

Interaction of the Neutron and the Proton.

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$$H_0 = 4\pi\kappa^2 c^2 \vec{U} \vec{U}^T + 4\pi c^2 \operatorname{div} \vec{U}^T \operatorname{div} \vec{U} + \frac{1}{4\pi\kappa} \operatorname{curl} \vec{U} \operatorname{curl} \vec{U} + \frac{1}{4\pi} \vec{U} \vec{U}$$

$$H' = -\left(\frac{g}{\kappa}\right) \left\{ -U_0 + \rho_1 (\vec{\sigma} \vec{U}) + \rho_2 (\vec{\sigma} \vec{F}) + \rho_3 (\vec{\sigma} \vec{G}) \right\} \bar{\Psi} \Psi + \text{comp. conj.}$$

$$\operatorname{div} \vec{F} + \kappa U_0 = 4\pi g \tilde{\Psi} \vec{\sigma} \Psi \quad U_0 = 4\pi\kappa c \operatorname{div} \vec{U}^T + \frac{4\pi g}{\kappa} \tilde{\Psi} \vec{\sigma} \Psi$$
$$\operatorname{curl} \vec{U} - \kappa \vec{G} = 4\pi g \tilde{\Psi} \rho_3 \vec{\sigma} \Psi$$

$$\vec{U}^T = -\frac{\vec{F}}{4\pi\kappa c}$$

$$H' = \left(\frac{g}{\kappa}\right) \left\{ \operatorname{div} 4\pi c \operatorname{div} \vec{U}^T \cdot \tilde{\Psi} \vec{\sigma} \Psi - \vec{U}^T \cdot \tilde{\Psi} \rho_1 \vec{\sigma} \Psi + 4\pi\kappa c \vec{U}^T \cdot \tilde{\Psi} \rho_2 \vec{\sigma} \Psi - \frac{1}{\kappa} \operatorname{curl} \vec{U} \cdot \tilde{\Psi} \rho_3 \vec{\sigma} \Psi - \frac{4\pi g}{\kappa} \tilde{\Psi} \rho_3 \vec{\sigma} \Psi \cdot \tilde{\Psi} \rho_3 \vec{\sigma} \Psi + \tilde{\Psi} \vec{\sigma} \Psi \cdot \tilde{\Psi} \vec{\sigma} \Psi \right\}$$

$$\vec{U} = 4\pi\kappa c^2 \vec{U}^T - 4\pi c^2 \operatorname{div} \operatorname{grad} \operatorname{div} \vec{U}^T - \frac{1}{4\pi\kappa} \operatorname{curl} \operatorname{curl} \vec{U}^T - \frac{g}{\kappa} 4\pi c \operatorname{grad} \tilde{\Psi} \vec{\sigma} \Psi + 4\pi g c \tilde{\Psi} \rho_3 \vec{\sigma} \Psi$$
$$\frac{1}{c} \frac{\partial \vec{U}}{\partial t} = -\kappa \vec{F} - \operatorname{grad} U_0 + \frac{4\pi g}{\kappa} \operatorname{grad} (\tilde{\Psi} \vec{\sigma} \Psi) - \frac{g}{\kappa} 4\pi c \operatorname{grad} \tilde{\Psi} \vec{\sigma} \Psi + 4\pi g c \tilde{\Psi} \rho_3 \vec{\sigma} \Psi$$

$$\frac{1}{c} \frac{\partial \vec{U}}{\partial t} + \operatorname{grad} U_0 + \kappa \vec{F} = -\frac{4\pi g}{\kappa} \operatorname{grad} (\tilde{\Psi} \vec{\sigma} \Psi) + 4\pi g \tilde{\Psi} \rho_3 \vec{\sigma} \Psi$$

$$-\vec{U}^T = \frac{1}{4\pi\kappa} \operatorname{curl} \operatorname{curl} \vec{U} + \frac{1}{4\pi} \vec{U} - \frac{g}{\kappa} \tilde{\Psi} \rho_1 \vec{\sigma} \Psi - \frac{g}{\kappa^2} \operatorname{curl} (\tilde{\Psi} \rho_3 \vec{\sigma} \Psi)$$

4πκ

$$\frac{1}{c} \frac{\partial \vec{F}}{\partial t} = \operatorname{curl} \vec{G} - \frac{4\pi g}{\kappa} \operatorname{curl} (\tilde{\Psi} \rho_3 \vec{\sigma} \Psi) + \kappa \vec{U} - \frac{4\pi g}{\kappa} \tilde{\Psi} \rho_3 \vec{\sigma} \Psi - \frac{4\pi g}{\kappa} \operatorname{curl} \tilde{\Psi} \rho_3 \vec{\sigma} \Psi$$

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$$\frac{1}{c} \frac{\partial U}{\partial t} + \text{grad } U_0 + \kappa F = 4\pi g \tilde{\Psi} \rho \tilde{\sigma} \tilde{\Omega} \Psi$$

$$\frac{1}{c} \frac{\partial F}{\partial t} - \text{curl } G - \kappa U = -4\pi g \tilde{\Psi} \rho \tilde{\sigma} \tilde{\Omega} \Psi$$

$$- \frac{4\pi g}{\kappa} \text{curl } \tilde{\Psi} \rho \tilde{\sigma} \tilde{\Omega} \Psi, ?$$

$$\text{div } F + \kappa U_0 = 4\pi g \tilde{\Psi} \tilde{\Omega} \Psi$$

$$+ \text{curl } \text{curl } U - \kappa G = -4\pi g \tilde{\Psi} \rho \tilde{\sigma} \tilde{\Omega} \Psi.$$

$$\frac{1}{c} \frac{\partial U}{\partial t} + \frac{1}{c} \frac{\partial}{\partial t} \text{grad } U_0 + \kappa \frac{1}{c} \frac{\partial F}{\partial t} + \text{grad } \text{div } U - \Delta U$$

$$- \kappa \text{curl } G = 4\pi g \left\{ \frac{1}{c} \frac{\partial}{\partial t} \tilde{\Psi} \rho \tilde{\sigma} \tilde{\Omega} \Psi - \text{curl } \tilde{\Psi} \rho \tilde{\sigma} \tilde{\Omega} \Psi \right\}$$

$$\text{or } \frac{1}{c} \frac{\partial U}{\partial t} - \Delta U + \kappa U$$

$$= 4\pi g \left\{ +\kappa \tilde{\Psi} \rho \tilde{\sigma} \tilde{\Omega} \Psi + \text{curl}(\tilde{\Psi} \rho \tilde{\sigma} \tilde{\Omega} \Psi) \right.$$

$$\left. + \frac{1}{c} \frac{\partial}{\partial t} \tilde{\Psi} \rho \tilde{\sigma} \tilde{\Omega} \Psi - \frac{1}{c} \frac{\partial}{\partial t} \tilde{\Psi} \tilde{\Omega} \Psi - \text{div } \tilde{\Psi} \rho \tilde{\sigma} \tilde{\Omega} \Psi \right\}$$

$$\text{grad} \left( \frac{1}{c} \frac{\partial U_0}{\partial t} + \text{div } U \right) = \text{grad } 4\pi g \left\{ \frac{1}{c} \frac{\partial}{\partial t} \tilde{\Psi} \tilde{\Omega} \Psi \right.$$

$$\left. + \text{div } \tilde{\Psi} \rho \tilde{\sigma} \tilde{\Omega} \Psi \right\}$$

$$\left( \frac{1}{c} \frac{\partial}{\partial t} - \Delta + \kappa \right) (\text{curl } U) \cong 4\pi g \left\{ \text{curl curl}(\tilde{\Psi} \rho \tilde{\sigma} \tilde{\Omega} \Psi) \right\}$$

$$+ \frac{g}{\kappa} \text{curl } U \cong \int \frac{e^{-\kappa r}}{r} \text{curl curl}(\tilde{\Psi} \rho \tilde{\sigma} \tilde{\Omega} \Psi) dv$$

$$- \frac{g}{\kappa^2} \text{curl } U \cdot \tilde{\Psi} \rho \tilde{\sigma} \tilde{\Omega} \Psi = - \frac{g^2}{\kappa^2} \int \frac{e^{-\kappa r}}{r} \text{curl curl}(\tilde{\Psi} \rho \tilde{\sigma} \tilde{\Omega} \Psi)$$

$$\times \tilde{\Psi} \rho \tilde{\sigma} \tilde{\Omega} \Psi$$

$$\text{grad div} - \Delta$$

$$\text{curl curl}(\tilde{\Psi} \rho \tilde{\sigma} \tilde{\Omega} \Psi) +$$

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$$\text{curl curl } \vec{G} - \kappa \text{curl } \vec{U} = \frac{4\pi g}{c} \text{curl } \vec{T}$$

$$= -4\pi g \text{curl } \tilde{\Phi} \rho_1 \tilde{\sigma} \tilde{\Omega} \Psi - \frac{8\pi g}{\kappa} \text{curl}^2 \tilde{\Phi} \rho_1 \tilde{\sigma} \tilde{\Omega} \Psi$$

$$\frac{1}{c} \frac{\partial \vec{U}}{\partial t} + \text{grad } U_0 + \kappa \vec{T} = 4\pi g \tilde{\Phi} \rho_1 \tilde{\sigma} \tilde{\Omega} \Psi = 4\pi g \vec{S}$$

$$\text{curl } \vec{U} - \kappa \vec{G} = -4\pi g \tilde{\Phi} \rho_1 \tilde{\sigma} \tilde{\Omega} \Psi = -4\pi g \vec{T}$$

$$\frac{1}{c} \frac{\partial \vec{T}}{\partial t} - \text{curl } \vec{G} - \kappa \vec{U} = -4\pi g \tilde{\Phi} \rho_1 \tilde{\sigma} \tilde{\Omega} \Psi = -4\pi g \vec{M}$$

$$d\vec{w} \vec{T} + \kappa U_0 = 4\pi g \tilde{\Phi} \tilde{\Omega} \Psi = 4\pi g \vec{M}_0$$

$$L = \frac{1}{4\pi} (\tilde{T} \tilde{T} - \tilde{G} \tilde{G} + \tilde{U}_0 U_0 - \tilde{U} \tilde{U})$$

$$+ \frac{g}{\kappa} (\tilde{S} \tilde{S} - \tilde{T} \tilde{T} - \tilde{U}_0 \tilde{M}_0 + \tilde{U} \tilde{M})$$

$$+ \frac{g}{\kappa} (\tilde{S} \tilde{S} - \tilde{T} \tilde{T} - U_0 \tilde{M}_0 + U \tilde{M})$$

$$( + \frac{4\pi g}{\kappa} (\tilde{S} \tilde{S} - \tilde{T} \tilde{T} + \tilde{M}_0 \tilde{M}_0 - \tilde{M} \tilde{M}) ) ?$$

$$- \frac{4\pi g}{\kappa} (\tilde{M} \tilde{S} - \tilde{G} \tilde{G})$$

$$+ \frac{4\pi g}{\kappa} (\tilde{T} \tilde{T} - \tilde{S} \tilde{S}) + \frac{4\pi g}{\kappa} (\tilde{M}_0 \tilde{M}_0 - \tilde{M} \tilde{M})$$

$$L = \frac{1}{4\pi \kappa^2} \left( \frac{1}{c} \frac{\partial \vec{U}}{\partial t} + \text{grad } U_0 - 4\pi g \vec{S} \right) \left( \frac{1}{c} \frac{\partial \vec{U}}{\partial t} + \text{grad } U_0 - 4\pi g \vec{S} \right)$$

$$- \frac{1}{4\pi \kappa^2} (\text{curl } \tilde{U} + 4\pi g \tilde{T}) (\text{curl } \tilde{U} + 4\pi g \tilde{T})$$

$$+ \frac{1}{4\pi} (\tilde{U}_0 U_0 - \tilde{U} \tilde{U}) - \frac{g}{\kappa} (\tilde{U}_0 \tilde{M}_0 - \tilde{U} \tilde{M})$$

$$- \frac{g}{\kappa} (U_0 \tilde{M}_0 - U \tilde{M})$$

$$U^\dagger = \frac{\partial L}{\partial \frac{\partial \vec{U}}{\partial t}} = \frac{1}{4\pi \kappa^2 c} \left( \frac{1}{c} \frac{\partial \vec{U}}{\partial t} + \text{grad } U_0 - 4\pi g \vec{S} \right) = - \frac{\tilde{T}}{4\pi \kappa c}$$

$$\therefore \frac{\partial \vec{U}}{\partial t} = 4\pi \kappa^2 c U^\dagger - c \text{grad } U_0 + 4\pi g c \vec{S}$$

$$U^\dagger = 0$$

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$$\begin{aligned}
 H = & U^{\dagger} (4\pi\kappa^2 c^2 \tilde{U}^{\dagger} - c \text{grad} U_0 + 4\pi g c S) \\
 & + \tilde{U}^{\dagger} (4\pi\kappa^2 c^2 U^{\dagger} - c \text{grad} \tilde{U}_0 + 4\pi g c \tilde{S}) \\
 & - 4\pi\kappa^2 c^2 \tilde{U}^{\dagger} \tilde{U}^{\dagger} + \frac{1}{4\pi\kappa^2} (\text{curl} \tilde{U} + 4\pi g \tilde{T}) (\text{curl} U + 4\pi g T) \\
 & - \frac{1}{4\pi} (\tilde{U}_0 U_0 - \tilde{U} U) + \frac{g}{\kappa} (\tilde{U}_0 M_0 - \tilde{U} M) + \frac{g}{\kappa} (U_0 \tilde{M}_0 - U \tilde{M})
 \end{aligned}$$

$$\text{curl} U - \kappa G = -4\pi g T$$

$$-4\pi\kappa c \text{div} \tilde{U}^{\dagger} + \kappa U_0 = 4\pi g M_0$$

$$U_0 = 4\pi c \text{div} \tilde{U}^{\dagger} + \frac{4\pi g}{\kappa} M_0$$

$$\begin{aligned}
 H = & 4\pi\kappa^2 c^2 \tilde{U}^{\dagger} U^{\dagger} - c U^{\dagger} \text{grad} (4\pi c \text{div} \tilde{U}^{\dagger} + \frac{4\pi g}{\kappa} M_0) \\
 & + 4\pi g c U^{\dagger} S - c \tilde{U}^{\dagger} \text{grad} (4\pi c \text{div} U^{\dagger} + \frac{4\pi g}{\kappa} \tilde{M}_0) \\
 & + 4\pi g c \tilde{U}^{\dagger} \tilde{S} + \frac{1}{4\pi\kappa^2} \text{curl} \tilde{U} \text{curl} U + \frac{g}{\kappa^2} \text{curl} U \cdot \tilde{T}
 \end{aligned}$$

$$+ \frac{g}{\kappa^2} \text{curl} \tilde{U} \cdot T + \frac{4\pi g^2}{\kappa^2} T \cdot T - \frac{1}{4\pi} (\tilde{U}_0 U_0 - \tilde{U} U)$$

$$- 4\pi (c \text{div} \tilde{U}^{\dagger} + \frac{g}{\kappa} M_0) (c \text{div} U^{\dagger} + \frac{g}{\kappa} \tilde{M}_0) + \frac{1}{4\pi} \tilde{U} U$$

$$+ \frac{4\pi g c}{\kappa} \text{div} U^{\dagger} \cdot M_0 + \frac{4\pi g^2}{\kappa^2} \tilde{M}_0 M_0 - \frac{g}{\kappa} \tilde{U} M$$

$$+ \frac{4\pi g c}{\kappa} \text{div} \tilde{U}^{\dagger} \cdot \tilde{M}_0 + \frac{4\pi g^2}{\kappa^2} \tilde{M}_0 \tilde{M}_0 - \frac{g}{\kappa} U \tilde{M}$$

$$H = 4\pi\kappa^2 c^2 \tilde{U}^{\dagger} U^{\dagger} + \frac{g}{4\pi\kappa^2} \text{div} U^{\dagger} \text{div} \tilde{U}^{\dagger} + \frac{1}{4\pi} \tilde{U} U$$

$$+ 4\pi g c (U^{\dagger} S + \tilde{U}^{\dagger} \tilde{S}) + \frac{g}{\kappa^2} (\text{curl} U \cdot \tilde{T} + \text{curl} \tilde{U} \cdot T)$$

$$+ \frac{4\pi g c}{\kappa} (\text{div} U^{\dagger} \cdot M_0 + \text{div} \tilde{U}^{\dagger} \cdot \tilde{M}_0) - \frac{g}{\kappa} (\tilde{U} M + U \tilde{M})$$

$$+ \frac{4\pi g^2}{\kappa^2} (T \cdot T + \tilde{M}_0 M_0)$$

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a)  $\frac{g}{\kappa^2} \text{curl } \mathbf{U} \cdot \tilde{\mathbf{T}}$

$$\frac{1}{c^2} \frac{\partial \mathbf{U}}{\partial t} = 4\pi \kappa^2 c^2 \tilde{\mathbf{U}}^T + 4\pi c^2 \text{grad div } \tilde{\mathbf{U}}^T + 4\pi g c S$$

$\frac{1}{c^2} \frac{\partial}{\partial t}$

$$- \frac{4\pi g c}{\kappa^2} \text{grad } M_0$$

$$- \tilde{\mathbf{U}}^T = \cancel{e \text{div}} \frac{1}{4\pi \kappa^2} \text{curl curl } \mathbf{U} + \frac{1}{4\pi} \mathbf{U}$$

$$+ \frac{g}{\kappa^2} \text{curl } \mathbf{T} - \frac{g}{\kappa} \mathbf{M}$$

$$\frac{1}{c^2} \ddot{\mathbf{U}} = -\text{curl curl } \mathbf{U} - \kappa^2 \mathbf{U} - 4\pi g \text{curl } \mathbf{T} + 4\pi g \kappa \mathbf{M}$$

$$+ \frac{1}{4\pi c^2} \text{grad div } \mathbf{U} + \frac{4\pi g}{\kappa} \text{grad div } \mathbf{M}$$

$$+ \frac{4\pi g}{c} \frac{\partial S}{\partial t} - \frac{4\pi g}{\kappa c} \frac{\partial}{\partial t} \text{grad } M_0$$

$$\frac{1}{c^2} \frac{\partial \mathbf{U}}{\partial t} - \Delta \mathbf{U} + \kappa^2 \mathbf{U} = 4\pi g \left\{ -\text{curl } \mathbf{T} + \kappa \mathbf{M} \right.$$

$$\left. - \frac{1}{\kappa} \text{grad div } \mathbf{M} + \frac{4\pi g}{c} \frac{\partial S}{\partial t} - \frac{4\pi g}{\kappa c} \frac{\partial}{\partial t} \text{grad } M_0 \right\}$$

$$\frac{g}{\kappa^2} \text{curl } \mathbf{U} \cdot \tilde{\mathbf{T}} = \frac{g^2}{\kappa^2} \left( \frac{e^{-\kappa r}}{r} (-\text{curl curl } \mathbf{T}) \right) dv \cdot \tilde{\mathbf{T}}(z)$$

$T_x = T_y = 0$

z-comp.  $-\frac{\partial}{\partial y} \left( \frac{\partial T_z}{\partial y} \right) + \frac{\partial}{\partial x} \left( \frac{\partial T_z}{\partial x} \right) = -\frac{\partial^2 T_z}{\partial x^2} - \frac{\partial^2 T_z}{\partial y^2}$

x-comp.  $\frac{\partial}{\partial y} \left( \frac{\partial T_z}{\partial y} \right) = \frac{\partial}{\partial z} \left( \frac{\partial T_z}{\partial x} \right) = \frac{\partial T_z}{\partial x \partial z}$

$$\frac{g^2}{\kappa^2} \left( \frac{\partial^2 T_z}{\partial x^2} + \frac{\partial^2 T_z}{\partial y^2} \right) \frac{e^{-\kappa r}}{r} \tilde{\mathbf{T}}$$

$$\left[ -\text{curl}_1 \text{curl}_1 \left( \frac{e^{-\kappa r}}{r} \right) \right]_{T(z)} \left| T(1) T(2) \right.$$

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$\cdot r^3$

⑧

$$T(1) \cdot \text{grad}_1 \cdot (T(2) \cdot \tilde{g}) \frac{e^{-\kappa r}}{r}$$

$$d\tilde{w} \left( \frac{e^{-\kappa r}}{r} \tilde{T}(2) \right) \cdot d\tilde{w} ( \quad )$$

$$+ (\tilde{T}(1) \text{grad}_1) (\tilde{T}(2) \text{grad}_2)$$

$$T(1) \Delta_1 \left( \frac{e^{-\kappa r}}{r} \tilde{T}(2) \right) = T(1) \left\{ + \kappa^2 \frac{e^{-\kappa r}}{r} \tilde{T}(2) - 4\pi \delta \right\} T(2)$$

$$= \kappa^2 T(1) \frac{e^{-\kappa r}}{r} T(2) - 4\pi \delta T(1) T(2)$$

$$\Delta \frac{e^{-\kappa r}}{r} = \kappa^2 \frac{e^{-\kappa r}}{r} - 4\pi \delta$$

$$- \frac{g^2}{\kappa^2} \int \int \text{curl curl } T(1) \, dv_1 \, dv_2 \cdot \frac{e^{-\kappa r}}{r} \tilde{T}(2)$$

$$= - \frac{g^2}{\kappa^2} \int \int \left\{ \text{div}_1 T(1) \cdot dv_1 \, dv_2 \cdot \text{div}_2 \frac{e^{-\kappa r}}{r} \tilde{T}(2) \right.$$

$$\left. + T(1) T(2) \, dv_1 \, dv_2 \cdot \Delta \frac{e^{-\kappa r}}{r} \right\}$$

$$= - \frac{g^2}{\kappa^2} \int \int (T(1) \text{grad}_1) (\tilde{T}(2) \text{grad}_2) \frac{e^{-\kappa r}}{r} \, dv_1 \, dv_2$$

$$+ g^2 \int \int T(1) \tilde{T}(2) \frac{e^{-\kappa r}}{r} \, dv_1 \, dv_2$$

$$- \frac{4\pi g^2}{\kappa^2} \int \int T(1) \tilde{T}(2) \delta(1,2) \, dv_1 \, dv_2$$

b)  $\frac{4\pi g c}{\kappa} \text{div } \tilde{U}^T \cdot \tilde{M}_0$

$$-\tilde{U}^T = + c^2 \text{curl curl } \tilde{U}^T + \frac{g c^2}{\kappa^2} \text{curl curl } S^T + \kappa c^2 \tilde{U}^T$$

$$- c^2 \text{grad div } \tilde{U}^T + g c S^T - \frac{g c}{\kappa} \text{grad } M_0 + \frac{g}{\kappa^2} \text{curl } \frac{\partial S^T}{\partial t}$$

$$- \frac{g}{\kappa} \frac{\partial M}{\partial t}$$

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$$\frac{1}{c^2} \frac{\partial^2 \tilde{U}^T}{\partial t^2} - \Delta \tilde{U}^T + \kappa^2 \tilde{U}^T \cong \frac{g}{\kappa c} \text{grad} M_0$$

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta + \kappa^2\right) \text{div} \tilde{U}^T \cong \frac{g}{\kappa c} \Delta M_0$$

$$\Delta M_0 = \kappa^2 M_0 - 4\pi$$

$$\frac{4\pi g c}{\kappa} \text{div} \tilde{U}^T \cdot \tilde{M}_0 \cong \frac{g}{\kappa^2} \iint \Delta M_0(1) \cdot \tilde{M}_0(2) \frac{e^{-\kappa r}}{r} dv$$

$$= \frac{g^2}{\kappa^2} \iint \{M_0(1) \tilde{M}_0(2)\} \Delta \cdot \frac{e^{-\kappa r}}{r} dv$$

$$= g^2 \iint \{M_0(1) \tilde{M}_0(2)\} \frac{e^{-\kappa r}}{r} dv - \frac{4\pi g^2}{\kappa} \int M_0(1) \tilde{M}_0(2) \delta(1,2) dv$$

$$= (\vec{\sigma}^{(1)} \text{grad}) (\vec{\sigma}^{(2)} \text{grad}) \frac{e^{-\kappa r}}{r}$$

$$= (\vec{\sigma}^{(1)} \text{grad}) \left\{ \underbrace{(\vec{\sigma}^{(2)} \vec{r})}_{-\kappa} \frac{e^{-\kappa r}}{r^2} - (\vec{\sigma}^{(2)} \vec{r}) \frac{e^{-\kappa r}}{r^3} \right\}$$

$$= \kappa^2 \frac{(\vec{\sigma}^{(1)} \vec{\sigma}^{(2)})}{r}$$

$$\cong \kappa^2 (\vec{\sigma}^{(1)} \vec{r}) (\vec{\sigma}^{(2)} \vec{r}) \frac{e^{-\kappa r}}{r^3} - \kappa (\vec{\sigma}^{(1)} \vec{\sigma}^{(2)}) \frac{e^{-\kappa r}}{r^2}$$

$$+ 2\kappa (\vec{\sigma}^{(1)} \vec{r}) (\vec{\sigma}^{(2)} \vec{r}) \frac{e^{-\kappa r}}{r^4} + \kappa (\vec{\sigma}^{(1)} \vec{r}) (\vec{\sigma}^{(2)} \vec{r}) \frac{e^{-\kappa r}}{r^4}$$

$$- (\vec{\sigma}^{(1)} \vec{\sigma}^{(2)}) \frac{e^{-\kappa r}}{r^3} + 3(\vec{\sigma}^{(1)} \vec{r}) (\vec{\sigma}^{(2)} \vec{r}) \frac{e^{-\kappa r}}{r^5}$$

$$= \kappa^2 (\vec{\sigma}^{(1)} \vec{r}) (\vec{\sigma}^{(2)} \vec{r}) \frac{e^{-\kappa r}}{r^3} - \kappa (\vec{\sigma}^{(1)} \vec{\sigma}^{(2)}) \frac{e^{-\kappa r}}{r^2}$$

$$+ \frac{3(\vec{\sigma}^{(1)} \vec{r}) (\vec{\sigma}^{(2)} \vec{r})}{r^2} \frac{e^{-\kappa r}}{r^2}$$

$$= 0 \quad \left\{ 4\kappa - \frac{1}{r} \right\} \frac{e^{-\kappa r}}{r^2}$$

s: state,

$$\frac{(\vec{\sigma}^{(1)} \vec{r}) (\vec{\sigma}^{(2)} \vec{r})}{r^2} = \iint \frac{x^i}{r^2} \sigma_x^{(1)} \sigma_x^{(2)} + \dots = \frac{\vec{\sigma}^{(1)} \vec{\sigma}^{(2)}}{3}$$

$$\{ (\sigma_x^{(1)} \cos \theta \sin \theta \cos \varphi) (\sigma_x^{(2)} \sin \theta \cos \varphi) + (\dots) + (\dots) \} 4 = 0$$

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$$L = \tilde{F}F - (\tilde{G} - \tilde{T})(G - T) - \tilde{U}U + \tilde{U}_0 U_0$$

$$\kappa F = \kappa S \left( \frac{1}{c} \frac{\partial \tilde{U}}{\partial t} + \text{grad } U_0 \right) - (\tilde{G} - \tilde{T})(G - T)$$

$$\kappa G = \kappa T + \text{curl } U \quad - \tilde{G}\tilde{T} - \tilde{G}\tilde{T}$$

$$\delta L = \delta U$$

$$\frac{\delta L}{\delta \tilde{U}} = -\frac{1}{\kappa c} \tilde{F}$$

$$H = -\frac{1}{\kappa c} \tilde{F} \cdot \frac{\delta U}{\delta t} \mp \quad - L$$

$$= -\frac{1}{\kappa} \tilde{F} (F - S + \dots)$$

$$+ (\kappa \tilde{T} + \text{curl } \tilde{U}) (\kappa T + \text{curl } U)$$

$$L = (\tilde{F} + \tilde{S})(F + S) - (\tilde{G} + \tilde{T})(G + T) - \tilde{U}U + \tilde{U}_0 U_0$$

$$- (M\tilde{U} - M_0\tilde{U}_0) + \text{comp. conj.}$$

$$= \frac{1}{\kappa^2} \left( \frac{1}{c} \frac{\partial \tilde{U}}{\partial t} + \text{grad } \tilde{U}_0 \right) \left( \frac{1}{c} \frac{\partial U}{\partial t} + \text{grad } U_0 \right) - \frac{1}{\kappa^2} \text{curl } \tilde{U} \text{curl } U$$

$$- \tilde{U}U + \tilde{U}_0 U_0 - (M\tilde{U} - M_0\tilde{U}_0) + \text{comp. conj.}$$

$$\delta L = \delta \tilde{U} \cdot \left\{ \frac{1}{\kappa c} \left( \frac{\partial (F+S)}{\partial t} \right) - \frac{1}{\kappa} \text{curl}(G+T) - \kappa U - M \right\}$$

$$+ \delta U_0 \left\{ \frac{1}{\kappa} \text{div}(F+S) + U_0 + M_0 \right\} +$$

$$\left. \begin{aligned} \frac{1}{\kappa c} \frac{\partial F}{\partial t} - \text{curl } G - \kappa U &= \kappa M - \frac{1}{c} \frac{\partial S}{\partial t} + \text{curl } T \\ \text{div } T + \kappa U_0 &= -\kappa M_0 - \text{div } S \end{aligned} \right\}$$

$$\frac{1}{c} \frac{\partial \tilde{U}}{\partial t} + \text{grad } \tilde{U}_0 + \kappa \tilde{F} = \kappa S$$

$$\text{curl } U - \kappa G = \kappa T$$

$$U^\dagger = -\frac{1}{\kappa c} (\tilde{F} + \tilde{S}) = \frac{1}{\kappa c} \left( \frac{1}{c} \frac{\partial \tilde{U}}{\partial t} + \text{grad } \tilde{U}_0 \right)$$

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$$H = \frac{1}{2c} (\tilde{F} + \tilde{U}) \frac{\partial}{\partial t} (\tilde{U} + \tilde{S}) + (\tilde{F} + \tilde{S})$$

$$- (\tilde{G} + \tilde{T})(\tilde{G} + \tilde{T}) + \tilde{U}\tilde{U} - \tilde{U}_0\tilde{U}_0 + (M\tilde{U} - M_0\tilde{U}_0)$$

$$+ \text{conj. conj.}$$

$$= -\tilde{U}^\dagger (\kappa^2 c^2 \tilde{U}^\dagger - c \text{grad } \tilde{U}_0) + \tilde{U}^\dagger (\kappa^2 c^2 \tilde{U}^\dagger - c \text{grad } U_0)$$

$$- \kappa^2 c^2 \tilde{U}^\dagger \tilde{U}^\dagger - \dots$$

$$= \kappa^2 c^2 \tilde{U}^\dagger \tilde{U}^\dagger - c(\tilde{U}^\dagger \text{grad } \tilde{U}_0 + \tilde{U}^\dagger \text{grad } U_0) - \text{curl } \tilde{U} \text{curl } U$$

$$+ \tilde{U}\tilde{U} - \tilde{U}_0\tilde{U}_0 + (M\tilde{U} - M_0\tilde{U}_0) + \text{conj. conj.}$$

$$\frac{1}{c} \frac{\partial \vec{F}}{\partial t} - \text{curl } \vec{G} - (\kappa \vec{U} + \text{grad } M) = 0 \quad \text{div } \vec{F} + (\kappa U_0 - \frac{1}{c} \frac{\partial M}{\partial t}) = 0$$

$$\frac{1}{c} \frac{\partial U}{\partial t} +$$

$$\frac{1}{c} \frac{\partial \vec{F}}{\partial t} - \text{curl } \vec{G} = \kappa \vec{U} + \text{grad } S$$

$$\text{div } \vec{F} = -\kappa U_0 + \frac{1}{c} \frac{\partial S}{\partial t}$$

$$\frac{1}{c} \frac{\partial \vec{U}}{\partial t} + \text{grad } U_0 + \kappa \vec{F} = 0$$

$$\text{curl } \vec{U} - \kappa \vec{G} = 0$$

$$\frac{1}{c} \frac{\partial}{\partial t} (\kappa \vec{U} + \text{grad } S) + \text{grad} (\kappa U_0 - \frac{1}{c} \frac{\partial S}{\partial t}) + \kappa \vec{F} = 0$$