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 京都大学基礎物理学研究所 湯川記念館史料室

E03020P13 ①

between the Neutron and the Proton

$$H_U = -4\pi c^2 \vec{U}^+ \vec{U}^T - \frac{1}{4\pi} (\text{grad } \vec{U} \text{ grad } U + \lambda^2 \vec{U} U) \\ + 4\pi c^2 \sum_i \vec{B}_i^+ \vec{B}_i^T + \frac{1}{4\pi} \sum_i (\text{grad } \vec{B}_i \text{ grad } B_i + \lambda^2 \vec{B}_i B_i)$$

$$H_M = \tilde{\Psi} \left\{ \frac{1}{2M} \left[ \vec{p} - \frac{g}{2c} \{ (\tau_1 - i\tau_2) \vec{B} + (\tau_1 + i\tau_2) \vec{B} \} \right]^2 + \not{D} \right. \\ \left. + \frac{g}{2} \{ (\tau_1 - i\tau_2) \vec{U} + (\tau_1 + i\tau_2) \vec{U} \} \Psi \right.$$

$$\bar{H} = \int H_U dV + \int H_M dV = \bar{H}_U + \bar{H}_M$$

$$i\hbar \frac{\partial U}{\partial t} = U \bar{H} - \bar{H} U = -i\hbar \cdot 4\pi c^2 \vec{U}^T$$

$$\frac{\partial U}{\partial t} = -4\pi c^2 \vec{U}^T \quad \frac{\partial \vec{U}}{\partial t} = -4\pi c^2 U^T$$

$$i\hbar \frac{\partial \vec{U}^T}{\partial t} = (-i\hbar) \left\{ \frac{1}{4\pi} (\Delta U - \lambda^2 U) \right\} = -\frac{g}{2} \tilde{\Psi} (\tau_1 - i\tau_2) \Psi$$

$$\Delta U - \frac{1}{c^2} \frac{\partial^2 U}{\partial t^2} - \lambda^2 U = -\frac{4\pi g}{2} \tilde{\Psi} (\tau_1 - i\tau_2) \Psi$$

$$\Delta \vec{U} - \frac{1}{c^2} \frac{\partial^2 \vec{U}}{\partial t^2} - \lambda^2 \vec{U} = -\frac{4\pi g}{2} \tilde{\Psi} (\tau_1 + i\tau_2) \Psi$$

$$\frac{\Delta \vec{B}}{\partial t} = 4\pi c^2 \vec{B}^T \quad \frac{\partial \vec{B}}{\partial t} = -4\pi c^2 \vec{B}^T$$

$$\Delta \vec{B} - \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} - \lambda^2 \vec{B} = -\frac{4\pi g}{2c} \tilde{\Psi} (\tau_1 - i\tau_2) \vec{I} \Psi$$

$$\vec{I} = \frac{1}{i\hbar} \{ \vec{B}^T \bar{H}_M - \bar{H}_M \vec{B}^T \}$$

$$\text{div } \vec{B} + \frac{1}{c} \frac{\partial U}{\partial t} = 0$$

$$\text{div } \vec{B} + \frac{1}{c} \frac{\partial \vec{U}}{\partial t} = 0$$

$$U = \sum a_\sigma \varphi_\sigma$$

$$\vec{U} = \sum \tilde{a}_\sigma \tilde{\varphi}_\sigma$$

$$U^T = \sum a_\sigma^+ \tilde{\varphi}_\sigma$$

$$\vec{U}^T = \sum \tilde{a}_\sigma^+ \varphi_\sigma$$

$$\left( \Delta - \lambda^2 + \frac{v^2}{c^2} \right) \varphi_\sigma = 0$$

$$\varphi = \sum b_\sigma \text{grad } \varphi_\sigma$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

$$\text{div } \vec{B}_{1,2} = 0$$

$$\vec{B}_3 = \text{grad } \varphi$$

$$\vec{B} = \sum_{\sigma} b_{\sigma} \text{grad} \phi_{\sigma} + \sum_{\sigma} b_{\sigma} \text{grad} \phi_{\sigma}$$

$$\text{div} \vec{B} + \frac{1}{c} \frac{\partial U}{\partial t} = 0 \rightarrow b_{\sigma} \Delta \phi_{\sigma} + \frac{1}{c} \dot{a}_{\sigma} \phi_{\sigma} = 0$$

$$\left(\frac{v_{\sigma}^2}{c^2} - \lambda\right) b_{\sigma} = \frac{\dot{a}_{\sigma}}{c} = -4\pi c \tilde{a}_{\sigma}^+$$

$$\left(\frac{v_{\sigma}^2}{c^2} - \lambda\right) \dot{b}_{\sigma} = \frac{\dot{a}_{\sigma}}{c}$$

$$\dot{a}_{\sigma} + v_{\sigma}^2 a_{\sigma} = T_{\sigma}$$

$$\tilde{b}_{\sigma} = 4\pi c \tilde{b}_{\sigma}^+$$

$$\begin{aligned} a_{\sigma} &= \frac{T_{\sigma} - \dot{a}_{\sigma}}{v_{\sigma}^2} \\ &= \frac{T_{\sigma} - c \left(\frac{v_{\sigma}^2}{c^2} - \lambda\right) 4\pi c^2 \tilde{b}_{\sigma}^+}{v_{\sigma}^2} \\ &= \frac{T_{\sigma}}{v_{\sigma}^2} - \frac{4\pi c^2 \tilde{b}_{\sigma}^+}{v_{\sigma}^2} \left(\frac{v_{\sigma}^2}{c^2} - \lambda\right) \end{aligned}$$

$$\bar{H}_U = -(4\pi c^2)^2 \sum_{\sigma} \tilde{a}_{\sigma}^+ a_{\sigma}^+ - \sum_{\sigma} v_{\sigma}^2 \tilde{a}_{\sigma} a_{\sigma}$$

$$+ \sum_{\sigma} \left(\frac{v_{\sigma}^2}{c^2} - \lambda\right) \left\{ (4\pi c^2)^2 \sum_{i,j} \tilde{b}_{\sigma}^{(i)} \tilde{b}_{\sigma}^{(j)} + \sum_{i,j} v_{\sigma}^2 b_{\sigma}^{(i)} b_{\sigma}^{(j)} \right\}$$

$$\iint \tilde{\Psi} \frac{g}{2} [(\tau, i\pi) \cup (\tau, +i\pi) \cup] \Psi dV = \sum_{\sigma} (\tilde{a}_{\sigma} \tilde{T}_{\sigma} + a_{\sigma} T_{\sigma})$$

$$\bar{H}_U = -c^2 \sum_{\sigma} \left(\frac{v_{\sigma}^2}{c^2} - \lambda\right) \tilde{b}_{\sigma} \tilde{b}_{\sigma} - \sum_{\sigma} \frac{(T_{\sigma} - 4\pi c^2 \left(\frac{v_{\sigma}^2}{c^2} - \lambda\right) \tilde{b}_{\sigma}^+)(T_{\sigma} - 4\pi c^2 \left(\frac{v_{\sigma}^2}{c^2} - \lambda\right) \tilde{b}_{\sigma}^+)}{v_{\sigma}^2}$$

$$+ \sum_{\sigma} v_{\sigma}^2 \left(\frac{v_{\sigma}^2}{c^2} - \lambda\right) \tilde{b}_{\sigma} b_{\sigma} + \sum_{\sigma} \left(\frac{v_{\sigma}^2}{c^2} - \lambda\right) (4\pi c^2)^2 \tilde{b}_{\sigma}^+ \tilde{b}_{\sigma}^+$$

$$c^2 \left(\frac{v_{\sigma}^2}{c^2} - \lambda\right) + v_{\sigma}^2$$

$$(4\pi c^2) \left(\frac{v_{\sigma}^2}{c^2} - \lambda\right) \left\{ 1 - \frac{c^2 \left(\frac{v_{\sigma}^2}{c^2} - \lambda\right)}{v_{\sigma}^2} \right\}$$

$$\sum_{\sigma} (\tilde{a}_{\sigma} T_{\sigma} + a_{\sigma} \tilde{T}_{\sigma}) = \frac{T_{\sigma} - 4\pi c^2 \left(\frac{v_{\sigma}^2}{c^2} - \lambda\right) \tilde{b}_{\sigma}^+}{v_{\sigma}^2} T_{\sigma} + \frac{T_{\sigma} - 4\pi c^2 \left(\frac{v_{\sigma}^2}{c^2} - \lambda\right) \tilde{b}_{\sigma}^+}{v_{\sigma}^2} \tilde{T}_{\sigma}$$

$$\bar{H}_U + \sum_{\sigma} (\tilde{a}_{\sigma} T_{\sigma} + a_{\sigma} \tilde{T}_{\sigma}) = \sum_{\sigma} c^2 \lambda \left(\frac{v_{\sigma}^2}{c^2} - \lambda\right) \tilde{b}_{\sigma} b_{\sigma} + \sum_{\sigma} (4\pi c^2)^2 \left(\frac{v_{\sigma}^2}{c^2} - \lambda\right) \frac{c^2 \lambda}{v_{\sigma}^2} \tilde{b}_{\sigma}^+ \tilde{b}_{\sigma}^+$$

$$+ \sum_{\sigma} \frac{\tilde{T}_{\sigma} T_{\sigma}}{v_{\sigma}^2}$$

$$\iiint \text{grad } \tilde{\varphi}_p \text{ grad } \varphi_\sigma dV = \iiint \varphi_p \left( \frac{v_\sigma^2}{c^2} - \lambda \right) \varphi_\sigma dV = 4\pi c^2 \left( \frac{v_\sigma^2}{c^2} - \lambda \right)$$

$$b_p^\dagger b_\sigma - b_\sigma b_p^\dagger = \frac{-i\hbar}{4\pi c^2 \left( \frac{v_\sigma^2}{c^2} - \lambda \right)}$$

~~$$b_\sigma = \frac{1}{\sqrt{2} \sqrt{4\pi c^2 \left( \frac{v_\sigma^2}{c^2} - \lambda \right)}} (c_\sigma + i d_\sigma), \quad b_\sigma^\dagger = \frac{1}{\sqrt{2}} (c_\sigma^\dagger - i d_\sigma^\dagger)$$

$$b_\sigma = c_\sigma - i d_\sigma, \quad b_\sigma^\dagger = c_\sigma^\dagger + i d_\sigma^\dagger$$~~

$$b_\sigma = \frac{1}{\sqrt{2} \sqrt{4\pi c^2 \left( \frac{v_\sigma^2}{c^2} - \lambda \right)}} (c_\sigma + i d_\sigma)$$

$$b_\sigma^\dagger = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{4\pi c^2 \left( \frac{v_\sigma^2}{c^2} - \lambda \right)}} (c_\sigma^\dagger - i d_\sigma^\dagger)$$

$$\tilde{b}_\sigma = \dots (c_\sigma - i d_\sigma)$$

$$\tilde{b}_\sigma^\dagger = \dots (c_\sigma^\dagger + i d_\sigma^\dagger)$$

$$c_\sigma^\dagger c_\sigma - c_\sigma c_\sigma^\dagger = -i\hbar$$

etc.

$$c^2 \lambda^2 \left\{ \sum_\sigma \frac{1}{\left( \frac{v_\sigma^2}{c^2} - \lambda \right)} b_\sigma b_\sigma + \frac{(4\pi c^2)}{v_\sigma^2} \left( \frac{v_\sigma^2}{c^2} - \lambda \right) b_\sigma^\dagger b_\sigma^\dagger \right\}$$

$$= \frac{c^2 \lambda^2}{2} \left\{ \frac{1}{4\pi c^2} \frac{1}{\lambda} (c_\sigma^2 + d_\sigma^2) + \frac{(4\pi c^2)}{v_\sigma^2} (c_\sigma^{\dagger 2} + d_\sigma^{\dagger 2}) \right\}$$

$$\left( \frac{\sqrt{v_\sigma} c_\sigma b}{\sqrt{4\pi c^2}}, \frac{p}{\sqrt{v_\sigma}} c_\sigma^\dagger \right)$$

$$\frac{1}{2} (p^2 + q^2) =$$

$$= \frac{c^2 \lambda^2}{2} \sum \frac{\hbar v_\sigma}{v_\sigma}$$

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{1}{2M} \left[ \vec{p} - \frac{q}{2c} \{ (\tau_1 - i\tau_2) \vec{B} + (\tau_1 + i\tau_2) \vec{B} \} \right]^2 \Psi + \frac{\hbar}{2} \mathcal{D} \Psi + \frac{q}{2} \{ (\tau_1 - i\tau_2) \vec{U} + (\tau_1 + i\tau_2) \vec{U} \} \Psi$$

$$-i\hbar \frac{\partial \tilde{\Psi}}{\partial t} = \frac{1}{2M} \left[ -\vec{p} - \frac{q}{2c} \{ (\tau_1 + i\tau_2) \vec{B} + (\tau_1 - i\tau_2) \vec{B} \} \right]^2 \tilde{\Psi} + \frac{\hbar}{2} \mathcal{D} \tilde{\Psi} + \frac{q}{2} \{ (\tau_1 + i\tau_2) \vec{U} + (\tau_1 - i\tau_2) \vec{U} \} \tilde{\Psi}$$

$$i\hbar \frac{\partial (\Psi \tilde{\Psi})}{\partial t} = \frac{1}{2M} \left( \tilde{\Psi} \vec{p} \Psi - \Psi \vec{p} \tilde{\Psi} \right)$$

$$\Leftrightarrow \frac{1}{2M} \left( \tilde{\Psi} (\tau_1 - i\tau_2) \vec{p} \Psi - (\vec{p} \tilde{\Psi}) (\tau_1 - i\tau_2) \Psi \right)$$