ~~expressiou~~ expression (8) for its interaction with the U-field, whereas the ordinary force of Wigner or Bartlett type appears only as the fourth order effect, as can be obtained by calculation similar to that for like particles in § 4.

§ 4. Deduction of the Force between Like Particles.

Next, the interaction between like particles can be deduced as fourth order process due to the interaction of the heavy particles with the U-field. If there are two neutrons, for example, in the absence of heavy quanta, they can interact through three intermediate stages, as shown in the scheme:

$$N_1+N_2 \begin{array}{l} \nearrow P_1+N_2+U_- \begin{array}{l}\nearrow N_1'+N_2+U_++U_- \\ \searrow P_1+P_2+U_-+U_-' \end{array} \\ \searrow N_1+P_2+U_-' \begin{array}{l}\nearrow P_1+P_2+U_-+U_-' \\ \searrow N_1+N_2'+U_++U_-' \end{array} \end{array} \begin{array}{l} \searrow N_1'-P_2+U_- \\ \nearrow P_1+N_2'+U_-' \end{array} \searrow N_1'+N_2', \qquad (19)$$

where ~~N_1~~ N_1 and N_2 denote the first and second particles in the neutron states, P_1 and P_2 those in the proton states and U_+ and U_- the quanta emitted with positive and negative charges respectively, different states being distinguished from each other by undashed and dashed ~~latters~~ letters. If we denote the initial states of the neutrons by n and m and the final states by n' and m' respectively, the corresponding matrix element of the perturbation energy becomes

$$H^{(4)n'm'}_{nm}=\int\cdots\int K(\vec r)\tilde u_{n'}(\vec r_1)u_n(\vec r_1)u_{m'}(\vec r_2)u_m(\vec r_2)dv_1dv_2, \qquad (20)$$

with

$$K(\vec r)=-(2\pi)^2(g\hbar c)^4\left\{4\sum_{k,l}\frac{e^{i(\vec l-\vec k)\vec r}}{E_kE_l^3(E_k+E_l)}+2\sum_{k,l}\frac{e^{i(\vec l-\vec k)\vec r}}{E_k^2E_l^2(E_k+E_l)}\right\}. \qquad (21)$$

It should be noticed that the ~~expkession~~ expression (20) is true so far as the momenta of the heavy particles due to the recoil can be neglected. This condition is not fulfilled very accurately owing to relatively large mass of the heavy quantum, but the conclusions reached in this section will be valid always.

The summation in (21) can be replaced by integration and we obtain after some calculations

$$K(\vec{r}) = -\frac{2g^4}{\pi\hbar c}\cdot\frac{1}{r^2}\int_0^\infty \frac{k\sin kr\cos kr\cdot dk}{(k^2+\kappa^2)^{\frac{3}{2}}}$$

$$= -\frac{g^4}{\hbar c}\frac{iH_0^{(1)}(2i\kappa r)}{r} = K(r) \tag{22}$$

g^4 κ^2

where $H_0^{(1)}$ is the Hankel function of zero order. This gives ordinary attractive force between two neutrons with the potential $K(r)$. The range of force in this case is only half of that between unlike particles, as $K(r)$ has the asymptotic form

$$K(r) = \frac{g^4}{\hbar c}\frac{e^{-2\kappa r}}{\sqrt{\pi\kappa r^3}}\left\{1+O\left(\frac{1}{\kappa r}\right)\right\} \tag{23}$$

for large r. Exactly the same result can be obtained for the case of two protons.

Relative magnitude of the forces between like and unlike particles is given by

$$\frac{|K(r)|}{|J(r)|} = \frac{g^2}{\hbar c}e^{\kappa r}iH_0^{(1)}(2i\kappa r) \tag{24}$$

which varies with κr as follows[9], if we omit the constant factor $g^2/\hbar c$.

κr	0.05	0.1	0.25	0.5	1.0	1.5
$e^{i\kappa r}\cdot iH_0^{(1)}(2i\kappa r)$	1.62	1.23	0.76	0.44	0.20	0.1

If we take $\kappa = 5\times10^{12}\,\mathrm{cm}^{-1}$, $g^2/\hbar c$ becomes about 1/10, and the like particle force, thus obtained is smaller by a factor about 10 than the unlike particle force. The ratio (24) becomes larger, however, if we take κ larger, so that it is not certain whether or not we have to introduce neutral heavy quanta also, in order to account for the approximate equality of two forces assumed in the current theory. Even if such neutral quanta exist, they can not easily be detected, but may have some bearing on the so-called "neutro-electric effect" discovered by Kikuchi, Husimi and Aoki[10], just as the charged quanta have on

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(9) Jahnke-Emde, Tables of Functions, 2nd Ed. p. 286.

(10) Kikuchi, Husimi and Aoki, Proc. Phys.-Math. Soc. Japan 18, 727, 1936; Kikuchi and Aoki, ibid. 19, 734, 1937; Takeda, ibid. 19, 835, 1937.

the β-disintegration. Such speculations are, of course, premature at present.

§ 5. Collision of High Energy Heavy Quanta with Atomic Nuclei.

If we assume that the hard component of the cosmic ray consists mainly of high energy heavy quanta with positive or negative charge above considered, the problem of collision of them with atoms or molecules becomes of primary importance. First, they will be scattered by the orbital electrons and the atomic nuclei owing to the electromagnetic interaction, resulting eventually in the dissipation of energy by excitation or ionization of atoms or molecules. The energy ~~ions~~ loss by Bremsstrahlung is far smaller than that by ionization, as the former is proportional to the square of the specific charge of the heavy quantum, which is about 200^{-2} of that of the electron. Thus, the behaviour of the heavy quantum in these respects resembles the proton rather than the electron.

Next, we have to consider the processes due to the non-electromagnetic interaction of the heavy quantum with the heavy particle, while we can neglect altogether those due to the non-electromagnetic interaction with the light particle, which is much smaller. Various processes are possible and, in general, any number of heavy particles, heavy quanta and protons can be emitted after collision of a heavy quantum with the nucleus. Among them, only two simple cases will be dealed with in detail.

The first is the elastic scattering of the heavy quantum of very high energy ~~K_0~~ E_0 by a heavy particle, e.g. a free neutron[11], initially at rest. The calculation can be performed in a manner similar to that for Compton scattering of the light quantum by a free electron. The differeniall cross section that the heavy quantum with positive charge is scattered into the solid angle $d\Omega$ in the direction θ, φ comes out to be

$$d\sigma=\frac{1}{4}\left(\frac{g^2}{Mc^2}\right)^2\left(\frac{E}{E_0}\right)^2\left\{\left(1+\frac{2Mc^2}{E_0}\right)^2+\frac{E}{E_0}\right\}d\Omega \qquad (25)$$

for E_0 and E large compared with $\overset{m_u c^2}{\cancel{Mc^2}}$, where E is the energy of the scattered quantum and ~~E~~ M is the mass of the heavy particle. Integrating (25) with respect to angles, the total cross section becomes

(11) The neutron scatters the quantum with positive charge only, whereas the proton scatters that with negative charge only, if the electromagnetic interaction ~~in the latter case~~ is ignored.

$$\sigma=\pi\left(\frac{g^2}{Mc^2}\right)^2\left\{\frac{(2+\gamma)^2}{\gamma^2(1+2\gamma)}+\frac{1+\gamma}{(1+2\gamma)^2}\right\} \tag{26}$$

where $\gamma=E_0/Mc^2$. The length $\frac{g^2}{Mc^2}$, which has a value about 2×10^{-15} cm for $g^2/\hbar c=1/10$, can be called the "radius of the heavy particle", if we assume its mass to be originated by the U-field surrounding it, just as e^2/mc^2 is called the electron radius. The cross section (26) has a value of the order of $10^{-29}\,\mathrm{cm}^2$ or smaller, so that the contribution of this process to the absorption of the high energy quanta will be negligible compared with that of other processes. The cross section increases as E_0 decreases, but is always small.

The second is ~~in~~ the absorption of the heavy quantum, with the negative charge for instance, by the nucleus with subsequent emission of a heavy particle, e.g. ~~or~~ a neutron. After some calculations similar to those for the photoelectric effect, the cross section reduces to

$$\sigma=64\pi\left(\frac{\hbar}{m_U c}\right)\left(\frac{g^2}{m_U c^2}\right)\left(\frac{c}{v}\right)\left(\frac{m_U c^2}{E_0}\right)^2\left(\frac{I}{E_0}\right)^{\frac{3}{2}}\left(1-\frac{I}{E_0}\right)^{\frac{3}{2}}\left(1-\frac{4I}{E_0}\right)^2, \tag{27}$$

if we assume the kinetic energy E_0 of the heavy quantum to be small compared with Mc^2, where v is the velocity of it and I is the energy necessary for taking the nentron out of the nucleus.

Now, the factor $64\pi(h/m_U c)\,(g^2/m_U c^2)$ has a value about 0.75×10^{-26} cm^2 and the remaining factors increase rapidly ~~with~~ as E_0 decreases, so that the cross section becomes fairly large for small E_0. The coefficient of absorption of the heavy quantum due to this process alone is given approximately by $\tau=NZ\sigma$, where Z is the atomic number and N the number of atoms per unit volume of the absorber. Thus, the absorption coefficient τ in Pb is $0.02\,\mathrm{cm}^{-1}$ for E_0 of the order of 10^8eV and $4\,\mathrm{cm}^{-1}$ for E_0 about 10^7eV. For still smaller values of E_0, τ is so large that the heavy quanta slowed down by ionization will soon be absorbed into matter. Quantitative comparison of these results with the experiment is difficult, but it is certain that there are no serious discrepancy between them.

§ 6. Concluding Remark.

There are many problems which were not dealed with in this paper, such as the interaction of the heavy quantum with the light particle and that with the electromagnetic field. The former is responsible for the β-distinegration as already shown in I. The latter will be important in relation to another problem of the spin and the

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magnetic moment of the heavy quantum, which will in turn have a close connection with the anomalous magnetic moment of the heavy particle. All these subjects will be discussed successively in subsequent papers.

In conclusion, the authors desire to ~~expre~~ express their heartfelt thanks to Dr. Y. Nishina and Prof. S. Kikuchi for continued support and encouragement.

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Note added in proof: Very recently, a more evidence for the existence of the new particle was obtained by Street and Stevenson (Phys. Rev. 52, 1003, 1937). They estimated the mass to be about 130 (±25%) times the electron mass. If we accept this values for m_U, the range of nuclear force becomes $\frac{1}{\kappa} = \frac{\hbar}{m_U c} \cong \frac{1}{130}\frac{\hbar}{mc}$, which is approximately equal to the electron radius $\frac{e^2}{mc^2}$, since $\frac{e^2}{\hbar c} \cong \frac{1}{137}$.