$$L = \frac{\mu c^2}{2}\left(\vec{F}^2 - \vec{G}^2 - K^2 - \vec{U}^2 + U_4^2\right) \qquad \kappa = \frac{\mu c}{\hbar}$$

$$-\frac{1}{c}\frac{\partial \vec{F}}{\partial t} + \mathrm{curl}\,\vec{G} - \mathrm{grad}\,K = +\kappa \vec{U} \qquad \mathrm{grad}\,U_4 + \frac{1}{c}\frac{\partial \vec{U}}{\partial t} = \kappa \vec{F}$$

$$+\mathrm{div}\,\vec{F} + \frac{1}{c}\frac{\partial K}{\partial t} = +\kappa U_4 \qquad -\mathrm{curl}\,\vec{U} = \kappa \vec{G}$$

$$\mathrm{div}\,\vec{U} + \frac{1}{c}\frac{\partial U_4}{\partial t} = -\kappa K$$

$$L = \frac{\mu c^2}{2\kappa}\Big\{\left(\mathrm{grad}\,U_4 + \frac{1}{c}\frac{\partial \vec{U}}{\partial t}\right)\vec{F} + \mathrm{curl}\,\vec{U}\cdot\vec{G} + \left(\mathrm{div}\,U + \frac{1}{c}\frac{\partial U_4}{\partial t}\right)K$$
$$- \kappa \vec{U}\vec{U} + \kappa U_4 U_4\Big\}$$

$$\delta L \sim \frac{\mu c^2}{\kappa}\Big\{\delta U_4\left(\mathrm{div}\,\vec{F} + \frac{1}{c}\frac{\partial K}{\partial t} + \kappa U_4\right) + \delta U\left(-\frac{1}{c}\frac{\partial \vec{F}}{\partial t} + \mathrm{curl}\,\vec{G}\right.$$
$$\left. - \mathrm{grad}\,K - \kappa \vec{U}\right)\Big\}$$

$$\frac{\partial L}{\partial \frac{\partial U_x}{\partial t}} = \mu c^2 \cdot F_x \cdot \frac{1}{\kappa c} = \hbar F_x$$

$$\frac{\partial L}{\partial \frac{\partial U_4}{\partial t}} = \mu c^2 \cdot K \cdot \frac{1}{\kappa c} = \hbar K.$$

$$U_x F_x' - F_x' U_x = i\delta(\vec{r}, \vec{r}').$$
$$U_4 K - K U_4 = i\delta(\vec{r}, \vec{r}').$$

$$H = \hbar \vec{F}\frac{\partial \vec{U}_x}{\partial t} + \hbar K \frac{\partial U_4}{\partial t} - L$$
$$= \vec{F}\left(\mu c^2\left(\vec{F} - \frac{1}{\kappa}\mathrm{grad}\,U_4\right)\right) + K\mu c^2\left(-K - \frac{1}{\kappa}\mathrm{div}\,\vec{U}\right) - L$$
$$= \frac{\mu c^2}{2}\Big\{\vec{F}^2 - K^2 - \frac{2}{\kappa}\vec{F}\,\mathrm{grad}\,U_4 - \frac{2}{\kappa}K\,\mathrm{div}\,\vec{U}$$
$$+ \frac{1}{\kappa^2}(\mathrm{curl}\,\vec{U})^2 + \vec{U}^2 - U_4^2\Big\}$$

$$\vec{U} = \sum(u_1\vec{e}_1 + u_2\vec{e}_2 + u_3\vec{e}_3)\,e^{i(\vec{k}\vec{r} - ck_4 t)} \qquad \vec{k} = k\vec{e}_3.$$
$$U_4 = \sum u_4\, e^{i(\vec{k}\vec{r} - \omega t)} \qquad [\vec{e}_1, \vec{e}_2] = \vec{e}_3$$
$$\vec{F} = \sum(f_1\vec{e}_1 + f_2\vec{e}_2 + f_3\vec{e}_3)\,e^{i(\vec{k}\vec{r} - ck_4 t)} \qquad [\vec{e}_2, \vec{e}_3] = \vec{e}_1$$
$$[\vec{e}_3, \vec{e}_1] = \vec{e}_2$$
$$K = \sum f_4\, e^{i(\vec{k}\vec{r} - \omega t)} \qquad \vec{G} = \sum(g_1\vec{e}_1 + g_2\vec{e}_2 + g_3\vec{e}_3)$$

$$\kappa \tilde{u}_3 = -ik_4 f_3 + ik \tilde{f}_4$$
$$\kappa \tilde{u}_4 = -ik \tilde{f}_3 + ik_4 \tilde{f}_4$$

$$u_3 \tilde{f}_3 - \tilde{f}_3 u_3 = \frac{i}{2}, \rightarrow \quad u_3 \tilde{u}_3 - \tilde{u}_3 u_3 = -i \frac{k_4}{\kappa} \cdot \left(\frac{i}{2}\right) = \frac{k_4}{2\kappa}$$

$$u_4 \tilde{f}_4 - \tilde{f}_4 u_4 = \frac{i}{2},$$

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$(\mathrm{curl}\,\vec{U})_x = \sum \{ u_1 (e_{1z} ik_y - e_{1y} ik_z) + u_2 (e_{2z} ik_y - e_{2y} ik_z) \} e^{i(\vec{k}\vec{r} - ck_4 t)}$

$\mathrm{curl}\,\vec{U} = \sum ik(u_1 \vec{e}_2 - u_2 \vec{e}_1) e^{\cdots}$

$\sum ik_4 (f_1 \vec{e}_1 + f_2 \vec{e}_2 + f_3 \vec{e}_3) e + \sum ik(g_1 \vec{e}_2 - g_2 \vec{e}_1) e - \sum i\vec{k} f_4 e = \sum \varkappa (u_1 \vec{e}_1 + u_2 \vec{e}_2 + u_3 \vec{e}_3)$

$ik_4 f_1 - ikg_2 = \varkappa u_1$ $\qquad \sum ik f_3 - \sum ik_4 f_4 = \sum \varkappa u_4$

$ik_4 f_2 + ikg_1 = \varkappa u_2$ $\quad \rightarrow \quad ik_4 f_1 = \varkappa (1 + \frac{k^2}{\varkappa^2}) u_1 = \frac{k_4^2}{\varkappa} u_1$

$ik_4 f_3 - ikf_4 = \varkappa u_3$ $\qquad \boxed{i\varkappa f_1 = k_4 u_1}$

$ikf_3 - ik_4 f_4 = \varkappa u_4$ $\qquad \boxed{i\varkappa f_2 = k_4 u_2}$

$iku_4 - ik_4 u_3 = \varkappa f_3$

$-ik_4 u_1 = \varkappa f_1$ $\quad \rightarrow \quad ik_4 f_3 - ikf_4 = \varkappa u_3$

$-ik_4 u_2 = \varkappa f_2$ $\qquad ikf_3 - ik_4 f_4 = \varkappa u_4$

$+ iku_2 = \varkappa g_1$

$-iku_1 = \varkappa g_2$

$iku_3 - ik_4 u_4 = -\varkappa f_4$

$\vec{U} = \sum'_{\vec{k}, k_4 > 0} (u_1 \vec{e}_1 + u_2 \vec{e}_2 + u_3 \vec{e}_3) e^{i(\vec{k}\vec{r} - ck_4 t)} + (\tilde{u}_1 \vec{e}_1 + \tilde{u}_2 \vec{e}_2 + \tilde{u}_3 \vec{e}_3) e^{-i(\vec{k}\vec{r} - ck_4 t)}$

$\vec{U}_x \vec{\tilde{F}}'_x - \vec{F}' \vec{U} = \sum'_{\vec{k}, k_4} (u_{1x} \tilde{f}_{1x} + u_{2x} \tilde{f}_{2x} + u_{3x} \tilde{f}_{3x} + \cancel{\tilde{u}_1 f_1 + \tilde{u}_2 f_2 + \tilde{u}_3 f_3} \cdots) e^{i\vec{k}(\vec{r} - \vec{r}')} + \sum (\tilde{u}_{1x} f_{1x} + \cdots) e^{-i\vec{k}(\vec{r} - \vec{r}')}$

$u_{1x} \tilde{f}_{1x} + u_{2x} \tilde{f}_{2x} + u_{3x} \tilde{f}_{3x} - (\tilde{f}_{1x} u_{1x} + \tilde{f}_{2x} u_{2x} + \tilde{f}_{3x} u_{3x}) = \frac{i}{2}$

$u_1 \tilde{f}_1 - \tilde{f}_1 u_1 = \frac{i}{2}$ etc. $\qquad k_4 (u_1 \tilde{u}_1 - \tilde{u}_1 u_1) = \frac{\varkappa}{2}$

$u_4 \tilde{f}_4 - \tilde{f}_4 u_4 = \frac{i}{2}$ etc. $\qquad k_4 (u_2 \tilde{u}_2 - \tilde{u}_2 u_2) = \frac{\varkappa}{2}$

$\bar{H} = \frac{\mu c^2}{2} \{ \sum'_{k, k_4} (\tilde{f}_1 f_1 + \tilde{f}_2 f_2 + \tilde{f}_3 f_3 - \tilde{f}_4 f_4) - \sum \frac{2ik}{\varkappa} (\tilde{f}_3 u_4 - \tilde{f}_3 \tilde{u}_4) - \frac{2ik_4}{\varkappa} (\tilde{f}_4 u_3 - f_4 \tilde{u}_3)$

$+ \sum' (f_1 \tilde{f}_1 + f_2 \tilde{f}_2 + f_3 \tilde{f}_3 - f_4 \tilde{f}_4)$

$+ \sum \frac{k^2}{\varkappa^2} (\tilde{u}_1 u_1 + \tilde{u}_2 u_2) + \frac{k^2}{\varkappa^2} \sum (u_1 \tilde{u}_1 + u_2 \tilde{u}_2) + \sum (\tilde{u}_1 u_1 + \tilde{u}_2 u_2 + \tilde{u}_3 u_3 - \tilde{u}_4 u_4)$

$+ \sum (u_1 \tilde{u}_1 + \cdots)$

$= \frac{\mu c^2}{2} \{ \sum' (\frac{k_4^2}{\varkappa^2} + \frac{k^2}{\varkappa^2} + 1)(\tilde{u}_1 u_1 + \tilde{u}_2 u_2 + u_1 \tilde{u}_1 + u_2 \tilde{u}_2) + \cdots \}$

③

$$+\frac{\mu c^2}{2}\sum\{\tilde{f}_3 f_3 - \tilde{f}_4 f_4 + \tilde{f}_3 f_3 - \tilde{f}_4 f_4 - \frac{2ik}{\varkappa}(\tilde{f}_3 u_4 - f_3 \tilde{u}_4 + \tilde{f}_4 u_3 - f_4 \tilde{u}_3)$$
$$+ \tilde{u}_3 u_3 - \tilde{u}_4 u_4 + u_3\tilde{u}_3 - u_4\tilde{u}_4\}$$

$$= \mu c^2 \sum{}' \frac{k_4^2}{\varkappa^2}(\tilde{u}_1 u_1 + \tilde{u}_2 u_2 + u_1\tilde{u}_1 + u_2\tilde{u}_2) + \overline{H}'$$

$$u_1\tilde{u}_1 - \tilde{u}_1 u_1 = \frac{\varkappa}{2k_4}$$

$$\cancel{u_1 = \frac{1}{2}(p_1+iq_1)} \rightarrow \cancel{(p_1-iq_1)(p_1+iq_1)-(p_1+iq_1)(p_1-iq_1)=}$$
$$\cancel{\tilde{u}_1 = \frac{1}{2}(p_1-iq_1)}$$
$$i(p_1q_1 - q_1p_1) = \frac{\varkappa}{2k_4}$$

$$u_1 = \frac{1}{2}\sqrt{\frac{\varkappa}{k_4}}(p_1+iq_1)$$
$$\tilde{u}_1 = \frac{1}{2}\sqrt{\frac{\varkappa}{k_4}}(p_1-iq_1) \rightarrow (p_1q_1 - q_1p_1) = -i$$

$$\frac{k_4^2}{\varkappa^2}(\tilde{u}_1 u_1 + u_1\tilde{u}_1) = \frac{k_4}{2\varkappa}(p_1^2 + q_1^2)$$
$$= \frac{k_4}{\varkappa}(n_1+\tfrac{1}{2})$$

$$= \hbar\sum_{\vec{k},\,k_4>0}{}' k_4 c\,(n_1+\tfrac{1}{2})+(n_2+\tfrac{1}{2}) + \overline{H}'$$

$$\overline{H}' = \frac{\mu c^2}{2}\sum_{\vec{k},\,k_4>0}\{\tilde{f}_3 f_3 + \tilde{f}_4 f_4 + f_3\tilde{f}_3 - f_4\tilde{f}_4 - \frac{2ik}{\varkappa}(\tilde{f}_3 u_4 - f_3\tilde{u}_4 + \tilde{f}_4 u_3$$
$$- f_4\tilde{u}_3) + \tilde{u}_3 u_3 - \tilde{u}_4 u_4 + u_3\tilde{u}_3 - u_4\tilde{u}_4\}$$

$$\left.\begin{matrix} ik_4 f_3 - ikf_4 = \varkappa u_3 \\ ikf_3 - ik_4 f_4 = \varkappa u_4 \end{matrix}\right) \qquad \left.\begin{matrix} -ik_4u_3 + iku_4 = \varkappa f_3 \\ -iku_3 + ik_4u_4 = \varkappa f_4 \end{matrix}\right)$$

$$\sqrt{\frac{k_4+\varkappa}{2\varkappa}}\,\alpha_3 \mp \sqrt{\frac{k_4-\varkappa}{2\varkappa}}\,\alpha_4 = u_3$$
$$+\sqrt{\frac{k_4-\varkappa}{2\varkappa}}\,\alpha_3 \mp \sqrt{\frac{k_4+\varkappa}{2\varkappa}}\,\alpha_4 = u_4$$

$$-ik_4\left(\sqrt{\frac{k_4+\varkappa}{2\varkappa}}\alpha_3 - \sqrt{\frac{k_4-\varkappa}{2\varkappa}}\alpha_4\right) + ik\left(\sqrt{\frac{k_4-\varkappa}{2\varkappa}}\alpha_3 - \sqrt{\frac{k_4+\varkappa}{2\varkappa}}\alpha_4\right)$$
$$= \varkappa\left(-i\sqrt{\frac{k_4+\varkappa}{2\varkappa}}\,\alpha_3 - i\sqrt{\frac{k_4-\varkappa}{2\varkappa}}\,\alpha_4\right)$$
$$= \varkappa\left(\sqrt{\frac{k_4+\varkappa}{2\varkappa}}\beta_3 - \sqrt{\frac{k_4-\varkappa}{2\varkappa}}\beta_4\right)$$

$$-i\sqrt{\frac{k_4+\varkappa}{2\varkappa}}(k_4 - (k_4+\varkappa)$$
$$-i\sqrt{\frac{k_4-\varkappa}{2\varkappa}}(k_4 - k_4+\varkappa$$

$$-ik_4\left(\sqrt{\frac{k_4+\varkappa}{2\varkappa}}\alpha_3 - \sqrt{\frac{k_4-\varkappa}{2\varkappa}}\alpha_4\right)$$
$$+ik_4\left(\sqrt{\frac{k_4-\varkappa}{2\varkappa}}\alpha_3 - \sqrt{\frac{k_4+\varkappa}{2\varkappa}}\alpha_4\right) = \varkappa\left(-i\sqrt{\frac{k_4-\varkappa}{2\varkappa}}\alpha_3 - i\sqrt{\frac{k_4+\varkappa}{2\varkappa}}\alpha_4\right)$$
$$= \varkappa\left(\sqrt{\cdots}\,\beta_3 - \sqrt{\cdots}\,\beta_4\right)$$

$$-i\alpha_3 = \beta_3 \qquad i\alpha_4 = \beta_4$$

$$\left.\begin{aligned} i\alpha_3 &= \sqrt{\frac{k_4+\varkappa}{2\varkappa}}\,u_3 - \sqrt{\frac{k_4-\varkappa}{2\varkappa}}\,u_4 \\ \alpha_4 &= +\sqrt{\frac{k_4-\varkappa}{2\varkappa}}\,u_3 - \sqrt{\frac{k_4+\varkappa}{2\varkappa}}\,u_4 \end{aligned}\right\}$$

$$\alpha_3\tilde{\beta}_3 - \tilde{\beta}_3\alpha_3 = \left(\sqrt{\frac{k_4+\varkappa}{2\varkappa}}u_3 - \sqrt{\frac{k_4-\varkappa}{2\varkappa}}u_4\right)\left(\sqrt{\frac{k_4+\varkappa}{2\varkappa}}\tilde{f}_3 - \sqrt{\frac{k_4-\varkappa}{2\varkappa}}\tilde{f}_4\right)$$

$$-(\quad)(\quad)$$

$$= \frac{k_4+\varkappa}{2\varkappa}\cdot\frac{i}{2} + \frac{k_4-\varkappa}{2\varkappa}\cdot\frac{i}{2} = \frac{k_4}{2\varkappa}i$$

$i\tilde{\alpha}_3 = \tilde{\beta}_3 : \cancel{\tilde{\beta}_3 = -i\tilde{\alpha}_3}$

$$\alpha_3\tilde{\alpha}_3 - \tilde{\alpha}_3\alpha_3 = +\frac{k_4}{2\varkappa}$$

$$\alpha_3\tilde{\beta}_4 - \tilde{\beta}_4\alpha_3 = \left(\sqrt{\frac{k_4+\varkappa}{2\varkappa}}u_3 - \sqrt{\frac{k_4-\varkappa}{2\varkappa}}u_4\right)\left(\sqrt{\frac{k_4-\varkappa}{2\varkappa}}\tilde{f}_3 - \sqrt{\frac{k_4+\varkappa}{2\varkappa}}\tilde{f}_4\right)$$

$$-(\quad)(\quad) = \frac{k}{2}\frac{i}{2} + \frac{k}{2}\frac{i}{2} = \frac{k\,i}{2}$$

$$\alpha_4\tilde{\beta}_3 - \tilde{\beta}_3\alpha_4 = \frac{k\,i}{2\varkappa}$$

$$\alpha_4\tilde{\beta}_4 - \tilde{\beta}_4\alpha_4 = \frac{k_4 i}{2\varkappa}$$

$\tilde{\beta}_4 = i\tilde{\alpha}_4$

$$\alpha_3\tilde{\alpha}_4 - \tilde{\alpha}_4\alpha_3 = -\frac{k}{2\varkappa}$$

$$\alpha_4\tilde{\alpha}_3 - \tilde{\alpha}_3\alpha_4 = +\frac{k}{2\varkappa}$$

$$\alpha_4\tilde{\alpha}_4 - \tilde{\alpha}_4\alpha_4 = -\frac{k_4}{2\varkappa}$$

$$H' = \frac{\mu c^2}{2}\sum_{\vec{k},\,k_4>0}\Bigl\{\left(\sqrt{\frac{k_4+\varkappa}{2\varkappa}}\tilde{\beta}_3 - \sqrt{\frac{k_4-\varkappa}{2\varkappa}}\tilde{\beta}_4\right)\left(\sqrt{\frac{k_4+\varkappa}{2\varkappa}}\beta_3 - \sqrt{\frac{k_4-\varkappa}{2\varkappa}}\beta_4\right) + (\beta_3 \quad \beta_4)(\tilde{\beta}_3 \quad \tilde{\beta}_4)$$

$$+\left(\sqrt{\frac{k_4-\varkappa}{2\varkappa}}\tilde{\beta}_3 - \sqrt{\frac{k_4+\varkappa}{2\varkappa}}\tilde{\beta}_4\right)\left(\sqrt{\frac{k_4-\varkappa}{2\varkappa}}\beta_3 - \sqrt{\frac{k_4+\varkappa}{2\varkappa}}\beta_4\right)$$

$$+(\quad \beta_3 \quad \beta_4)(\quad \tilde{\beta}_3 \quad \tilde{\beta}_4)$$

$$-\frac{2ik}{\varkappa}\Bigl\{\left(\sqrt{\frac{k_4+\varkappa}{2\varkappa}}\tilde{\beta}_3 - \sqrt{\frac{k_4-\varkappa}{2\varkappa}}\tilde{\beta}_4\right)\left(\sqrt{\frac{k_4-\varkappa}{2\varkappa}}\alpha_3 - \sqrt{\frac{k_4+\varkappa}{2\varkappa}}\alpha_4\right)$$

$$-\left(\sqrt{\frac{k_4+\varkappa}{2\varkappa}}\beta_3 - \sqrt{\frac{k_4-\varkappa}{2\varkappa}}\beta_4\right)\left(\sqrt{\frac{k_4-\varkappa}{2\varkappa}}\tilde{\alpha}_3 - \sqrt{\frac{k_4+\varkappa}{2\varkappa}}\tilde{\alpha}_4\right)$$

$$+\left(\sqrt{\frac{k_4-\varkappa}{2\varkappa}}\tilde{\beta}_3 - \sqrt{\frac{k_4+\varkappa}{2\varkappa}}\tilde{\beta}_4\right)\left(\sqrt{\frac{k_4+\varkappa}{2\varkappa}}\alpha_3 - \sqrt{\frac{k_4-\varkappa}{2\varkappa}}\alpha_4\right)$$

$$-\left(\sqrt{\frac{k_4-\varkappa}{2\varkappa}}\beta_3 - \sqrt{\frac{k_4+\varkappa}{2\varkappa}}\beta_4\right)\left(\sqrt{\frac{k_4+\varkappa}{2\varkappa}}\tilde{\alpha}_3 - \sqrt{\frac{k_4-\varkappa}{2\varkappa}}\tilde{\alpha}_4\right)\Bigr\}$$

$+$

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$$\bar{H}' = \frac{\mu c^2}{2} \sum_{\vec{k},\, k_4>0} \{ \text{[illegible]}$$

$$+ \tilde{\alpha}_3 \alpha_3 + \alpha_3 \tilde{\alpha}_3 - \tilde{\alpha}_4 \alpha_4 - \alpha_4 \tilde{\alpha}_4$$

$$- \frac{2ik^2}{\varkappa^2} (\tilde{\beta}_3 \alpha_3 - \beta_3 \tilde{\alpha}_3 + \tilde{\beta}_4 \alpha_4 - \beta_4 \tilde{\alpha}_4)$$

$$+ \frac{2ik}{\varkappa} \cdot \frac{k_4}{\varkappa} (\tilde{\beta}_3 \alpha_4 - \beta_3 \tilde{\alpha}_4 - \tilde{\beta}_4 \alpha_3 + \beta_4 \tilde{\alpha}_3) \}$$

$\beta_3 = i\alpha_3$

$\tilde{\beta}_3 = +i\tilde{\alpha}_3$

$\beta_4 = i\alpha_4$

$\tilde{\beta}_4 = -i\tilde{\alpha}_4$

$$\frac{k}{\varkappa} \tilde{\beta}_3 \alpha_3 + \tilde{\beta}_4 \quad \alpha_3$$

$$- \beta_3 \tilde{\alpha}_3 - \beta_4 \tilde{\alpha}_4 \quad -i$$

$$\bar{H}' = \mu c^2 \sum_{\vec{k},\, k_4>0} \{ \tilde{\alpha}_3 \alpha_3 + \alpha_3 \tilde{\alpha}_3 - \tilde{\alpha}_4 \alpha_4 - \alpha_4 \tilde{\alpha}_4$$

$$+ \frac{k^2}{\varkappa^2} (\tilde{\alpha}_3 \alpha_3 + \alpha_3 \tilde{\alpha}_3 - \tilde{\alpha}_4 \alpha_4 - \alpha_4 \tilde{\alpha}_4)$$

$$- \frac{k k_4}{\varkappa^2} (\tilde{\alpha}_3 \alpha_4 + \alpha_3 \tilde{\alpha}_4 + \tilde{\alpha}_4 \alpha_3 + \alpha_4 \tilde{\alpha}_3)$$

$$= \mu c^2 \frac{k_4}{\varkappa} \sum \Big\{ \frac{k_4}{\varkappa} \{ \tilde{\alpha}_3 \alpha_3 + \alpha_3 \tilde{\alpha}_3 - \tilde{\alpha}_4 \alpha_4 - \alpha_4 \tilde{\alpha}_4 \}$$

$$- \frac{k}{\varkappa} \{ \tilde{\alpha}_3 \alpha_4 + \alpha_3 \tilde{\alpha}_4 + \tilde{\alpha}_4 \alpha_3 + \alpha_4 \tilde{\alpha}_3 \} \Big\}$$

$$\alpha_3 \tilde{\alpha}_3 - \tilde{\alpha}_3 \alpha_3 = \frac{k_4}{2\varkappa} \qquad \alpha_4 \tilde{\alpha}_3 - \tilde{\alpha}_3 \alpha_4 = \frac{k}{2\varkappa}$$

$$\alpha_3 \tilde{\alpha}_4 - \tilde{\alpha}_4 \alpha_3 = -\frac{k}{2\varkappa} \qquad \alpha_4 \tilde{\alpha}_4 - \tilde{\alpha}_4 \alpha_4 = -\frac{k_4}{2\varkappa}$$

$$\cancel{\alpha_3 = \frac{1}{2} p_3 + i q_3}$$

$$\alpha_4 = p_4 + i q_4 \qquad p$$

$$\alpha_3 = \frac{1}{2}\sqrt{\frac{k_4}{\varkappa}} (p_3 + i q_3) \rightarrow \quad p_3 q_3 - q_3 p_3 = -i$$

$$\alpha_4 = \frac{1}{2}\sqrt{\frac{k_4}{\varkappa}} (p_4 - i q_4) \rightarrow \quad p_4 q_4 - q_4 p_4 = -i$$

$$\cancel{p_4 + iq}$$

$$(p_4 \mp i q_4)(p_3 \mp i q_3) - (p_3 \mp i q_3)(p_4 \mp i q_4) = \frac{2k}{k_4}$$

$$(p_4 p_3 - p_3 p_4) + (q_3 q_4 - q_4 q_3) = \frac{2k}{k_4}$$

$$\mp i (p_4 q_3 - q_3 p_4) \pm i (p_3 q_4 - q_4 p_3)$$

$$(p_3 q_3 - q_3 p_4) - (p_3 q_4 - q_4 p_3) = 0$$

$$(p_4 p_3 - p_3 p_4) + (q_3 q_4 - q_4 q_3) = \frac{2k}{k_4}$$

$$\frac{k_4}{\varkappa}\tilde{\alpha}_3\alpha_3 - \frac{k}{\varkappa}\tilde{\alpha}_3\alpha_4 - \frac{k}{\varkappa}\tilde{\alpha}_4\alpha_3 - \frac{k_4}{\varkappa}\tilde{\alpha}_4\alpha_4$$

$$= \left(\sqrt{\frac{k_4+\varkappa}{2\varkappa}}\tilde{\alpha}_3 - \sqrt{\frac{k_4-\varkappa}{2\varkappa}}\tilde{\alpha}_4\right)\left(\sqrt{\frac{k_4+\varkappa}{2\varkappa}}\alpha_3 - \sqrt{\frac{k_4-\varkappa}{2\varkappa}}\alpha_4\right)$$

$$+ \left(\sqrt{\frac{k_4-\varkappa}{2\varkappa}}\tilde{\alpha}_3 - \sqrt{\frac{k_4+\varkappa}{2\varkappa}}\tilde{\alpha}_4\right)\left(\sqrt{\frac{k_4-\varkappa}{2\varkappa}}\alpha_3 - \sqrt{\frac{k_4+\varkappa}{2\varkappa}}\alpha_4\right)$$

$$k\left(\sqrt{\frac{k_4+\varkappa}{2\varkappa}} - \sqrt{\frac{k_4-\varkappa}{2\varkappa}}\right)$$

$$\sqrt{\frac{k_4+\varkappa}{2\varkappa}}$$

1. Langrangian.

$$L = \left(\frac{1}{c}\frac{\partial \vec{V}^*}{\partial t} + \mathrm{grad}\, U^*\right)\left(\frac{1}{c}\frac{\partial \vec{V}}{\partial t} + \mathrm{grad}\, U\right) - \mathrm{curl}\, \vec{V}^* \mathrm{curl}\, \vec{V}$$
$$- \left(\mathrm{div}\, \vec{V}^* + \frac{1}{c}\frac{\partial U^*}{\partial t}\right)\left(\mathrm{div}\, \vec{V} + \frac{1}{c}\frac{\partial U}{\partial t}\right) - \varkappa^2(\vec{V}^*\vec{V} - U^*U)$$

$$\delta L \sim \delta \vec{V}^*\Big(-\frac{1}{c^2}\frac{\partial^2 \vec{V}}{\partial t^2} - \frac{1}{c}\mathrm{grad}\frac{\partial U}{\partial t} - \mathrm{curl}\,\mathrm{curl}\, \vec{V} + \mathrm{grad}\,\mathrm{div}\, \vec{V}$$
$$+ \frac{1}{c}\mathrm{grad}\frac{\partial U}{\partial t} - \varkappa^2 \vec{V}\Big)$$
$$+ \delta U^*\Big(-\frac{1}{c}\cancel{\mathrm{grad}}\frac{\partial \vec{V}}{\partial t} - \mathrm{div}\,\mathrm{grad}\, U + \frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} + \frac{1}{c}\mathrm{div}\frac{\partial \vec{V}}{\partial t}$$
$$+ \varkappa^2 U\Big) + \cdots$$

2. Canonical Variables

$$V_x^\dagger = \frac{\partial L}{\partial \frac{\partial V_x}{\partial t}} = \frac{1}{c^2}\frac{\partial V_x^*}{\partial t} + \frac{1}{c}\mathrm{grad}\, U^* \quad \text{etc.}$$

$$U^\dagger = \frac{\partial L}{\partial \frac{\partial U}{\partial t}} = -\frac{1}{c^2}\frac{\partial U^*}{\partial t} - \frac{1}{c}\mathrm{div}\, \vec{V}^*$$

3. Hamiltonian

$$H = \left(\frac{1}{c^2}\frac{\partial \vec{V}^*}{\partial t} + \frac{1}{c}\mathrm{grad}\, U^*\right)\frac{\partial \vec{V}}{\partial t} - \left(\frac{1}{c^2}\frac{\partial U^*}{\partial t} + \frac{1}{c}\mathrm{div}\, \vec{V}^*\right)\frac{\partial U}{\partial t}$$
$$+ \frac{\partial V^*}{\partial t}\left(\frac{1}{c^2}\frac{\partial \vec{V}}{\partial t} + \frac{1}{c}\mathrm{grad}\, U\right) - \frac{\partial U^*}{\partial t}\left(\frac{1}{c^2}\frac{\partial U}{\partial t} + \frac{1}{c}\mathrm{div}\, \vec{V}\right)$$
$$- L$$
$$= c^2 \vec{V}^\dagger(\vec{V}^{\dagger *} - \frac{1}{c}\mathrm{grad}\, U^*) - c^2 U^\dagger(U^{\dagger *} - \frac{1}{c}\mathrm{div}\, \vec{V}^*)$$
$$+ c^2(\vec{V}^\dagger - \frac{1}{c}\mathrm{grad}\, U^*)\vec{V}^{\dagger *} \cancel{- c(\vec{V} - \frac{1}{c}\mathrm{grad}\, U)U^{*\dagger *}}$$
$$- c^2(U^\dagger - \frac{1}{c}\mathrm{div}\, \vec{V}^*)U^{\dagger *}$$
$$- c^2\vec{V}^\dagger\vec{V}^{\dagger *} + \mathrm{curl}\, \vec{V}^* \mathrm{curl}\, \vec{V} + c^2 U^\dagger U^{*\dagger}$$
$$+ \varkappa^2(\vec{V}^*\vec{V} - U^*U),$$
$$= c^2(\vec{V}^\dagger\vec{V}^{\dagger *} - U^\dagger U^{\dagger *}) - c(\vec{V}^\dagger \mathrm{grad}\, U + \mathrm{grad}\, U^* \cdot \vec{V}^{\dagger *})$$
$$+ c(U^\dagger \mathrm{div}\, \vec{V} + \mathrm{div}\, \vec{V}^* \cdot U^{*\dagger}) + \mathrm{curl}\, \vec{V}^* \mathrm{curl}\, \vec{V}$$
$$+ \varkappa^2(\vec{V}^*\vec{V} - U^*U).$$

4. Gauge Invariance.

$$\left.\begin{array}{l} U \to U + \frac{1}{c}\frac{\partial \chi}{\partial t}. \\ \vec{V} \to \vec{V} \mp \cancel{\frac{1}{c}\frac{\partial \chi}{\partial t}}\ \mathrm{grad}\,\chi. \\ \mathrm{div}\,\vec{V}^{*} + \frac{1}{c}\frac{\partial U}{\partial t} \to \mathrm{div}\,\vec{V} + \frac{1}{c}\frac{\partial U}{\partial t} - \varkappa^2 \chi. \end{array}\right) \text{ non-invariant}$$

$$\left.\begin{array}{l} \varkappa \vec{F} = \frac{1}{c}\frac{\partial \vec{V}}{\partial t} + \mathrm{grad}\,U \\ \varkappa \vec{G} = \mathrm{curl}\,\vec{V} \end{array}\right\} \text{invariant}$$

$$\mathrm{div}\,\vec{V} + \frac{1}{c}\frac{\partial U}{\partial t} = \varkappa M$$

L : invariant.

$$L = \left(\frac{1}{c}\frac{\partial \vec{V}^{*}}{\partial t} + \mathrm{grad}\,U^{*}\right)\left(\qquad\right) - \mathrm{curl}\,\vec{V}^{*}\,\mathrm{curl}\,\vec{V} - \varkappa^2 M^{*}M$$
$$- \varkappa^2\left(\vec{V}^{*}\vec{V} - U^{*}U\right)$$

$$\delta L = \varkappa\,\delta\vec{V}^{*}\,\frac{1}{c}\cdot\left(-\frac{\partial \vec{F}}{\partial t} - \mathrm{curl}\,\vec{G}\right) - \varkappa^2\,\delta M^{*}\cdot M$$
$$-\varkappa\vec{V})$$
$$+ \varkappa\,\delta U^{*}\left(-\mathrm{div}\,\vec{F} + \varkappa U\right) + \cdots\cdots$$