~~The~~ Constitution of the Deuteron

The Scattering of ~~the~~ Neutron by ~~the~~ Proton

$$\left\{\frac{\hbar^2}{M}\Delta + (E-V)\right\}\psi = 0.$$

$$V = g^2\frac{e^{-\varkappa r}}{r}P$$

$$\psi = \frac{\chi(r)}{r}P_l^m(\cos\theta)e^{im\varphi}$$

$$\left\{\frac{d^2\chi}{dr^2} - \frac{l(l+1)}{r^2}\chi + \frac{M}{\hbar^2}\left(E + (-1)^l g^2\frac{e^{-\varkappa r}}{r}\right)\chi = 0.\right.$$

$l=0$: $$\frac{d^2\chi}{dr^2} + \left(A + B\frac{e^{-\varkappa r}}{r}\right)\chi = 0. \qquad A = \frac{ME}{\hbar^2} \qquad B = \frac{Mg^2}{\hbar^2}.$$

$\varkappa r = x$ $$\frac{d^2\chi}{dx^2} + \left(a + b\frac{e^{-x}}{x}\right)\chi = 0. \qquad a = \frac{ME}{\varkappa^2\hbar^2} \qquad b = \frac{Mg^2}{\varkappa\hbar^2}$$

$$\cancel{\chi = \int_C e^{ipx}u(p)\,dp} \qquad \alpha^2 = a.$$

$$\chi = e^{-\alpha x}u(x).$$

$$\frac{d\chi}{dx} = -\alpha\, e^{-\alpha x}u(x) + e^{-\alpha x}u'(x)$$

$$\frac{d^2\chi}{dx^2} = \alpha^2 e^{-\alpha x}u - 2\alpha e^{-\alpha x}u' + e^{-\alpha x}u''$$

$$\frac{d^2u}{dx^2} - 2\alpha\frac{du}{dx} + \frac{be^{-x}}{x}u = 0.$$

$$u(x) = \int_C e^{ipx}f(p)\,dp \qquad \frac{e^{(ip-1)x}}{x}$$

$$\cancel{\int_C\left(-p^2 - 2\alpha pi + \frac{be^{-x}}{x}\right)e^{ipx}f(p)\,dp}$$

$$\cancel{= \int_C(-p^2 - 2\alpha pi)e^{ipx}f(p)\,dp}$$

$$\int_C\{x(-p^2 - 2\alpha pi) + be^{-x}\}e^{ipx}f(p)\,dp$$

$$= \int_C\left\{(ip^2 - 2\alpha p)\frac{de^{ipx}}{dp}f(p) + be^{ipx}\sum_{n=0}\frac{(-1)^n}{n!}\frac{d^n f(p)}{dp^n}\right\}dp$$

$$= [(ip^2 - 2\alpha p)e^{ipx}f(p)] - \int_C e^{ipx}\left\{\frac{d}{dp}\{(ip^2 - 2\alpha p)f(p)\} + b\sum\right\}dp.$$

e^{-x} 1. にとれば.

$$f(p) = (p + 2\alpha i)^{\beta} p^{\gamma}$$

$$\frac{d}{dp}\, i\,(p + 2\alpha i)^{\beta+1} p^{\gamma+1} = \cancel{i(\beta+1)(\gamma+1) f(p)}.$$

$$= i\{(\beta+1)p + (\gamma+1)(p + 2\alpha i)\}(\ )^{\beta}(\ )^{\gamma}$$

$\cancel{(\beta+1)(\gamma+1)}$

$\cancel{\beta+\gamma+2 = bi}$.

$$= -b(\ )^{\beta}(\ )^{\gamma}.$$

$$\beta = -\gamma - 2.\quad 2\alpha(\gamma+1) = b.$$

$\cancel{f(p) = e^{\delta p} g(p)}$:

$E > 0$, $\alpha > 0$ (real) ; $\gamma + 1 = \frac{b}{2\alpha} > 0$. real.

$$\beta + 1 = \frac{-b}{2\alpha}.$$

$y = \cancel{\ } \dfrac{e^{-x}}{x}$

$$\frac{d\chi}{dx} = \frac{(1-x)}{\cancel{(x+1)}} e^{-x} \frac{d\chi}{dy}$$

$$\frac{d^2\chi}{dx^2} = \cancel{(x+1)e^{-2x}} \frac{d^2\chi}{dy^2} + 2(x+2)e^{x}\frac{d\chi}{dy} \cancel{*} (x+1)e^{x}.$$

$\cancel{(1-x)^2 e^{-2x}}$

$$\frac{d\chi}{dx} = -\frac{e^{-x}(1+x)}{x^2}\frac{d\chi}{dy}$$

$$\frac{d^2\chi}{dx^2} = \left(\frac{e^{-x}(1+x)}{x^2}\right)^2 \frac{d^2\chi}{dy^2} + \left\{\frac{1+x}{x^2} + \frac{2(1+x)}{x^3} - \frac{1}{x^2}\right\} e^{-x}\frac{d\chi}{dy}$$

$$= e^{-2x}\frac{(1+x)^2}{x^4}\cdot\frac{d^2\chi}{dy^2} + \frac{\cancel{x} + x^2 + 2x + x^2 - \cancel{x}}{x^3}$$

$$\parallel \quad \frac{2(x+1)}{x^2} e^{-x}\frac{d\chi}{dy}.$$

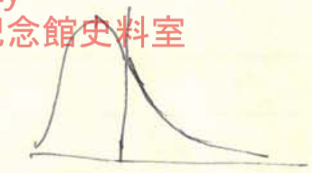
$$e^{x}\cdot x^2\left\{\frac{d^2\chi}{dx^2} + \left(a + \frac{b\,e^{-x}}{x}\right)\chi\right\} = e^{-x}\frac{(1+x)^2}{x^3}\frac{d^2\chi}{dy^2} + (e^{x}x^2 a + b\cancel{x})\chi.$$

$$= y\cdot\left(\frac{1+x}{x}\right)^2 \frac{d^2\chi}{dy^2} + (y\cdot x^2 a + b)\chi = 0.$$

x

$a < 0 \quad a + b\frac{e^{-x_0}}{x_0} = 0 \quad \boxed{x_0 e^{x_0} = \frac{b}{a'}}$

$= -a' \lessgtr 0 \; (a' > 0)$



$$\chi = C \sin kx \qquad x < x_0$$

$$= e^{-\alpha x} \qquad \alpha^2 = a'. \qquad x > x_0.$$

$$C \sin kx_0 = e^{-\alpha x_0}$$

$$kC \cos kx_0 = -\alpha e^{-\alpha x_0}.$$

$$\therefore \tan kx_0 = -\frac{k}{\alpha} \qquad \boxed{\tan^2 kx_0 = \frac{k^2}{a'}}$$

$$C^2 = \left(1 + \frac{\alpha^2}{k^2}\right) e^{-2\alpha x_0}.$$

$$-k^2 C \sin kx =$$

$$\{a'\} = \int_0^\infty \chi\left(\frac{d^2\chi}{dx^2} + \frac{be^{-x}}{x}\right)\chi \, dx = \cancel{-k^2C^2 \sin^2 kx}$$

$$C^2 \int_0^{x_0} \left(-k^2 + \frac{be^{-x}}{x}\right) \sin^2 kx \, dx$$

$$+ \int_{x_0}^\infty \left(-k^2 + \frac{be^{-x}}{x}\right) e^{-\alpha x} \, dx.$$

$$\chi = xe^{-\alpha x}. \qquad \frac{d\chi}{dx} = (1 - \alpha x)e^{-\alpha x}$$

$$\frac{d^2\chi}{dx^2} = \{(1 - \alpha x)(-\alpha) - \alpha\} e^{-\alpha x} = \cancel{\{(1-\alpha) + \alpha^2 x\}}$$

$$= \alpha(\alpha x - 2) e^{-\alpha x}$$

$$= \alpha\left(\alpha - \frac{2}{x}\right)\chi.$$