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Linearization of the U-field equations

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$\mathbb{E}+i\mathbb{H}=\mathbb{F}$

$$\mathrm{curl}\,\mathbb{F} = \frac{4\pi i}{c}\ldots\frac{\partial \mathbb{F}}{\partial t}$$

$$\mathrm{div}\,\mathbb{F} = 4\pi\rho$$

$$\kappa\mathbb{F} = -\mathrm{grad}\,V - \frac{1}{c}\frac{\partial \mathbb{A}}{\partial t} + i\,\mathrm{curl}\,\mathbb{A}.$$

$$0 = \kappa M = \mathrm{div}\,A + \frac{1}{c}\frac{\partial V}{\partial t}$$

$i\,\mathrm{div}$ $\qquad -\frac{1}{c}\frac{\partial}{\partial t}$

$$\left.\begin{aligned}
&\mathrm{curl}\,\mathbb{F} - \frac{i}{c}\frac{\partial \mathbb{F}}{\partial t} - i\,\mathrm{grad}\,M = \frac{4\pi i \mathbb{I}}{c} - i\kappa' A\\
&\mathrm{div}\,\mathbb{F} + \frac{1}{c}\frac{\partial M}{\partial t} = 4\pi\rho - \kappa' V.\\
&-\mathrm{grad}\,V - \frac{1}{c}\frac{\partial \mathbb{A}}{\partial t} + i\,\mathrm{curl}\,\mathbb{A} = \kappa\mathbb{F}.\\
&\mathrm{div}\,\mathbb{A} + \frac{1}{c}\frac{\partial V}{\partial t} = +\kappa M.
\end{aligned}\right\}$$

$$-\frac{1}{c}\frac{\partial}{\partial t}\mathrm{curl}\,\mathbb{A} + i\,\mathrm{curl}\,\mathrm{curl}\,\mathbb{A} - \frac{i}{c}\frac{\partial}{\partial t}(i\,\mathrm{curl}\,\mathbb{A}) \mp i\,\mathrm{grad}\,\mathrm{div}\,\mathbb{A} + \mathrm{grad}\,\frac{i}{c}\frac{\partial V}{\partial t} + \frac{i}{c^2}\frac{\partial^2 \mathbb{A}}{\partial t^2}$$

$$\pm\frac{i}{c}\,\mathrm{grad}\,\frac{\partial V}{\partial t} = \frac{4\pi i\kappa\mathbb{I}}{c} - i\kappa\kappa'\mathbb{A}.$$

$$\kappa\left(\Delta\mathbb{A} - \frac{1}{c^2}\frac{\partial^2\mathbb{A}}{\partial t^2}\right) = -\frac{4\pi\kappa\mathbb{I}}{c} + \kappa\kappa'\mathbb{A}.$$

$$-\Delta V - \frac{1}{c}\frac{\partial}{\partial t}\mathrm{div}\,\mathbb{A} \cancel{\kappa i} + \frac{1}{c}\frac{\partial}{\partial t}\mathrm{div}\,\mathbb{A} + \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = 4\pi\kappa\rho - \kappa\kappa' V.$$

$$\frac{1}{c}\mathrm{div}\frac{\partial \mathbb{F}}{\partial t} + \Delta M \pm \frac{1}{c}\frac{\partial}{\partial t}\mathrm{div}\,\mathbb{F} - \frac{1}{c^2}\frac{\partial^2 M}{\partial t^2} = -\frac{4\pi\,\mathrm{div}\,\mathbb{I}}{c} + \kappa'\mathrm{div}\,\mathbb{A}$$

$$-\frac{4\pi}{c}\frac{\partial\rho}{\partial t} + \frac{\kappa'}{c}\frac{\partial V}{\partial t}.$$

$$\Delta M - \frac{1}{c^2}\frac{\partial^2 M}{\partial t^2} = -\frac{4\pi}{c}\left(\mathrm{div}\,\mathbb{I} + \frac{\partial\rho}{\partial t}\right) + \kappa'\kappa M$$

$2\times 8 = (\overset{3}{\mathbb{F}}, \overset{4}{V}, \mathbb{A}), \overset{1}{M}) \times 2$

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$$H = \iiint\{\tilde{\mathbb{F}}\mathbb{F} \pm \tilde{M}M\}$$

$$\frac{\partial \tilde{V}}{\partial x}\cdot\frac{\partial V}{\partial x}$$

$\mathrm{div}\qquad \kappa\mathbb{F}$

$$L = \iiint\left\{\cancel{\mathrm{curl}}\left(-\mathrm{grad}\,\tilde{V} - \frac{1}{c}\frac{\partial\tilde{\mathbb{A}}}{\partial t} + i\,\mathrm{curl}\,\tilde{\mathbb{A}}\right)\left(-\mathrm{grad}\,V - \frac{1}{c}\frac{\partial\mathbb{A}}{\partial t} - i\,\mathrm{curl}\,\mathbb{A}\right)\right.$$

$$-\cancel{\left(\mathrm{div}\,\mathbb{F} + \frac{1}{c}\frac{\partial M}{\partial t}\right)}\left(\mathrm{div}\,\tilde{\mathbb{A}} + \frac{1}{c}\frac{\partial\tilde{V}}{\partial t}\right)\left(\mathrm{div}\,\mathbb{A} + \frac{1}{c}\frac{\partial V}{\partial t}\right). \qquad \kappa\kappa'\tilde{\mathbb{A}}\mathbb{A}$$

$$\tilde{V}V$$

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Vacuum

$$\mathbb{L} = \frac{1}{8\pi}\iiiint \Big[\big(-\text{grad}\,\tilde{V} - \frac{1}{c}\frac{\partial \tilde{\mathbb{A}}}{\partial t} + i\,\text{curl}\,\tilde{\mathbb{A}}\big) \times \mathbb{F}$$

$$- \big(\text{div}\,\tilde{\mathbb{A}} + \frac{1}{c}\frac{\partial \tilde{V}}{\partial t}\big) \times M \quad + \quad \varkappa\varkappa' \overbrace{(\tilde{\mathbb{A}}\,\mathbb{A} + \tilde{V} V)}^{dv}$$

$$=$$

$$\text{div}\,\mathbb{F} + \frac{1}{c}\frac{\partial M}{\partial t} + \varkappa' V = 0,$$

$$\left.\begin{aligned} -i\quad \frac{1}{c}\frac{\partial \mathbb{F}}{\partial t} + i\,\text{curl}\,\mathbb{F} + \text{grad}\,M - \varkappa' \mathbb{A} = 0 \\ \text{curl}\,\mathbb{F} - \frac{i}{c}\frac{\partial \mathbb{F}}{\partial t} - i\,\text{grad}\,M + i\varkappa' \mathbb{A} = 0 \end{aligned}\right\}$$

$$\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) F_y +$$

$$- A_x \left(\frac{\partial F_y}{\partial z} - \frac{\partial F_z}{\partial y}\right)$$

$$= A_x (\text{curl}\,F)_x$$

$$V^{\dagger} = -\frac{\varkappa}{8\pi}\tilde{M} \qquad \tilde{V}^{\dagger} = -\frac{\varkappa}{8\pi c} M.$$

$$\mathbb{A}^{\dagger} = -\frac{\varkappa}{8\pi c}\tilde{\mathbb{F}} \qquad \tilde{\mathbb{A}}^{\dagger} = -\frac{\varkappa}{8\pi c}\mathbb{F}.$$

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$$\mathbb{H} = \iiint H\,dv$$

$$H = V^{\dagger}\frac{\partial V}{\partial t} + \mathbb{A}^{\dagger}\frac{\partial \mathbb{A}}{\partial t} + \frac{\partial \tilde{V}}{\partial t}\tilde{V}^{\dagger} + \frac{\partial \tilde{\mathbb{A}}}{\partial t}\tilde{\mathbb{A}}^{\dagger}$$

$$- L_{\varkappa \frac{\tilde{M}}{8\pi}} \quad (M - \quad (\varkappa M - \text{div}\,A)$$

$$= -\frac{\varkappa}{8\pi c}\tilde{M}\frac{\partial V}{\partial t} - \frac{\varkappa}{8\pi c}\tilde{\mathbb{F}}\frac{\partial \mathbb{A}}{\partial t} \cancel{- \frac{\varkappa}{8\pi c}\frac{\partial \tilde{V}}{\partial t} M} - \frac{\varkappa}{8\pi c}\frac{\partial \tilde{\mathbb{A}}}{\partial t}\mathbb{F}$$

$$+ \frac{\varkappa}{8\pi}(\text{div}\,\tilde{\mathbb{A}})\varkappa M + \cancel{\frac{\varkappa}{8\pi c}\frac{\partial \tilde{V}}{\partial t} M}$$

$$= -\frac{\varkappa^2 \tilde{M} M}{8\pi} + \frac{\varkappa^2}{8\pi}(\tilde{M}\,\text{div}\,\mathbb{A} + \text{div}\,\tilde{\mathbb{A}}\cdot M)$$

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$$\cancel{L = \frac{1}{4\pi c^2}\Big(\frac{\partial \tilde{V}}{\partial t}\frac{\partial V}{\partial t} -}$$

$$\bar{L} = \iiint L\,dv.$$

$$L = -\frac{1}{4\pi}\Big(\frac{1}{c^2}\frac{\partial \tilde{U}}{\partial t}\frac{\partial U}{\partial t} - \operatorname{grad}\tilde{U}\operatorname{grad}U - \kappa^2\tilde{U}U\Big)$$

$$+\frac{1}{4\pi}\Big(\frac{1}{c^2}\frac{\partial \vec{\tilde{V}}}{\partial t}\frac{\partial \vec{V}}{\partial t} - \sum_i \operatorname{grad}\tilde{V}_i\operatorname{grad}V_i - \kappa^2\vec{\tilde{V}}\vec{V}\Big)$$

$$U^\dagger = \frac{-1}{4\pi c^2}\frac{\partial \tilde{U}}{\partial t} \qquad \tilde{U}^\dagger = \frac{-1}{4\pi c^2}\frac{\partial U}{\partial t}$$

$$\vec{V}^\dagger = \frac{1}{4\pi c^2}\frac{\partial \vec{\tilde{V}}}{\partial t} \qquad \vec{\tilde{V}}^\dagger = \frac{1}{4\pi c^2}\frac{\partial \vec{V}}{\partial t}.$$

$$\bar{H} = \iiint H\,dv$$

$$H = U^\dagger\frac{\partial U}{\partial t} + \frac{\partial \tilde{U}}{\partial t}\tilde{U}^\dagger + \vec{V}^\dagger\frac{\partial \vec{V}}{\partial t} + \frac{\partial \vec{\tilde{V}}}{\partial t}\vec{\tilde{V}}^\dagger$$

$$-L$$

$$= -\frac{1}{4\pi}\Big(\frac{1}{c^2}\frac{\partial \tilde{U}}{\partial t}\frac{\partial U}{\partial t} + \operatorname{grad}\tilde{U}\operatorname{grad}U + \kappa^2\tilde{U}U\Big)$$

$$+\frac{1}{4\pi}\Big(\frac{1}{c^2}\frac{\partial \vec{\tilde{V}}}{\partial t}\frac{\partial \vec{V}}{\partial t} + \sum_i \operatorname{grad}\tilde{V}_i\operatorname{grad}V_i + \kappa^2\vec{\tilde{V}}\vec{V}\Big)$$

$$= -4\pi c^2\tilde{U}^\dagger U^\dagger - \frac{1}{4\pi}\operatorname{grad}\tilde{U}\operatorname{grad}U - \kappa^2\tilde{U}U$$

$$+4\pi c^2\vec{\tilde{V}}^\dagger\vec{V}^\dagger + \frac{1}{4\pi}\Big(\sum_i \operatorname{grad}\tilde{V}_i\operatorname{grad}V_i + \kappa^2\vec{\tilde{V}}\vec{V}\Big)$$

$$U U^{\dagger\prime} - U^{\dagger\prime} U = i\hbar\delta$$

$$i\hbar\frac{\partial U}{\partial t} = U\bar{H} - \bar{H}U = -4\pi c^2 i\hbar\,\tilde{U}^\dagger\ldots$$

etc

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$$\mathrm{curl}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} \qquad \mathrm{div}\,\mathbf{E} = 0$$

$$\mathrm{curl}\,\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} \qquad \mathrm{div}\,\mathbf{H} = 0$$

$$\mathbf{E} = -\mathrm{grad}\,V - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} + \mathrm{curl}\,\mathbf{B}$$

$$\mathbf{H} = \mathrm{curl}\,\mathbf{A} + \mathrm{grad}\,U + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$M = \frac{1}{c}\frac{\partial V}{\partial t} + \mathrm{div}\,\mathbf{A}$$

$$N = \frac{1}{c}\frac{\partial U}{\partial t} + \mathrm{div}\,\mathbf{B}$$

$$\mathrm{curl}\,\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} - \mathrm{grad}\,N = \varkappa^2 \mathbf{B}$$

$$\mathrm{curl}\,\mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} - \mathrm{grad}\,M = -\varkappa^2 \mathbf{A}$$

$$-\Delta \mathbf{A} + \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2}$$

$$\mathrm{curl}\,\mathrm{curl}\,\mathbf{A} + \cancel{\frac{1}{c}\frac{\partial}{\partial t}\mathrm{curl}\,\mathbf{B}} + \cancel{\frac{1}{c}\,\mathrm{grad}\frac{\partial V}{\partial t}} + \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2}$$

$$- \cancel{\frac{1}{c}\frac{\partial}{\partial t}\mathrm{curl}\,\mathbf{B}} - \cancel{\frac{1}{c}\,\mathrm{grad}\frac{\partial V}{\partial t}} - \mathrm{grad}\,\mathrm{div}\,\mathbf{A}$$

$$= -\varkappa^2 \mathbf{A}$$

~~curl curl~~

$$-\cancel{\frac{1}{c}\,\mathrm{curl}\frac{\partial \mathbf{A}}{\partial t}} + \mathrm{curl}\,\mathrm{curl}\,\mathbf{B} + \cancel{\frac{1}{c}\,\mathrm{curl}\frac{\partial \mathbf{A}}{\partial t}}$$

$$+ \cancel{\frac{1}{c}\,\mathrm{grad}\frac{\partial U}{\partial t}} + \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} - \cancel{\frac{1}{c}\frac{\partial}{\partial t}\mathrm{grad}\,U} - \mathrm{grad}\,\mathrm{div}\,\mathbf{B}$$

$$= -\varkappa^2 \mathbf{B}$$

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$$\cancel{\varkappa}\ -\operatorname{grad} V - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} + \operatorname{curl} \mathbf{B} = \varkappa \mathbf{E}$$

$$\operatorname{grad} U + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} + \operatorname{curl} \mathbf{A} = \varkappa \mathbf{H}$$

$$\frac{1}{c}\frac{\partial V}{\partial t} + \operatorname{div} \mathbf{A} = \varkappa M$$

$$\frac{1}{c}\frac{\partial U}{\partial t} + \operatorname{div} \mathbf{B} = \varkappa N$$

$$\operatorname{curl} \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} + \operatorname{grad} N = -\varkappa \mathbf{B}$$

$$\operatorname{curl} \mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} - \operatorname{grad} M = -\varkappa \mathbf{A}$$

$$\cancel{H = \frac{\varkappa^2}{8\pi}(\mathbf{E}^2 + \mathbf{H}^2) + M^2 + N^2}$$

$$\cancel{\operatorname{grad}}\quad \operatorname{div} \mathbf{E} + \frac{1}{c}\frac{\partial M}{\partial t} = -\varkappa V$$

$$\operatorname{div} \mathbf{H} + \frac{1}{c}\frac{\partial N}{\partial t} = +\varkappa U$$

$$-\operatorname{div}\operatorname{grad} V - \cancel{\frac{1}{c}\frac{\partial}{\partial t}\operatorname{div}\mathbf{A}\ \operatorname{grad}} - \frac{1}{c}\frac{\partial}{\partial t}\operatorname{div}\mathbf{A}$$

$$+\frac{1}{c}\frac{\partial^2 V}{\partial t^2} + \frac{1}{c}\frac{\partial}{\partial t}\operatorname{div}\mathbf{A} = -\varkappa^2 V.$$

$$\operatorname{div}\operatorname{grad} U + \frac{1}{c}\frac{\partial}{\partial t}\operatorname{div}\mathbf{B} = \frac{1}{c}\frac{\partial^2 U}{\partial t^2} - \frac{1}{c}\frac{\partial}{\partial t}\operatorname{div}\mathbf{B}$$

$$= \varkappa^2 U$$

$$H = \frac{\varkappa^2}{8\pi}\left\{(\mathbf{E}^2 + \mathbf{H}^2) - (M^2 + N^2) + \mathbf{A}^2 + \mathbf{B}^2 - (V^2 + U^2)\right\}$$

$$= \frac{1}{8\pi}\left\{\left(-\operatorname{grad} V - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} + \operatorname{curl}\mathbf{B}\right)^2 + \left(\operatorname{grad} U + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} + \operatorname{curl}\mathbf{A}\right)^2\right.$$

$$-\left(\frac{1}{c}\frac{\partial V}{\partial t} + \operatorname{div}\mathbf{A}\right)^2 - \left(\frac{1}{c}\frac{\partial U}{\partial t} + \operatorname{div}\mathbf{B}\right)^2$$

$$\left.+\varkappa^2(\mathbf{A}^2 + \mathbf{B}^2 - V^2 - U^2)\right\}$$

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$$L = \frac{\varkappa^2}{8\pi}\{E^2 + H^2 - M^2 - N^2 - A^2 - B^2 + V^2 + U^2\}$$

$$\frac{\partial L}{\partial \frac{\partial A}{\partial t}} = -\frac{\varkappa}{4\pi c}E, \qquad \frac{\partial L}{\partial \frac{\partial V}{\partial t}} = -\frac{\varkappa}{4\pi c}M$$

$$\frac{\partial L}{\partial \frac{\partial B}{\partial t}} = \frac{\varkappa}{4\pi c}H \qquad \frac{\partial L}{\partial \frac{\partial U}{\partial t}} = \frac{\varkappa}{4\pi c}N$$

$$H = -\frac{\varkappa}{4\pi c}E\frac{\partial A}{\partial t} + \frac{\varkappa}{4\pi c}H\frac{\partial B}{\partial t} - \frac{\varkappa}{4\pi c}M\frac{\partial V}{\partial t} + \frac{\varkappa}{4\pi c}N\frac{\partial U}{\partial t}$$

$$- L$$

$$= \frac{\varkappa^2}{8\pi}\{E^2 + H^2 - M^2 - N^2 + A^2 + B^2 - V^2 - U^2\}$$

$$A_x\left(-\frac{\varkappa}{4\pi c}E_x'\right) - \left(-\frac{\varkappa}{4\pi c}E_x'\right)A_x = i\hbar\,\delta(\vec{r}, \vec{r}')$$

$$A_x E_x' - E_x' A_x = \cancel{2i\hbar} = -\frac{4\pi\hbar c}{\varkappa}\, i\,\delta(\vec{r}, \vec{r}')$$

$$\varkappa = \frac{m_u c}{\hbar} \qquad = \cancel{-4\pi m_u c^2\, i\,\delta(\vec{r}, \vec{r}')}$$

$$\frac{m^2 l^4 t^{-2}}{m} = m l^4 t^{-2} \qquad = -\frac{4\pi\hbar^2}{m_u}\, i\,\delta(\vec{r}, \vec{r}')$$

$$\varkappa^2 E^2 \to$$

$$\frac{\varkappa^2}{8\pi}(E_x^2 + A_x^2) = \sum_{mn} u_m u_n\left(a \sum_{mn}\left(\frac{\varkappa^2}{8\pi}a_{xn}^2 + 2\pi c^2 b_{xn}^2\right)\right.$$

$$A_x = \sum_n a_{xn} u_n \qquad = 4\pi c^2 \sum \frac{1}{2}\left(b_{xn}^2 + \frac{\varkappa^2}{16\pi^2 c^2} a_{xn}^2\right)$$

$$\times \quad E_x = -\frac{4\pi c}{\varkappa}\sum b_{xn} v_n \qquad = 4\pi c^2 \sum_n \left(n_{xn} + \frac{1}{2}\right)\hbar \frac{\varkappa}{4\pi c}.$$

$$\frac{\varkappa^2}{4\pi c} =$$

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~~$E = \sum e_n E_n$~~ $\quad H = \sum$

~~$A = \sum a a_n A_n$~~ $\quad B = \sum$

$A = \sum a_n A_n \quad H = \sum h_n A_n$

$E = \sum e_n E_n \quad B = \sum b_n B_n$

$M = \sum m_n M_n \quad N = \sum n_n N_n$

$V = \sum v_n V_n \quad U = \sum u_n U_n$

$- \mathrm{grad}\, V_n - \frac{1}{c} a.$

$\omega_1 = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$ $\omega_2 = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$ $\omega_3 = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$

$\pi_1 = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$ $\pi_2 = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$ $\pi_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\pi_1\pi_2 - \pi_2\pi_1 = +2i\pi_3$ etc.

$\pi_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & +i & 0 \end{pmatrix}$ $\pi_2 = \begin{pmatrix} 0 & 0 & +i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}$ $\pi_3 = \begin{pmatrix} 0 & -i & 0 \\ +i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$

$+i\begin{pmatrix} 0 & 0 & 0 \\ +i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + i\begin{pmatrix} 0 & -i & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = 2i\pi_3$

$S^{-1}\sigma S = 0$ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

$\begin{pmatrix} ia_{21} & -ia_{22} \\ ia_{11} & ia_{12} \end{pmatrix} = \begin{pmatrix} a_{11} & -a_{12} \\ +a_{21} & -a_{22} \end{pmatrix}$

$-ia_{21} = a_{11}$ $\quad -ia_{22} = a_{12}$

$ia_{11} = +a_{21}$ $\quad ia_{12} = a_{22}$

$(\sigma_y + \sigma_z)$

$(\sigma_y + \sigma_z)^{-1}\sigma_y(\sigma_y + \sigma_z)$ $\quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ i & -1 \end{pmatrix}$

$= (\sigma_y + \sigma_z)\sigma_z$

$S = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i & 0 \\ i & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\quad S^{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i & 0 \\ i & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$S^{-1}\pi_3 S = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}$ $\quad S^{-1}\pi_1 S = \frac{1}{2}\begin{pmatrix} 1 & -i & 0 \\ i & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}\begin{pmatrix} 1 & -i & 0 \\ i & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$= \frac{1}{2}\begin{pmatrix} 1 & -i & 0 \\ i & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ -1 & -i & 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & +i \\ -1 & -i & 0 \end{pmatrix}$

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$$S^{-1}\pi_2 S = \frac{1}{2}\begin{pmatrix} 1 & -i & 0 \\ i & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & i & 0 \\ i & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & -i & 0 \\ i & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & -1 & 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & 0 & i \\ 0 & 0 & -1 \\ -i & -1 & 0 \end{pmatrix}$$

$$\pi_1' = \frac{1}{2}\begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \pi_2' = \frac{1}{2}\begin{pmatrix} 0 & i & 0 \\ -i & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} \qquad \pi_3' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$\omega_1{}^{\cancel{\pi_1'^2}},\ \omega_2,\ \omega_3$

$$\overset{p}{\mathrm{curl}} = \cancel{\vec{\pi}\,\mathrm{grad}\ \pi_x \frac{\partial}{\partial x} + \pi_z \frac{\partial}{\partial y}} + \pi$$

$$-\frac{1}{\hbar}\vec{\pi}\,\vec{p}$$

$\omega_1 \vec{\pi}$