Generalization of Maxwell's scheme

Maxwell の Field Equations

$$\mathrm{curl}\,\mathbb{H} = \frac{4\pi \mathbb{I}}{c} + \frac{1}{c}\frac{\partial \mathbb{E}}{\partial t} \qquad \mathrm{div}\,\mathbb{E} = 4\pi\rho$$

$$\mathrm{curl}\,\mathbb{E} = -\frac{1}{c}\frac{\partial \mathbb{H}}{\partial t} \qquad \mathrm{div}\,\mathbb{H} = 0$$

略して

$$\mathbb{H} = \mathrm{curl}\,\mathbb{A} \qquad \mathbb{E} = -\mathrm{grad}\,V - \frac{1}{c}\frac{\partial \mathbb{A}}{\partial t}$$

$$-M_0 = \mathrm{div}\,\mathbb{A} + \frac{1}{c}\frac{\partial V}{\partial t}$$

とおくと.

$$\Delta \mathbb{A} - \frac{1}{c^2}\frac{\partial^2 \mathbb{A}}{\partial t^2} = -\frac{4\pi \mathbb{I}}{c} - \mathrm{grad}\,M_0$$

$$\Delta V - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = -4\pi\rho + \frac{1}{c}\frac{\partial M_0}{\partial t}$$

これらの式は

$$\left\{-\begin{pmatrix}1&&&\\&1&&\\&&1&\\&&&1\end{pmatrix}\frac{p_t}{c} + \begin{pmatrix}&i&&\\i&&&\\&&&i\\&&-i&\end{pmatrix}p_x + \begin{pmatrix}&&i&\\&&&-i\\i&&&\\&i&&\end{pmatrix}p_y + \begin{pmatrix}&&&i\\&&i&\\&-i&&\\i&&&\end{pmatrix}p_z\right\}\begin{pmatrix}V\\A_x\\A_y\\A_z\end{pmatrix}$$

$$= i\hbar\begin{pmatrix}-\frac{1}{c}\frac{\partial V}{\partial t} - \mathrm{div}\,A\\ E_x + iH_x\\ E_y + iH_y\\ E_z + iH_z\end{pmatrix}$$

or

$$\left\{-\frac{p_t}{c} + \begin{pmatrix}&-i&&\\i&&&\\&&&i\\&&-i&\end{pmatrix}p_x + \begin{pmatrix}0&0&i&0\\0&0&0&-i\\-i&0&0&0\\0&i&0&0\end{pmatrix}p_y + \begin{pmatrix}&&&i\\&&i&\\&-i&&\\-i&&&\end{pmatrix}p_z\right\}\begin{pmatrix}iV\\A_x\\A_y\\A_z\end{pmatrix}$$

$$= i\hbar\begin{pmatrix}-i\left(\frac{1}{c}\frac{\partial V}{\partial t} + \mathrm{div}\,A\right)\\ E_x + iH_x\\ E_y + iH_y\\ E_z + iH_z\end{pmatrix} = i\hbar\begin{pmatrix}+iM_0\\M_x\\M_y\\M_z\end{pmatrix} \qquad \mathbb{M} = \mathbb{E} + i\mathbb{H}$$

$$\left(-\frac{p_t}{c} + \beta_x p_x + \beta_y p_y + \beta_z p_z\right)\begin{pmatrix}iV\\A_x\\A_y\\A_z\end{pmatrix} = i\hbar\begin{pmatrix}iM_0\\M_x\\M_y\\M_z\end{pmatrix}$$

$$\beta_x\beta_y = \begin{pmatrix}&&1&\\&&&1\\&-1&&\\-1&&&\end{pmatrix} = -i\beta_z$$

$$\beta_y\beta_x = i\beta_z \qquad \text{etc}$$

$$\beta_5 = \begin{pmatrix}&&1&\\&&&1\\1&&&\\&1&&\end{pmatrix}$$

$$\begin{pmatrix}0&i\sigma_y\\i\sigma_y&0\end{pmatrix} \quad \begin{pmatrix}0&i\sigma_z\\-i\sigma_z&0\end{pmatrix} \quad \begin{pmatrix}0&i\sigma_x\\-i\sigma_x&0\end{pmatrix}$$

(b)

$$\mathrm{curl}\,\mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi \mathbf{I}}{c} + \lambda' \mathbf{A} \qquad \mathrm{div}\,\mathbf{E} = 4\pi\rho$$

$$\mathrm{curl}\,\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} = \lambda'' \mathbf{E} \qquad \mathrm{div}\,\mathbf{H} = 0$$

$$\mathrm{curl}\,\mathbf{A} = \lambda \mathbf{H}$$

~~$\frac{1}{c}\frac{\partial V}{\partial t}$~~ − grad

$$-\mathrm{grad}\,V - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} = \lambda \mathbf{E}.$$

$\left(\frac{1}{c}\frac{\partial V}{\partial t} + \mathrm{div}\,\mathbf{A}\right)$: scalar

$\mathbf{E} + i\mathbf{H}$: space vector

~~cur~~ M_0 $\quad \mathrm{curl}\,\mathbf{M} - \frac{i}{c}\frac{\partial \mathbf{M}}{\partial t} = \frac{4\pi i \mathbf{I}}{c} + \lambda' \mathbf{A}$

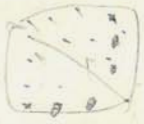
M_0 $\quad \mathrm{div}\,\mathbf{M} = 4\pi\rho. + \lambda' V$

~~curl M~~ =

$$i\,\mathrm{curl}\,\mathbf{A} - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \mathrm{grad}\,V = \lambda'' \mathbf{M}.$$

div ~~div M~~ div $\quad \frac{1}{c}\frac{\partial V}{\partial t} + \mathrm{div}\,\mathbf{A} = \lambda'' M_0$

curl $- \frac{i}{c}\frac{\partial}{\partial t}$



$\mathbf{M}, \mathbf{A}$: space vector
M_0, A_0 : scalar

A_0 $\mathbf{A}$: Four Vector
$\mathbf{M}$: ~~space vector~~ self adjoint tensor
M_0 : scalar

div $\quad \mathrm{curl}\,\mathbf{M} - \frac{i}{c}\frac{\partial \mathbf{M}}{\partial t} - i\,\mathrm{grad}\,M_0 = \frac{4\pi i\,\mathbf{I}}{c} + 4i\lambda' \mathbf{A}$

$\frac{1}{c}\frac{\partial}{\partial t}$ $\quad \mathrm{div}\,\mathbf{M} + \frac{1}{c}\frac{\partial M_0}{\partial t} = 4\pi\rho + \lambda' A_0$

curl $\quad \mathrm{curl}\,\mathbf{A} + \frac{i}{c}\frac{\partial \mathbf{A}}{\partial t} + i\,\mathrm{grad}\,A_0 = -i\lambda'' \mathbf{M}$

$$\mathrm{div}\,\mathbf{A} + \frac{1}{c}\frac{\partial A_0}{\partial t} = \lambda'' M_0$$

~~$\lambda'' = 0$~~ → $\lambda'' = 1 \quad \lambda' = 0 \qquad M_0 = 0. \rightarrow$ Maxwell.

(c)

$$i\,\mathrm{div}\left(\mathrm{curl}\,A + \frac{i}{c}\frac{\partial A}{\partial t} + i\,\mathrm{grad}\,A_0\right) + \frac{i}{c}\frac{\partial}{\partial t}\left(\mathrm{div}\,A + \frac{1}{c}\frac{\partial A_0}{\partial t}\right)$$

$$= 4\pi\lambda''\rho - \lambda'\lambda'' A_0$$

$\frac{h}{mc}$

$$\Delta A_0 - \frac{1}{c^2}\frac{\partial^2 A_0}{\partial t^2} - \lambda'\lambda'' A_0 = -4\pi\lambda''\rho.$$

ΔA -

$$\mathrm{curl}\left(\mathrm{curl}\,A + \frac{i}{c}\frac{\partial A}{\partial t} + i\,\mathrm{grad}\,A_0\right) - \frac{i}{c}\frac{\partial}{\partial t}\left(\mathrm{curl}\,A + \frac{i}{c}\frac{\partial A}{\partial t} + i\,\mathrm{grad}\,A_0\right) - \mathrm{grad}\left(\mathrm{div}\,A + \frac{1}{c}\frac{\partial A_0}{\partial t}\right)$$

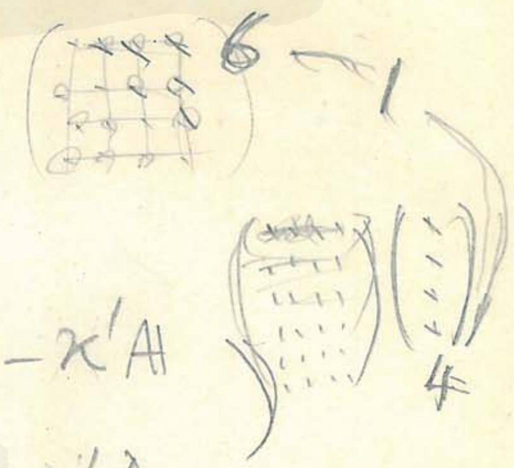
$-i\lambda''$
$$= \frac{4\pi\lambda''I}{c} - \lambda'\lambda'' A.$$

λ'' curl

$$-\Delta A + \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = \frac{4\pi\lambda'' I}{c} - \lambda'\lambda'' A$$

$(-i\lambda'')$ $-iE$

$$M = E + iH.$$



$$\mathrm{curl}\,\underset{F}{E} + \frac{1}{c}\frac{\partial \underset{G}{H}}{\partial t} = 0$$

div
$$\mathrm{curl}\,H - \frac{1}{c}\frac{\partial E}{\partial t} - \mathrm{grad}\,M_0 = \frac{4\pi I}{c} - \kappa' A$$

$\frac{1}{c}\frac{\partial}{\partial t}$ grad A
$$\mathrm{div}\,E + \frac{1}{c}\frac{\partial M_0}{\partial t} = 4\pi\rho - \kappa' A_0$$

$$\mathrm{div}\,H = 0$$

Complex.

It should be observed further that ... are complex so that

$$\mathrm{curl}\,A = \kappa H$$

$$\frac{1}{c}\frac{\partial A}{\partial t} + \mathrm{grad}\,A_0 = -\kappa E$$

$$\mathrm{div}\,A + \frac{1}{c}\frac{\partial A_0}{\partial t} = \kappa M_0$$

$\kappa = 1$, $\kappa' = 0$ $M_0 = 0$ $\rightarrow$ Maxwell.

$\circ\ \mathrm{curl}\,\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} = 0$ $\qquad \mathrm{div}\,\mathbf{H} = 0$

$\mathrm{curl}\,\mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} - \mathrm{grad}\,M_0 \doteq \frac{4\pi \mathbf{I}}{c} - \varkappa' \mathbf{A}$

$\mathrm{div}\,\mathbf{E} + \frac{1}{c}\frac{\partial M_0}{\partial t} = 4\pi\rho - \varkappa' A_0$

curl $\circ\ \mathrm{curl}\,\mathbf{A} = \varkappa \mathbf{H}$ $\qquad \mathrm{grad}\,A_0 + \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} = -\varkappa \mathbf{E}.$

$\mathrm{curl}\,\mathrm{curl}\,\mathbf{A} = \varkappa \frac{4\pi \mathbf{I}}{c}$ ~~...~~

$- \frac{1}{c}\frac{\partial}{\partial t}\left(\mathrm{grad}\,A_0 + \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}\right) + \varkappa\,\mathrm{grad}\,M_0 = \varkappa\varkappa' \mathbf{A}.$

~~div grad A =~~

div $\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A} - \mathrm{grad}\left(\mathrm{div}\,\mathbf{A} + \frac{1}{c}\frac{\partial A_0}{\partial t} - \varkappa M_0\right) \pm \varkappa\varkappa' \mathbf{A}$

$= -\varkappa \frac{4\pi \mathbf{I}}{c}.$

$\mathrm{div}\left(\mathrm{grad}\,A_0 + \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}\right) = -\varkappa\left(4\pi\rho - \varkappa' A_0 - \frac{1}{c}\frac{\partial M_0}{\partial t}\right)$

$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)A_0 + \frac{1}{c}\frac{\partial}{\partial t}\left(\mathrm{div}\,\mathbf{A} + \frac{1}{c}\frac{\partial A_0}{\partial t} - \varkappa M_0\right)$

$\frac{1}{c}\frac{\partial}{\partial t}$ $\qquad - \varkappa\varkappa' A_0 = -\varkappa\, 4\pi\rho.$

Supplementary Condition: $\boxed{\mathrm{div}\,\mathbf{A} + \frac{1}{c}\frac{\partial A_0}{\partial t} - \varkappa M_0 = 0}$

$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A} - \varkappa\varkappa' \mathbf{A} = -\varkappa \frac{4\pi \mathbf{I}}{c}.$

$\varkappa = 1;\quad \varkappa' = 0,\quad M_0 = 0 \rightarrow$ Maxwell's case

$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \varkappa\varkappa'\right)M_0$ ~~(div A + ... M_0)~~ $\quad - \frac{4\pi}{c}\left(\mathrm{div}\,\mathbf{I} + \frac{\partial \rho}{\partial t}\right)$

$- \varkappa'\left(\mathrm{div}\,\mathbf{A} + \frac{1}{c}\frac{\partial A_0}{\partial t} - \varkappa M_0\right) =$ ~~$\frac{4\pi \mathbf{I}}{c}$~~

~~$\frac{\partial}{\partial t}\left(\mathrm{div}\,\mathbf{A} + \frac{1}{c}\frac{\partial A_0}{\partial t} - \varkappa M_0\right) =$~~

$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \varkappa\varkappa'\right)M_0 = -\frac{4\pi}{c}\left(\mathrm{div}\,\mathbf{I} + \frac{\partial \rho}{\partial t}\right)$

$$-\frac{1}{c}\frac{\partial}{\partial t}\operatorname{div}E - \Delta M_0 = \Delta \operatorname{div}E - \varkappa \operatorname{div}A$$

$$\frac{1}{c}\frac{\partial}{\partial t}\operatorname{div}E + \frac{1}{c^2}\frac{\partial^2 M_0}{\partial t^2} = 4\pi\rho - \frac{\varkappa'}{c}\frac{\partial A_0}{\partial t}$$

$$\frac{\partial}{\partial t}\left(\operatorname{div}A + \frac{1}{c}\frac{\partial A_0}{\partial t} - \varkappa M_0\right) = 0.$$

function fluctuation

Electromagnetic Field のある場合

$$-i\hbar\frac{\partial}{\partial x} - \frac{e}{c}A_x$$

$$\operatorname{curl}\vec{B} = \quad -i\hbar\frac{\partial B_z}{\partial y} - \frac{e}{c}A_y B_z + i\hbar\frac{\partial B}{}$$

$$-i\hbar\frac{\partial B_z}{\partial y} - \frac{e}{c}A_y B_z + i\hbar\frac{\partial B_y}{\partial z} + \frac{e}{c}A_z B_y$$

$$\operatorname{curl}\operatorname{curl}\vec{B} = \left(-i\hbar\frac{\partial}{\partial y} - \frac{e}{c}A_y\right)\left(-i\hbar\frac{\partial B_y}{\partial x} - \frac{e}{c}A_x B_y + i\hbar\frac{\partial B_x}{\partial y} + \frac{e}{c}A_y B_x\right)$$

$$-\left(-i\hbar\frac{\partial}{\partial z} - \frac{e}{c}A_z\right)\left(-i\hbar\frac{\partial B_x}{\partial z} - \frac{e}{c}A_z B_x + i\hbar\frac{\partial B_z}{\partial x} + \frac{e}{c}A_x B_z\right)$$

$$= \left(-i\hbar\frac{\partial}{\partial x}\right)\left(-i\hbar\frac{\partial B_y}{\partial y}\right) - \left(-i\hbar\frac{\partial}{\partial y}\right)\left(-i\hbar\frac{\partial B_x}{\partial y}\right)$$

$$+\left(-i\hbar\frac{\partial}{\partial x}\right)\left(-i\hbar\frac{\partial B_z}{\partial z}\right) - \left(-i\hbar\frac{\partial}{\partial z}\right)\left(-i\hbar\frac{\partial B_x}{\partial z}\right)$$

$$+\left(-i\hbar\frac{\partial}{\partial x}\right)\left(-i\hbar\frac{\partial B_x}{\partial x}\right) - \left(-i\hbar\frac{\partial}{\partial x}\right)\left(-i\hbar\frac{\partial B_x}{\partial x}\right)$$

$$+ (-i\hbar)\left(\frac{e}{c}\right) \quad A_x\frac{\partial B_y}{\partial y} + \frac{\partial A_x}{\partial y}B_y - A_y\frac{\partial B_x}{\partial y} - \frac{\partial A_y}{\partial y}B_x$$

$$+ \quad A_y\frac{\partial B_y}{\partial x} -$$

$$= \left(-i\hbar\frac{\partial}{\partial y} - \frac{e}{c}A_y\right)\left(i\hbar\frac{\partial B_x}{\partial y} \mp \frac{e}{c}A_y B_x\right) + \left(-i\hbar\frac{\partial}{\partial y} - \frac{e}{c}A_y\right)\left(i\hbar\frac{\partial B_y}{\partial x} - \frac{e}{c}A_x B_y\right)$$

$$-\left(-i\hbar\frac{\partial}{\partial z} - \frac{e}{c}A_z\right)\left(-i\hbar\frac{\partial B_x}{\partial z} \mp \frac{e}{c}A_z B_x\right) + \left(-i\hbar\frac{\partial}{\partial z} - \frac{e}{c}A_z\right)\left(-i\hbar\frac{\partial B_z}{\partial x} - \frac{e}{c}A_x B_z\right)$$

$$-\left(-i\hbar\frac{\partial}{\partial x} - \frac{e}{c}A_x\right)\left(-i\hbar\frac{\partial B_x}{\partial x} - \frac{e}{c}A_x B_x\right)$$

$$+\left(-i\hbar\frac{\partial}{\partial x} - \frac{e}{c}A_x\right)\left(-i\hbar\frac{\partial B_x}{\partial x} - \frac{e}{c}A_x B_x\right)$$

$$= - \qquad + \left(-i\hbar\frac{\partial}{\partial x} - \frac{e}{c}A_x\right)\left\{\left(-i\hbar\frac{\partial B_y}{\partial y} - \frac{e}{c}A_y B_y\right) + (\quad) + (\quad)\right\}$$

$$-(-i\hbar)\left(\frac{e}{c}\right)\frac{\partial A_x}{\partial y}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_y}{\partial x}\right)B_y \ldots$$

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{B} - \mathrm{grad}\left(\mathrm{div}\,\vec{B} + \frac{1}{c}\frac{\partial B_0}{\partial t} - \varkappa M_0\right) - \varkappa\varkappa'\vec{B}$$

$$(-i\hbar)^{-1}\left(-\frac{e}{c}\right)[\vec{H}\vec{B}] = 0.$$

$$\frac{(-i\hbar)^2}{2M}$$

$$-\frac{e\hbar}{2Mc}\left\{H_x\begin{pmatrix}0&0&+i\\0&0&0\\-i&0&0\end{pmatrix} + H_y\begin{pmatrix}0&0&i\\0&0&0\\+i&0&0\end{pmatrix} + H_z\begin{pmatrix}0&0&i\\-i&0&0\\0&0&0\end{pmatrix}\right\}\begin{pmatrix}B_x\\B_y\\B_z\end{pmatrix}$$

$$\begin{pmatrix}H_yB_z - H_zB_y\\H_zB_x - H_xB_z\\H_xB_y - H_yB_x\end{pmatrix}$$

$$\begin{pmatrix}0&0&0\\0&0&i\\0&-i&0\end{pmatrix} + H_y\begin{pmatrix}0&0&-i\\0&0&0\\+i&0&0\end{pmatrix} + H_z\begin{pmatrix}0&i&0\\-i&0&0\\0&0&0\end{pmatrix}\Bigg\}\begin{pmatrix}B_x\\B_y\\B_z\end{pmatrix}$$