On the Interaction of Elementary Particles. III.

Jan. 6, 1937.

Linear Equations for the New Field.

8
$$\begin{cases} \frac{1}{c}\frac{\partial \vec{G}}{\partial t} + \mathrm{curl}\,\vec{F} = 0 \quad (1) & \mathrm{div}\,\vec{G} = 0 \quad (2) \\ \boxed{\frac{1}{c}\frac{\partial \vec{F}}{\partial t} - \mathrm{curl}\,\vec{G} - \kappa\vec{K} = 0} \quad (3) & \boxed{\mathrm{div}\,\vec{F} + \kappa K_0 = 0} \quad (4) \end{cases}$$

7 6
$$\begin{cases} \frac{1}{c}\frac{\partial K_0}{\partial t} + \mathrm{div}\,\vec{K} = 0 \quad (5) & \\ \boxed{\frac{1}{c}\frac{\partial \vec{K}}{\partial t} + \mathrm{grad}\,K_0 + \kappa\vec{F} = 0} \quad (6) & \boxed{\mathrm{curl}\,\vec{K} - \kappa\vec{G} = 0} \quad (7) \end{cases}$$

$(\vec{G}\ \vec{F})$: six vector $\qquad (K_0, \vec{K})$: four vector

之等の式から iteration によって、各 field quantities が d'Alembertian を出る.

例へば.
(1) $\frac{1}{c}\frac{\partial}{\partial t}\times(1) - \mathrm{curl}\times(3)$ は

$$\frac{1}{c^2}\frac{\partial^2 \vec{G}}{\partial t^2} + \frac{1}{c}\frac{\partial}{\partial t}\mathrm{curl}\,\vec{F} - \mathrm{curl}\,\frac{1}{c}\frac{\partial \vec{F}}{\partial t} + \mathrm{grad}\,\mathrm{div}\,\vec{G} - \Delta\vec{G} + \kappa\,\mathrm{curl}\,\vec{K} = 0$$

or (7) により

$$= \frac{1}{c^2}\frac{\partial^2 \vec{G}}{\partial t^2} - \Delta\vec{G} + \kappa^2\vec{G} = 0.$$

Potential の導入.

(2): $\vec{G} = \mathrm{curl}\,\vec{U} \qquad$ (1): $\mathrm{curl}\left(\frac{1}{c}\frac{\partial \vec{U}}{\partial t} + \vec{F}\right) = 0$

∴ $\vec{F} = -\frac{1}{c}\frac{\partial \vec{U}}{\partial t} - \mathrm{grad}\,U_0. \qquad \vec{U}, U_0$: four vector.

(7): $\mathrm{curl}(\vec{K} - \kappa\vec{U}) = 0$: $\vec{K} = \kappa\vec{U} + \mathrm{grad}\,S$

(6): $\frac{\kappa}{c}\frac{\partial \vec{U}}{\partial t} + \mathrm{grad}\left(\frac{1}{c}\frac{\partial S}{\partial t} + K_0\right) - \frac{\kappa}{c}\frac{\partial \vec{U}}{\partial t} - \kappa\,\mathrm{grad}\,U_0 = 0.$

∴ $K_0 = \kappa U_0 - \frac{1}{c}\frac{\partial S}{\partial t} + \cancel{c(t)}.$

$$\left.\begin{aligned} &(5):\ \frac{\kappa}{c}\frac{\partial U_0}{\partial t} - \frac{1}{c^2}\frac{\partial^2 S}{\partial t^2} + \frac{1}{c}\frac{\partial c(t)}{\partial t} + \kappa\,\mathrm{div}\,\vec{U} + \Delta S = 0. \\ &(4):\ -\frac{1}{c}\frac{\partial}{\partial t}\mathrm{div}\,\vec{U} - \Delta U_0 + \kappa^2 U_0 - \frac{\kappa}{c}\frac{\partial S}{\partial t} + \cancel{\kappa c(t)} = 0 \\ &(3):\ -\frac{1}{c^2}\frac{\partial^2 \vec{U}}{\partial t^2} - \frac{1}{c}\frac{\partial}{\partial t}\mathrm{grad}\,U_0 - \mathrm{grad}\,\mathrm{div}\,\vec{U} + \Delta\vec{U} \end{aligned}\right\}$$

$$-\kappa^2\vec{U} - \kappa\,\mathrm{grad}\,S = 0.$$

or
$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \kappa^2\right)\vec{U} = \mathrm{grad}\left(\frac{1}{c}\frac{\partial U_0}{\partial t} + \mathrm{div}\,\vec{U} + \kappa S\right)$$
$$(\quad 〃 \quad)\,U_0 = -\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{c}\frac{\partial U_0}{\partial t} + \mathrm{div}\,\vec{U} + \kappa S\right) \cancel{+\kappa c(t)}$$
$$(\quad 〃 \quad)\,S = -\kappa\left(\frac{1}{c}\frac{\partial U_0}{\partial t} + \mathrm{div}\,\vec{U} + \kappa S\right) \cancel{-\frac{1}{c}\frac{\partial c(t)}{\partial t}}$$

~~$C(t)=0$?~~

∴ $(U_0, \vec{U}, S)$ はそれぞれ

は $\boxed{\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \varkappa^2\right)\begin{Bmatrix}\vec{U}\\ U_0\\ S\end{Bmatrix} = 0}$

及び supplementary condition

(S) $\boxed{\frac{1}{c}\frac{\partial U_0}{\partial t} + \operatorname{div}\vec{U} + \varkappa S = 0}$

を仮定しておけばよい。

$$\delta(\operatorname{curl}\tilde{\vec{U}})_x(\operatorname{curl}\vec{U})_x = \delta\left(\frac{\partial \tilde{U}_y}{\partial z} - \frac{\partial \tilde{U}_z}{\partial y}\right) \times \left(\frac{\partial U_y}{\partial z} - \frac{\partial U_z}{\partial y}\right) \simeq -\delta\tilde{U}_y\left(\frac{\partial^2 U_y}{\partial z^2} - \frac{\partial^2 U_z}{\partial y \partial z}\right) - \delta\tilde{U}_z\left(\frac{\partial^2 U_z}{\partial y^2} - \frac{\partial^2 U_y}{\partial z \partial y}\right)$$

$$= -(\delta\tilde{\vec{U}}, \Delta\vec{U}) + \tilde{\vec{U}}\operatorname{grad}\cdot\operatorname{div}\vec{U}$$

$$\boxed{\begin{aligned} \vec{F} &= -\frac{1}{c}\frac{\partial\vec{U}}{\partial t} - \operatorname{grad} U_0 \qquad & K_0 &= \varkappa U_0 - \frac{1}{c}\frac{\partial S}{\partial t}, \\ \vec{G} &= \operatorname{curl}\vec{U} & \vec{K} &= \varkappa\vec{U} + \operatorname{grad} S \end{aligned}}$$

Lagrangian for the New Field.

$$L = \tilde{\vec{F}}\vec{F} - \tilde{\vec{G}}\vec{G} - \tilde{\vec{K}}\vec{K} + \tilde{K}_0 K_0$$

$$= \left(\frac{1}{c}\frac{\partial\tilde{\vec{U}}}{\partial t} + \operatorname{grad}\tilde{U}_0\right)\left(\frac{1}{c}\frac{\partial\vec{U}}{\partial t} + \operatorname{grad}U_0\right) - \operatorname{curl}\tilde{\vec{U}}\operatorname{curl}\vec{U}$$

$$- (\varkappa\tilde{\vec{U}} + \operatorname{grad}\tilde{S})(\varkappa\vec{U} + \operatorname{grad}S) + \left(\varkappa\tilde{U}_0 - \frac{1}{c}\frac{\partial\tilde{S}}{\partial t}\right)\left(\varkappa U_0 - \frac{1}{c}\frac{\partial S}{\partial t}\right)$$

$$\begin{aligned}\delta L = {} & \delta\tilde{U}_0\left(-\frac{1}{c}\frac{\partial}{\partial t}\operatorname{div}\vec{U} - \Delta U_0 + \varkappa^2 U_0 - \frac{\varkappa}{c}\frac{\partial S}{\partial t}\right) \\ & + \delta\tilde{\vec{U}}\Big(-\frac{1}{c^2}\frac{\partial^2\vec{U}}{\partial t^2} - \operatorname{grad}\frac{1}{c}\frac{\partial U_0}{\partial t} + \Delta\vec{U} - \operatorname{grad}\operatorname{div}\vec{U} \\ & \qquad - \varkappa^2\vec{U} - \varkappa\operatorname{grad}S\Big) \\ & + \delta\tilde{S}\left(\Delta S + \varkappa\operatorname{div}\vec{U} + \frac{\varkappa}{c}\frac{\partial U_0}{\partial t} - \frac{1}{c^2}\frac{\partial^2 S}{\partial t^2}\right) \\ = {} & 0.\end{aligned}$$

supplementary condition (S) を assume すれば、$\delta L = 0$ から $U_0, \vec{U}, S$ に対する wave equation が出る。

若しくは $L = \tilde{\vec{F}}\vec{F} - \tilde{\vec{G}}\vec{G} - \tilde{\vec{K}}\vec{K} + \tilde{K}_0 K_0 - \left(\frac{1}{c}\frac{\partial\tilde{U}_0}{\partial t} + \operatorname{div}\tilde{\vec{U}} + \varkappa\tilde{S}\right) \times \left(\frac{1}{c}\frac{\partial U_0}{\partial t} + \operatorname{div}\vec{U} + \varkappa S\right)$

とおけば、 $= \delta\tilde{U}_0\left(-\frac{1}{c}\frac{\partial}{\partial t}\operatorname{div}\vec{U} - \Delta U_0 + \varkappa^2 U_0 - \frac{\varkappa}{c}\frac{\partial S}{\partial t} + \frac{1}{c}\frac{\partial^2 U_0}{c\,\partial t^2} + \frac{1}{c}\frac{\partial}{\partial t}\operatorname{div}\vec{U} + \frac{\varkappa}{c}\frac{\partial S}{\partial t}\right) + \cdots$

で正しい wave equation が出る。

$$\frac{1}{c}\frac{\partial U_0}{\partial t}+\mathrm{div}\,\vec{U}+\varkappa S=0$$

此の條件の意味を考へて見ると、

$\vec{F},\vec{G},\ K_0,\vec{K}$ は

$$\left.\begin{aligned} U_0 &= U_0'+\frac{1}{c}\frac{\partial \chi}{\partial t}\\ \vec{U} &= \vec{U}'-\mathrm{grad}\,\chi \\ S &= S'+\varkappa\chi \end{aligned}\right\}$$

$$\left(\Delta-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\varkappa^2\right)\chi=0$$

なる transformation に対して invariant である。これは generalized gauge transformation と考へられる

この時には Langrangian ~~數~~ も矢張り invariant である ~~………~~,

$$\because\quad \frac{1}{c}\frac{\partial U_0}{\partial t}+\mathrm{div}\,\vec{U}+\varkappa S=\left(\frac{1}{c}\frac{\partial U_0'}{\partial t}+\mathrm{div}\,\vec{U}'+\varkappa S'\right)-\left(\Delta-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\varkappa^2\right)\chi$$

$$\frac{\partial L}{\partial \frac{\partial U_0}{\partial t}}=-\frac{1}{c}\left(\frac{1}{c}\frac{\partial \tilde{U}_0}{\partial t}+\mathrm{div}\,\tilde{\vec{U}}+\varkappa\tilde{S}\right)=\tilde{U}_0^{\dagger}=$$

$$\frac{\partial L}{\partial \frac{\partial U_x}{\partial t}}=\frac{1}{c}\left(\frac{1}{c}\frac{\partial \tilde{U}_x}{\partial t}+\frac{\partial \tilde{U}_0}{\partial x}\right)=U_x^{\dagger}=-\tilde{F}_x/c$$

$$\frac{\partial L}{\partial \frac{\partial S}{\partial t}}=\frac{1}{c}\left(\frac{1}{c}\frac{\partial \tilde{S}}{\partial t}-\varkappa\tilde{U}_0\right)=S^{\dagger}=-\frac{\tilde{K}_0}{c}$$

$$L=c^2\dot{\tilde{\vec{U}}}_x^{\dagger}\dot{\vec{U}}_x^{\dagger}-\mathrm{curl}\,\tilde{\vec{U}}\;\mathrm{curl}\,\vec{U}+c^2S^{\dagger}\tilde{S}^{\dagger}-(\varkappa\tilde{\vec{U}}+\mathrm{grad}\,\tilde{S})(\varkappa\vec{U}+\mathrm{grad}\,S)$$
$$-\;c^2\dot{\tilde{U}}_0^{\dagger}\dot{U}_0^{\dagger}$$

$$U_0=\sum_{\vec{k}\,(k_0>0)}\left\{a^{(+)}_{0,\vec{k}}\,e^{i(\vec{k}\vec{r}-ck_0t)}+a^{(-)}_{0,\vec{k}}\,e^{-i(\vec{k}\vec{r}-ck_0t)}\right\}$$

$$\vec{U}_0=\sum_{\vec{k}\,(k_0>0)}\left\{\left(u^{(+)}_{1,\vec{k}}\vec{e}_{1,\vec{k}}+u^{(+)}_{2,\vec{k}}\vec{e}_{2,\vec{k}}+u^{(+)}_{3,\vec{k}}\vec{e}_{3,\vec{k}}\right)e^{i(\vec{k}\vec{r}-ck_0t)}\right.$$
$$\left.-\left(u^{(-)}_{1,\vec{k}}\vec{e}_{1,\vec{k}}+u^{(-)}_{2,\vec{k}}\vec{e}_{2,\vec{k}}+u^{(-)}_{3,\vec{k}}\vec{e}_{3,\vec{k}}\right)e^{-i(\vec{k}\vec{r}-ck_0t)}\right\}$$

$$S=\sum_{\vec{k}\,(k_0>0)}\left\{s^{(+)}_{\vec{k}}\,e^{i(\vec{k}\vec{r}-ck_0t)}+s^{(-)}_{\vec{k}}\,e^{-i(\vec{k}\vec{r}-ck_0t)}\right\}$$

$\vec{e}_1\,\vec{e}_2$

u_{1k}　　u'_{1k}

$$\frac{1}{c}\frac{\partial U_0}{\partial t} = -c \tilde{U}_0^{+} - g \,\mathrm{div}\, \vec{U} - \varkappa S$$

$$c U_0^{+} = \left(\frac{1}{c}\frac{\partial \tilde{U}_0}{\partial t} + \ldots + \varkappa \tilde{S}\right)$$

$$= \sum\left\{ k_0 \tilde{u}_{0,k}^{(+)} e^{-i(\vec{k}\vec{r}-ck_0t)} - k_0 \tilde{u}_{0,k}^{(-)} e^{+i(\vec{k}\vec{r}-ck_0t)} \right\}$$

$$+ i \sum\left\{ (k \tilde{u}_{1k}^{(+)} e^{-i(\vec{k}\vec{r}-ck_0t)} - k \tilde{u}_{1k}^{(-)} e^{+i(\vec{k}\vec{r}-ck_0t)}) \right\}$$

$$+ \varkappa \sum\left\{ \tilde{s}_{0k}^{(+)} e^{-i(\vec{k}\vec{r}-ck_0t)} + \tilde{s}_{0k}^{(-)} e^{i(\vec{k}\vec{r}-ck_0t)} \right\}$$

$$= \sum\left\{ (-ik_0 \tilde{u}_{0k}^{(+)} + ik \tilde{u}_{1k}^{(+)} + \varkappa \tilde{s}_k^{(+)}) e^{-i(\vec{k}\vec{r}-ck_0t)} \right.$$

$$\left. + (ik_0 \tilde{u}_{0k}^{(-)} - ik \tilde{u}_{1k}^{(-)} + \varkappa \tilde{s}_k^{(-)}) e^{i(\vec{k}\vec{r}-ck_0t)} \right\}$$

$$c \vec{U}^{+} = \left(\frac{1}{c}\frac{\partial \tilde{\vec{U}}}{\partial t} + \mathrm{grad}\, \tilde{U}_0\right) \qquad \frac{\partial U}{\partial t} =$$

$$= \sum\left\{ +ik_0 (\tilde{u}_{1k}^{(+)} \vec{e}_{1k} + \tilde{u}_{2k}^{(+)} \vec{e}_{2k} + \tilde{u}_{3k}^{(+)} \vec{e}_{3k}) e^{-i(\vec{k}\vec{r}-ck_0t)} \right.$$

$$\left. - ik_0 (\tilde{u}_{1k}^{(-)} \vec{e}_{1k} + \qquad) e^{i(\vec{k}\vec{r}-ck_0t)} \right\}$$

$$+ \sum\left\{ -ik \tilde{u}_{1k}^{(+)} \vec{e}_{1k} e^{-i(\vec{k}\vec{r}-ck_0t)} + ik \tilde{u}_{1k}^{(-)} \vec{e}_{1k} e^{i(\quad)} \right\}$$

$$= \quad i(k_0 - k)$$

$$c S^{+} = \frac{1}{c}\frac{\partial \tilde{S}}{\partial t} - \varkappa \tilde{U}_0$$

$$= \sum (ik_0 \tilde{S}_k^{(+)} e^{-i(\vec{k}\vec{r}-ck_0t)} - ik_0 \tilde{S}_k^{(-)} e^{+i(\vec{k}\vec{r}-ck_0t)})$$

$$- \sum (\varkappa \tilde{u}_{0k}^{(+)} e^{-i(\quad)} + \varkappa \tilde{u}_{0k}^{(-)} e^{+i(\quad)})$$

$$\mathrm{curl}\, \vec{U} = \sum (i u_{1k}^{(+)} [\vec{k}, \vec{e}_{1k}] + i u_{2k}^{(+)} [\vec{k}, \vec{e}_{2k}] + i u_{3k}^{(+)} [\vec{k}, \vec{e}_{3k}])$$

$$e \qquad + \ldots \qquad \vec{k} = k \vec{e}_{1k}$$

$$k_y e_{i2} \frac{\partial}{\partial y} \qquad = \sum (ik u_{2k}^{(+)} \vec{e}_{3k} - ik u_{3k}^{(+)} \vec{e}_{2k}) e^{i(\vec{k}\vec{r} - k_0 ct)}$$

$$- \sum (ik u_{2k}^{(-)} \vec{e}_{3k} - ik u_{3k}^{(-)} \vec{e}_{2k}) e^{-i(\quad)}$$

$$\varkappa \vec{U} + \mathrm{grad}\, S = \sum (\varkappa u_{1k}^{(+)} \vec{e}_{1k} + \varkappa u_{2k}^{(+)} \vec{e}_{2k} + \varkappa u_{3k}^{(+)} \vec{e}_{3k}) e^{i(\vec{k}\vec{r} - k_0 ct)} + (\quad)$$

$$+ \sum (ik s_k^{(+)} \vec{e}_{1k} e^{i(\vec{k}\vec{r} - ck_0 t)} - ik s_k^{(-)} \vec{e}_{1k} e^{\cdots})$$

$$H = U_0^\dagger \frac{\partial U_0}{\partial t} + \vec{U}^\dagger \frac{\partial \vec{U}}{\partial t} + S^\dagger \frac{\partial S}{\partial t} + \tilde{U}_0^\dagger \frac{\partial \tilde{U}_0}{\partial t} + \tilde{\vec{U}}^\dagger \frac{\partial \tilde{\vec{U}}}{\partial t} + \tilde{S}^\dagger \frac{\partial \tilde{S}}{\partial t} - L$$

$$= c^2 U_0^\dagger \left(-\hat{U}_0^\dagger - \tfrac{1}{c}\operatorname{div}\vec{U} - \tfrac{\kappa}{c} S\right) + c^2 \vec{U}^\dagger \left(\hat{\tilde{\vec{U}}}{}^\dagger - \tfrac{1}{c}\operatorname{grad} U_0\right)$$

$$+ c^2 S^\dagger \left(\hat{S}^\dagger + \tfrac{\kappa}{c} U_0\right)$$

$$+ c^2 \tilde{U}_0^\dagger \left(-U_0^\dagger - \tfrac{1}{c}\operatorname{div}\tilde{\vec{U}} - \tfrac{\kappa}{c}\tilde{S}\right) + c^2 \tilde{\vec{U}}^\dagger \left(\vec{U}^\dagger - \tfrac{1}{c}\operatorname{grad}\tilde{U}_0\right)$$

$$+ c^2 \tilde{S}^\dagger \left(S^\dagger + \tfrac{\kappa}{c}\tilde{U}_0\right)$$

$$- c^2 \vec{U}^\dagger \tilde{\vec{U}}^\dagger + \operatorname{curl}\tilde{\vec{U}}\,\operatorname{curl}\vec{U} - c^2 S^\dagger \tilde{S}^\dagger + (\kappa\tilde{\vec{U}} + \operatorname{grad}\tilde{S})(\kappa\vec{U} + \operatorname{grad} S)$$

$$+ c^2 \tilde{U}_0^\dagger U_0^\dagger$$

$$= -c^2 \tilde{U}_0^\dagger U_0^\dagger + c^2 \tilde{\vec{U}}^\dagger \vec{U}^\dagger + c^2 \tilde{S}^\dagger S^\dagger +$$

$v_k \quad v_k$

$$\bar{H} = \sum \Big\{\left(-ik_0 \tilde{u}_{0k}^{(+)} + ik\tilde{u}_{1k}^{+} + \kappa \tilde{s}_k^{(+)}\right)\left(-ik_0 u_{0k}^{(+)}\right) + \left(ik_0 \tilde{u}_{0k}^{(-)} - ik\tilde{u}_{1k}^{(-)} + \kappa\tilde{s}_k^{(-)}\right)$$

$$\times \left(ik_0 u_{0k}^{(-)}\right)\Big\} \ \cancel{+\text{comp. conj}} + \text{comp conj.}$$

$$+ \sum \Big\{\Big(ik_0 \tilde{u}_{1k}^{(+)} u_{1k}^{(+)} + ik_0 \tilde{u}_{2k}^{(+)} u_{2k}^{(+)} + ik_0 \tilde{u}_{3k}^{(+)} u_{3k}^{(+)}$$

$$- ik\tilde{u}_{1k}^{(+)} u_{1k}^{(+)}\Big)\cancel{(ik_0)} + \Big(-ik_0\tilde{u}_{1k}^{(-)} u_{1k}^{(-)} - ik_0 \tilde{u}_{2k}^{(-)} u_{2k}^{(-)} - ik_0 \tilde{u}_{3k}^{(-)} u_{3k}^{(-)}$$

$$+ ik\tilde{u}_{1k}^{(-)} u_{1k}^{(-)}\Big)\cancel{(+ik_0)}\Big\} \ \cancel{+\text{comp conj}} + \text{comp conj} +$$

$$+ \sum \Big\{\left(ik_0 \tilde{S}_k^{(+)} - \cancel{ik_0}\,\kappa\tilde{u}_{0k}^{(+)}\right)\left(\cancel{i k_0}\, s_k^{(+)}\right)$$

$$+ \left(-ik_0 \tilde{S}_k^{(-)} - \kappa\tilde{u}_{0k}^{(-)}\right)\left(+ik_0\, s_k^{(-)}\right)\Big\} + \text{comp. conj.} - L.$$

$$\cancel{= \sum k_0 - k_0^2 \tilde{u}_{0k}^{(+)} u_{0k}^{(+)} - \quad + \text{conj co}}$$

$$= \sum k_0^2 \left(-\tilde{u}_{0k}^{(+)} u_{0k}^{(+)} + \tilde{u}_{1k}^{(+)} u_{1k}^{(+)} + \tilde{u}_{2k}^{(+)} u_{2k}^{(+)} + \tilde{u}_{3k}^{(+)} u_{3k}^{(+)} + \tilde{S}_k^{(+)} S_k^{(+)}\right)$$

$$\cancel{+\text{conj. comp.}} \qquad + (-) + \text{comp. conj}$$

$$+ \sum \Big\{k_0 k\,\tilde{u}_{1k}^{(+)} u_{0k}^{(+)} - \cancel{i} k_0 \kappa\,\tilde{S}_k^{(+)} u_{0k}^{(+)} - k_0 k\,\tilde{u}_{1k}^{(+)} u_{1k}^{(+)}$$

$$+ ik_0 \kappa\,\tilde{u}_{0k}^{(+)} s_k^{(+)}\Big\} \ \cancel{+\text{conj comp.}} + (-) + \text{comp. conj.}$$

$$-\bar{L},$$

165.50
8.275
157.22

$$\bar{L} = c^2 \tilde{\vec{U}}^{+} \tilde{\vec{U}}^{+} - c^2 U_0^{+} \tilde{U}_0^{+} - \text{curl}\,\tilde{\vec U}\,\text{curl}\,\vec U + (\kappa \tilde{S} \ldots)(\ldots) - (\kappa \tilde{\vec U} + \text{grad}\,\tilde S)(\kappa \vec U + \text{grad}\, S)$$

$$= -\sum (-ik_0 \tilde u_{0k}^{(+)} + ik \tilde u_{1k}^{(+)} + \kappa \tilde S_k^{(+)})(ik_0 u_{0k}^{(+)} - iku_{1k}^{(+)} + \kappa S_k^{(+)})$$

$$+ \sum (k_0^2 (\tilde u_{1k}^{(+)} u_{1k}^{(+)}) + \tilde u_{2k}^{(+)} u_{2k}^{(+)} + \tilde u_{3k}^{(+)} u_{3k}^{(+)}) \quad {+k^2} \quad +\sum k_0^2$$

$$+ \sum -2k_0 k\, \tilde u_{1k}^{(+)} u_{1k}^{(+)}$$

$$- \sum k^2 (\tilde u_{2k}^{(+)} u_{2k}^{(+)} + \tilde u_{3k}^{(+)} u_{3k}^{(+)})$$

$$+ \sum (ik_0 \tilde S_k^{(+)} - \kappa \tilde u_{0k}^{(+)})(-ik_0 S_k^{(+)} - \kappa u_{0k}^{(+)})$$

$$- \sum \{(\kappa \tilde u_{1k}^{(+)} - ik \tilde S_k^{(+)})(\kappa u_{1k}^{(+)} + ik S_k^{(+)}) + \kappa^2 \tilde u_{2k}^{(+)} u_{2k}^{(+)} + \kappa^2 \tilde u_{3k}^{(+)} u_{3k}^{(+)}\}$$

$$\bar{H} = \sum \{ -k_0^2 (\tilde u_{0k}^{(+)} u_{0k}^{(+)}) + 2k_0^2 (\tilde u_{1k}^{(+)} u_{1k}^{(+)} + \tilde u_{2k}^{(+)} u_{2k}^{(+)} + \tilde u_{3k}^{(+)} u_{3k}^{(+)}) + k_0^2 \tilde S_k^{(+)} S_k^{(+)} \}$$

$$+ \sum \{ \cancel{k_0 k\, \tilde u_{1k}^{(+)} u_{0k}^{(+)}} - \cancel{ik_0 \kappa \tilde S_k^{(+)} u_{0k}^{(+)}} - \cancel{k_0 k\, \tilde u_{1k}^{(+)} u_{1k}^{(+)}} + \cancel{ik_0\kappa \tilde u_{0k}^{(+)} S_k^{(+)}} + \cancel{k_0 k\, \tilde u_{0k}^{(+)} u_{1k}^{(+)}} + \cancel{ik_0 \kappa \tilde u_{0k}^{(+)} S_k^{(+)}} - \cancel{k_0 k\, \tilde u_{1k}^{(+)} u_{1k}^{(+)}} - \cancel{ik_0\kappa \tilde S_k^{(+)} u_{0k}^{(+)}} \}$$

$$+ \sum \{ \cancel{k_0 k\, \tilde u_{0k}^{(+)} u_{1k}^{(+)}} + k^2 \tilde u_{1k}^{(+)} u_{1k}^{(+)} + \kappa^2 \tilde S_k^{(+)} S_k^{(+)} - ik_0 \kappa \tilde u_{0k}^{(+)} S_k^{(+)}$$
$$\cancel{+ k_0 k\, \tilde u_{1k}^{(+)} u_{0k}^{(+)}} + ik\kappa \tilde u_{1k}^{(+)} S_k^{(+)} - ik\kappa \tilde S_k^{(+)} u_{1k}^{(+)} + \cancel{ik_0 \kappa \tilde S_k^{(+)} u_{0k}^{(+)}} \; \tilde S_k^{(+)} \tilde u_{0k}^{(+)} \}$$

$$- \cancel{k^2 \tilde u_{1k}^{(+)} u_{1k}^{(+)}} + \cancel{2k_0 k\, \tilde u_{1k}^{(+)} u_{1k}^{(+)}}$$

$$- \cancel{ik_0 \kappa \tilde u_{0k}^{(+)} S_k^{(+)}} + ik_0 \kappa \tilde S_k^{(+)} u_{0k}^{(+)} - \kappa^2 \tilde u_{0k}^{(+)} u_{0k}^{(+)}$$

$$+ \kappa^2 \tilde u_{1k}^{(+)} u_{1k}^{(+)} + ik\kappa \tilde u_{1k}^{(+)} S_k^{(+)} - ik\kappa \tilde S_k^{(+)} u_{1k}^{(+)} + k^2 \tilde S_k^{(+)} S_k^{(+)}$$

$$\cancel{+ \kappa^2 \tilde u_{2k}^{(+)} u_{2k}^{(+)} + \kappa^2 \tilde u_{3k}^{(+)} u_{3k}^{(+)}}$$

$$= \sum (k_0^2 + \kappa^2)(-\tilde u_{0k}^{(+)} u_{0k}^{(+)} + \tilde S_k^{(+)} S_k^{(+)})$$

$$+ \sum 2ik\kappa\, (\tilde u_{1k}^{(+)} S_k^{(+)} - \tilde S_k^{(+)} u_{1k}^{(+)})$$

$$+ 2\sum k_0^2 (\tilde u_{2k}^{(+)} u_{2k}^{(+)} + \tilde u_{3k}^{(+)} u_{3k}^{(+)} + u_{1k}^{(+)} \tilde u_{1k}^{(+)} + u_{3k}^{(+)} \tilde u_{3k}^{(+)})$$

Vertauschungsrelationen:

$$U_0 U_0^{+\prime} - U_0^{+\prime} U_0 = i\hbar\delta. \quad \text{etc}$$

一般に

$$\sum\left(a_k^{(+)} e^{i(\vec{k}\vec{r}-ck_0t)} + a_k^{(-)} e^{-i(\vec{k}\vec{r}-ck_0t)}\right)\sum\left(\tilde{b}_k^{(+)} e^{-i\vec{k}\vec{r}'\cdots} + \tilde{b}_k^{(-)} e^{+\cdots}\right)$$

$$-\sum\left(b_k \qquad\right)\sum\left(a_k \qquad\right) + (-)$$

$$= \sum\left(a_k^{(+)}\tilde{b}_k^{(+)} - \tilde{b}_k^{(+)} a_k^{(+)}\right) e^{i\{\vec{k}(\vec{r}-\vec{r}')-ck_0(t-t')\}} + \sum_{f}\left(a_k^{(+)}\tilde{b}_k^{(-)} - \tilde{b}_k^{(-)} a_k^{(-)}\right) e^{-i\vec{k}(\vec{r}-\vec{r}')\cdots}$$

$$+ \cdots \qquad + (-) \cdots$$

之が δ-function x iℏ であらねばならぬ.
t=t' においては

$$a_k^{(+)}\tilde{b}_k^{(+)} - \tilde{b}_k^{(+)} a_k^{(+)} = i\hbar/2 \quad \text{etc}$$

$$a_k^{(-)}\tilde{b}_k^{(-)} - \tilde{b}_k^{(-)} a_k^{(-)} = i\hbar/2 \quad \text{etc.}$$

その他の pair は Vertauschbar でなければならぬ.

$$u_{0k}^{(+)}\left(-ik_0\tilde{u}_{0k}^{(+)} + ik\tilde{u}_{1k}^{(+)} + \varkappa\tilde{s}_k^{(+)}\right) - \left(-ik_0\tilde{u}_{0k}^{(+)} + ik\tilde{u}_{1k}^{(+)} + \varkappa\tilde{s}_k^{(+)}\right)u_{0k}^{(+)} = \frac{i\hbar c}{2\epsilon}$$

$$u_{1k}^{(+)}\left(i(k_0-k)\tilde{u}_{1k}^{(+)}\right) - \left(i(k_0-k)\tilde{u}_{1k}^{(+)}\right)u_{1k}^{(+)} = \frac{i\hbar c}{2\epsilon}.$$

$$u_{2k}^{(+)}\left(ik_0\tilde{u}_{2k}^{(+)}\right) - \left(ik_0\tilde{u}_{2k}^{(+)}\right)u_{2k}^{(+)} = \frac{i\hbar c}{2\epsilon}. \quad \text{etc}$$

$$s_k^{(+)}\left(ik_0\tilde{s}_k^{(+)} - \varkappa\tilde{u}_{0k}^{(+)}\right) - \left(ik_0\tilde{s}_k^{(+)} - \varkappa\tilde{u}_{0k}^{(+)}\right)s_k^{(+)} = \frac{i\hbar c}{2\epsilon} \quad \text{etc.}$$

$$\cancel{k_0}\,u_{2k}^{(+)}\tilde{u}_{2k}^{(+)} - \tilde{u}_{2k}^{(+)}u_{2k}^{(+)} = \frac{\hbar c}{2k_0\epsilon}.$$

$$\tilde{u}_{2k}^{(+)}u_{2k}^{(+)} \cancel{u_{2k}} = \frac{\hbar c}{2k_0\epsilon}\cdot n_{2k}^{(+)}$$

$$\therefore\ 2\sum k_0^2\left(\tilde{u}_{2k}^{(+)}u_{2k}^{(+)} + u_{2k}^{(+)}\tilde{u}_{2k}^{(+)}\right) = \cancel{\frac{\hbar k_0}{c}}\,n\,\frac{\hbar k_0 c}{2}\cdot\left(n_{2k}^{(+)} + \frac{1}{2}\right).$$

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$$L = c^2 \vec{U}^\dagger \tilde{\vec{U}}^\dagger - \mathrm{curl}\,\tilde{\vec{U}}\,\mathrm{curl}\,\vec{U} - (\varkappa\tilde{\vec{U}} + \mathrm{grad}\,\tilde{S})(\varkappa\vec{U} + \mathrm{grad}\,S) - c^2 U_0^\dagger \tilde{U}_0^\dagger$$
$$+ c^2 S^\dagger \tilde{S}^\dagger$$

$$H = \vec{U}^\dagger \frac{\partial\vec{U}}{\partial t} + \tilde{\vec{U}}^\dagger \frac{\partial\tilde{\vec{U}}}{\partial t} - c^2 \vec{U}^\dagger\tilde{\vec{U}}^\dagger + \mathrm{curl}\,\tilde{\vec{U}}\,\mathrm{curl}\,\vec{U} + (\varkappa\tilde{\vec{U}} + \mathrm{grad}\,\tilde{S})(\varkappa\vec{U} + \mathrm{grad}\,S)$$
$$+ U_0^\dagger \frac{\partial U_0}{\partial t} + \tilde{U}_0^\dagger \frac{\partial\tilde{U}_0}{\partial t} + c^2 U_0^\dagger \tilde{U}_0^\dagger + S^\dagger \frac{\partial S}{\partial t} + \tilde{S}^\dagger \frac{\partial\tilde{S}}{\partial t} - c^2 S^\dagger \tilde{S}^\dagger$$
$$= \vec{U}^\dagger(c^2\tilde{\vec{U}}^\dagger - c\,\mathrm{grad}\,U_0) + \tilde{\vec{U}}^\dagger(c^2 U^\dagger - c\,\mathrm{grad}\,\tilde{U}_0) - c^2 U^\dagger \tilde{U}^\dagger$$
$$+ \mathrm{curl}\,\tilde{\vec{U}}\,\mathrm{curl}\,\vec{U} + (\varkappa\tilde{\vec{U}} + \mathrm{grad}\,\tilde{S})(\varkappa\vec{U} + \mathrm{grad}\,S)$$
$$+ U_0^\dagger(c^2\tilde{U}_0^\dagger - c\,\mathrm{div}\,\vec{U} - \varkappa c S) + \tilde{U}_0^\dagger(-c^2 U_0^\dagger - c\,\mathrm{div}\,\tilde{\vec{U}} - \varkappa c\tilde{S})$$
$$+ c^2 U_0^\dagger \tilde{U}_0^\dagger + S^\dagger(c^2\tilde{S}^\dagger + \varkappa c U_0) + \tilde{S}^\dagger(c^2 S^\dagger + \varkappa c\tilde{U}_0) - c^2 S^\dagger \tilde{S}^\dagger$$
$$= c^2\tilde{\vec{U}}^\dagger\vec{U}^\dagger - c(\vec{U}^\dagger\mathrm{grad}\,U_0 + \tilde{\vec{U}}^\dagger\mathrm{grad}\,\tilde{U}_0) + \mathrm{curl}\,\tilde{\vec{U}}\,\mathrm{curl}\,\vec{U}$$

$$\vec{U} = \vec{U}_1 + \vec{U}_2, \qquad \mathrm{curl}\,\vec{U}_1 = 0 \qquad \mathrm{div}\,\vec{U}_2 = 0,$$
$$\vec{U}^\dagger = \vec{U}_1^\dagger + \vec{U}_2^\dagger, \qquad \mathrm{curl}\,\vec{U}_1^\dagger = 0 \qquad \mathrm{div}\,\vec{U}_2^\dagger = 0.$$

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \varkappa^2\right)\vec{U} = 0: \quad \int \vec{U}_1\vec{U}_2\,dv \propto \int \vec{U}_1 \Delta\vec{U}_2\,dv = \int \vec{U}_1(\mathrm{curl}\,\mathrm{curl}\,\vec{U}_2$$
$$+ \mathrm{grad}\,\mathrm{div}\,\vec{U}_2)\,dv = 0 \quad \text{etc}$$
$$\int \vec{U}_2\,\mathrm{grad}\,S\,dv = -\int S\,\mathrm{div}\,\vec{U}_2\,dv = 0$$

$$+ c(\mathrm{div}\,\vec{U}_1^\dagger \cdot U_0 + \mathrm{div}\,\tilde{\vec{U}}_1^\dagger \cdot \tilde{U}_0)$$
$$\bar{H} = \int\{c^2(\tilde{\vec{U}}_1^\dagger\vec{U}_1^\dagger + \tilde{\vec{U}}_2^\dagger\vec{U}_2^\dagger) - c(\vec{U}_1^\dagger\mathrm{grad}\,U_0 + \tilde{\vec{U}}_1^\dagger\,\mathrm{grad}\,\tilde{U}_0)$$
$$+ \mathrm{curl}\,\tilde{\vec{U}}_2\,\mathrm{curl}\,\vec{U}_2 + (\varkappa\tilde{\vec{U}}_1 + \mathrm{grad}\,\tilde{S})(\varkappa\vec{U}_1 + \mathrm{grad}\,S) + \varkappa^2\tilde{\vec{U}}_2\vec{U}_2$$
$$+ c^2\tilde{U}_0^\dagger U_0^\dagger - cU_0^\dagger(\mathrm{div}\,\vec{U}_1 + \varkappa S) - c\tilde{U}_0^\dagger(\mathrm{div}\,\tilde{\vec{U}}_1 + \varkappa\tilde{S})$$
$$+ c^2\tilde{S}^\dagger S^\dagger + \varkappa c(S^\dagger U_0 + \tilde{S}^\dagger\tilde{U}_0)\}\,dv$$

$$k^2\tilde{\vec{U}}_2\vec{U}_2$$
$$= \int\{c^2\tilde{\vec{U}}_2^\dagger\vec{U}_2^\dagger + \tilde{\vec{U}}_2\left(\frac{-1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{U}_2\}\,dv.$$
$$+ \int\{c^2\tilde{\vec{U}}_1^\dagger\vec{U}_1^\dagger + (\varkappa\tilde{\vec{U}}_1 + \mathrm{grad}\,\tilde{S})(\varkappa\vec{U}_1 + \mathrm{grad}\,S)$$
$$+ c^2(\tilde{U}_0^\dagger U_0^\dagger + \tilde{S}^\dagger S^\dagger)\}\,dv. \qquad \frac{\partial U_0^\dagger}{\partial t}\cdot U_0$$

$\varkappa S = -\mathrm{div}\,\vec{U}_1$;

$\varkappa S^\dagger = -\mathrm{div}\,\vec{U}^\dagger$;

$$U_0^\dagger = 0$$

$$-i\hbar\frac{\partial U_0^\dagger}{\partial t} = \bar{H}U_0^\dagger - U_0^\dagger\bar{H} = i\hbar c(\mathrm{div}\,\vec{U}_1^\dagger + \varkappa c S^\dagger) \qquad \frac{\partial U_0^\dagger}{\partial t} = 0.$$
or
$$\frac{1}{c}\frac{\partial U_0^\dagger}{\partial t} + \mathrm{div}\,\vec{U}^\dagger + \varkappa S^\dagger = 0.$$

$$\bar{H} = \int \left\{ c^2 \tilde{\vec{U}}_2^+ \vec{U}_2^+ + U_2 \left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) U_2 \right\} dv$$

$$+ \int \left\{ c^2 \tilde{\vec{U}}_1^+ \vec{U}_1^+ + (\varkappa \tilde{\vec{U}}_1 + \mathrm{grad}\, \tilde{S})(\varkappa \vec{U}_1 + \mathrm{grad}\, S) + c^2 \tilde{S}^+ S^+ \right\}$$

$\vec{U}_1 = \vec{U}_1' - \mathrm{grad}\, X$　　　　$\mathrm{div}\, \vec{U}_1^* + \varkappa S = 0.$

$S = S' + \varkappa X.$　　$c^2 \tilde{\vec{U}}_1^+ \vec{U}_1^+ + \frac{1}{\varkappa^2} (\varkappa^2 \tilde{\vec{U}}_1 - \Delta \tilde{\vec{U}}_1)(\varkappa^2 \vec{U}_1 - \Delta \vec{U}_1) + c^2 \tilde{S}^+ S^+$

$-\frac{1}{c}\frac{\partial X}{\partial t} = U_0$

$$c^2 (\tilde{\vec{U}}_1^+ \vec{U}_1^+) + \frac{1}{\varkappa^2} \left(\frac{1}{c^2} \frac{\partial^2 \tilde{\vec{U}}_1}{\partial t^2} \right) \left(\frac{1}{c^2} \frac{\partial^2 \vec{U}_1}{\partial t^2} \right) + \frac{c^2}{\varkappa^2} \tilde{\vec{U}}_1^+ \Delta \vec{U}_1^+$$

$$= -\frac{c^2}{\varkappa^2} \tilde{\vec{U}}_1^+ \left(\frac{1}{c^2} \frac{\partial^2 \vec{U}_1^+}{\partial t^2} \right) + \frac{1}{\varkappa^2} \left(\frac{1}{c^2} \frac{\partial^2 \tilde{\vec{U}}_1}{\partial t^2} \right) \left(\frac{1}{c^2} \frac{\partial^2 \vec{U}_1}{\partial t^2} \right).$$

$$= \frac{c^2 k_0^2}{\varkappa^2} \tilde{\vec{U}}_1^+ \vec{U}_1^+ + \frac{k_0^4}{\varkappa^2} \tilde{\vec{U}}_1 \vec{U}_1$$

$$L = \tilde{\vec{F}}\vec{F} - \tilde{G}G + \tilde{K}_0 K_0 - \tilde{K}K - \tilde{M}M.$$

$$\vec{F} = -\frac{1}{c}\frac{\partial \vec{U}}{\partial t} - \mathrm{grad}\, U_0 \qquad K_0 = \varkappa U_0 - \frac{1}{c}\frac{\partial S}{\partial t}$$

$$\vec{G} = \mathrm{curl}\, \vec{U} \qquad \vec{K} = \varkappa \vec{U} + \mathrm{grad}\, S.$$

$$M = \frac{1}{c}\frac{\partial U_0}{\partial t} + \mathrm{div}\, \vec{U} + \varkappa S.$$

$$U_0^\dagger = -\frac{1}{c}\tilde{M} \qquad U_0(\vec{r},t)U_0^\dagger(\vec{r}',t) - U_0^\dagger(\vec{r}',t)U_0(\vec{r},t) = i\hbar\delta(\vec{r},\vec{r}')$$

$$\vec{U}^\dagger = -\frac{1}{c}\tilde{\vec{F}} \qquad \vec{U}(\vec{r},t)\vec{U}_0^\dagger(\vec{r}',t) - \vec{U}_0^\dagger(\vec{r}',t)U_0(\vec{r},t) = i\hbar\delta(\vec{r},\vec{r}')$$

$$S^\dagger = -\frac{1}{c}\tilde{K}_0 \qquad S(\vec{r},t)S^\dagger(\vec{r}',t) - S^\dagger(\vec{r}',t)S(\vec{r},t) = i\hbar\delta(\vec{r},\vec{r}')$$

$$L = c^2\vec{U}^\dagger\tilde{\vec{U}}^\dagger - \mathrm{curl}\,\tilde{\vec{U}}\,\mathrm{curl}\,\vec{U} + c^2 S^\dagger\tilde{S}^\dagger - (\varkappa\tilde{\vec{U}} + \mathrm{grad}\,\tilde{S})(\varkappa\vec{U} + \mathrm{grad}\,S)$$
$$- c^2 U_0^\dagger\tilde{U}_0^\dagger.$$

$$H = U_0^\dagger\frac{\partial U_0}{\partial t} + \vec{U}^\dagger\frac{\partial \vec{U}}{\partial t} + S^\dagger\frac{\partial S}{\partial t} + \text{comp. conj.} - L$$
$$= -cU_0^\dagger(c\tilde{U}_0^\dagger + \mathrm{div}\,\vec{U} + \varkappa S) + c\vec{U}^\dagger(c\tilde{\vec{U}}^\dagger - \mathrm{grad}\,U_0) + cS^\dagger(c\tilde{S}^\dagger + \varkappa U_0)$$
$$- c\tilde{U}_0^\dagger(cU_0^\dagger + \mathrm{div}\,\tilde{\vec{U}} + \varkappa\tilde{S}) + c\tilde{\vec{U}}^\dagger(c\vec{U}^\dagger - \mathrm{grad}\,\tilde{U}_0) + c\tilde{S}^\dagger(cS^\dagger + \varkappa\tilde{U}_0)$$
$$- c^2\vec{U}^\dagger\tilde{\vec{U}}^\dagger + \mathrm{curl}\,\tilde{\vec{U}}\,\mathrm{curl}\,\vec{U} - c^2 S^\dagger\tilde{S}^\dagger + (\varkappa\tilde{\vec{U}} + \mathrm{grad}\,\tilde{S})(\varkappa\vec{U} + \mathrm{grad}\,S)$$
$$+ c^2 U_0^\dagger\tilde{U}_0^\dagger$$
$$= -c^2\tilde{U}_0^\dagger U_0^\dagger - cU_0^\dagger(\mathrm{div}\,\vec{U} + \varkappa S) - c\tilde{U}_0^\dagger(\mathrm{div}\,\tilde{\vec{U}} + \varkappa\tilde{S})$$
$$+ c^2\tilde{\vec{U}}^\dagger\vec{U}^\dagger - c\vec{U}^\dagger\mathrm{grad}\,U_0 - c\tilde{\vec{U}}^\dagger\mathrm{grad}\,\tilde{U}_0 + \mathrm{curl}\,\tilde{\vec{U}}\,\mathrm{curl}\,\vec{U}$$
$$+ c^2\tilde{S}^\dagger S^\dagger + \varkappa c S^\dagger U_0 + \varkappa c\tilde{S}^\dagger\tilde{U}_0 + (\varkappa\tilde{\vec{U}} + \mathrm{grad}\,\tilde{S})(\varkappa\vec{U} + \mathrm{grad}\,S).$$

$M = 0$ なる supplementary condition を assume する.

or $\quad M = -c\tilde{U}_0^\dagger = 0.$

$$H\tilde{U}_0^\dagger - \tilde{U}_0^\dagger H = +\frac{i\hbar}{c}(\mathrm{div}\,\tilde{\vec{U}}^\dagger + \varkappa\tilde{S}^\dagger)$$

なお又 $\dot{M} = 0$, or $\mathrm{div}\,\tilde{\vec{U}}^\dagger + \varkappa\tilde{S}^\dagger = 0$ とすれば.

$$H(\mathrm{div}\,\tilde{\vec{U}}^\dagger + \varkappa\tilde{S}^\dagger) - (\mathrm{div}\,\tilde{\vec{U}}^\dagger + \varkappa\tilde{S}^\dagger)H = 0$$

$$\because \int dv'\left\{\left(\frac{\partial\tilde{U}_z'}{\partial y'} - \frac{\partial\tilde{U}_y'}{\partial z'}\right)\left(\frac{\partial\tilde{U}_x^\dagger}{\partial x} + \frac{\partial\tilde{U}_y^\dagger}{\partial y} + \frac{\partial\tilde{U}_z^\dagger}{\partial z}\right) - (\quad)(\quad)\right\}\left(\frac{\partial U_z'}{\partial y'} - \frac{\partial U_y'}{\partial z'}\right)$$
$$= \int dv'\, i\hbar\left(\frac{\partial\delta(\vec{r},\vec{r}')}{\partial y'\partial z} - \frac{\partial\delta(\vec{r},\vec{r}')}{\partial z'\partial y}\right)\left(\frac{\partial U_z'}{\partial y'} - \frac{\partial U_y'}{\partial z'}\right) = \cancel{-i\hbar\int dv'\left(\frac{\partial^2 U_z'}{\partial y'\partial z} - \frac{\partial}{}\right)}\;0,$$

$$\int dv'\left\{\left(\varkappa\tilde{U}_x' + \frac{\partial\tilde{S}'}{\partial x'}\right)(\mathrm{div}\,\tilde{\vec{U}}^\dagger + \varkappa\tilde{S}^\dagger) - (\quad)(\quad)\right\}\left(\varkappa U_x' + \frac{\partial S'}{\partial x'}\right)$$
$$= +i\hbar\int\varkappa\left(\frac{\partial\delta}{\partial x} + \frac{\partial\delta}{\partial x'}\right)(\quad) = 0.$$

$$\therefore\ U_0^+ = \tilde{U}_0^+ = 0$$
$$\operatorname{div}\vec{U}^+ + \varkappa S^+ = \operatorname{div}\tilde{\vec{U}}^+ + \varkappa\tilde{S}^+ = 0$$

なる supplementary condition を assume する。

更に

$$\left.\begin{aligned} U_0 &= U_0' + \frac{1}{c}\frac{\partial\chi}{\partial t} \\ \vec{U} &= \vec{U}' - \operatorname{grad}\chi \\ S &= S' + \varkappa\chi \end{aligned}\right\} \quad \left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \varkappa^2\right)\chi = 0$$

なる gauge transformation に対して $\vec{F}, \vec{G}, K_0, \vec{K}, M$ 等は凡て invariant,
従って L もまた invariant。この χ を適当にとる事により $U_0 = 0$ に reduce し得る。

すると、$U_0^+ = \tilde{U}_0^+ = 0$ の條件は

$$\operatorname{div}\vec{U} + \varkappa S = \operatorname{div}\tilde{\vec{U}} + \varkappa\tilde{S} = 0$$

に reduce される。

而して Hamiltonian は

$$H = c^2\tilde{\vec{U}}^+\vec{U}^+ + \operatorname{curl}\tilde{\vec{U}}\operatorname{curl}\vec{U} + \frac{c^2}{\varkappa^2}\operatorname{div}\tilde{\vec{U}}^+\cdot\operatorname{div}\vec{U}^+$$
$$+ \frac{1}{\varkappa^2}(\varkappa^2\tilde{\vec{U}} - \operatorname{grad}\operatorname{div}\tilde{\vec{U}})(\varkappa^2\vec{U} - \operatorname{grad}\operatorname{div}\vec{U})$$

$$\vec{U} = \vec{U}_1 + \vec{U}_2 \qquad \operatorname{curl}\vec{U}_1 = 0 \qquad \operatorname{div}\vec{U}_2 = 0$$
$$\vec{U}^+ = \vec{U}_1^+ + \vec{U}_2^+ \qquad \operatorname{curl}\vec{U}_1^+ = 0 \qquad \operatorname{div}\vec{U}_2^+ = 0$$

～ ~~grad div~~

とおけば、

$$\int \vec{U}_1^+\vec{U}_2^+ dv = 0 \text{ 故に}$$

$$\bar{H} = \int dv\{c^2\tilde{\vec{U}}_1^+\vec{U}_1^+ + c^2\tilde{\vec{U}}_2^+\vec{U}_2^+ + \operatorname{curl}\tilde{\vec{U}}_2\operatorname{curl}\vec{U}_2 + \frac{c^2}{\varkappa^2}\operatorname{div}\tilde{\vec{U}}_1^+\operatorname{div}\vec{U}_1^+$$
$$+ \varkappa^2\tilde{\vec{U}}_2\vec{U}_2 + \frac{1}{\varkappa^2}(\varkappa^2\tilde{\vec{U}}_1 - \Delta\tilde{\vec{U}}_1)(\varkappa^2\vec{U}_1 - \Delta\vec{U}_1)$$
$$= \int dv\left\{c^2\tilde{\vec{U}}_1^+\vec{U}_1^+ - \frac{c^2}{\varkappa^2}\tilde{\vec{U}}_1^+\Delta\vec{U}_1^+ + \frac{1}{\varkappa^2}(\varkappa^2\tilde{\vec{U}}_1 - \Delta\tilde{\vec{U}}_1)(\varkappa^2\vec{U}_1 - \Delta\vec{U}_1)\right\}$$
$$+ \int dv\{c^2\tilde{\vec{U}}_2^+\vec{U}_2^+ - \tilde{\vec{U}}_2\Delta\vec{U}_2 + \varkappa^2\tilde{\vec{U}}_2\vec{U}_2\}$$

$$\vec{U}_1 = \sum_{k,(k_0>0)}\{u_{1k}\vec{e}_1 e^{i(\vec{k}\vec{r} - ck_0t)} + v_{1k}\vec{e}_1 e^{-i(\vec{k}\vec{r} - ck_0t)}\}$$
$$\vec{U}_1^+ = \sum_{k,(k_0>0)}\{u_{1k}^+\vec{e}_1 e^{-i(\vec{k}\vec{r} - ck_0t)} + v_{1k}^+\vec{e}_1 e^{+i(\vec{k}\vec{r} - ck_0t)}\}$$

$$k_0 = +\sqrt{k^2 + \varkappa^2} \qquad k\vec{e}_1 = \vec{k}$$

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$$\vec{U}_2 = \sum_k \{ (u_{2k}\vec{e}_2 + u_{3k}\vec{e}_3) e^{i(\vec{k}\vec{r} - ck_0 t)} + (v_{2k}\vec{e}_2 + v_{3k}\vec{e}_3) e^{-i(\vec{k}\vec{r} - ck_0 t)} \}$$

$$\vec{U}_2^{+} = \sum_k \{ (u_{2k}^{+}\vec{e}_2 + u_{3k}^{+}\vec{e}_3) e^{-i(\vec{k}\vec{r} - ck_0 t)} + (v_{2k}^{+}\vec{e}_2 + v_{3k}^{+}\vec{e}_3) e^{+i(\vec{k}\vec{r} - ck_0 t)} \}$$

e_3, $e_1 \perp e_2$

$$u_{1k} u_{1k}^{+} - u_{1k}^{+} u_{1k} = \frac{i\hbar}{2} \quad \text{etc.}$$

$$\bar{H} = \sum_k \Big[\Big\{ \frac{k_0^2}{\kappa^2} c^2 \tilde{u}_{1k}^{+} u_{1k}^{+} + \frac{k_0^4}{\kappa^2} \tilde{u}_{1k} u_{1k} \Big\} + (c^2 \tilde{u}_{2k}^{+} u_{2k}^{+} + k_0^2 \tilde{u}_{2k} u_{2k})$$
$$+ (\qquad \qquad \}] + (v) \Big]$$

$$c\vec{U}^{+} = \frac{1}{c} \frac{\partial \vec{U}}{\partial t} ; \quad c u_{1k}^{+} = \tilde{u}_{1k}(-ik_0) \quad \text{etc.}$$

$$H = \sum_k \Big\{ \frac{k_0^4}{\kappa^2} (u_{1k}\tilde{u}_{1k} + \tilde{u}_{1k} u_{1k}) + k_0^2 (u_{2k}\tilde{u}_{2k} + \tilde{u}_{2k} u_{2k})$$
$$+ k_0^2 (u_{3k}\tilde{u}_{3k} + \tilde{u}_{3k} u_{3k}) + (v) \Big\}$$

$$u_{1k}\tilde{u}_{1k} - \tilde{u}_{1k} u_{1k} = \frac{-\hbar c}{2 k_0} \quad \text{etc}$$

$$= \sum_k \Big\{ \frac{k_0^3 c}{\kappa^2} (n_{1k} + \tfrac{1}{2}) + \frac{k_0 c}{2} (n_{2k} + \tfrac{1}{2}) + \frac{k_0 c}{2} (n_{3k} + \tfrac{1}{2})$$
$$+ (v) \Big\}$$

$$S^{\prime} S^{+\prime} - S^{+} S^{\prime} = i\hbar\delta.$$

$$\frac{1}{\kappa^2} \{ (\mathrm{div}\, \vec{U}^{\prime}) (\mathrm{div}\, \vec{U}^{+\prime}) - (\mathrm{div}\, \vec{U}^{+\prime}) (\mathrm{div}\, \vec{U}) \} = i\hbar\delta,$$

supplementary conditions.

$$U_0^\dagger = -\frac{1}{c}\tilde{M} = 0$$

$$\bar{H}U_0^\dagger - U_0^\dagger\bar{H} = \int(c\vec{U}^\dagger \operatorname{grad} U_0 + \varkappa c S^\dagger U_0)dv \cdot U_0^\dagger - U_0^\dagger\int(\quad)dv = 0$$

$$\text{or}\quad \operatorname{div}\vec{U}^\dagger + \varkappa S^\dagger = 0$$

$$\bar{H}(\operatorname{div}\vec{U}^\dagger + \varkappa S^\dagger) - (\operatorname{div}\vec{U}^\dagger + \varkappa S^\dagger)\bar{H} = 0.$$

$$H = c^2\tilde{\vec{U}}^\dagger\vec{U}^\dagger - c\vec{U}^\dagger \operatorname{grad} U_0 - c\tilde{\vec{U}}^\dagger \operatorname{grad}\tilde{U}_0 + \operatorname{curl}\tilde{\vec{U}}\operatorname{curl}\vec{U}$$
$$+ c^2\tilde{S}^\dagger S^\dagger + \varkappa c S^\dagger U_0 + \varkappa c\tilde{S}^\dagger\tilde{U}_0 + (\varkappa\tilde{\vec{U}} + \operatorname{grad}\tilde{S})(\varkappa\vec{U} + \operatorname{grad} S)$$

$$\bar{H} = \int dv\{c^2\tilde{\vec{U}}^\dagger\vec{U}^\dagger + \varkappa^2\tilde{\vec{U}}\vec{U} + \operatorname{curl}\tilde{U}\operatorname{curl}\vec{U} + c^2\tilde{S}^\dagger S^\dagger$$
$$-\varkappa\tilde{S}\operatorname{div}\vec{U} - \varkappa\operatorname{div}\tilde{\vec{U}}\cdot S - \tilde{S}\Delta S\}$$

curl curl = grad div − Δ

$$\vec{U} = \vec{U}_1 + \vec{U}_2 \qquad \operatorname{curl}\vec{U}_1 = 0 \quad \operatorname{div}U = \operatorname{div}U_1$$
$$\operatorname{div}\vec{U}_2 = 0 \quad \operatorname{curl}U = \operatorname{curl}U_2$$

$$\bar{H} = \int dv\{(c^2\tilde{\vec{U}}_1^\dagger\vec{U}_1^\dagger + \varkappa^2\tilde{\vec{U}}_1\vec{U}_1 + c^2\tilde{S}^\dagger S^\dagger - \varkappa\tilde{S}\operatorname{div}\vec{U}_1 - \varkappa\operatorname{div}\tilde{\vec{U}}_1\cdot S$$
$$-\tilde{S}\Delta S) + (c^2\tilde{\vec{U}}_2^\dagger\vec{U}_2^\dagger + \varkappa^2\tilde{\vec{U}}_2\vec{U}_2 - \tilde{\vec{U}}\Delta\vec{U})\}$$

$$\vec{U}_1\vec{U}_1^{\dagger\prime} - \vec{U}_1^{\dagger\prime}\vec{U}_1 = i\hbar\delta,$$
$$\vec{U}_2\vec{U}_2^{\dagger\prime} - \vec{U}_2^{\dagger\prime}\vec{U}_2 = i\hbar\delta.$$
$$S S^{\dagger\prime} - S^{\dagger\prime}S = i\hbar\delta,$$

$$U_0 = \sum_k\{u_{0k}e^{i(\vec{k}\vec{r}-ck_0t)} + v_{0k}e^{-i(\vec{k}\vec{r}-ck_0t)}\}$$
$$S = \sum_k\{s_k e^{\cdots} + t_k e^{\cdots}\}$$

$$\vec{U}_1^\dagger = +\frac{1}{c}\left(\frac{1}{c}\frac{\partial\tilde{\vec{U}}_1}{\partial t} + \operatorname{grad}\tilde{U}_0\right) \quad \text{etc.} \qquad \operatorname{curl}\vec{U}_1^\dagger = 0 \qquad \operatorname{div}\vec{U}_1^\dagger = \frac{1}{c}\operatorname{div}\left(\frac{1}{c}\frac{\partial\tilde{\vec{U}}_1}{\partial t} + \operatorname{grad}\tilde{U}_0\right)$$

$$\vec{U}_2^\dagger = +\frac{1}{c^2}\frac{\partial\tilde{\vec{U}}_2}{\partial t} \qquad \operatorname{div}\vec{U}_2^\dagger = 0 \qquad \operatorname{curl}\vec{U}_2^\dagger = \frac{1}{c}\operatorname{curl}\left(\frac{1}{c}\frac{\partial\tilde{\vec{U}}_2}{\partial t}\right)$$

$$S^\dagger = -\frac{1}{c}\left(\varkappa\tilde{U}_0 - \frac{1}{c}\frac{\partial\tilde{S}}{\partial t}\right) = \frac{1}{c}\left(\frac{1}{c}\frac{\partial\tilde{S}}{\partial t} - \varkappa\tilde{U}_0\right)$$

$$\vec{U}_1 = \sum_k\{u_{1k}\vec{e}_1 e^{i(\vec{k}\vec{r}-ck_0t)} + v_{1k}\vec{e}_1 e^{-i(\vec{k}\vec{r}-ck_0t)}\} \qquad k_0 = +\sqrt{k^2+\varkappa^2}$$

$$\vec{U}_1^\dagger = \sum_k\{u_{1k}^\dagger\vec{e}_1 e^{-i(\vec{k}\vec{r}-ck_0t)} + v_{1k}^\dagger\vec{e}_1 e^{i(\vec{k}\vec{r}-ck_0t)}\} \qquad k\vec{e}_1 = \vec{k},$$

$$\vec{U}_2 = \sum_k\{(u_{2k}\vec{e}_2 + u_{3k}\vec{e}_3)e^{i(\vec{k}\vec{r}-ck_0t)} + (v_{2k}\vec{e}_2 + v_{3k}\vec{e}_3)e^{-i(\vec{k}\vec{r}-ck_0t)}\}$$

$$\vec{U}_2^\dagger = \sum_k\{(u_{2k}^\dagger\vec{e}_2 + u_{3k}^\dagger\vec{e}_3)e^{-i(\vec{k}\vec{r}-ck_0t)} + (v_{2k}^\dagger\vec{e}_2 + v_{3k}^\dagger\vec{e}_3)e^{i(\vec{k}\vec{r}-ck_0t)}\}$$

$$u_{1k}u_{1k}^\dagger - u_{1k}^\dagger u_{1k} = \frac{i\hbar}{2}\delta, \qquad u_{2k}u_{2k}^{\dagger\prime} - u_{2k}^{\dagger\prime}u_{2k} = \frac{i\hbar}{2}\delta \qquad \text{etc}$$

$$u_{1k}^\dagger = \frac{1}{c}(ik_0\tilde{u}_{1k} + ik\tilde{u}_{0k}), \qquad s_k^\dagger = \frac{1}{c}(ik_0\tilde{s}_k - \varkappa\tilde{u}_{0k})$$

$$u_{2k}^\dagger = \frac{1}{c}ik_0\tilde{u}_{2k} \qquad \text{etc.}$$

$$\left(\frac{(\cancel{k_0^2}-k^2)}{k_0}u_{1k}+\frac{ikx}{k_0}s_k\right)\left(\frac{k^2}{k_0}\tilde{u}_{1k}-i\frac{kx}{k_0}\tilde{s}_k\right)+$$

$$=\frac{x}{k_0}(xu_{1k}+iks_k)(x\tilde{u}_{1k}-ik\tilde{s}_k)+x^2\tilde{u}_{1k}u_{1k}$$

$$(xs_k-iku_{1k})(x\tilde{s}_k+ik\tilde{u}_{1k})$$

$$+\frac{k}{k_0}(-ixs_k-ku_{1k})(ix\tilde{s}_k-k\tilde{u}_{1k})-ikx\tilde{s}_ku_{1k}$$

$$+ikx\tilde{u}_{1k}s_k+k^2\tilde{s}_ks_k$$

$$=xk_0\,u_{1k}\tilde{u}_{1k}+x^2\tilde{u}_{1k}u_{1k}+xks_k\tilde{s}_k+k^2\tilde{s}_ks_k.$$

$$-ikx\left(\frac{x}{k_0}+1\right)\tilde{s}_ku_{1k}+ikx\left(\frac{x}{k_0}+1\right)\tilde{u}_{1k}s_k$$

$$(x\tilde{u}_{1k}-ik\tilde{s}_k)(xu_{1k}+iks_k)$$

$$xu_{1k}+iks_k=\sqrt{\frac{k_0}{x}}x_k \qquad \cancel{\frac{k_0^2}{x}x_k\tilde{x}_k+}$$

$$ku_{1k}+ixs_k=\sqrt{\frac{k_0}{x}}y_k \qquad \frac{x}{k_0}x_k\tilde{x}_k+\tilde{x}_kx_k.$$

$$+\frac{x}{k_0}y_k\tilde{y}_k.$$

$$x_k\tilde{y}_k-\tilde{y}_kx_k=(xu_{1k}+iks_k)(k\tilde{u}_{1k}-ix\tilde{s}_k)-(\quad)(\quad)$$

$$=xk(u_{1k}\tilde{u}_{1k}-\cdots)$$

$$+kx(s_k\tilde{s}_k-\cdots)$$

$$\alpha_k=\sqrt{\frac{k_0+x}{2x}}u_{1k}+i\sqrt{\frac{k_0-x}{2x}}s_k \qquad \sqrt{\frac{k_0+x}{2x}}\alpha_k=\frac{k_0+x}{2x}u_{1k}+i\frac{k}{2x}s_k$$

$$\beta_k=+i\sqrt{\frac{k_0-x}{2x}}u_{1k}\pm\sqrt{\frac{k_0+x}{2x}}s_k \qquad +i\sqrt{\frac{k_0-x}{2x}}\beta_k=+\frac{k_0-x}{2x}u_{1k}+i\frac{k}{2x}s_k$$

$-i$

$$\sqrt{\frac{k_0+x}{2x}}\alpha_k+i\sqrt{\frac{k_0-x}{2x}}\beta_k=\frac{1}{x}(ku_{1k}+iks_k) \qquad \sqrt{\frac{k_0+x}{2x}}$$

$$i\sqrt{\frac{k_0-x}{2x}}\alpha_k+\sqrt{\frac{k_0+x}{2x}}\beta_k=\frac{1}{x}(\;iku_{1k}$$

$$\bar{H} = \sum_k \{ (c^2 \ldots - ik\varkappa \tilde{s}_k u_{1k} + ik\varkappa \tilde{u}_{1k} s_k + k^2 \tilde{s}_k s_k) + (c^2 \tilde{u}_{2k}^{+} u_{2k}^{+} + k_0^2 \tilde{u}_{2k} u_{2k}) + (c^2 \tilde{u}_{3k}^{+} u_{3k}^{+} + k_0^2 \tilde{u}_{3k} u_{3k}) \} + \Sigma(v, t)$$

$$u_{1k}(ik_0 \tilde{u}'_{1k} - ik\tilde{u}'_{0k}) + (ik_0 \tilde{u}_{1k} - ik\tilde{u}_{0k}) u_{1k} = \frac{i\hbar}{2}\delta$$

$u_{1k}\tilde{u}_{0k} \ldots u_{1k} = 0.$

$\bar{H} =$

$$\bar{H} = \sum \{ (k_0 u_{1k} - k u_{0k})(k_0 \tilde{u}_{1k} - k\tilde{u}_{0k}) + \varkappa^2 \tilde{u}_{1k} u_{1k} + (ik_0 s_k - \varkappa u_{0k})(ik_0 \tilde{s}_k - \varkappa \tilde{u}_{0k}) - ik\varkappa \tilde{s}_k u_{1k} + ik\varkappa \tilde{u}_{1k} s_k + k^2 \tilde{s}_k s_k \}$$

$$+ \sum k_0^2 (\tilde{u}_{2k}^{+} u_{2k}^{+} + \tilde{u}_{2k} u_{2k}) + \sum k_0^2 (u_{3k} \tilde{u}_{3k}^{+} + \tilde{u}_{3k} u_{3k}).$$

$$= \sum \{ (k_0 u_{1k} - \frac{k^2 u_{1k} - ik\varkappa s_k}{k_0})(k_0 \tilde{u}_{1k} - \frac{k\tilde{u}_{1k} + ik\varkappa \tilde{s}_k}{k_0}) + \varkappa^2 \tilde{u}_{1k} u_{1k} + (ik_0 s_k - \frac{k\varkappa u_{1k} - i\varkappa^2 s_k}{k_0})(ik_0 \tilde{s}_k - \frac{k\varkappa \tilde{u}_{1k} + i\varkappa^2 \tilde{s}_k}{k_0}) - ik\varkappa \tilde{s}_k u_{1k} + ik\varkappa \tilde{u}_{1k} s_k + k^2 \tilde{s}_k s_k \} + \sum_2 + \sum_3 + \Sigma(v, t)$$

$(\because \frac{1}{c}\frac{\partial U_0}{\partial t} + \operatorname{div}\vec{U} + \varkappa S = 0 : -ik_0 u_{0k} + ik u_{1k} + \varkappa s_k = 0.$

$$u_{0k} = \frac{k u_{1k} - i\varkappa s_k}{k_0} \quad \text{et c.})$$

$$\bar{H} = \sum \{ \frac{\varkappa^2}{k_0^2}(\varkappa u_{1k} + ik s_k)(\varkappa \tilde{u}_{1k} - ik\tilde{s}_k) + \frac{k^2}{k_0^2}(\varkappa s_k - ik u_{1k})(\varkappa \tilde{s}_k + ik\tilde{u}_{1k}) + (\varkappa \tilde{u}_{1k} - ik\tilde{s}_k)(\varkappa u_{1k} + ik s_k) \}$$

$(k_0^2 - k^2)$

$3k_0^2 k^2 + 2k_0$

$$= \sum \{ \frac{\varkappa^4 + k^4 + \varkappa^2 k_0^2}{k_0^2} u_{1k} \tilde{u}_{1k} + \frac{2\varkappa^2 k^2 + k_0^2 k^2}{k_0^2} \tilde{s}_k s_k - 2ik\varkappa \tilde{s}_k u_{1k} + 2ik\varkappa \tilde{u}_{1k} s_k \}$$

$(k_0^2 - k_0^2)^2 + k^4 + (k_0^2 - k^2) k_0^2 =$

$ik u_0^{+}$

$$\bar{H} = \int dv \{ -c^2 \tilde{U}_0^\dagger U_0 - c \tilde{U}_0 (\mathrm{div}\,\vec{U}_1 + \varkappa S) - c U_0 (\mathrm{div}\,\tilde{\vec{U}}_1 + \varkappa \tilde{S})$$
$$+ c^2 \tilde{\vec{U}}_1^\dagger \vec{U}_1^\dagger + c(\mathrm{div}\,\vec{U}_1^\dagger + \varkappa S^\dagger) U_0 + c(\mathrm{div}\,\tilde{\vec{U}}_1^\dagger + \varkappa \tilde{S}^\dagger)\tilde{U}_0 + c^2 \tilde{S}^\dagger S^\dagger$$
$$+ \varkappa^2 \tilde{\vec{U}}_1 \vec{U}_1 - \varkappa \tilde{S}\,\mathrm{div}\,\vec{U}_1 - \varkappa\,\mathrm{div}\,\tilde{\vec{U}}_1 \cdot S - \tilde{S} \Delta S.$$
$$+ \int dv (U_2, U_2^\dagger \quad)$$

$$\bar{H} = \int dv \{ c^2 \tilde{\vec{U}}_1^\dagger \vec{U}_1^\dagger + c^2 \tilde{S}^\dagger S^\dagger + \varkappa^2 \tilde{\vec{U}}_1 \vec{U}_1 - \varkappa \tilde{S}\,\mathrm{div}\,\vec{U}_1 - \varkappa\,\mathrm{div}\,\tilde{\vec{U}}_1 \cdot S - \tilde{S} \Delta S \}$$

$$= \sum_k \{ c^2 \tilde{u}_{1k}^\dagger u_{1k}^\dagger + c^2 \tilde{s}_k^\dagger s_k^\dagger + \varkappa^2 \tilde{u}_{1k} u_{1k} - ik\varkappa \tilde{s}_k u_{1k} + ik\varkappa \tilde{u}_{1k} s_k + k^2 \tilde{s}_k s_k \}$$

$$= \quad \tilde{s}_k^\dagger s_k^\dagger$$

$$c u_{1k}^\dagger = ik_0 \tilde{u}_{1k} - ik \tilde{u}_{0k} \qquad c s_k^\dagger = ik_0 \tilde{s}_k - \varkappa \tilde{u}_{0k}$$
$$u_{1k} u_{1k}^{\dagger\prime} - u_{1k}^{\dagger\prime} u_{1k} = \frac{i\hbar}{2}\delta. \qquad s_k s_k^{\dagger\prime} - s_k^{\dagger\prime} s_k = \frac{i\hbar}{2}\delta.$$

$$-i \quad ik_0 \tilde{u}_{0k} - ik\tilde{u}_{1k} + \varkappa \tilde{s}_k = 0.$$

$$\tilde{u}_{0k} = \frac{k\tilde{u}_{1k} + i\varkappa \tilde{s}_k}{k_0}$$

$$c u_{1k}^\dagger = \frac{i(k_0^2 - k^2)\tilde{u}_{1k} + k\varkappa \tilde{s}_k}{k_0} = \frac{\varkappa}{k_0}(i\varkappa \tilde{u}_{1k} + k\tilde{s}_k).$$

$$c s_k^\dagger = \frac{i(k_0^2 - \varkappa^2)\tilde{s}_k - k\varkappa \tilde{u}_{1k}}{k_0} = \frac{k}{k_0}(ik\tilde{s}_k - \varkappa \tilde{u}_{1k}).$$
$$= \frac{ik}{k_0}(k\tilde{s}_k + i\varkappa \tilde{u}_{1k})$$

$$\bar{H} = \sum_k \Big[\frac{\varkappa^2}{k_0^2} \{ (k\tilde{s}_k + i\varkappa u_{1k})(k\tilde{s}_k + i\varkappa \tilde{u}_{1k}) + \frac{k^2}{k_0^2}(ks_k - i\varkappa \tilde{u}_{1k})(k\tilde{s}_k + i\varkappa \tilde{u}_{1k})$$
$$+ \cancel{(\varkappa \tilde{u}_{1k} - ik\tilde{s}_k)(\varkappa u_{1k} + iks_k)}$$
$$+ (k\tilde{s}_k + i\varkappa \tilde{u}_{1k})(ks_k - i\varkappa u_{1k})$$
$$= \cancel{\sum_k \Big[k^2 s_k \tilde{s}_k + \frac{\varkappa^2 - k^2}{k_0^2} i\varkappa k(u_{1k}\tilde{s}_k - s_k \tilde{u}_{1k})} + \varkappa^2 u_{1k} \tilde{u}_{1k},$$
$$+ k^2 \tilde{s}_k s_k - i\varkappa k(\tilde{s}_k u_{1k} - \tilde{u}_{1k} s_k) + \varkappa^2 \tilde{u}_{1k} u_{1k} \Big]$$

$$u_{1k}(i\varkappa \tilde{u}_{1k} - k\tilde{s}_k) - (i\varkappa \tilde{u}_{1k} - k\tilde{s}_k) u_{1k} = \frac{i\hbar c k_0}{2\varkappa}\delta$$
$$s_{1k}(i\varkappa \tilde{u}_{1k} - k\tilde{s}_k) - (i\varkappa \tilde{u}_{1k} - k\tilde{s}_k) s_{1k} = 0$$
$$u_{1k}(ik\tilde{s}_k - \varkappa \tilde{u}_{1k}) - (ik\tilde{s}_k - \varkappa \tilde{u}_{1k}) u_{1k} = 0$$
$$s_{1k}(\qquad) - (\qquad \qquad \gamma_{1k} = \frac{i\hbar c k_0}{2k}$$

$$\bar{H} = \sum_k \{ (\cancel{-} k s_k \text{[illegible]}) + (k\tilde{s}_k + i\kappa \tilde{u}_{1k})(k s_k - i\kappa u_{1k}) \}$$

$$s_{1k}(ik\tilde{s}_k - \kappa \tilde{u}_{1k}) - (ik\tilde{s}_k - \kappa\tilde{u}_{1k}) s_{1k} = \frac{i\hbar c k_0}{2k},$$

$-i$

k

$$s_k(k\tilde{s}_k + i\kappa\tilde{u}_{1k}) - (\qquad) s_k = \frac{\hbar c k_0}{2k}$$

$$u_{1k}(\qquad \text{〃} \qquad) - (\qquad) u_{1k} = 0$$

$-i\kappa u_{1k}$

$$(k s_k - i\kappa u_{1k})(k\tilde{s}_k + i\kappa\tilde{u}_{1k}) - (\qquad)(\qquad)$$

$$= \frac{\hbar c k_0}{2}.$$

$\bar{H} =$

§2. Interaction of Heavy Quanta with the Electro-Magnetic Field.

$$\overline{L} = \iiint L \, dv$$

$$L = L_U + L_E$$

$$L_U = \tilde{\vec{F}}\vec{F} - \tilde{\vec{G}}\vec{G} - \tilde{\vec{K}}\vec{K} + \tilde{K}_0 K_0 - \tilde{M}M$$

6
30

$$L_E = \frac{1}{8\pi}(\vec{E}^2 - \vec{H}^2)$$

この場合

$$\begin{cases} \vec{F} = -\left(\frac{1}{c}\frac{\partial}{\partial t} + \frac{ie}{\hbar c}A_0\right)\vec{U} - \left(\mathrm{grad} - \frac{ie}{\hbar c}\vec{A}\right)U_0 \\ \vec{G} = \left(\mathrm{grad} - \frac{ie}{\hbar c}\vec{A}\right)\times\vec{U} \\ \vec{K} = \varkappa\vec{U} + \left(\mathrm{grad} - \frac{ie}{\hbar c}\vec{A}\right)S \\ K_0 = \varkappa U_0 - \left(\frac{1}{c}\frac{\partial}{\partial t} + \frac{ie}{\hbar c}A_0\right)S \\ M = \left(\frac{1}{c}\frac{\partial}{\partial t} + \frac{ie}{\hbar c}A_0\right)U_0 + \left(\mathrm{grad} - \frac{ie}{\hbar c}\vec{A}\right)\vec{U} + \varkappa S \end{cases}$$

$$\frac{e}{\hbar c}A_0 = A_0'$$

$$\frac{e}{\hbar c}\vec{A} = \vec{A}'$$

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4 注意. L_U は

1) §2, (2), (3) 式が compatible にならぬ.

$$\frac{1}{c}\frac{\partial\vec{G}}{\partial t} + \mathrm{curl}\,\vec{F} = 0 \qquad \mathrm{div}\,\vec{G} = 0$$

$$\frac{1}{c}\frac{\partial\vec{F}}{\partial t} - \mathrm{curl}\,\vec{G} - \varkappa\vec{K} = 0 \qquad \mathrm{div}\,\vec{F} + \varkappa K_0 = 0$$

$$\frac{1}{c}\frac{\partial\vec{U}}{\partial t} + \mathrm{grad}\,U_0 + \varkappa\vec{F} =$$

§2. Interaction of Heavy Quanta with the Electromagnetic Field.

$$\left.\begin{aligned}
&\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{ie}{\hbar c}A_0\right)\vec{F}-\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)\times\vec{G}-\varkappa\vec{U}=0\\
&\qquad\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)\vec{F}+\varkappa U_0=0\\
&\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{ie}{\hbar c}A_0\right)\vec{U}+\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)U_0+\varkappa\vec{F}=0\\
&\qquad\left(\mathrm{Grad}-\frac{ie}{\hbar c}\vec{A}\right)\times\vec{U}-\varkappa\vec{G}=0
\end{aligned}\right\}$$

$$\left.\begin{aligned}
&\partial_t\vec{F}-\vec{\partial}\times\vec{G}-\varkappa\vec{U}=0 \qquad &\vec{\partial}\vec{F}+\varkappa U_0=0\\
&\partial_t\vec{U}+\vec{\partial}U_0+\varkappa\vec{F}=0 \qquad &\vec{\partial}\times\vec{U}-\varkappa\vec{G}=0
\end{aligned}\right\}$$

$$(\partial_t^2-\vec{\partial}^2)\vec{F}-\vec{\partial}_t(\vec{\partial}\times\vec{G})-\varkappa\partial_t\vec{U}-\varkappa\vec{\partial}\cdot U_0=0$$

$$(\partial_t^2-\vec{\partial}^2+\varkappa^2)\vec{F}-\partial_t(\vec{\partial}\times\vec{G})=0$$

$$\cancel{\varkappa(\partial_t^2-\vec{\partial}^2)\vec{G}-\varkappa\,\partial_t(\vec{\partial}\times\vec{U})+\vec{\partial}_t\,\partial}\;\varkappa\,\vec{\partial}\times(\partial_t\vec{F})+\varkappa^2(\vec{\partial}\times\vec{U})$$

$$+\varkappa\{\partial_t^2+\vec{\partial}\times(\vec{\partial}\times\vec{G})\}-\partial_t^2(\vec{\partial}\times\vec{U})\pm\varkappa\,\vec{\partial}\times(\partial_t\vec{F})$$

$$\cancel{\varkappa}\{\partial_t^2-\vec{\partial}^2+\varkappa^2\}\vec{G}+\underbrace{\vec{\partial}\,\vec{\partial}\,\vec{G}}_{+\vec{\partial}(\vec{\partial}\vec{G})}-\vec{\partial}\times(\partial_t\vec{F})-\left\{\cancel{\frac{1}{\varkappa}\partial_t^2(\vec{\partial}\times\vec{U})}\ \partial_t^2\vec{G}\right\}=0$$

$$(\partial_t^2-\vec{\partial}^2+\varkappa^2)\vec{F}-\frac{1}{\varkappa}\partial_t(\vec{\partial}\times(\vec{\partial}\times\vec{U}))=0$$

$$(\quad\text{''}\quad)\vec{G}+\frac{1}{\varkappa}\{\vec{\partial}\,\vec{\partial}\,(\vec{\partial}\times\vec{U})-\partial_t^2(\vec{\partial}\times\vec{U})$$
$$+\vec{\partial}\times(\partial_t^2\vec{U})+\vec{\partial}\times(\vec{\partial}\,U_0)\}=0$$

§3. Linear Field Equations of Proca Type.

curl $\vec{U}$

$\frac{1}{c}\frac{\partial}{\partial t}$

$$\left.\begin{array}{ll} \frac{1}{c}\frac{\partial \vec{F}}{\partial t} - \mathrm{curl}\,\vec{G} - \varkappa \vec{U} = 0 & \mathrm{div}\,\vec{F} + \varkappa U_0 = 0 \\ \frac{1}{c}\frac{\partial \vec{U}}{\partial t} + \mathrm{grad}\,U_0 + \varkappa \vec{F} = 0 & \mathrm{curl}\,\vec{U} - \varkappa \vec{G} = 0. \end{array}\right\} \quad (1)$$

$$\varkappa\,\frac{1}{c}\frac{\partial}{\partial t}\,\mathrm{curl}\,\vec{F} - \mathrm{grad}\,\mathrm{div}\,\vec{G} + \Delta \vec{G} - \varkappa\,\mathrm{curl}\,\vec{U}$$

$$= \frac{-1}{\varkappa}\frac{\partial}{c\,\partial t}\,\mathrm{curl}\left(\frac{1}{c}\frac{\partial \vec{U}}{\partial t}\right) - \frac{1}{\varkappa}\cancel{\mathrm{grad}\,\mathrm{div}\,\mathrm{curl}\,\vec{U}} + \Delta \vec{G} - \varkappa^2 \vec{G}$$

$$= -\frac{1}{c^2}\frac{\partial^2 \vec{G}}{\partial t^2} + \Delta \vec{G} - \varkappa^2 \vec{G} = 0.$$

$$\frac{1}{c^2}\frac{\partial^2 \vec{F}}{\partial t^2} - \mathrm{curl}\,\frac{1}{c}\frac{\partial}{\partial t}\frac{\mathrm{curl}\,\vec{U}}{\varkappa} + \varkappa\,\underline{\mathrm{grad}\,U_0} + \varkappa^2 \vec{F} = 0$$

$$- \mathrm{grad}\,\mathrm{div}\,\vec{F}$$

or $$\frac{1}{c^2}\frac{\partial^2 \vec{F}}{\partial t^2} - \Delta \vec{F} - \cancel{\mathrm{curl}\,\mathrm{curl}\left(\vec{F} + \frac{1}{\varkappa c}\frac{\partial \vec{U}}{\partial t}\right)} + \varkappa^2 \vec{F} = 0.$$

$$\frac{1}{c^2}\frac{\partial^2 \vec{U}}{\partial t^2} + \mathrm{grad}\,\frac{1}{c}\frac{\partial U_0}{\partial t} + \varkappa(\mathrm{curl}\,\vec{G} + \varkappa \vec{U})$$

$$= \frac{1}{c^2}\frac{\partial^2 \vec{U}}{\partial t^2} + \frac{1}{\varkappa}\frac{1}{c}\frac{\partial}{\partial t}\,\mathrm{grad}\,\mathrm{div}\,\vec{F} + \varkappa\,\mathrm{curl}\,\mathrm{curl}\,\vec{U} \qquad + \varkappa^2 \vec{U}$$

$$= \frac{1}{c^2}\frac{\partial^2 \vec{U}}{\partial t^2} - \Delta \vec{U} + \cancel{\mathrm{grad}\,\mathrm{div}\left(\vec{U} - \frac{1}{\varkappa}\frac{\partial \vec{F}}{\partial t}\right)} + \varkappa^2 \vec{U} = 0$$

$$\frac{1}{c}\frac{\partial}{\partial t}\,\mathrm{div}\,\vec{U} + \Delta U_0 + \varkappa\,\mathrm{div}\,\vec{F}$$

$$= \Delta U_0 - \frac{1}{c^2}\frac{\partial^2 U_0}{\partial t^2} - \varkappa^2 U_0 + \cancel{\frac{1}{c}\frac{\partial}{\partial t}\left(\mathrm{div}\,\vec{U} + \frac{1}{c}\frac{\partial U_0}{\partial t}\right)} = 0$$

$$\mathrm{div}\,\vec{U} + \frac{1}{c}\frac{\partial U_0}{\partial t} = 0$$

$$\frac{1}{c}\frac{\partial \vec{G}}{\partial t} + \mathrm{curl}\,\vec{F} = 0 \qquad \mathrm{div}\,\vec{G} = 0 \quad (2)$$

之等は (1)から必然的に follow する.

又 (1) 中から U_0, $\vec{G}$ を eliminate すると

$$\left.\begin{array}{l} \frac{\varkappa}{c}\frac{\partial \vec{F}}{\partial t} - \mathrm{curl}\,\mathrm{curl}\,\vec{U} - \varkappa^2 \vec{U} = 0 \\ \frac{\varkappa}{c}\frac{\partial \vec{U}}{\partial t} - \mathrm{grad}\,\mathrm{div}\,\vec{F} + \varkappa^2 \vec{F} = 0 \end{array}\right)\!\!\Big)$$

Electromagnetic Field が在ると、(2) は (1) と compatible でない。

何故 (1) は

$$\left(\frac{1}{c}\frac{\partial}{\partial t}-\frac{ie}{\hbar c}A_0\right)\vec{F}-\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\times\vec{G}-\varkappa\vec{U}=0$$

$$\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\vec{F}+\varkappa U_0=0$$

$$\left(\frac{1}{c}\frac{\partial}{\partial t}-\frac{ie}{\hbar c}A_0\right)\vec{U}+\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)U_0+\varkappa\vec{F}=0 \qquad (3)$$

$$\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\times\vec{U}-\varkappa\vec{G}=0$$

上から sum すれば

$$\varkappa\left\{\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\vec{U}+\left(\frac{1}{c}\frac{\partial}{\partial t}-\frac{ie}{\hbar c}A_0\right)U_0\right\}$$

$$=\left\{\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\left(\frac{1}{c}\frac{\partial}{\partial t}-\frac{ie}{\hbar c}A_0\right)-\left(\frac{1}{c}\frac{\partial}{\partial t}-\frac{ie}{\hbar c}A_0\right)\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\right\}\vec{F}$$

$$-\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\cdot\left\{\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\times\vec{G}\right\}$$

$$\varepsilon=\frac{ie}{\hbar c}$$

$$=\frac{ie}{\hbar c}\left(\vec{E}\cdot\vec{F}+\vec{H}\cdot\vec{G}\right) \qquad \varkappa=\frac{m_0c}{\hbar}$$

$$\varkappa\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}-\frac{ie}{\hbar c}A_0\right)\vec{G}+\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\times\vec{F}\right\}$$

$$=\left(\frac{1}{c}\frac{\partial}{\partial t}-\frac{ie}{\hbar c}A_0\right)\left\{\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\times\vec{U}\right\}-\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\times$$

$$\times\left(\frac{1}{c}\frac{\partial}{\partial t}-\frac{ie}{\hbar c}A_0\right)\vec{U}-\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\times\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)U_0$$

$$=\frac{ie}{\hbar c}\left(\vec{E}\times\vec{U}-\vec{H}U_0\right)$$

$$\left(\frac{1}{c}\frac{\partial}{\partial t}-\frac{ie}{\hbar c}A_0\right)\left(\frac{\partial U_z}{\partial y}+\frac{ie}{\hbar c}A_yU_z\right) \qquad \left(\frac{\partial}{\partial z}+\frac{ie}{\hbar c}A_z\right)\left(\frac{\partial U_0}{\partial y}+\frac{ie}{\hbar c}A_yU_0\right)$$

$$-\left(\frac{\partial}{\partial y}+\frac{ie}{\hbar c}A_y\right)\left(\frac{1}{c}\frac{\partial U_z}{\partial t}-\frac{ie}{\hbar c}A_0U_z\right) \qquad -\left(\frac{\partial}{\partial y}+\frac{ie}{\hbar c}A_y\right)\left(\frac{\partial U_0}{\partial z}+\frac{ie}{\hbar c}A_zU_0\right)+$$

$$=-\frac{ie}{\hbar c}\frac{\partial A_z}{\partial y}U_0$$

$$+\frac{1}{c}\frac{\partial A_y}{\partial t}U_z+\frac{ie}{\hbar c}\frac{\partial A_0}{\partial y}U_z \qquad -\frac{ie}{\hbar c}H_xU_0$$

$$E_yU_z$$

$$\kappa\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)G = \left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\left\{\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\times\vec{U}\right\}$$
$$= \frac{ie}{\hbar c}\vec{H}\vec{U}$$

$$\left(\frac{\partial}{\partial x}+\frac{ie}{\hbar c}A_x\right)\left\{\frac{\partial U_z}{\partial y}+\frac{ie}{\hbar c}A_y U_z\right\}$$

$$L = \tilde{F}_i F_i - \tilde{G}_i G_i + \tilde{U}_i U_i + \tilde{U}_0 U_0$$

where

~~the suffix i which appears twice, n~~

~~the summation suffix i appearing twice denotes the summ~~

~~the summation for i, which appearing twice, is omitted.~~

The suffix i stands for 1, 2, or 3 and the summation with respect to ~~it~~ is omitted.

$$U_i^\dagger = \frac{\partial L}{\partial\frac{\partial U_i}{\partial t}} = -\frac{1}{\kappa c}\tilde{F}_i \qquad \frac{\partial L}{\partial\frac{\partial U_0}{\partial t}} = 0. \qquad \frac{1}{\kappa}\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)\tilde{\vec{F}}-\frac{ie}{\hbar c}A_0\tilde{\vec{U}}$$

$$\frac{1}{c}\frac{\partial\tilde{\vec{U}}}{\partial t} = -\kappa\tilde{\vec{F}}-\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)\tilde{U}_0-\frac{ie}{\hbar c}A_0\tilde{\vec{U}} = \kappa^2 c\vec{U}^\dagger+\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)$$

$$L = \kappa^2c^2\,\vec{U}^\dagger_i\tilde{\vec{U}}^\dagger_i - \frac{1}{\kappa^2}\left\{\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)\times\tilde{\vec{U}}\right\}\left\{\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\times\vec{U}\right\}$$
$$-\tilde{\vec{U}}_i\vec{U}_i + c^2\left\{\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)\tilde{\vec{U}}^\dagger\right\}\left\{\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\vec{U}^\dagger\right\}$$

$$H = \vec{U}^\dagger\frac{\partial\vec{U}}{\partial t}+\tilde{\vec{U}}^\dagger\frac{\partial\tilde{\vec{U}}}{\partial t} - L$$
$$= \kappa^2c^2\,\tilde{\vec{U}}^\dagger\vec{U}^\dagger - c^2\tilde{\vec{U}}^\dagger\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)\vec{U}^\dagger - \frac{ie}{\hbar}\tilde{\vec{U}}^\dagger A_0\tilde{\vec{U}}$$
$$- c^2\vec{U}^\dagger\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\tilde{\vec{U}}^\dagger + \frac{ie}{\hbar}\vec{U}^\dagger A_0\vec{U}.$$
$$-\frac{1}{\kappa^2}\left\{\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)\times\tilde{\vec{U}}\right\}\left\{\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\times\vec{U}\right\}$$
$$-\tilde{\vec{U}}\vec{U} + c^2\left\{\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)\tilde{\vec{U}}^\dagger\right\}\left\{\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec{A}\right)\vec{U}^\dagger\right\}$$