§4. Interaction of the U-field with the Heavy Particle.

$$\bar{H}_U = \iiint \{ \varkappa^2 c^2 \tilde{U}^{+} U^{+} + c^2 \operatorname{div} \tilde{U}^{+} \operatorname{div} U^{+} + \frac{1}{\varkappa^2} \operatorname{curl} \tilde{U} \operatorname{curl} U + \tilde{U} U \} dv,$$

$$U = \sum_k \sum_j u_{jk} \vec{e}_{jk} e^{i\vec{k}\vec{r}}$$

$$U^{+} = \sum_k \sum_j u^{+}_{jk} \vec{e}_{jk} e^{-i\vec{k}\vec{r}}$$

$$U_0 = -\frac{1}{\varkappa} \operatorname{div} F = \frac{c}{\varkappa^2} \operatorname{div} \tilde{U}^{+}$$

$$= \sum_k \frac{ikc}{\varkappa^2} \tilde{u}^{+}_{jk} e^{+i\vec{k}\vec{r}}$$

$$G = \operatorname{curl} U = \sum_k \sum_j (i k)(u_{2k} \vec{e}_{3k} - u_{3k} \vec{e}_{2k}) e^{i\vec{k}\vec{r}}$$

$$F = \varkappa c \tilde{U}^{+} = -\varkappa c \sum \tilde{u}^{+}_{jk} \vec{e}_{jk} e^{i\vec{k}\vec{r}}$$

$$\bar{H}_1 = g_1 \quad \vec{\alpha}\, u_{jk} e_{jk}$$

$$\frac{1}{c}\frac{\partial \vec{F}}{\partial t} - \operatorname{curl} \vec{G} - \varkappa \vec{U} = \varkappa \vec{M} \qquad \operatorname{div} F + \varkappa U_0 = \varkappa M_0$$

$$\frac{1}{c}\frac{\partial \vec{U}}{\partial t} + \operatorname{grad} U_0 + \varkappa \vec{F} = \vec{S} \qquad \operatorname{curl} \vec{H} - \varkappa \vec{G} = \varkappa \vec{T}$$

$$\bar{H} = \bar{H}_U + \bar{H}_M + \bar{H}_1 + \bar{H}_2$$

$$\bar{H}_U = \iiint \{ \varkappa^2 c^2 U^{+} \tilde{U}^{+} + c^2 \operatorname{div} U^{+} \operatorname{div} \tilde{U}^{+} + \frac{1}{\varkappa^2} \operatorname{curl} \tilde{U} \operatorname{curl} U + \hat{U} U \} dv$$

$$\bar{H}' = (\vec{M}\tilde{\vec{U}} - M_0 \tilde{U}_0) + (\vec{S}\tilde{\vec{F}} - \vec{T}\tilde{\vec{G}}) + \text{comp. conj.}$$

$$\bar{L} = \tilde{F} F - \tilde{G} G - \tilde{U} U + \tilde{U}_0 U_0 - (\vec{M}\tilde{\vec{U}} - M_0 \tilde{U}_0) + (\vec{S}\tilde{\vec{F}} - \vec{T}\tilde{\vec{G}})$$

$$- \text{comp. conj.} + \bar{L}_M$$

$$\frac{1}{\varkappa^2}\left(\frac{1}{c}\frac{\partial \tilde{\vec{U}}}{\partial t} + \operatorname{grad} \tilde{U}_0\right)\left(\frac{1}{c}\frac{\partial \vec{U}}{\partial t} + \operatorname{grad} U_0\right) - \frac{1}{\varkappa^2} \operatorname{curl} \tilde{U} \operatorname{curl} U - \tilde{U} U + \tilde{U}_0 U_0$$

$$-(\vec{M}\tilde{\vec{U}} - M_0 \tilde{U}_0) + \frac{\vec{S}}{\varkappa}\left(\frac{1}{c}\frac{\partial \tilde{\vec{U}}}{\partial t} + \operatorname{grad} \tilde{U}_0\right) - \frac{\vec{T}}{\varkappa}(\operatorname{curl} \tilde{\vec{H}})$$

$$= \frac{1}{\varkappa^2}\left(\frac{1}{c}\frac{\partial \tilde{\vec{U}}}{\partial t} + \operatorname{grad} \tilde{U}_0 - \frac{\tilde{S}}{\varkappa}\right)\left(\frac{1}{c}\frac{\partial \vec{U}}{\partial t} + \operatorname{grad} U_0 - \frac{S}{\varkappa}\right) \to$$

$$\frac{1}{\varkappa}\frac{1}{c}\frac{\partial \vec{F}}{\partial t} - \frac{1}{\varkappa}\,\mathrm{curl}\,G - U - M = 0$$

$$\frac{1}{\varkappa}\,\mathrm{div}\,\vec{F} + U_0 + M_0 = 0$$

$$\frac{1}{c^2}\frac{\partial^2 F}{\partial t^2} - \mathrm{curl}\frac{\partial \vec{G}}{c\,\partial t} - \varkappa\frac{1}{c}\frac{\partial \vec{U}}{\partial t} - \mathrm{grad\,div}\,F - \varkappa\,\mathrm{grad}\,U_0$$
$$= \varkappa\left(\frac{1}{c}\frac{\partial \vec{M}}{\partial t} + \mathrm{grad}\,M_0\right)$$

$$\frac{1}{c^2}\frac{\partial^2 F}{\partial t^2} - \Delta F - \mathrm{curl}\,\mathrm{curl}\,F - \mathrm{curl}\,\frac{1}{c}\frac{\partial \vec{G}}{\partial t} + \varkappa^2 F - \varkappa^2 S =$$

$$\mathrm{curl}\left(\mathrm{curl}\,F + \frac{1}{c}\frac{\partial \vec{G}}{\partial t}\right) = \frac{1}{\varkappa}\,\mathrm{curl}\left(-\frac{1}{c}\frac{\partial \vec{U}}{\partial t} - \mathrm{grad}\,U_0 + \varkappa\,\mathrm{curl}\,S\right.$$
$$\left. + \mathrm{curl}\frac{\partial U}{c\,\partial t} + \varkappa\frac{1}{c}\frac{\partial \vec{T}}{\partial t}\right)$$
$$= \mathrm{curl}\left(\mathrm{curl}\,\vec{S} + \frac{1}{c}\frac{\partial \vec{T}}{\partial t}\right)$$

$$\frac{1}{c^2}\frac{\partial^2 F}{\partial t^2} - \Delta F + \varkappa^2 F = \varkappa\left(\frac{1}{c}\frac{\partial \vec{M}}{\partial t} + \mathrm{grad}\,M_0\right) + \varkappa^2 S$$
$$+ \frac{1}{\varkappa}\,\mathrm{curl}\left(\mathrm{curl}\,\vec{S} + \frac{1}{c}\frac{\partial \vec{T}}{\partial t}\right) = -S'$$

$$-\mathrm{curl}\left(\frac{1}{c}\frac{\partial \vec{F}}{\partial t}\right) + \mathrm{curl}\,\mathrm{curl}\,\vec{G} + \varkappa\,\mathrm{curl}\,\vec{U} = \varkappa\,\mathrm{curl}\,\vec{M}$$

$$-\frac{1}{c}\frac{\partial}{\partial t}\left\{\frac{1}{\varkappa}\mathrm{curl}\frac{\partial U}{c\,\partial t} - \varkappa\,\mathrm{curl}\,\vec{S}\right\} + \underbrace{\mathrm{grad\,div}\,\vec{G}}_{\varkappa\,\mathrm{div}\,T} - \Delta\vec{G} + \varkappa^2\vec{G}$$
$$-\varkappa^2 T = -\varkappa\,\mathrm{curl}\,\vec{M}.$$

$$\frac{1}{c}\frac{\partial \vec{G}}{\partial t} - \frac{1}{c}\frac{\partial \vec{T}}{\partial t}$$

$$\frac{1}{c^2}\frac{\partial^2 G}{\partial t^2} - \Delta G + \varkappa^2 G = -\varkappa\,\mathrm{curl}\,M + \varkappa^2 T$$
$$+ \frac{1}{c}\frac{\partial}{\partial t}\left(\mathrm{curl}\,\vec{S} + \frac{1}{c}\frac{\partial \vec{T}}{\partial t}\right) - \mathrm{grad\,div}\,T$$
$$= -T'$$

$$\frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} + \mathrm{grad}\,\frac{1}{c}\frac{\partial U_0}{\partial t} + \kappa\frac{1}{c}\frac{\partial \vec{F}}{\partial t} + \mathrm{curl\,curl}\,U - \kappa\,\mathrm{curl}\,G$$
$$= \kappa\left(\frac{1}{c}\frac{\partial S}{\partial t} - \mathrm{curl}\,T\right)$$

$$\frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} - \Delta U + \kappa^2 U + \mathrm{grad}\left(\frac{1}{c}\frac{\partial U_0}{\partial t} + \mathrm{div}\,U\right) = \kappa^2 M + \kappa\left(\frac{1}{c}\frac{\partial S}{\partial t} - \mathrm{curl}\,T\right)$$

$$\frac{1}{c}\frac{\partial U_0}{\partial t} + \mathrm{div}\,U = -\left(\frac{1}{c}\frac{\partial M_0}{\partial t} + \mathrm{div}\,M\right)$$

$$\frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} - \Delta U + \kappa^2 U = \underbrace{\mathrm{grad}\left(\frac{1}{c}\frac{\partial M_0}{\partial t} + \mathrm{div}\,M\right) - \kappa^2 M + \kappa\left(\cancel{\frac{1}{c}\frac{\partial S}{\partial t} - \mathrm{curl}\,T}\right)}_{=\vec{M}'}$$

$$\frac{1}{c}\frac{\partial}{\partial t}\mathrm{div}\,U + \Delta U_0 + \kappa\,\mathrm{div}\,F = \kappa\,\mathrm{div}\,S$$

$$-\frac{1}{c^2}\frac{\partial^2 U_0}{\partial t^2} + \Delta U_0 - \kappa^2 U_0 - \kappa^2 M_0 = \kappa\,\mathrm{div}\,S + \frac{1}{c}\frac{\partial}{\partial t}\left(\frac{\partial M_0}{\partial t} + \mathrm{div}\,M\right)$$

$$\frac{1}{c^2}\frac{\partial^2 U_0}{\partial t^2} - \Delta U_0 + \kappa^2 U_0 = \underbrace{-\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{c}\frac{\partial M_0}{\partial t} + \mathrm{div}\,M\right) - \kappa^2 M_0 + \kappa\,\mathrm{div}\,S}_{=M_0'}$$

Heavy Particle が $(\vec{r}_1, \vec{r}_2)$ にあり、その energy diff. を negligible とすると、

$$\bar{H}' = \int\Big\{\tilde{\vec{M}}(\vec{r}_1)\vec{M}'(\vec{r}_2) - \tilde{M}_0(r_1)M_0'(r_2) - \tilde{\vec{S}}(\vec{r}_1)\vec{S}'(\vec{r}_2) + \tilde{\vec{T}}(r_1)\vec{T}'(r_2)$$
$$+ \vec{M}(\vec{r}_2)\tilde{\vec{M}}'(\vec{r}_1) - M_0(r_2)\tilde{M}'(r_1) - \vec{S}(r_2)\tilde{\vec{S}}'(r_2) + T(r_2)\tilde{T}'(r_1)\Big\}$$
$$\times \frac{e^{-\kappa r}}{r}\,dv$$

$$= \int g_1\Big\{\alpha^{(1)}\left(-\mathrm{grad\,div}\,M^{(2)} + \kappa^2 M^{(2)} + \kappa\,\mathrm{curl}\,T^{(2)}\right) - \left(\kappa^2 M_0^{(2)} - \kappa\,\mathrm{div}\,S^{(2)}\right)\Big\}$$
$$+ \alpha^{(2)}\left(-\mathrm{grad\,div}\,\tilde{M}^{(1)} + \kappa^2\tilde{M}^{(1)} + \mathrm{curl}\,\tilde{T}^{(1)}\right) + \left(\kappa^2\tilde{M}_0^{(2)} - \kappa\,\mathrm{div}\,\tilde{S}^{(2)}\right)\Big\}$$
$$+ \quad g_2\Big\{$$
$$- \rho_3^{(1)}\vec{\sigma}^{(1)}\rho_3^{(1)}\vec{\sigma}^{(2)}\Big\}$$

なお 両方とも静止しているから

$$= \int 2g_1^2\kappa^2\left(\alpha^{(1)}\alpha^{(2)} - 1\right) + 2g_2^2\kappa^2\left\{\rho_2^{(1)}\vec{\sigma}^{1}\rho_2^{(2)}\vec{\sigma}^{2}\right.$$

$$\cong 2g_1^2\kappa^2(-1) - 2g_2^2\kappa^2\,\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}$$