On the Interaction of Elementary Particles, III.

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can be accounted for ~~by as the pr... magnetic as the contribut~~ as due to the U-field surrounding the heavy particle. ~~The theoretic~~ Thus, the apparent ~~The~~ calculated ~~result~~ magnetic moments of the proton and the neutron ~~comes~~ comes out to be $2. \frac{e\hbar}{2Mc}$ and $-2 \frac{e\hbar}{2Mc}$, which is in fair agreement with the experimental value ..., ... $\frac{e\hbar}{2Mc}$.

(1)

On the Interaction of Elementary Particles. III.

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§1. Introduction and Summary

In this paper, which is the continuation of the previous papers under the same heading[1], the authors want to deal first with the problem of ~~Maxwell's field equation~~ linearization of wave equations for the U-field. Since the heavy quanta obey Bose statistics and have zero or integer spin, a generalization of Maxwell's field equations ~~rathers~~ rather than that of Dirac's will be adequate to our purpose. Owing to the presence of the mass and charge on the one hand and the fact that the field variables are complex on the other, ~~the linearized equations obtained are very complicated,~~ complicated linear differential equations ~~for~~ connecting two scalars, two six vectors and two four vectors are necessitated for the complete description of the field. ~~(§2)~~ (§2.)
In the presence of the electromagnetic field, these equation can be modified in the usual manner, from which we can conclude that the heavy quantum has the spin 1 and the magnetic moment $\frac{e\hbar}{2m_0c}$, where m_0 is the mass of the heavy quantum.
The deviation of the magnetic moments of the proton and the neutron from the normal values $\frac{e\hbar}{2Mc}$ and 0 respectively, can be ~~account~~ where M is the mass of the heavy particle, can be accounted for respectively as due to the U-field partial splitting up of the proton into the neutron and the heavy quantum of positive charge ~~and~~ that of the neutron into the proton and the heavy quantum of negative charge. Thus, the apparent magnetic moments of them turn out to be about ~~2.~~ $3\cdot\frac{e\hbar}{2Mc}$ and ~~-~~ $-\frac{e\hbar}{2Mc}$ respectively which is in fair agreement with the experimental values[2].

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1) Yukawa, Proc. Phys.-Math. Soc. Japan 17, 48, 1935; Yukawa and Sakata, ibid. 20, ~~1938~~ 50, 472, 1936. These will be referred to as I and II respectively.
2) Experimental values of Rabi, Kellogg and Zacharias is $2.85\frac{e\hbar}{2Mc}$ and $-0.85\frac{e\hbar}{2Mc}$ respectively, † whereas ~~those~~ the value of Estermann, Simpson and Stern (Phys. Rev. 52, 535, 1937) is $2.46\frac{e\hbar}{2Mc}$ for proton.

In this ~~calculat~~ estimation ~~it is assumed that~~ it is assumed that "the radius of the ~~n~~heavy particle" is determined, so as to give ~~the~~ the whole mass of the heavy particle (~~is due~~ can be attributed to the self energy due to the U-field surrounding it, ~~and~~ so that "the radius of the heavy particle" becomes about $\frac{g^2}{Mc^2}$. (P.S.)

§2. Linearization of the Equations for the U-field.

The quadratic equations

$$\left.\begin{aligned}&\left(\Delta-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\varkappa^2\right)U=0\\&\left(\Delta-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\varkappa^2\right)\tilde{U}=0\end{aligned}\right\}\qquad(1)$$

for the U-field in vacuum, can be decomposed into the linear equations

$$\cancel{\operatorname{curl}\vec{B}=\varkappa\vec{K}}\qquad\cancel{\operatorname{grad}B_0+\frac{1}{c}\frac{\partial\vec{B}}{\partial t}=-\varkappa\vec{F}}$$

$$\operatorname{curl}\vec{V}=\varkappa\vec{K}\qquad\operatorname{grad}U+\frac{1}{c}\frac{\partial\vec{V}}{\partial t}=-\varkappa\vec{F}$$

$$\operatorname{curl}\vec{K}-\frac{1}{c}\frac{\partial\vec{F}}{\partial t}-\operatorname{grad}S+\varkappa'\vec{V}=0$$

$$\operatorname{div}\vec{F}+\frac{1}{c}\frac{\partial S}{\partial t}+\varkappa'U=0.$$

$$\left.\begin{aligned}&\operatorname{curl}\vec{V}=\varkappa\vec{K}\qquad&&\operatorname{curl}\vec{K}-\frac{1}{c}\frac{\partial\vec{F}}{\partial t}-\operatorname{grad}S=-\varkappa'\vec{V}\\&\operatorname{grad}U+\frac{1}{c}\frac{\partial\vec{V}}{\partial t}=-\varkappa\vec{F}\qquad&&\operatorname{div}\vec{F}+\frac{1}{c}\frac{\partial S}{\partial t}=-\varkappa'U\end{aligned}\right\}(2)$$

and

$$\left.\begin{aligned}&\operatorname{curl}\tilde{\vec{V}}=\varkappa\tilde{\vec{K}}\qquad&&\operatorname{curl}\tilde{\vec{K}}-\frac{1}{c}\frac{\partial\tilde{\vec{F}}}{\partial t}-\operatorname{grad}\tilde{S}=-\varkappa'\tilde{\vec{V}}\\&\operatorname{grad}\tilde{U}+\frac{1}{c}\frac{\partial\tilde{\vec{V}}}{\partial t}=-\varkappa\tilde{\vec{F}}\qquad&&\operatorname{div}\tilde{\vec{F}}+\frac{1}{c}\frac{\partial\tilde{S}}{\partial t}=-\varkappa'\tilde{U},\end{aligned}\right\}(3)$$

where U and $\vec{V}$ are the time and space components of a four vector respectively ~~and~~, $\vec{K},\vec{F}$ ~~form~~ a six vector and S ~~is~~ a scalar, the symbol ~ denoting the conjugate complex quantities. $U,\vec{V}$ (and $\tilde{U}\tilde{\vec{V}}$) ~~they~~ correspond to always the ~~scalar~~ four potential, ~~electric and magnetic fields~~ ~~and~~, whereas S (and $\tilde{S}$) ~~there are~~ ~~is~~ no $\vec{K},\vec{F}$ (and $\tilde{\vec{K}},\tilde{\vec{F}}$) to counterpart ~~for~~ in the case of the electromagnetic field. As in the latter case, we have assume further, two supplementary conditions

$$\left.\begin{aligned}&\frac{1}{c}\frac{\partial U}{\partial t}+\operatorname{div}\vec{V}-\varkappa S=0\\&\frac{1}{c}\frac{\partial\tilde{U}}{\partial t}+\operatorname{div}\tilde{\vec{V}}-\varkappa\tilde{S}=0\end{aligned}\right\}\qquad\begin{aligned}&(4)\\&(5)\end{aligned}\ (4)$$

$\frac{\partial}{\partial x} - \frac{ie}{\hbar c}V$

$-\frac{i}{\hbar}$ $i\hbar\frac{\partial}{\partial t} + \frac{e}{c}A_0$ $\quad -i\hbar\frac{\partial}{\partial x} + \frac{e}{c}A_0$ $\quad \frac{i}{\hbar}$ $\quad \frac{\partial}{\partial x} + \frac{ei}{\hbar c}A_0$

for the four vector $(U, \vec{V})$ ~~and~~ $\vec{\tilde{U}}$ and the scalar S, as well as for $(\tilde{U}, \vec{\tilde{V}})$ and $\tilde{S}$.

~~Combining~~ the equations (2) and (4) (or (3) and (5)), we can easily show that ~~the all~~ each of the quantities appearing in these equations satisfy the quadratic equation of the type (1), if we put $\kappa = \kappa'$.
In a special case, in which all the quantities are real and $\kappa = 1$, $\kappa' = 0$ and $M_0 = 0$, the above equations ~~reduce~~ to Maxwell's field equations.
~~In the following~~ ~~Hereafter~~, Hence, we ~~consider the case~~ assume tentatively ~~the equations (2) to (5) for~~ (2), (3), (4), (5) for $\kappa = \kappa'$ ~~is ; take~~ the correct ~~fiel~~ equation ~~for~~ ~~describing~~ the field of the heavy quanta.

§ 3. Interaction of the ~~U-field~~ Heavy Quanta with the Electromagnetic Field.

In the presence of the electromagnetic field, $(U, \vec{V})$, $(\vec{K}, \vec{F}, \vec{K})$ and S behave as wave functions for the particle with negative charge, since they involve the operators, which ~~include~~ decrease the number of the quanta with negative charge by one, ~~and increase~~ as well as those, which increase the number of the quanta with positive charge by one.[3)] Hence, ~~the equation~~ the ~~operat~~ differentiation operators in (2) $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ and $\frac{\partial}{\partial t}$ should be replaced by

$$\frac{\partial}{\partial x} + \frac{ie}{\hbar c}A_x, \quad \frac{\partial}{\partial y} + \frac{ie}{\hbar c}A_y, \quad \frac{\partial}{\partial z} + \frac{ie}{\hbar c}A_z$$

and

$$\frac{\partial}{\partial t} - \frac{ie}{\hbar c}A_0$$

respectively, where A_0 and $\vec{A}$ are the scalar and vector potentials for the electromagnetic field.
Similarly, ~~the equat.~~ ~~the corresponding operators~~ $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ and $\frac{\partial}{\partial t}$ in (3) should be replaced by

$$\frac{\partial}{\partial x} - \frac{ie}{\hbar c}A_x, \quad \ldots \quad \text{and} \quad \frac{\partial}{\partial t} + \frac{ie}{\hbar c}A_0$$

respectively.

3) ~~Compare~~ See the expressions (10) in II.