On the Interaction of Elementary Particles. III.

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§1. Introduction and Summary

In two previous papers[1]), ~~a theory~~ the interaction of elementary particles was discussed ~~on the hypothesis of~~ by introducing a new field of force. On quantizing this field, we obtained ~~the~~ new quanta each with the elementary charge either positive or negative and the mass intermediate between those of the electron and the proton, ~~satisfying~~ obeying the Bose statistics. ~~In I~~ This field was ~~considered~~ assumed to ~~be~~ ~~have two potentials of four vector~~ described by two four vector functions conjugate ~~complex~~ to each other in I, whereas it was described by ~~two~~ scalar functions in II. Neither of ~~these~~ two formulations was ~~satisfactory~~ ~~sufficient~~ found ample ~~adequate~~ ~~enough~~ for the derivation of the ~~nuclear~~ interaction forces, and the magnetic moments of the heavy particles. ~~In this paper~~, These two formulations were adopted for the sake of their simplicity, but ~~none~~ neither of them was ~~found adequate to the derivation of the correct~~ ~~considered to be perfect adequate~~ ~~found adequate to the [illegible] devoid of~~ sufficient for the derivation ~~of all sorts of nucl[illegible] into~~ of the ~~correct~~ complete ~~in~~ expressions for the interaction of heavy particles and their anomalous magnetic moments.

In this paper, we ~~want to start~~ begin ~~from the problem of linearization~~ with the construction of the ~~of the new field equations and to show that~~ equations and show ~~that we arrive at~~ ~~for~~ the ~~new~~ field ~~equations~~ considered as a generalization ~~extension~~ of

1) Yukawa, Proc. Phys.-Math. Soc. Japan 17, 48, 1935; Yukawa and Sakata, ibid. 19, 1084, 1937. ~~See also~~ These papers will be referred to as I and II respectively. See also Yukawa, ~~Proc.~~ ibid. 19, 712, 1937.

† ~~On~~ ~~Quite,~~ Very ~~recently,~~ after Meanwhile, ~~we noticed~~ it came to our notice that Proca[3] had developed a method of linearization of wave equations for ~~the electron~~ ~~to~~ the electron ~~in order to c~~ as an extension of the ~~non-negative~~ scalar theory of Pauli and Weisskopf.[4] The formulation of Proca ~~is~~ differs from ours only slightly. (See Appendix). Very recently, Kemmer[5] and Bhabha[6] ~~different~~ discussed the ~~problems~~ nature of nuclear force, anomalous magnetic moment[6] of the heavy particle[6] and cosmic ray ~~by the~~ using Proca's scheme.

The new field equations can be derived from the Lagrangian, so that the canonical variables describing the field ~~is obt~~ are ~~obtained~~ determined in the usual way ~~In~~ and the Hamiltonian. ~~The quantum theory~~ ~~these~~ ~~Thus, the quantum~~ above scheme we can go over into the quantum theory by constructing the commutation relations and equations of motions for these variables, are obtained,

the result of ~~in accord~~ agree with the current theory both in sign and magnitude can be obtained ~~by showing~~ the constants ~~suitably~~ theory

3) Proca, Jour. d. Phys. 7, 347, 1936. See further Durandin and Erschow, Sow. Phys. 12, 466, 1937.
4) Pauli and Weisskopf, Helv. Phys. 7, 709, 1934.
5) Kemmer, Nature 141, 116, 1938; Bhabha, ibid. 141, 117, 1938.
6) The problem of magnetic moment of the heavy particle was discussed also by Fröhlich and Heitler, Nature 141, 37, 1938.

Finally, ~~we~~ the possible forms of the interaction between the U-field and the heavy particle ~~was~~ is considered ~~and the~~, and the ~~interaction~~ force between the neutron and the proton was deduced. It was found that ~~the~~ the ~~correct~~ the combination of the exchange forces of Majorana and Heisenbergs, which ~~is in agreement~~, which is in accord ~~with the correct~~ is ~~expected~~ accept in the current

The field is thus described by ~~two~~ ~~a pair vectors~~ and two pairs of vectors, complex conjugate to each other. If we write down the Maxwell's equations for the electromagnetic field. (§2). ~~These equations are shown to be a special case of the~~ It is interesting that ~~this~~ system of equations written in spinor form reduces ~~are~~ ~~correspond~~ to a special case of Dirac's ~~the~~ wave equations[2] ~~with the sp~~ for the particle with the spin larger than $\frac{1}{2}$, [3](see Appendix). These four and six vectors can be derived from ~~the potentials, consist~~ which consist of ~~two~~ four vectors and two scalars complex conjugate to each other, which ~~all of which~~ and satisfying quadratic equations ~~of the type~~ with the proper mass term. ~~The La~~ ~~The Lagrangian~~ for the new field ~~the Hamiltonian and the commutation relat for the new field is~~ The Lagrangian ~~is expressed in terms of~~, the canonical conjugate variables to the potentials, the commutation relations and the Hamiltonian for the new field is determined successively. (§2). ~~In V By decomposing the field into Fourier components, we find that, the each~~ ~~harmonic~~ ~~monochromatic wave three independent each monochromatic wave cons~~ We can decompose the field into Fourier components, each of which consists of three ~~independent waves~~ ~~two of~~ components. This in its turn indicates that the quanta accompanying this field have the spin 1. (§3) In the presence of the electromagnetic field, the Lagrangian for the U-field is transformed in the usual manner and it follows that the quantum has the magnetic moment of the magnitude $\frac{e\hbar}{2M_U c}$. It is shown further that (§4) ~~The~~ the origin of the ~~magnetic~~ anomalous magnetic moments of the neutron and the proton can be attributed to the ~~partial existence~~ virtual ~~state with existence~~ contribution of the heavy quantum in the intermediate states. (§4). ~~The~~

~~The deduction of the nuclear force from~~ by the help of these field equations The ~~modification~~ problem modification of these field equations due to the presence of the heavy particle and the deduction of the nuclear force

2) Dirac, Proc. Roy. Soc. (A) 155, 447, 1937. See also Sakata and Yukawa, Proc. Phys.-Math. Soc. Japan 19, 91, 1937.

† Hereafter, the arrow → ~~above the quantity~~ variables denoting the space vector will ~~sometimes~~ often, be omitted in order to avoid the unnecessary complication of the formulae.

will be dealed with in the subsequent paper.

§2. Linearized Equations for the ~~New~~ U Field in Vacuum

We consider two three dimensional vectors $\vec{F}$ and $\vec{G}$ forming a six vector in four dimensional space, corresponding to the electric and magnetic vectors in electrodynamics and assume them to satisfy linear field equations of Maxwellian type, which, however, ~~become~~ result in the quadratic equations

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \varkappa^2\right)\left\{\begin{matrix}\vec{G}\\ \vec{F}\end{matrix}\right\} = 0 \qquad (1)$$

by iteration, instead of the d'Alembertian equations, where

$$\varkappa = \frac{m_U c}{\hbar}.$$

Such a system of linear equations can be constructed only by introducing, further, a four ~~vec~~ vector with the time component U_0 and the space components $\vec{U}$. ~~Now~~, We assume, now, ten linear equations

$$\frac{1}{c}\frac{\partial \vec{F}}{\partial t} - \mathrm{curl}\,\vec{G} - \varkappa \vec{U} = 0 \qquad \mathrm{div}\,\vec{F} + \varkappa U_0 = 0 \quad \begin{matrix}(2)\\(2a)\end{matrix}$$

$$\frac{1}{c}\frac{\partial \vec{U}}{\partial t} + \mathrm{grad}\,U_0 + \varkappa \vec{F} = 0 \qquad \mathrm{curl}\,\vec{U} - \varkappa \vec{G} = 0 \quad (3)$$

~~where the symbol~~ arrow → above denoting the vector is omitted ~~for simp~~ for simplicity. ~~for the above~~ ~~ten~~ Exactly the same set of equations ~~are~~ is assumed for the six vector $(\tilde{F}, \tilde{G})$ and for the four vector $(\tilde{K}_0, \tilde{K})$, which are complex conjugate to (F, G) and (U_0, U) respectively.

~~In the case of the zero rest mass, i.e. $\varkappa = 0$, the first ~~set~~ four equations reduce~~ From (2) and (3), we can easily derive the quadratic equations (1) for (F, G) and also for (U_0, U). ~~There are~~ ~~These are, moreover,~~ ~~six~~ ~~five~~ ~~identities~~ Furthermore, five linear equations

$$\frac{1}{c}\frac{\partial U_0}{\partial t} + \mathrm{div}\,U = 0 \quad \Big\} \ (3)$$

$$\frac{1}{c}\frac{\partial G}{\partial t} + \mathrm{curl}\,F = 0 \qquad \mathrm{div}\,G = 0$$

~~to immediately follows~~ from (2)

~~between~~ can be derived from (2) and (3)

$$\boxed{\begin{matrix} k = \frac{1}{2} \\ l = 1 \end{matrix}}$$

(2) can be derived ~~by varying~~ the Lagrangian

~~by taking~~ U, U₀, (from U and U₀ as independent variables and using the relations), the field equations can be derived from the

$$\bar{L} = \iiint L\,dv \qquad (4)$$

with

$$L = \tilde{F}F - \tilde{G}G - \tilde{U}U + \tilde{U}_0 U_0, \qquad (5)$$

~~considering~~ by variating ~~U, U_0, $\tilde{U}$, $\tilde{U}_0$~~

The variables which are canonically conjugate to U, U_0, $\tilde{U}$ and $\tilde{U}_0$ can ~~$\vec{U}$, U_0~~ be defined in the usual way by the relations

$$\left.\begin{aligned} U_0^{\dagger} &= \frac{\partial L}{\partial \frac{\partial U_0}{\partial t}} = 0 \\ U_x^{\dagger} &= \frac{\partial L}{\partial \frac{\partial U_x}{\partial t}} = \frac{-\tilde{F}_x}{c} \quad \text{etc.} \end{aligned}\right\} \qquad (6)$$

and the relations complex conjugate to them. The Hamiltonian ~~Next, the Ham for the U-field in vacuum~~ becomes thus

$$\bar{H} = \iiint H\,dv \qquad (7)$$

with

$$H = U_0^{\dagger}\frac{\partial U_0}{\partial t} + \vec{U}^{\dagger}\frac{\partial \vec{U}}{\partial t} + \text{comp. conj.} - L$$

$$\cancel{= c\,\tilde{U}_0^{\dagger} U_0^{\dagger} +}$$

$$= \kappa^2 c^2\, \tilde{U}^{\dagger} U^{\dagger} - \kappa^2 c\,(U^{\dagger} \operatorname{grad} U_0 + \tilde{U}^{\dagger} \operatorname{grad} \tilde{U}_0)$$

$$+ \frac{1}{\kappa^2} \operatorname{curl} \tilde{U} \operatorname{curl} U + \tilde{U} U - \tilde{U}_0 U_0. \qquad (8)$$

In the quantum theory ~~U_x~~, the canonical variables U_x, U_y, U_z, $U_x^{\dagger}, U_y^{\dagger}, U_z^{\dagger}$ ~~etc.~~ and ~~their~~ the variable complex conjugate to them should satisfy the commutation relations

$$\left.\begin{aligned} &U_x(\vec{r},t)\,U_x^{\dagger}(\vec{r}',t) - U_x^{\dagger}(\vec{r}',t)\,U_x(\vec{r},t) = i\hbar\,\delta(\vec{r},\vec{r}') \\ &U_x(\vec{r},t)\,U_y^{\dagger}(\vec{r}',t) - U_y^{\dagger}(\vec{r}',t)\,U_x(\vec{r},t) = \cancel{i\hbar\,\delta(\vec{r},\vec{r}')}\ 0 \\ &U_x(\vec{r},t)\,\tilde{U}_x^{\dagger}(\vec{r}',t) - \tilde{U}_x^{\dagger}(\vec{r}',t)\,U_x(\vec{r},t) = 0 \\ &\qquad \text{etc.,} \end{aligned}\right\} \qquad (9)$$

corresponding to the Bose statistics.

$$\frac{\partial U_x}{\partial t} = \kappa^2 c^2 \tilde{U}_x^\dagger - \frac{c^2}{\cancel{\kappa^2}} \cancel{\mathrm{div}}\ \mathrm{grad}\ \mathrm{div}\ \tilde{U}^\dagger$$

$$\cancel{= \frac{c}{\kappa^2}(c\tilde{U}_x^\dagger}$$

or $$\frac{\partial U_x}{\partial t} = \kappa c F_x - \frac{c}{\cancel{\kappa}}\ \mathrm{grad}\ U_0$$

curl curl U

$$\frac{\partial \tilde{U}_x^\dagger}{\partial t} = \cancel{\kappa^2 c^2} \frac{1}{\kappa^2} \mathrm{curl}\ \tilde{U} \cdot \mathrm{curl}\ U - U_x$$

$$= -\frac{1}{\kappa^2}\ \mathrm{curl}_x(\kappa G) - U_x$$

$$-\frac{1}{\kappa c}\frac{\partial}{\partial t}\bar{H} = -\frac{1}{\kappa}\ \mathrm{curl}\ G - U_x$$

As shown by (6), ~~the canonical conjugate to U_0 vanishes~~ The variables $U_0^\dagger$, $\tilde{U}_0^\dagger$, which are canonically conjugate to U_0 and $\tilde{U}_0$ respectively, vanish identically, so that ~~it can~~ they can not be taken into the quantum theory as ~~canonically~~ canonically conjugate to U_0 and $\tilde{U}_0$ respectively. Hence, we have to eliminate U_0 and $\tilde{U}_0$ themselves ~~from~~ by using the conditions

$$\operatorname{div}\vec{F} + \kappa U_0 = 0$$

or

$$U_0 = \frac{c}{\kappa}\operatorname{div}\tilde{U}^\dagger$$

and

$$\tilde{U}_0 = \frac{c}{\kappa}\operatorname{div}U^\dagger \qquad (10)$$

which follow from (3) and the relation $\operatorname{div}\vec{F} + \kappa U_0 = 0$. The Hamiltonian (4), now, ~~takes~~ takes the form

$$H_U = \kappa^2 c^2\, U^\dagger\tilde{U}^\dagger - \frac{c^2}{\kappa^2}\left(U^\dagger \operatorname{grad}\operatorname{div}\tilde{U}^\dagger + \tilde{U}^\dagger\operatorname{grad}\operatorname{div}U^\dagger\right) + \frac{1}{\kappa^2}\operatorname{curl}\tilde{U}\operatorname{curl}U + \tilde{U}U - \frac{c^2}{\kappa^2}\operatorname{div}U^\dagger\operatorname{div}\tilde{U}^\dagger,$$

so that we obtain

$$\bar{H}_U = \iiint\left\{\kappa^2 c^2\, U^\dagger\tilde{U}^\dagger + \frac{c^2}{\kappa^2}\operatorname{div}U^\dagger\operatorname{div}\tilde{U}^\dagger + \frac{1}{\kappa^2}\operatorname{curl}\tilde{U}\operatorname{curl}U + \tilde{U}U\right\}dv \qquad (12)$$

by partial integration.

The ~~quan~~ field equations in the quantum theory, which can be obtained ~~by differentiat~~ from

$$i\hbar\frac{\partial U_x}{\partial t} = U_x\bar{H} - \bar{H}U_x \quad \text{etc.}$$

$$i\hbar\frac{\partial U_x^\dagger}{\partial t} = U_x^\dagger\bar{H} - \bar{H}U_x^\dagger \quad \text{etc.}; \qquad (13)$$

~~take become~~ by using the commutation relations (9). They are

$-ik\, {}^{+}_{(}k_i u^{\dagger}_{ik}$ ~~curl~~ $e_{2k}\, e^{i\vec{k}\vec{r}-ick_0 t)}$

$= i[\vec{k}\, e_{2k}] = ik\vec{e}_{3k}$

$ik\vec{e}_{1k}$

$ik(\vec{e}_{3k} u_{2k} - \vec{e}_{2k} u_{3k})$

$$-ick_0 u_{ik} = \varkappa^2 c^2 \tilde{u}^{\dagger}_{ik} + c^2 k^2 \delta_{i1} \tilde{u}^{\dagger}_{1k}$$

$$-ick_0 \tilde{u}^{\dagger}_{ik} = -\frac{k^2}{\varkappa^2} \cancel{\mathrm{curl}}\, \delta^{x}_{i\geq 1}\,(1-\delta_{i1})\, u_{ik} - u_{ik}$$

$$ic\varkappa\, u^{\dagger}_{ik} = -\tilde{u}_{ik}$$

$$u^{\dagger}_{ik} = +\frac{i}{c\varkappa}\tilde{u}_{ik}$$

$$H = \varkappa^2 c^2 \tilde{u}^{\dagger}_{ik} u^{\dagger}_{ik} + c^2 k^2 \tilde{u}^{\dagger}_{1k} u^{\dagger}_{1k} + \frac{k^2}{\varkappa^2}(\tilde{u}_{2k} u_{2k} + \tilde{u}_{3k} u_{3k})$$

$$+ \tilde{u}_{ik} u_{ik}$$

$$= k_0^2 c^2 \tilde{u}^{\dagger}_{1k} u^{\dagger}_{1k} + \tilde{u}_{1k} u_{1k}$$

$$+ \sum_{i=2,3} \left(\varkappa^2 c^2 \tilde{u}^{\dagger}_{jk} u^{\dagger}_{jk} + \frac{k_0^2}{\varkappa^2} \tilde{u}_{jk} u_{jk}\right)$$

$$=$$

$$-ick_0 \tilde{u}^{\dagger}_{ik} = +\frac{k^2}{\varkappa^2}(1-\delta_{i1}) u_{ik} - u_{ik}$$

$$= -\frac{k_0^2}{\varkappa^2} u_{ik} + \delta_{i1} \frac{k^2}{\varkappa^2} u_{ik}$$

$$\left.\begin{aligned} &-\cancel{ick_0}\,\dot{u}_{ik} = 4\pi \varkappa^2 c^2 \tilde{u}^{\dagger}_{ik} + 4\pi k^2 c^2 \delta_{i1} \tilde{u}^{\dagger}_{1k} \\ &-\cancel{ick_0}\,\dot{\tilde{u}}^{\dagger}_{ik} = -\frac{k_0^2}{4\pi\varkappa^2} u_{ik} + \frac{k^2}{4\pi\varkappa^2}\delta_{i1} u_{ik} \end{aligned}\right\} (18)$$

$$\left(c^2 k_0^2 \tilde{u}^{\dagger}_{ik} = k_0^2 c^2 \tilde{u}^{\dagger}_{ik} + \left(\frac{k_0^2 k^2 c^2}{\varkappa^2} - k^2 c^2 - \frac{k^4 c^2}{\varkappa^2}\right)\delta_{i1} u_{ik}\right)$$

$$\cancel{ick_0}\,\dot{u}^{\dagger}_{ik} = -\frac{k_0^2}{\varkappa^2}\tilde{u}_{ik} + \frac{k^2}{\varkappa^2}\delta_{i1}\tilde{u}_{ik}$$

$$-\cancel{ick_0}\,\dot{u}^{\dagger}_{ik} = \frac{k_0^2 - k^2\delta_{i1}}{\varkappa^2}\tilde{u}_{ik}$$

$$\frac{\partial U_x}{\partial t} = 4\pi\varkappa^2 c^2 \tilde{U}_x^{+} - 4\pi c^2 \,\mathrm{grad\,div}\, \vec{U}^{\dagger} \quad \text{etc},$$

which reduce to

$$\frac{\partial \tilde{U}_x^{+}}{\partial t} = -\frac{1}{4\pi\varkappa^2}\,\mathrm{curl}_x(\mathrm{curl}\,\vec{U}) - \frac{1}{4\pi} U_x \quad \text{etc}, \qquad (14)$$

$$\frac{1}{c}\frac{\partial U_x}{\partial t} + \mathrm{grad}\, U_0 + \varkappa \Gamma_x = 0 \quad \text{etc}.$$

$$\frac{1}{c}\frac{\partial \Gamma_x}{\partial t} - \mathrm{curl}_x \vec{G} - \varkappa \vec{U}_x = 0, \qquad (15)$$

If we consider the relations (6) ~~and~~, (10) and the relation

$$\mathrm{curl}\, \vec{U} - \varkappa G = 0, \quad (16)$$

(13) reduce to

which ~~are the same~~ constitute (the field equations (12), , together with (11) and (16),

§3. Representation of U-Field in Vacuum by Normal Coordinates:

If we consider the field in a unit cube, we can change the field variables into new ones by Fourier transformation

$$~~U = \sum \{u_{ik} e^{ik}~~$$

$$\vec{U} = \sum_{\vec{k}} \sum_{i=1,2,3} \{u_{ik}\vec{e}_{ik} e^{i(\vec{k}\vec{r}-\cancel{ck_0 t})} + v_{ik}\vec{e}_{ik} e^{-i(\vec{k}\vec{r}-\cancel{ck_0 t})}\},$$

~~where~~

$$\vec{U}^{\dagger} = \sum_{\vec{k}} \sum_{i=1,2,3} \{u_{ik}^{\dagger}\vec{e}_{ik} e^{-i(\vec{k}\vec{r}-\cancel{ck_0 t})} - v_{ik}^{\dagger}\vec{e}_{ik} e^{+i(\vec{k}\vec{r}-\cancel{ck_0 t})}\}, \qquad (17)$$

where the suffix k stands for the vector $\vec{k}$ and $\vec{e}_{1k}$ is a unit vector parallel to $\vec{k}$, while $\vec{e}_{2k}$, $\vec{e}_{3k}$ are perpendicular to $\vec{e}_{1k}$ and ~~perpendicular~~ to each other, forming a right handed system. By inserting (17) into the field equations we obtain ~~linear simultaneous~~ equations of motions for the new variables u_{ik}, v_{ik}, $u_{ik}^{\dagger}$ and $v_{ik}^{\dagger}$, ~~and their complex conjugate which can be soluble only when~~ from which we obtain $\ddot{u}_{ik} = -k_0^2 c^2 u_{ik}$, where $k_0 = \sqrt{k^2 + \varkappa^2}$.

The first summation (17) $\sum_{\vec{k}}$ ~~in (17) should be performed for all $\vec{k}$ with the integer components multiplied by 2π, but with the restriction $k_0 = +\sqrt{k^2+\varkappa^2}$.~~

Further, we can deduce the ~~rel~~ commutation relations of the type

$$\tilde{u}^{+}_{jk} = \frac{1}{2}(p_{jk} - i p'_{jk})$$

$$u^{+}_{jk} = \frac{1}{2}(p_{jk} + i p'_{jk}) \frac{\sqrt{k_0}}{\sqrt{\varkappa^2 + \delta_{j,1} k^2}\,\sqrt{c}}$$

$$u_{jk} = \frac{1}{2}(q_{jk} + i q'_{jk}) \frac{\sqrt{\varkappa^2 + \delta_{j,1} k^2}\,\sqrt{c}}{\sqrt{k_0}}$$

$$q_{jk} p_{jk} - p_{jk} q_{jk} = i\hbar \quad \text{etc.}$$

$$\overline{H} = \sum \frac{1}{4} \frac{(k_0^2 - \delta_{j,1} k^2)}{k_0 c} (p_{jk}^2 + p'^2_{jk}) + \frac{(\varkappa^2 + \delta_{j,1} k^2)\, c}{k_0 \varkappa^2} (k_0^2 - \delta_{j,1} k^2) \frac{1}{4} (q_{jk}^2 + q'^2_{jk})$$

$$= \sum \frac{k_0 c}{4} (p_{jk}^2 + q_{jk}^2 + p'^2_{jk} + q'^2_{jk})$$

$$= \sum \frac{\hbar k_0 c}{\cancel{4}} \left(N_{jk} + \frac{1}{2}\right) + \cancel{N'_{jk} + \frac{1}{2}}$$

$$\boxed{\tilde{v}, v}$$

$$\overline{H}_u =$$

$$N_{jk} = \frac{1}{k_0 \hbar c} \cancel{p\, 4\pi c^2} (\varkappa^2 + \delta_{j,1} k^2) u \ldots$$

) Thus, ~~the summation with in (15), (18) with respect to $\vec{k}_0$~~. the

$$u_{jk}\tilde{u}_{lk'} - \tilde{u}_{lk'}u_{jk} = \frac{i\hbar}{2}\,\delta_{jl}\,\delta(\vec{k},\vec{k}') \qquad (19)$$

or

$$u_{jk}\tilde{u}_{jk} - \tilde{u}_{jk}u_{jk} = + \frac{k_0\hbar c}{2}\,\frac{\varkappa^2}{k_0^2 - k^2\delta_{j1}}\,\delta_{jl}\,\delta(\vec{k},\vec{k}') \qquad (20)$$

Finally, the Hamiltonian (12) can easily be transformed into the form

$$\bar{H} = \sum_k \sum_j \left\{ (\varkappa^2 + \delta_{j1}k^2)\frac{c^2}{4\pi}\,\tilde{u}^{+}_{jk}u^{+}_{jk} + \frac{k_0^2 - \delta_{j1}k^2}{\varkappa^2}\,\frac{1}{4\pi}\,\tilde{u}_{jk}u_{jk} \right\} + (v) \qquad (21)$$

or

$$= \sum_k \sum_j \left\{ \frac{1}{4\pi}\frac{(\varkappa^2 + \delta_{j1}k^2)(k_0^2 - k^2\delta_{j1})^2}{k_0^2\varkappa^2}\,\tilde{u}_{jk}\tilde{u}_{jk} + \frac{k_0^2 - k^2\delta_{j1}}{\varkappa^2}\,\tilde{u}_{jk}u_{jk} \right\} + (v) \qquad (22)$$

$$= \sum_k \sum_j (k_0^2 - k^2\delta_{j1})/\varkappa^2 \left\{ \frac{(\varkappa^2 + \delta_{j1}k^2)(k_0^2 - k^2\delta_{j1})}{k_0^2\varkappa^2}\,u_{jk}\tilde{u}_{jk} + \tilde{u}_{jk}u_{jk} \right\}$$

by using the relations (18)

$$\cancel{= \sum_k \sum_j \frac{k_0^2 - k^2\delta_{j1}}{\varkappa^2}\{u_{jk}\tilde{u}_{jk} + \tilde{u}_{jk}u_{jk}\} + (v)}$$

$$= \sum_k \sum_j \left\{ k_0\hbar c\left(N_{jk} + \tfrac{1}{2}\right) + k_0\hbar c\left(M_{jk} + \tfrac{1}{2}\right) \right\} \qquad (23)$$

where

$$\left.\begin{aligned} N_{jk} &= \frac{k_0^2 - k^2\delta_{j1}}{\hbar c\, k_0\varkappa^2}\,\tilde{u}_{jk}u_{jk} \\ M_{jk} &= \frac{k_0^2 - k^2\delta_{j1}}{\hbar c\, k_0\varkappa^2}\,\tilde{v}_{jk}v_{jk} \end{aligned}\right\} \qquad (24)$$

N_{jk}, M_{jk}, which are all commutative with each other and each has the eigenvalues 0, 1, 2, . . . , ~~so that the Hamiltonian~~ The variables N_{jk} denotes the number of the heavy quanta with the positive charge in the state of energy $E_k = \hbar c\sqrt{k^2+\varkappa^2}$ and momentum $\hbar c\vec{k}$, whereas M_{jk} that with negative charge in the state of energy E_k and momentum $-\hbar c\vec{k}$. The suffix $j = 1$ denotes the states represented by the ~~transverse~~ longitudinal wave, while $j = 2$ or 3 that represented by quantized the transverse ~~waves~~ wave,

Thus, the U-field, which is described the four vectors and ~~the six~~ two two vectors complex conjugate with each other ~~res~~ respectively, is accompanied by the quanta obeying Bose statistics, with the mass $m_U = \frac{\hbar\kappa}{c}$ and the charge either positive or negative, as in the case of the scalar field discussed in II. The suffix j, which can take three values, ~~indicates~~ indicates the extra degree of freedom ~~represen~~ fact that the quanta have each spin 1, ~~while~~ in contrast to ~~the~~ spin 0 in the case of the scalar field.

corresponding to the fa

(~~indicating~~ corre that ~~the quanta have each~~ spin 1 of the quanta)

) ~~The impossibility~~ the quantization of ~~this~~ the scalar field corresponding to Fermi statistics ~~can proved to the way similar way with the case of scalar field, which is impossible, the proof~~ is, which was shown by Pauli-Weisskopf (Helv. Phys. 7, , 1934), can be easily ~~but~~ be extended to ~~this case~~ and our case, as already shown by Erschow .. (loc. cit.).

$$\delta\left\{\left(\frac{\partial \tilde U_z}{\partial y}+\frac{ie}{\hbar c}A_y\tilde U_z-\frac{\partial \tilde U_y}{\partial z}+\frac{ie}{\hbar c}A_z\tilde U_y\right)\left(\frac{\partial U_z}{\partial y}-\frac{ie}{\hbar c}A_yU_z-\frac{\partial U_y}{\partial z}+\frac{ie}{\hbar c}A_zU_y\right)\right\}$$

$$=\frac{ie}{\hbar c}\delta A_y\left\{\tilde U_z\frac{\partial U_z}{\partial y}-\frac{\partial \tilde U_z}{\partial y}U_z-2\frac{ie}{\hbar c}A_y\tilde U_zU_z-\tilde U_z\frac{\partial U_y}{\partial z}+\frac{\partial \tilde U_y}{\partial z}U_z-\frac{ie}{\hbar c}\tilde U_zA_zU_y+\frac{ie}{\hbar c}\tilde U_yA_zU_z\right\}$$

$$L_U=\frac{1}{\varkappa^2}\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{ie}{\hbar c}A_0\right)\tilde U-\left(\mathrm{grad}+\frac{ie}{\hbar c}\vec A\right)\tilde U_0\right)\Big\}$$

$$\times\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}-\frac{ie}{\hbar c}A_0\right)U-\left(\mathrm{grad}-\frac{ie}{\hbar c}A\right)U_0\right\}$$

$$-\frac{1}{\varkappa^2}\left\{\left(\mathrm{grad}+\frac{ie}{\hbar c}A\right)\times\tilde U\right\}\left\{\left(\mathrm{grad}-\frac{ie}{\hbar c}A\right)\times U\right\}$$

$$-\tilde U U+\tilde U_0U_0.$$

$$L_E=\frac{1}{8\pi}\left\{\left(\frac{1}{c}\frac{\partial A}{\partial t}+\mathrm{grad}A_0\right)^2-(\mathrm{curl}\,H)^2\right\}$$

$$\delta L_U=$$

$$*\ -\frac{ie}{\hbar c}\delta A_0\,\frac{1}{\varkappa^2}\left\{\tilde U\frac{1}{c}\frac{\partial U}{\partial t}-\frac{1}{c}\frac{\partial\tilde U}{\partial t}U-2\frac{ie}{\hbar c}A_0\tilde UU+\tilde U\left(\mathrm{grad}-\frac{ie}{\hbar c}A\right)U_0\right.$$

$$\left.+\left(\mathrm{grad}+\frac{ie}{\hbar c}A\right)\tilde U_0\cdot U\right\}$$

$$-\frac{ie}{\hbar c}\delta A_x\frac{1}{\varkappa^2}\left\{\vec{\tilde U}\frac{\partial\vec U}{\partial x}-\frac{\partial\vec{\tilde U}}{\partial x}\cdot\vec U\right.$$

$$*\ =-\delta A_0\cdot\rho+\delta A\cdot I/c$$

$$\delta L_E=\frac{+1}{4\pi}\left\{\delta A_0\ \mathrm{div}\,E\right.$$

$$\left.\delta A\left(\frac{1}{c}\frac{\partial E}{\partial t}-\mathrm{curl}\,H\right)\right\}$$

grad

$$-\frac{ie}{\hbar c}\frac{1}{\varkappa^2}\delta A_y\left\{\tilde U_z\frac{\partial U_z}{\partial y}-\frac{\partial\tilde U_z}{\partial y}U_z-2\frac{ie}{\hbar c}A_y\tilde U_zU_z\right.$$

$$-\tilde U_z\frac{\partial U_y}{\partial z}+\frac{\partial\tilde U_y}{\partial z}U_z-\frac{ie}{\hbar c}\tilde U_zA_zU_y-\frac{ie}{\hbar c}\tilde U_yA_zU_z$$

$$+\tilde U_y\frac{\partial U_y}{\partial z}-\frac{\partial\tilde U_y}{\partial z}U_y-2\frac{ie}{\hbar c}A_z\tilde U_yU_y$$

$$\left.-\tilde U_y\frac{\partial U_z}{\partial y}+\frac{\partial\tilde U_z}{\partial y}U_y\right.$$

§ 4. Interaction of the Heavy Quanta with the Electromagnetic Field.

In the presence of the electromagnetic field with the scalar and the vector potentials A_0 and $\vec{A}$, grad and $\frac{1}{c}\frac{\partial}{\partial t}$ operating on the variable $\vec{F}$, $\vec{G}$, U and U_0 should be replaced by

$$\mathrm{Grad} - \frac{ie}{\hbar c}\vec{A} \quad \text{and} \quad \frac{1}{c}\frac{\partial}{\partial t} + \frac{ie}{\hbar c}A_0$$

respectively, corresponding to the fact that these variables involve the operator, which increases the number of the positive charged quanta by one and decreases that of the negative charged quanta by one. Similarly, grad and $\frac{1}{c}\frac{\partial}{\partial t}$ operating on $\tilde{F}$, $\tilde{G}$, $\tilde{U}$ and $\tilde{U}_0$, should be replaced by grad and $\frac{1}{c}\frac{\partial}{\partial t}$

$$\mathrm{Grad} + \frac{ie}{\hbar c}\vec{A} \quad \text{and} \quad \frac{1}{c}\frac{\partial}{\partial t} + \frac{ie}{\hbar c}A_0$$

respectively.

~~It should be~~ Thus, the field equations (2) and (3) become

$$\left.\begin{aligned}\left(\frac{1}{c}\frac{\partial}{\partial t} + \frac{ie}{\hbar c}A_0\right)\vec{F} - \left(\mathrm{Grad} - \frac{ie}{\hbar c}\vec{A}\right)\times\vec{G} - \kappa\vec{U} = 0\\ \left(\mathrm{Grad} - \frac{ie}{\hbar c}\vec{A}\right)\vec{F} + \kappa U_0 = 0.\end{aligned}\right\}\quad(25)$$

and

$$\left.\begin{aligned}\left(\frac{1}{c}\frac{\partial}{\partial t} + \frac{ie}{\hbar c}A_0\right)\vec{U} + \left(\mathrm{Grad} - \frac{ie}{\hbar c}\vec{A}\right)U_0 + \kappa\vec{F} = 0\\ \left(\mathrm{Grad} - \frac{ie}{\hbar c}\vec{A}\right)\times\vec{U} - \kappa\vec{G} = 0\end{aligned}\right\}\quad(26)$$

where the symbol $\times$ denotes the vector ~~multi product.~~ multiplication.

The Langrangian for the system consisting of the U-field and the electromagnetic field can be written in the form

$$\bar{L} = \iiint L\,dv, \qquad (27)$$

with

$$\left.\begin{aligned}L &= L_U + L_E\\ L_U &= \tilde{F}F - \tilde{G}G - \tilde{U}U + \tilde{U}_0U,\\ L_E &= \frac{1}{8\pi}(\vec{E}^2 - \vec{H}^2).\end{aligned}\right\}\quad(28)$$

$$-\frac{1}{\kappa^2}\delta\left\{\left(\frac{\partial \tilde U_z}{\partial y}+\frac{ie}{\hbar c}A_y\tilde U_z-\frac{\partial \tilde U_y}{\partial z}-\frac{ie}{\hbar c}A_z\tilde U_y\right)\left(\frac{\partial U_z}{\partial y}-\frac{ie}{\hbar c}A_yU_z-\frac{\partial U_y}{\partial z}+\frac{ie}{\hbar c}A_zU_y\right)\right.$$

$$=-\frac{1}{\kappa^2}\frac{ie}{\hbar c}\left\{\delta A_y\left\{\tilde U_z\frac{\partial U_z}{\partial y}-\frac{\partial \tilde U_z}{\partial y}U_z-2\frac{ie}{\hbar c}A_y\tilde U_zU_z-\tilde U_z\frac{\partial U_y}{\partial z}+\frac{\partial \tilde U_y}{\partial z}U_z\right.\right.$$

$$\left.+\frac{ie}{\hbar c}\tilde U_zA_zU_y+\frac{ie}{\hbar c}\tilde U_yA_zU_z\right\}$$

$$+\delta A_z\left\{\tilde U_y\frac{\partial U_y}{\partial z}-\tilde U_y\frac{\partial \tilde U_y}{\partial z}U_y-2\frac{ie}{\hbar c}A_z\tilde U_yU_y-\tilde U_y\frac{\partial U_z}{\partial y}+\frac{\partial \tilde U_z}{\partial y}U_y\right.$$

$$\left.\left.+\frac{ie}{\hbar c}\tilde U_yA_yU_z+\frac{ie}{\hbar c}\tilde U_zA_yU_y\right\}\right\}.$$

$$\delta A_y\cdot-\frac{1}{\kappa^2}\frac{ie}{\hbar c}\left\{\tilde U_z\frac{\partial U_z}{\partial y}-\frac{\partial \tilde U_z}{\partial y}U_z-\frac{2ie}{\hbar c}A_y\tilde U_zU_z-\tilde U_z\frac{\partial U_y}{\partial z}+\frac{\partial \tilde U_y}{\partial z}U_z\right.$$

$$+\frac{ie}{\hbar c}\tilde U_zA_zU_y+\frac{ie}{\hbar c}\tilde U_yA_zU_z$$

$$+\tilde U_x\frac{\partial U_x}{\partial y}-\frac{\partial \tilde U_x}{\partial y}U_x-\frac{2ie}{\hbar c}A_y\tilde U_xU_x-\tilde U_x\frac{\partial U_y}{\partial x}+\frac{\partial \tilde U_y}{\partial x}U_x$$

$$\left.+\frac{ie}{\hbar c}\tilde U_xA_xU_y+\frac{ie}{\hbar c}\tilde U_yA_xU_x\right\}$$

$$=\delta A_y\left(-\frac{1}{\kappa^2}\frac{ie}{\hbar c}\right)\cdot\left\{\tilde{\vec U}\frac{\partial \vec U}{\partial y}-\frac{\partial \tilde{\vec U}}{\partial y}\vec U-\frac{2ie}{\hbar c}\tilde{\vec U}\vec U\cdot A_y\right.$$

$$-(\vec U\,\mathrm{grad})U_y+(\tilde{\vec U}\,\mathrm{grad})\tilde U_y+\frac{ie}{\hbar c}(\tilde{\vec U}\vec A)U_y$$

$$\left.+\frac{ie}{\hbar c}\tilde U_y(\vec A\cdot\vec U)\right\}.$$

The equations ~~with~~ (25) can be derived from this Lagrangian by performing the variation with respect to ~~the~~ $\tilde{U}$ and U_0 and by using the relations (26). On the other hand, ~~the~~ Maxwellian equations for the electromagnetic field ~~which~~ can be ~~obtained~~ derived by performing the variation with respect to $\vec{A}$ and A_0 and by using the relations

$$\vec{E} = \cancel{\mathrm{curl}\, A} - \frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \mathrm{grad}\, A_0, \quad \vec{H} = \mathrm{curl}\, \vec{A}, \qquad (29)$$

~~become~~ Thus, we obtain

$$\cancel{\mathrm{curl}\, \vec{E}}$$

$$\mathrm{curl}\, \vec{H} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{I} \qquad \mathrm{div}\, \vec{E} = 4\pi\rho, \qquad (30)$$

where ~~the~~:

$$\rho = \frac{ie}{\cancel{\hbar c}\ \varkappa^2 \hbar c}\Big\{\tilde{U}\frac{1}{c}\frac{\partial U}{\partial t} - \frac{1}{c}\frac{\partial \tilde{U}}{\partial t}U - 2\frac{ie}{\hbar c}A_0\tilde{U}U - \tilde{U}\cdot\mathrm{grad}\, U_0 + \mathrm{grad}\,\tilde{U}_0\cdot U + \frac{ie}{\hbar c}(\tilde{U}A)U_0 + \frac{ie}{\hbar c}\tilde{U}_0(AU)\Big\},$$

$\frac{e\hbar}{m_0^2c^3}$ $\frac{m_0^2c^2}{\hbar^2}$

$$I_x = -\frac{ie}{\varkappa^2\hbar c}\Big\{\tilde{U}\cdot\frac{\partial U}{\partial x} - \frac{\partial \tilde{U}}{\partial x}\cdot U - \frac{2ie}{\hbar c}A_x\tilde{U}U - (\tilde{U}\,\mathrm{grad})U_x - (U\,\mathrm{grad})\tilde{U}_x + \frac{ie}{\hbar c}(\tilde{U}A)U_x + \frac{ie}{\hbar c}U_x(AU)\Big\}, \qquad (31)$$

which are the charge and current density due to the presence of heavy quanta.

~~By changing the~~ ~~in more~~ variables These expressions can be written by using normal coordinates as in § 3.

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{ie}{\hbar c}A_0\right)^2\vec{U}-\cancel{\left(\mathrm{grad}-\frac{ie}{\hbar c}A\right)}\left(\mathrm{grad}-\frac{ie}{\hbar c}A\right)U_0+\kappa\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{ie}{\hbar c}A_0\right)\vec{F}$$

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{ie}{\hbar c}A_0\right)$$

$$+\frac{1}{\kappa}\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)\times\left\{\left(\mathrm{grad}-\frac{ie}{\hbar c}A\right)\times\vec{U}\right\}$$

$$+\kappa^2\vec{U}$$

$$=\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{ie}{\hbar c}A_0\right)^2\vec{U}-\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)^2\vec{U}+\kappa^2\vec{U}$$

$$+\sum_{i=x,y,z}\left(\mathrm{grad}-\frac{ie}{\hbar c}\right)_i\left(\mathrm{grad}-\frac{ie}{\hbar c}A\right)U_i$$

$$+\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{ie}{\hbar c}A_0\right)\cdot\left(\mathrm{grad}-\frac{ie}{\hbar c}A\right)\cdot U_0$$

$$=\left\{\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{ie}{\hbar c}A_0\right)^2-\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)^2+\kappa^2\right\}\vec{U}$$

$$+\left(\mathrm{grad}-\frac{ie}{\hbar c}A\right)\left\{\left(\mathrm{grad}\cdot U-\frac{ie}{\hbar c}A\right)U+\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{ie}{\hbar c}A_0\right)U_0\right\}$$

$$+\frac{ie}{\hbar c}\left(H_z\cancel{E_x}U_y\cancel{+E_xU}-H_yU_z\pm E_xU_0\right)$$

$$\cancel{\left(\mathrm{grad}-\frac{ie}{\hbar c}A\right)}$$

$$\kappa\left\{\left(\mathrm{grad}-\frac{ie}{\hbar c}A\right)U+\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{ie}{\hbar c}A_0\right)U_0\right\}\qquad\frac{\partial}{\partial t}\qquad-\frac{ie}{\hbar c}\frac{\partial}{\partial x}G_z,\ A_xG_y$$

$$=\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{ie}{\hbar c}A_0\right)\vec{F}-\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)\left\{\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)\times\vec{G}\right\}$$

$$-\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{ie}{\hbar c}A_0\right)\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)\vec{F}$$

$$=-\frac{ie}{\hbar c}\left(\vec{E}\vec{F}-\vec{H}\vec{G}\right)$$

Next, we ~~can~~ introduce the ~~canonically~~ canonical conjugate variables to $\vec{U}, U_0$ etc., by the relations.

$$\left.\begin{aligned} U^{\dagger}_x &= \frac{\partial L}{\partial \frac{\partial U_x}{\partial t}} \left(= \frac{\partial L}{\partial F_x} \cdot \frac{\partial F_x}{\partial \frac{\partial U_x}{\partial t}} \right) = \tilde{F}_x \cdot \left(-\frac{1}{\varkappa c}\right) \\ &= \frac{1}{\varkappa^2 c} \left\{ \left(\frac{1}{c}\frac{\partial}{\partial t} - \frac{ie}{\hbar c} A_0 \right) \tilde{\vec{U}} + \left(\mathrm{grad} + \frac{ie}{\hbar c} \vec{A} \right) \tilde{U}_0 \right\} \end{aligned}\right\} \quad (32)$$

$$U^{\dagger}_0 = 0$$

in addition to ~~the canonical variables describing the~~ etc. ~~descri~~ for the electromagnetic field which can be introduced in the usual way. ~~These Hence, the~~ so that the Hamiltonian for the total system becomes

$$\bar{H} = \iiint H\,dv \qquad (33)$$

with

$$H = H_U + H_E$$

$$\left.\begin{aligned} H_U &= U^{\dagger}_x \frac{\partial U_x}{\partial t} + \tilde{U}^{\dagger} \frac{\partial \tilde{U}}{\partial t} - L_U \\ &= 4\pi \varkappa^2 c^2 \tilde{U}^{\dagger} U^{\dagger} - \frac{ie}{\hbar} U^{\dagger} A_0 U \\ &\quad - c U^{\dagger} \left(\mathrm{grad} - \frac{ie}{\hbar c} A \right) U_0 + \frac{ie}{\hbar} \tilde{U}^{\dagger} A_0 \tilde{U} - c \tilde{U}^{\dagger} \left(\mathrm{grad} + \frac{ie}{\hbar c} A \right) \tilde{U}_0 \\ &\quad + \frac{1}{4\pi\varkappa^2} \left\{ \left(\mathrm{grad} + \frac{ie}{\hbar c} A \right) \times \tilde{U} \right\} \left\{ \left(\mathrm{grad} - \frac{ie}{\hbar c} A \right) \times U \right\} \\ &\quad + \frac{1}{4\pi} \tilde{U} U - \frac{c^2}{4\pi\varkappa^2} \left\{ \left(\mathrm{grad} + \frac{ie}{\hbar c} A \right) \tilde{U}^{\dagger} \right\} \left\{ \left(\mathrm{grad} - \frac{ie}{\hbar c} A \right) U^{\dagger} \right\} \end{aligned}\right\} \quad (34)$$

$$H_E = \frac{1}{8\pi}(E^2 + H^2),$$

and $U_0, \tilde{U}_0$ should be eliminated by using the relations

~~We ass~~ In the quantum theory, ~~we assume~~ the canonical variables $U, \tilde{U}, U^{\dagger}$ and $\tilde{U}^{\dagger}$ should satisfy the commutation relations (9) in §2 ~~and the field equations the~~ and the field equations ~~for~~ can be obtained by constructing the equations of the type (13) ~~in §2~~.

~~The Hamiltonian~~ If we change the variables for the U-field by ~~the~~ Fourier transformation (17) in §2, the new variables satisfy the commutation relations (19) and the Hamiltonian ~~takes the form~~ takes the form similar to (23).

$$\overline{H} = \overline{H}_U + \cancel{\iiint H_E\,dv}$$

$$\cancel{\overline{H}_U = \iiint H_U\,dv} \qquad (35)$$

with $\cancel{\overline{H}_U = \iiint H_U\,dv}$

~~Instead of~~
~~We do not go here into the~~ of the complicated formulae.
~~Instead of saying the~~ In this way, we can deal with the problem of the interaction of the ~~heavy quanta~~ U-field with the electromagnetic field in any detail, but we consider here only the problem of the magnetic moment of the heavy quantum.
We can deduce ~~the~~ second order equations from (25) and (26) by iteration. Thus, we obtain, for example,

$$\left\{\left(\frac{1}{c}\frac{\partial}{\partial t} + \frac{ie}{\hbar c}A_0\right)^2 - \left(\mathrm{grad} - \frac{ie}{\hbar c}\vec{A}\right)^2 + \kappa^2\right\}\vec{U}$$

$$-\frac{ie}{\hbar c}\left(\vec{H}\times\vec{U} + \vec{E}U_0 + \vec{E}\vec{F} - \vec{H}\vec{G}\right) = 0. \qquad (35)$$

Now, when a heavy quantum ~~with the~~ with W, positive charge is present in the state ~~of with~~ the ~~kinetic~~ $m_U c^2$, W, ~~exten~~ energy small compare with the proper energy $m_U c^2$, the equations ~~is~~ (35) reduces, in the first approximation, to

$$-\frac{\hbar^2}{2m_U}\ \cancel{W - \frac{ie}{}eA_0}$$

$$\left\{-W + eA_0 + \frac{\hbar^2}{2m_U}\left(\mathrm{grad} - \frac{ie}{\hbar c}\vec{A}\right)^2\right\}\vec{U}$$

$$+\frac{ie\hbar}{2m_U c}\cdot\left(\vec{H}\times\vec{U} + \vec{E}\vec{F}\right) = 0, \qquad (36)$$

$$\}\ \vec{F}$$

because U_0 and G become small compared with F and U in this case. ~~The equations (36) shows that~~ ~~Similarly, we obtain the equations~~

$$\}\ \vec{F}$$

The equations (36) show that the heavy quantum with the positive charge has the magnetic moment ~~which can be expressed~~

by the ~~operator~~ operator vector with the components

$$U_x\left(-H_y U_z + H_z U_y\right), \quad U_z\left(-H_x U_y + H_y U_x\right), \quad U_y\left(-H_z U_x + H_x U_z\right)$$

$$\frac{ie\hbar}{2m_U c}\left(\quad\right), \qquad \to ie\hbar$$

$$\frac{e\hbar}{2m_U c}\cdot\vec{\sigma},$$

where $\vec{\sigma}$ is a vector with the components

$$H_x\begin{pmatrix}0&0&0\\0&0&i\\0&-i&0\end{pmatrix} + H_y\begin{pmatrix}0&0&i\\0&0&0\\-i&0&0\end{pmatrix} + H_z\begin{pmatrix}0&i&0\\-i&0&0\\0&0&0\end{pmatrix}\begin{pmatrix}U_x\\U_y\\U_z\end{pmatrix}$$

$$\vec{\sigma}\quad \begin{pmatrix}0&0&0\\0&0&i\\0&-i&0\end{pmatrix}, \quad \begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix}, \quad \begin{pmatrix}0&i&0\\-i&0&0\\0&0&0\end{pmatrix}. \qquad (37)$$

These matrices have each eigenvalues $1, 0, -1$, so that the ~~magnetic moment~~ magnitude of the magnetic moment becomes

$$\frac{e\hbar}{2m_U c}, \quad \cancel{\mu_0}$$

which is smaller than the magnetic moment of the electron by a factor $\frac{m}{m_U}$ $(\cong \cancel{100^{-1}})$.

has ~~the spin~~ magnetic moment of amount $\frac{e\hbar}{2Mc}$ in ~~the~~ nonrelativistic approximation ~~and~~ this value in itself is smaller ~~than~~ compared with that of the electron, but,

Now, as shown in the preceding section, the heavy quantum owning to the strong interaction interaction of the heavy quantum and the heavy particle, which is responsible for the nuclear force as shown in I, II and ~~the~~ §6 of this paper, the probability of the virtual presence of the heavy quanta in the neighborhood, is very much larger than that of the electrons. Thus, we can expect ~~a large~~ ~~larger~~ the contribution of the heavy ~~quanta~~ quantum to the extra magnetic moments of the neutron and the proton ~~than~~ is far larger than that of the electrons. ~~The di~~ Such an ~~idea was already~~ One ~~of~~ the present authors performed the estimation of extra moments on ~~these~~ this idea[2] and similar consideration was made by Fröhlich and Heitler[3] and by Bhabha[4] ~~indepen~~ independently.

owing to the small probability of the virtual ~~virtual~~ presence of the ~~small~~ light ~~ele~~ particles.

§ 5. Magnetic Moments of the Neutron and the Proton

As well known, ~~The~~ the anomalous magnetic moments of the ~~heavy~~ neutron and the proton particle, ~~which can not~~ which have the values are in apparent contradiction with the assumption that ~~the~~ it ~~must~~ satisfies the relativistic wave equations of Dirac type. Wick[)] has shown that the suggested ~~showed~~ that this ~~can be~~ discrepancy ~~might~~ be attributed to the presence of the electron anomaly and the neutrino as intermediate virtual state, but ~~in the magnetic of~~ the extra magnetic moment thus calculated obtained was found to be far too small ~~to account~~ compared with for the actual ~~experimental~~ value. ~~One of the present authors~~ of we, as ~~being~~ as we and ~~Fröhlich and Heitler~~ [)] ~~has showed recently~~ ~~The~~ Now, as shown the ~~result of~~ the preceding section ~~that~~ The heavy quantum has as shown in the magnetic moment of the magnitude $e\hbar$ ~~which~~ which smaller than the $2\mu c$, ~~almost~~, in nonrelativistic approximation, ~~suggest at once~~ ~~that its contribution~~ which is about idem 10 time the nuclear ~~proton~~ magneton. ~~this will~~ We ~~thus~~ arrived at ~~the~~ ~~so suggest its contributi~~ ~~once~~ thus ~~and we~~ arrive a this suggesting itself, at once that the anomalous ~~mo~~ of the magnetic moment of the heavy particle can be ~~attributed~~ explained by the ~~Thus the~~ idea suggest thus, so that we come at once to the suggestion, that the above anomaly ~~of the magnetic mo~~ can be explained by the virtual presence of the heavy quantum, ~~In fact~~ as already discussed by one of the present authors[1)] ~~and~~, by ~~Hei~~ Fröhlich and Heitler and by Bhabha[)].

~~Although the exact calculation calculation~~ treatment of this problem can be ~~dealed with~~ solved in ~~detail~~ exact, accurately, only if the form of interaction of the heavy particle with the U-field is determined in detail, which will be ~~shown~~ dealed with in § , so that we want to content ourselves with the following rough estimation. ~~The probability that~~ When a neutron is in a state n with the

[)] Taketani, Kagaku, I, 532, 1937

[)] Fröhlich and Heitler, Nature 141, 37, 1938.

[)] Bhabha, loc. cit.

[)] Wick, Rend. Lincei 21, 170, 1935.

energy E_n, the fraction of time, during which the neutron ~~of~~ is splitting virtually into a proton and a heavy quantum with the negative charge, is given by

$$\tau = \sum_m \frac{|\bar{H}'_{nm}|^2}{(E_n - E_m)^2}, \qquad (38)$$

where E_m is the energy of the virtual state m and $\bar{H}'_{nm}$ is the matrix element of the energy of interaction between the heavy particle and the U-field. Thus, the magnetic moment of the neutron due to the virtual presence of the heavy quantum is given roughly by

$$\mu \cong \frac{e\hbar}{2m_U c} \cdot \tau \cdot C, \qquad (39)$$

where C is a constant of order 1 depending on the correlation of the spin of the neutron and that of the heavy quantum. τ can be estimated by using the expression (14) in II for $\bar{H}'_{nm}$ and takes the form

$$\tau \cong \frac{g^2\hbar c}{\pi Mc} \cdot \cancel{k}\, dk \quad \frac{g^2}{\pi \cancel{\hbar c}\, Mc^2} \int_0^{K} dk, \qquad (40)$$

where k is the wave number of the heavy quantum. The upper limit K for the wave number can be determined by the assumption, that the self energy of the neutron due to the U-field surrounding it has an order of magnitude of Mc^2, where M is the neutron mass, ~~i.e. Thus, we obtain~~
Thus we obtain

$$Mc^2 \cong \sum_m \frac{|H'_{nm}|^2}{(E_n - E_m)} \cong \frac{g^2\hbar c}{\pi} \cdot \frac{1}{Mc^2} \int_0^K k\,dk \;*$$

~~Thus or Using A~~ $= \frac{g^2\hbar c K^2}{2\pi}$

or

$$K \cong Mc^2 \cdot \sqrt{\frac{2\pi}{g^2\hbar c}}. \qquad (41)$$

~~or~~ Inserting this value into ~~(39)~~ (40), ~~and (39) we have~~ ~~the magnetic~~ of the neutron ~~becomes~~ we obtain.

$$\cancel{\mu \cong} \cdot \tau \cong \sqrt{\frac{2}{\pi}\frac{g^2}{\hbar c}} \simeq \frac{1}{4},$$

so that

$$\mu \cong \frac{1}{4}\,\frac{e\hbar}{2m_\nu c} \cong 2 \cdot \cong 2 \cdot \frac{e\hbar}{2Mc}.$$

Thus, the sign of the magnetic moment of neutron is negative and its magnitude ~~which~~ is ~~the or~~ a roughly about twice the ~~nuclear~~ nuclear magneton in close agreement with ~~as required from~~ the experimental result. ~~The sign of~~ The additional magnetic moment of the proton can similarly be calculated estimated similarly and It is clear from the symmetry consideration, that according to our theory, the ~~magnetic~~ additional magnetic moment of the proton has the same magnitude and the ~~opposite~~ positive sign, with that of the neutron which is also in agreement with the experiment.

§6. Interaction of the U-Field with the Heavy Particle

In the preceding paper (II, §3), we found that the scalar U-field was not adequate for describing the interaction between the neutron and the proton, as the exchange force of Heisenberg type thus obtained had the sign opposite to the one required from the experimental result. Now, the U-field considered in this paper consist of two four vectors and two six vectors, so that there are ample spaces for various types of interaction between the U-field and the heavy particle, which in turn ~~will~~ result in various types of forces between the ~~heavy particle~~ neutron and the proton as second order effects ~~processes~~. Thus, in the presence of the heavy particle, the most general ~~field~~ equations for the U-field ~~takes & may take~~ may have ~~have~~ the form

$$\cancel{\frac{1}{c}\frac{\partial U}{\partial t} + \mathrm{grad}\, U_0 + \kappa F =} \qquad (39)$$

$$\left.\begin{array}{ll} \dfrac{1}{c}\dfrac{\partial \vec{F}}{\partial t} - \mathrm{curl}\,\vec{G} - \kappa\vec{U} = -4\pi g_1\vec{M} & \mathrm{div}\,\vec{F} + \kappa U_0 = 4\pi g_1 M_0 \\ \dfrac{1}{c}\dfrac{\partial \vec{U}}{\partial t} + \mathrm{grad}\, U_0 + \kappa\vec{F} = 4\pi g_2\vec{T} & \mathrm{curl}\,U - \kappa G = -4\pi g_2\vec{S} \end{array}\right\} \qquad (40)$$

if we ignore the electromagnetic field, where $\vec{M}$, M_0 are ~~the bilinear form of a four vector~~ the space and the time components of a ~~four~~ vector, ~~constructed by forming~~ which can be expressed by ~~the~~ a bilinear form for each of the wave functions of the heavy particle. ~~In the simplest~~ The simplest ~~form for~~ expressions for $\vec{M}$, M_0 ~~are~~ become, if we ~~assume~~ the heavy particle to satisfy the wave equations of Dirac's type,

$$\left.\begin{array}{l} \vec{M} = \tilde{\Psi}(\vec{\alpha}\tilde{Q})\Psi, \\ M_0 = \tilde{\Psi}\tilde{Q}\Psi, \end{array}\right\} \qquad (41)$$

where Ψ, $\tilde{\Psi}$ are the wave functions for the heavy particle each with eight components, $\vec{\alpha}$ ~~the~~ the Dirac matrices and $\tilde{Q}$ is the operator changing the ~~neutron proton into the neutron.~~ heavy particle from the ~~proton~~ proton to the neutron state. ~~Similarly~~ $\vec{S}$, $\vec{T}$ are ~~the~~ two space vectors forming a six vector, which can be constructed in a way similar to $\vec{M}$, M_0. The simplest ~~for~~ expressions for them corresponding to (40) are

$$\vec{T} = \tilde{\Psi}\rho_2\vec{\sigma}\tilde{Q}\Psi, \qquad \vec{S} = \tilde{\Psi}\rho_3\vec{\sigma}\tilde{Q}\Psi, \qquad (42)$$

where ρ's and $\vec{\sigma}$ has the same meaning as those ~~used~~ in the usual formulation of the Dirac's theory of the electron.

$\mathfrak{T}$: $\rho\alpha = \rho_3\rho_1\sigma$ $\left(\begin{matrix} 0 & 0 & 1 \\ \rho & \rho & \end{matrix}\right) - i\rho_3\rho_1\sigma$

$\mathfrak{S}$: $\beta\sigma = (\rho_3\sigma)$

(18)

g_1, g_2 are the constants with the dimension of the electric charge as the constant g in I and II and characterize the strengths of ~~the~~ two types of interactions above considered respectively.

~~The Lag.~~ The ~~above~~ field equations (39) can be derived from the Lagrangian $\bar{L}_U = \iiint L_U \, dv$ with

$$L_U = \frac{1}{4\pi}(\tilde{F}F - \tilde{G}G + \tilde{U}_0 U_0 - \tilde{U}U) + \frac{4\pi g_1^2}{\varkappa^2}(\tilde{T}T - \tilde{S}S) - \frac{4\pi g_1^2}{\varkappa^2}(\tilde{M}_0 M_0 - \tilde{M}M) \quad (43)$$

$$+ \frac{g_1}{2c}(\tilde{U}M - \tilde{U}_0 M_0 + U\tilde{M} - U_0\tilde{M}_0), \quad (44)$$

where U, U_0, $\tilde{U}_0$, and $\tilde{U}$ are considered as independent variables and ~~the~~ F, G, $\tilde{F}$ and $\tilde{G}$ are auxiliary considered as dependent variables defined by (40). ~~Thus,~~ Hence, the canonical ~~conjugate~~ variables canonically conjugate to U, $\tilde{U}$, etc. ~~are~~ become

$$U_x^\dagger = \frac{\partial L}{\partial \frac{\partial U_x}{\partial t}} = -\frac{\tilde{F}_x}{4\pi\varkappa c}, \quad \text{etc.,}$$

$$U_0^\dagger = 0, \quad \text{etc.} \quad (45)$$

~~and thus~~ so that we obtain

$$\frac{\partial \tilde{U}}{\partial t} = 4\pi\varkappa^2 c^2 U^\dagger - c \operatorname{grad} \tilde{U}_0 + 4\pi g c \tilde{\mathfrak{T}}, \quad \text{etc.} \quad (46)$$

by the help of the relations (40).

The Hamiltonian for the U field, thus, takes the form

$$\bar{H} = \iiint H \, dv \quad (47)$$

with

$$H = U^\dagger(4\pi\varkappa^2 c^2 \tilde{U}^\dagger - c \operatorname{grad} U_0 + 4\pi g_2 c \mathfrak{T})$$
$$+ \tilde{U}^\dagger(4\pi\varkappa^2 c^2 U^\dagger - c \operatorname{grad} \tilde{U}_0 + 4\pi g_2 c \tilde{\mathfrak{T}})$$
$$- 4\pi\varkappa^2 c^2 U^\dagger \tilde{U}^\dagger + \frac{1}{4\pi\varkappa^2}(\operatorname{curl} \tilde{U} + 4\pi g_2 \tilde{\mathfrak{S}})(\operatorname{curl} U + 4\pi g_2 \mathfrak{S})$$
$$- \frac{1}{4\pi}(\tilde{U}_0 U_0 - \tilde{U}U) + \frac{g_1}{\varkappa}(\tilde{U}_0 M_0 - \tilde{U}M + U_0\tilde{M}_0 - U\tilde{M}). \quad (48)$$
$$\cancel{-\frac{4\pi g_1^2}{\varkappa^2}(\tilde{T}T - \tilde{S}S) + \frac{4\pi g_1^2}{\varkappa^2}(\tilde{M}_0 M_0 - \tilde{M}M)}$$

In the quantum theory, U_x, $U_x^\dagger$ etc. should satisfy the commutation relations of the type (9), and U_0 and $\tilde{U}_0$ should be eliminated by using the relations

$$U_0 = 4\pi c \operatorname{div} \tilde{U}^\dagger + \frac{4\pi g}{\varkappa}\tilde{M}_0, \quad \tilde{U}_0 = 4\pi c \operatorname{div} U^\dagger + \frac{4\pi g}{\varkappa}\tilde{M}_0. \quad (49)$$

The Hamiltonian becomes, thus,

$$H_U = 4\pi\varkappa^2 c^2 \tilde{U}^\dagger U^\dagger + 4\pi c^2 \operatorname{div} U^\dagger \operatorname{div} \tilde{U}^\dagger + \frac{1}{4\pi\varkappa^2}\operatorname{curl}\tilde{U}\operatorname{curl} U$$

$H = H_U + H'$ with

$$H' = \ldots + \frac{1}{4\pi}\tilde{U}U + 4\pi g_2 c(U^{\dagger}\mathfrak{T} + \tilde{U}^{\dagger}\tilde{\mathfrak{T}}) + \frac{g_2}{\kappa^2}(\mathrm{curl}\,U\cdot\tilde{\mathfrak{S}} + \mathrm{curl}\,\tilde{U}\cdot\mathfrak{S})$$
$$+ \frac{4\pi g_1 c}{\kappa}(\mathrm{div}\,U^{\dagger}\cdot M_0 + \mathrm{div}\,\tilde{U}^{\dagger}\tilde{M}_0) - \frac{g_1}{\kappa}(\tilde{U}M + U\tilde{M})$$
$$+ \frac{4\pi}{\kappa^2}(g_1^2\tilde{M}_0 M_0 + g_2^2\tilde{\mathfrak{S}}\mathfrak{S}) - \frac{4\pi}{\kappa^2}\cancel{(g_1^2\tilde{M}M + g_2^2\tilde{T}T)} \qquad (50)$$

~~In~~ this expression, H_0 the terms ~~involving~~ ~~independent of the,~~ which do not involve the constants g_1, g_2, denote the energy of the U-field in vacuum as in § 2 and, H'_1 ~~the those~~ which ~~involve dependents on g_1, g_2 linearly~~ ~~denote~~ the energy of ~~interaction~~ interaction between the heavy particle and the U-field. The last two terms involving g_1^2 and g_2^2 depend ~~only~~ only on the wave functions of the heavy particle, so that they should be considered as ~~a sort of the self~~ the zero-range interaction between the ~~neutron~~ due to and the proton. This seems ~~to be~~ at first sight to be a ...

~~We shall show, however, that these terms compensate with those which ap.~~

~~Later on,~~ ~~we shall show, however that~~ These terms, however, ~~will~~ compensate with those which ~~will appear~~ are derived appear from the ~~calcular~~ second order ~~perturbation~~ effect ~~so that~~ ~~there there are only finite range we obtain~~ from the terms linear in g_1, g_2, as will be shown in the following calculations. ~~so that we obtain~~.

~~Now, The field equations of the type (39), (40), will are derived from the above Hamiltonian by using the commutation~~

Now, the ~~field~~ equations of motion for U_x, $U_x^{\dagger}$ etc., which are ~~correspond~~ equivalent to the field equations (39), (40), ~~can be~~ are derived from the above Hamiltonian in the usual manner and take the form

$$\dot{U} = 4\pi\kappa^2 c^2 \tilde{U}^{\dagger} - 4\pi c^2 \,\mathrm{grad}\,\mathrm{div}\,\tilde{U}^{\dagger} + 4\pi g_2 c\,\tilde{\mathfrak{T}} - \frac{4\pi g_1 c}{\kappa}\,\mathrm{grad}\,\tilde{M}_0 \qquad (51)$$

$$-\dot{\tilde{U}}^{\dagger} = \frac{1}{4\pi\kappa^2}\,\mathrm{curl}\,\mathrm{curl}\,U + \frac{1}{4\pi}U + \frac{g_2}{\kappa^2}\,\mathrm{curl}\,\mathfrak{S} - \frac{g_1}{\kappa}M$$

From these equations, we ~~obtain~~ can easily the ~~eq~~ quadratic ~~a~~ equations

$$
(52)\quad \left\{
\begin{aligned}
&\frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} - \Delta U + \kappa^2 U = 4\pi g_1\Big(\kappa M - \frac{1}{\kappa}\,\mathrm{grad\,div}\,M - \cancel{\frac{1}{\kappa c}\frac{\partial}{\partial t}}\,\mathrm{grad}\,M_0\Big) \\
&\quad - \frac{1}{\kappa}\,\mathrm{grad}\,\frac{1}{c}\frac{\partial M_0}{\partial t}\Big) + 4\pi g_2\Big(\frac{1}{c}\frac{\partial \mathbf{T}}{\partial t} - \mathrm{curl}\,\mathbf{S}\Big) \\
&\frac{1}{c^2}\frac{\partial^2 \tilde{U}^{\dagger}}{\partial t^2} - \Delta \tilde{U}^{\dagger} + \kappa^2 \tilde{U}^{\dagger} = \frac{g_1}{\kappa c}\Big(\mathrm{grad}\,M_0 + \frac{1}{c}\frac{\partial M}{\partial t}\Big) \\
&\quad - \frac{g_2}{\kappa}\Big(T + \frac{1}{\kappa^2}\,\mathrm{curl\,curl}\,T + \frac{1}{c}\frac{\partial T}{\partial t} - \frac{1}{\kappa^2 c}\,\mathrm{curl}\,\frac{1}{c}\frac{\partial S}{\partial t}\Big)
\end{aligned}
\right\} 5
$$

The ~~tot~~ complete Hamiltonian for the system ~~contain~~ consisting of the ~~neutron~~ U-field and the heavy particle is

$$\bar{H} = \iiint (H_U + H' + H_M)\,dv \tag{53}$$

with H_U, H' given by (50) and

$$H_M = \tilde{\Psi}\{\vec{\alpha} - i\hbar c\,\vec{\alpha}\,\mathrm{grad} + \beta(\tfrac{1+\tau_3}{2}M_N c + \tfrac{1-\tau_3}{2}M_P c)\}\Psi, \tag{54}$$

if we ignore the electromagnetic ~~field~~ interaction ~~of~~ between the ~~U-field~~ heavy quantum and the proton, where thus Ψ, $\tilde{\Psi}$ are quantised wave equations for the heavy particle ~~each with the eight components,~~ ~~as~~ satisfy the same commutation relations as in II, each with eight components). (We assume the ~~heavy particle is assumed~~ to satisfy ~~the Dirac's~~ Dirac's wave equations ~~as in II.~~ and use ~~the same~~ the ~~and a~~ same notations as in II.

~~The interaction between the U-field and the heavy particle expressed by H'~~

§7. Deduction of the ~~interaction~~ Force between the Neutron and the Proton

The interaction between the neutron and the proton, ~~which~~ is caused by ~~the~~ virtual absorption and emission of the ~~heavy~~ heavy quanta, can be calculated from the Hamiltonian (53) ~~by the straight~~ as second order effect by the straight forward application of the perturbation theory as in II. ~~This is performed to through by trans In this process~~ Namely, we first ~~transl~~ transform the variables for the U-field into (and further, we assume ~~the interaction~~ M, M_0, T, S etc to have the simplest forms ( ) and ( ).)

~~When two heavy particles are at the point~~

If ~~we~~ consider ~~two~~ the two heavy particles which have ~~with~~ the coordinates $\vec{r}_1$, ~~$\vec{r}_2$~~, ~~the momenta~~ the spin matrices, ($\rho_1^{(1)}$, ~~$\rho_2^{(1)}$, $\rho_3^{(1)}$~~) $\vec{\sigma}^{(1)}$ and ~~the exchange operator~~ Q_r, Q, isotopic spin matrices $\tau_1^{(1)}$ $\tau_2^{(1)}$ $\tau_3^{(1)}$ and another denoted by $\vec{r}_1$, $\vec{p}_1$ ($\rho_1^{(1)}$, $\rho_2^{(1)}$, $\rho_3^{(1)}$, $\vec{\sigma}^{(1)}$) ($\tau_1^{(1)}$ $\tau_2^{(1)}$ $\tau_3^{(1)}$) and $\vec{r}_2$, $\vec{p}_2$ ($\rho_1^{(2)}$, $\rho_2^{(2)}$, $\rho_3^{(2)}$, $\vec{\sigma}^{(2)}$) ($\tau_1^{(2)}$, $\tau_2^{(2)}$, $\tau_3^{(2)}$) respectively, ~~the~~ Hamiltonian for the system consisting of these particles and the U-field ~~can be considered as~~ can be written in the form

$$\overline{H} = \iiint H_U\,dv + \overline{H}_M + \overline{H}'.$$

with H_U as given by ( ) and

$$\overline{H}_M = \sum_{j=1,2}\left\{c\,\rho_1^{(j)}(\vec{\sigma}_j\vec{p}_j) + \rho_3^{(j)}\left(\frac{1+\tau_3^{(j)}}{2}M_N c^2 + \frac{1-\tau_3^{(j)}}{2}M_P c^2\right)\right\}$$

$$\overline{H}' = \sum_j 4\pi g_2 c\left\{U^{\dagger}(\vec{r}_j)\,\rho_2^{(j)}\vec{\sigma}^{(j)}\,\tilde{Q}_j + U^{\dagger}(\vec{r}_j)\,\rho_2^{(j)}\vec{\sigma}^{(j)}\,Q_j\right\}$$

$$\tilde{Q}_j = \frac{\tau_1^{(j)} - i\tau_2^{(j)}}{2} \qquad Q_j = \frac{\tau_1^{(j)} + i\tau_2^{(j)}}{2}$$

$$+\ \frac{g_2}{\kappa^2}\left(\mathrm{curl}_1 U(\vec{r}_1)\right)$$

8

where $\tilde{Q}_j = \dfrac{\tau_1^{(j)} - i\tau_2^{(j)}}{2}$, $Q_j = \dfrac{\tau_1^{(j)} + i\tau_2^{(j)}}{2}$

and curl_j, grad_j, div_j denote the differentiations with respect to r_j, the normal coordinates as in §2, ~~and~~ then express the perturbation energy ~~in these coordinate~~ due to the presence of the neutron and the proton in these coordinates and finally, after performing ~~a~~ rather compliated ~~int~~ integrations, we obtain the interaction energy. ~~The following~~ an alternative method, which is similar to that given in I, ~~and leads to the same result as above~~, ~~seems is more~~ seems to be more convenient in the present case. We confirmed that ~~the two~~ these two methods lead to exactly the same result, so that we ~~dec~~ want to enunciate ~~the le~~ only the latter in the following.

~~Now, when the energy of the heavy particle is small compared with the kinetic proper energy Mc^2, where M is the mass of the heavy particle,~~

Now, we want to deduce the interaction between the neutron and the proton by assuming for $\vec{M}, M_0$ and $\vec{T}, \vec{S}$ the forms given by (41) and (42) respectively. When ~~they are~~ simplest, as in the state of the kinetic energy small compared each of them, with the proper energy $M_N c^2$ or $M_P c^2$, the quantities

$$\vec{M} = \tilde{\Psi}(\vec{\alpha}\tilde{Q})\Psi \quad , \quad \vec{T} = -\tilde{\Psi}\rho_2\vec{\sigma}\tilde{Q}\Psi$$

~~are pro~~ are negligible compared with

$$M_0 = \tilde{\Psi}\tilde{Q}\Psi \qquad \vec{S} = \tilde{\Psi}\rho_3\vec{\sigma}\tilde{Q}\Psi \doteq \tilde{\Psi}\vec{\sigma}\tilde{Q}\Psi \quad (55)$$

~~Hence, In this case~~ Moreover, ~~in the~~ the difference of ~~energy of~~ the energies of the neutron and the proton is usually small compared with the proper energy $m_U c^2$ of the heavy quantum, so that the terms in (52) involving the time derivative can be neglected altogether. In this approximation, ~~Thus~~ the equations (52) reduce to

$$(56) \quad \cancel{\frac{1}{c^2}\frac{\partial^2}{\partial t^2}} - \Delta U + \varkappa^2 U = \cancel{\frac{4\pi g_1}{\varkappa}\ \varkappa M\ \mathrm{grad}} - \frac{4\pi g_2}{\varkappa}\,\mathrm{curl}\, S$$

$$-\Delta\tilde{U}^{+} + \varkappa^2\tilde{U}^{+} = \frac{g_1}{\varkappa c}\,\mathrm{grad}\, M_0$$

and correspondingly, ~~and~~ the interaction energy ~~of inter~~ between the ~~neutron and~~ heavy particle and the U-field becomes approximately

$$\overline{H}' = \iiint \{ ~~4\pi~~ H' dv \qquad + \frac{4\pi g_1^2}{\kappa^2}\tilde{S}S. \quad (56)$$

with
$$H' \cong ~~\frac{g_2}{\kappa^2}\{\text{curl}\, U\cdot~~ ~~\frac{4\pi g_1 c}{}~~ \frac{g_2}{\kappa^2}(\text{curl}\, U\cdot\tilde{S} + \text{curl}\,\tilde{U}\cdot S)$$
$$+ \frac{4\pi g_1 c}{\kappa}(\text{div}\, U^+ \cdot M_0 + \text{div}\,\tilde{U}^+\cdot\tilde{M}_0) + \frac{4\pi}{\kappa^2}g_1^2\tilde{M}_0 M_0$$

~~The secon~~ Now the particular solutions of (55), due to the presence of the heavy ~~p~~particle is given for U, U^+ by

$$U(\vec{r}_1) = g_2 \iiint \frac{e^{-\kappa r}}{r}\text{curl}_2 S(\vec{r}_2)\, dv_2$$
$$\tilde{U}^+(\vec{r}_1) = \frac{-g_1}{4\pi\kappa c}\,\text{grad}_1 \iiint \frac{e^{-\kappa r}}{r}\,\text{grad}_2 M_0(\vec{r}_2)\, dv_2, \qquad (57)$$

~~respective~~ where $\vec{r} = \vec{r}_1 - \vec{r}_2$

and curl_2, grad_2 denote the differentiations with respect to ~~$\vec{r}_2$~~ the coordinates $\vec{r}_2$. Inserting these expressions in (57), we obtain the ~~second order perturbation energy~~ energy of interaction between the heavy particles

$$\overline{H}' = [redacted] -\frac{g_2^2}{\kappa^2}\int\cdots\int \tilde{S}(\vec{r}_1)\,\text{curl}_1\left(\frac{e^{-\kappa r}}{r}\text{curl}_2 \tilde{S}(\vec{r}_2)\right) dv_1 dv_2$$

(58)
$$+ \frac{4\pi g_2^2}{\kappa^2}\int\cdots\int \tilde{S}(\vec{r}_1)\,\delta(\vec{r}_1,\vec{r}_2)\,S(\vec{r}_2)\,dv_1 dv_2$$
$$~~+\text{comp. conj}~~ + \frac{g_1^2}{\kappa^2}\int\cdots\int \tilde{M}_0(\vec{r}_1)\,\text{div}_1\left(\frac{e^{-\kappa r}}{r}\,\text{grad}_2 M_0(\vec{r}_2)\right)dv_1 dv_2$$
$$+ \frac{4\pi}{\kappa^2}g_1^2 ~~\tilde{M}_0 M_0~~ \int\cdots\int \tilde{M}_0(\vec{r}_1)\,\delta(\vec{r}_1,\vec{r}_2)\,M_0(\vec{r}_2)\,dv_1 dv_2$$

+ comp. conj., where curl_1, div_1 denote the differ. with respect to $\vec{r}_1$.

By performing the partial integration and by using the relation

$$~~(-\Delta_1 + \kappa^2)~~ (-\text{div}_1\,\text{grad}_1 + \kappa^2)\frac{e^{-\kappa r}}{r} = -4\pi\,\delta(\vec{r}_1,\vec{r}_2), \qquad ~~(59)~~ (60)$$

$\tilde{Q}Q + \tilde{Q}\tilde{Q} = \left(\frac{\tau_1 - i\tau_2}{2}\right)\left(\frac{\tau_1 + i\tau_2}{2}\right)$ (23)

$+(\quad)(\quad) = \frac{\tau_1(1)\tau_2(2) + \tau_2(1)\tau_2(2)}{2}$.

we obtain finally.

$$\bar{H}' = g_1^2 \int\cdot\cdot\int \{\tilde{M}_0(\vec{r}_1) M_0(\vec{r}_2) + \cancel{\tilde{M}_0(\vec{r}_2)\tilde{M}_0(\vec{r}_2)}\} \frac{e^{-\varkappa r}}{r} dv_1 dv_2$$

(61)
$$+ g_2^2 \int\cdot\cdot\int \left[\tilde{S}(\vec{r}_1) S(\vec{r}_2) - \frac{\{\tilde{S}(\vec{r}_1)\,\mathrm{grad}\}\{S(\vec{r}_2)\,\mathrm{grad}\}}{\kappa^2}\right]$$

$$\times \frac{e^{-\varkappa r}}{r} dv_1 dv_2$$

$+$ comp. conj.,

where grad denotes the differentiation with respect to $\vec{r}$ ($\vec{r}_1 - \vec{r}_2$) operating on $e^{-\varkappa r}/r$. If we insert the expressions ~~(41) and (42)~~ for M_0 and S ~~respectively, the interaction energy becomes finally~~

~~$$\bar{H}' = g_1^2 \int\cdot\cdot\int \tilde{\Psi}\tilde{Q}\Psi(\tilde{\Psi}(\vec{r}_1)\tilde{Q}\Psi(\vec{r}_1) \cdot \tilde{\Psi}(\vec{r}_2)\tilde{Q}\Psi(\vec{r}_2)) \frac{e^{-\varkappa r}}{r} dv_1 dv_2$$~~

~~(62)~~
~~$$+ g_2^2 \int\cdot\int \tilde{\Psi}(\vec{r}_1)\vec{\sigma}\Psi(\vec{r}_1) \cdot \Psi$$~~

in (61), we find that the energy of interaction ~~energy~~ between ~~the neutron and the proton~~ two heavy particles, whose coordinates, spin matrices and τ-isotopic spin matrices are expressed by

$$\vec{r}_1,\ \vec{\sigma}^{(1)},\ (\tau_1^{(1)}, \tau_2^{(1)}, \tau_3^{(1)})$$

and
$$\vec{r}_2,\ \vec{\sigma}^{(2)},\ (\tau_1^{(2)}, \tau_2^{(2)}, \tau_3^{(2)}),$$

$\vec{\sigma}^{(1)}$ grad div $\sigma^{(2)}$

is given by

$$H_{12}^{A} = \left\{g_1^2 + g_2^2(\vec{\sigma}^{(1)}\cdot\vec{\sigma}^{(2)}) - g_2^2\frac{(\vec{\sigma}^{(1)}\mathrm{grad})(\vec{\sigma}^{(2)}\mathrm{grad})}{\kappa^2}\right\} \frac{e^{-\varkappa r}}{r} \cdot \frac{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}}{2} \qquad (62)$$

~~The first term containing g_1^2 is~~

or
$$= \frac{(\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)})}{2}\left[g_1^2 + g_2^2\left\{(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}) - \frac{(\vec{\sigma}^{(1)}\vec{r})(\vec{\sigma}^{(2)}\vec{r})}{r^2}\right\}\frac{e^{-\varkappa r}}{r}\right.$$
$$\left. + g_2^2\left\{(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}) - \frac{3(\vec{\sigma}^{(1)}\vec{r})(\vec{\sigma}^{(2)}\vec{r})}{r^2}\right\}\left(\varkappa - \frac{1}{r}\right)\frac{e^{-\varkappa r}}{r^2}\right] \qquad (63)$$

~~This rather complicated expressions for the~~
As well known, this is a combination of exchange forces of Majorana and Heisenberg types ~~for the next~~ between the neutron and the proton.

When ~~the~~ a ~~neutron and a proton~~ the system consisting of a neutron and a proton ~~are~~ is in the S state, as in the case of the deuteron ~~in the~~, the wave function for the system is spherically symmetric with respect to $\vec{r}$, so that ~~the average value~~ we have the average value

$$\frac{\overline{(\vec{\sigma}^{(1)}\vec{r})(\vec{\sigma}^{(2)}\vec{r})}}{r^2} = \frac{1}{3}\vec{\sigma}^{(1)}\vec{\sigma}^{(2)} \qquad (64)$$

~~average~~. Thus in S state, the interaction energy (63) reduces to

$$H_{12} = \frac{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}}{2}\left\{g_1^2 + \frac{2g_2^2}{3}(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)})\right\}\frac{e^{-\varkappa r}}{r}$$

$$= P^H \left(g_1^2 - \frac{1}{3}g_2^2\right)\frac{e^{-\varkappa r}}{r} + \frac{4g_2^2}{3}P^M\frac{e^{-\varkappa r}}{r}, \qquad (65)$$

where

$$P^H = \frac{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}}{2}, \quad P^M = \frac{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}}{2}\cdot\frac{1+\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}}{2} \qquad (66)$$

are the exchange operators of Heisenberg and Majorana respectively. ~~This~~ The expression (65) shows that the Majorana force is attractive in S-state in accordance with the ~~usu~~ assumptions of the ~~oth~~ current theory, whereas the Heisenberg force ~~is attractive can be become~~ becomes attractive for ^3S state and repulsive for ^1S, if we ~~take g_1~~ assume the relation $g_1^2 > \frac{1}{3}g_2^2$ between two constants g_1, g_2, as required from the experiment. The relative magnitude of the Heisenberg and Majorana forces is

$$3g_1^2 - g_2^2 : 4g_2^2$$

~~so that~~ and already the simplest assumption $g_1 = g_2$

gives the ratio 1 : 4, which is not far from the ratio accepted in the current theory. Thus, our theory can deduce ~~reproduce~~ the exchange force between the neutron and the proton, which is a ~~correct in both in sign and magnitude~~ ~~a combinati~~ ~~state~~ combination correct both in sign and magnitude.†

In ~~order to~~ as well as to obtain the ordinary forces of Wigner and Bartlett type between the neutron and the proton, we have to ~~perform~~ calculate the fourth order ~~calculat~~ perturbation energy. ~~In order to obtain~~ ~~as well~~ as the short range forces between ~~the~~ two neutrons or two ~~neutrons~~ protons, ~~In II, we at~~ we found that It was shown in II, that these forces ~~are~~ was smaller by a factor 10 than the exchange force above obtained, so that ~~p we considered~~ the introduction of the neutral quanta might be necessitate in ~~order to the exhaustive~~ produce the or solution ~~in~~ solve the ~~whole~~ problem exhaustively. These conclusions seems to be true in the present case, ~~and~~ assumed in the It is ~~easily~~ not difficult to consider the ~~neutral~~ U-~~field~~ field ~~corresponding~~ accompanied ~~to the~~ by the heavy neutral quanta, which satisfies the ~~equation~~ linear equation similar ~~to the U-field~~ considered in this paper. The detailed ~~for~~ those discussions of ~~these~~ subject, however, will be postponed to the next paper.

† In ~~the P and higher state~~ the higher states, P, D etc., the expression H_{12} for the exchange force becomes more complicated, and ~~it~~ will be ~~a interesting~~ to ~~do~~ consider these cases in detail, ~~but~~ elsewhere

In the coordinate system, the annihilation is accompanied by the transition of

The light particle ~~jumps~~ from the neutrino state of the negative ~~energy~~ about $-\frac{m_0c^2}{2}$ ~~to the~~ and momentum $\hbar\vec{k} - \frac{m_0c}{2}$ to the electron state of the positive energy. ~~$\frac{m_0c^2}{2}$ and the momentum~~ $\hbar\vec{k}$

The conservation laws requires that the magnitude of the energies of the both state are equal to ~~about~~ $\frac{m_0c^2}{2}$ and the momenta are equal to ~~$\hbar\vec{k}$~~ each other, $\hbar\vec{k}$ say, and with the magnitude ~~$\hbar k$ about~~ $\hbar k \cong \frac{m_0c}{2}$.

§8. Creation and Annihilation of the Heavy Quanta

In the preceding sections, we considered the ~~contribution of the heavy quanta in the ia intermediate state~~ effects due to the virtual presence of the heavy quanta in the intermediate states. Now, ~~Now~~ The heavy quantum can be created, ~~only~~ if the energy greater than $m_U c^2$ is supplied, but ~~they~~ it will ~~be~~ soon be a~~bsorbed &~~ annihilated by collision with matter as discussed in §5, II. It should be noticed, further, that ~~even in the heavy quanta are a quantum~~ is anni ~~the sponta~~ the heavy quantum, ~~is annihilated in the~~ free space by emitting ~~a~~ disappear, ~~spontaneously~~ even, ~~heavy qu~~ a positive or negative electron and a neutrino or an antineutrino simultaneously according as the charges of the heavy quantum is positive or negative, as ~~the probe~~ already pointed out by Bhabha.[14] ~~the~~ The probability ~~frequency of occurrence~~ of this process is ~~comparatively small~~ ~~not large~~ ~~(not large~~ owing to the ~~although the~~ small interaction of the heavy quantum with the light particle, ~~the mean life~~ but it is ~~not so small as to make large enough for preventing the~~ (In this case, the conservation laws are not violated on account of the proper energy of the heavy quantum.)

Owing to the small interaction of the heavy quantum with the light particle, the (~~frequency~~ probability) of occurrence of the above process is ~~comparatively~~ so (small, ~~large~~) that the) ~~track~~ mean free path of the ~~hea~~ high speed heavy quantum is large compared with the dimension of the measuring apparatus, but, in some case, ~~at~~ is not ~~so~~ small ~~as to~~ enough, make the mean free path large compared with the ~~depth~~ height of the whole atomosphere, ~~away, when the~~ as will be shown presently.

We consider ~~s~~ a heavy quantum with the negative charge, for instance, ~~which is at rest~~ with respect to a certain coordinate ~~system, it can give~~ ~~w~~ ~~n~~ In this coordinate system, the energies of the electron and the antineutrino emitted by annihilation are both approximately equal to $\frac{m_U c^2}{2}$ and their momenta ~~are~~ have equal ~~in~~ magnitude and opposite

---

14) Bhabha, loc. cit.

In this coordinate system, $\hbar\vec{k}$, $\hbar\vec{k}'$ say, about $\frac{m_U c}{2}$ $\hbar k$, ~~$\hbar k'$~~ $\hbar k' \cong \frac{m_U c}{2}$ jumps the light particle ~~an neutrino~~ of the negative

If we

the dire ~~the~~ magnitude $(k, k+dk)$ in the solid

According of the ... theory, (27)

~~in~~ directions. The probability ~~of annihilation~~ per unit time ~~is given by~~ of annihilation go by emitting ~~a~~ the electron, with the momentum ~~between~~ in the range ~~(between~~ $\hbar\vec{k}$, ~~and~~ and $\hbar(\vec{k}+d\vec{k})$ ~~and the~~ ~~$W_0 = \frac{2\pi}{\hbar}$~~ neutrino with the momentum ~~between~~ & in the corresponding range is given by

$$\frac{2\pi}{\hbar}\sum |V|^2 \frac{k^2 dk}{(2\pi)^3 dE} d\Omega \qquad (\ )$$

in the direction ~~with~~ (within solid angle $d\Omega$. if we consider the whole ~~whole~~ system ~~to be~~ to be in the unit cube, where E is the total energy ~~with in~~ of ~~the~~ ~~final state~~ where V is the matrix element, of the energy of interaction of the U-field $V_{\vec{k}} =$ with the light particle, corresponding to the above transition). ~~If we~~ ~~In order to obtain an~~ ~~determine~~ the ~~avg~~ order of magnitude of the ~~trans~~ above probability, ~~it is sufficient~~ According to the scalar U-field theory, which is sufficient for the determination of the matrix element ~~&~~ ~~at least~~, can be written in the form

$$V = g'\hbar c\sqrt{\frac{2\pi}{m_\nu c^2}}\ \tilde{\varphi}\beta\psi,$$

~~The summation~~ $\sum$ means the summation with respect to the spin of the electron and the neutrino.

$k = \frac{m_\nu c}{2\hbar}$

since $E = m_\nu c^2 \cong 2\hbar k c$ in this case

$\frac{1}{2\hbar c}$

( ) reduces at once to

$$\frac{2\pi}{\hbar}|V|^2\frac{k^2 dk\left(\frac{m_\nu c^2}{2\hbar c}\right)^2 d\Omega}{(2\pi)^3\cdot 2\hbar c}$$

$$= \frac{\sum|V|^2 d\Omega}{(2\pi)^2(2\hbar c)^3\hbar}(m_\nu c^2)^2 \qquad \frac{m_\nu c}{32\pi^2\hbar^4}\sum|V|^2 d\Omega$$

where g' is the small constant ~~characterizing the interact~~ introduced in I which is smaller than g by a factor about 10^{-8}, and ψ, φ is the ~~eigenfunctions~~ ~~each with four components~~, of the electron and ~~the~~ wave neutrino, i.e. the ~~the neutrino in the negative energy state~~ anti emitted, which disappears after the ~~by the process~~ the above process after

The detailed discussion of these subjects, however, will be made in the next paper.

Konopinski-Uhlenbeck type, but will be a complicated combination of various types of forces.

In ~~this~~ connection, it ~~will~~ is be important to determine ~~the cross~~ how large ~~are~~ the contributions of other processes ~~to the~~ creation of ~~the~~ heavy quanta.

settled only if the cross sections of these processes are ~~calculated~~ determined.

~~It is not certain~~

~~The question~~ This, whether this discrepancy can be removed by taking the ~~processes~~ contributions of other process to the creation of the heavy quanta or not, can be

~~In any case, it will be~~ is important to

~~The detailed discussions will be made elsewhere.~~

In conclusion, it should be remarked that ~~the present~~

~~Thus, it is important for the~~ This question ~~problem~~ can be settled, only if ~~the contribution~~ the

~~above assumption for the interaction between~~

theory of the U-field interaction

~~The assum~~ the interaction ~~between~~ of the U-field with ~~and~~ the light particles as well as that between the heavy particle has ~~a~~ rather complicated form in our theory, so that the interaction between the light and heavy particle, which can derived as second order effect, and is responsible for the β-disintegration, ~~will be~~ can not be identified with neither ~~with the. the pure Fermi~~ that of pure Fermi type nor with that of

~~Finally~~ Using so that ( ) becomes

$$\frac{2\pi}{\hbar} \cdot \frac{g'^2 (\hbar c)^2 \cdot (2\pi)^3}{2\pi \, (2\pi \hbar c)^3 \cdot \hbar} d\Omega \sum |\tilde{\varphi}^* \rho \psi|^2$$

$$= \frac{g'^2}{\hbar c} \cdot \frac{d\Omega}{\hbar \cdot 16\pi} \cdot \sum |\tilde{\varphi}^* \rho \psi|^2$$

$$= \frac{g'^2}{\hbar c}.$$

On the other hand,

~~There are~~ many processes, which ~~can give rise to the~~ are connected with the creation of the heavy quantum, ~~such as~~ ~~the~~ such the creation of pair of ~~heavy~~ positive and negative quanta by γ-rays of energy larger than $2m_U c^2$, the emission of one or more heavy quanta by nuclear disintegration caused by high energy radiations of various sorts, etc. Among them, the creation ~~of~~ by γ-rays ~~seems to have~~ the largest cross section, ~~which~~ ~~and~~ is ~~about~~ roughly about $\left(\frac{m}{m_U}\right)^2$ times ~~the cross~~ that of the creation by γ-rays of ~~the~~ pair of electrons $\cong 10^{-4}$. ~~Thus, we can expect~~ Hence, if the primary cosmic ray consists exclusively of the ~~positive~~ and negative electrons, which ~~are~~ ~~consequently~~ ~~is~~ accompanied by about equal number of light quanta ~~give rise~~ ~~due the~~ ~~radiative~~, which is created by the radiative collision, the number of the secondary heavy quanta ~~can~~ ~~will~~ ~~be about~~ have should the order of magnitude ~~about~~ 10^{-4} ~~as~~ per one primary electron, provided that ~~already discussed by Bhabha~~, ~~whether~~ this Bhabha considered this value a little too small to account for the experimental results of Bowen ...) ~~and~~, ~~but~~ ~~The conclusion~~ Thus, whether or not ~~other~~ processes ~~of the complete solution~~ ~~of this~~ problem can be solved, when it is determined ~~however, only if the cross sections of creation of the heavy quanta by other processes~~ relative importance of other processes to the creation of heavy quanta ~~contribution~~ contribution may happen to be important

$\vec{r}, \ldots$ where p

~~Using~~ using this Hamiltonian ~~and the commutation relations for $\Psi, \tilde{\Psi}$~~, (21)

From this $\bar{H}$, ~~and the we obtain~~ the quantized wave equations ~~of motion~~ for them for the ~~quantized wave functions Ψ by, $\tilde{\Psi}$~~.

$$\left.\begin{aligned} \tilde{\Psi}^{(i)}(\vec{r},t)\,\Psi^{(j)}(\vec{r}',t) + \Psi^{(j)}(\vec{r}',t)\,\tilde{\Psi}(\vec{r},t) &= \delta_{ij}\,\delta(\vec{r},\vec{r}') \\ \Psi^{(i)}(\vec{r},t)\,\Psi^{(j)}(\vec{r}',t) + \quad &= 0 \\ \tilde{\Psi}^{(i)} \quad \tilde{\Psi}^{(j)} \quad + \quad &= 0, \end{aligned}\right\} \quad (52)$$

for ~~with~~ $i, j = 1, 2 \ldots 8$,

we obtain ~~the~~ quantized wave equations for the heavy particle

UQ

$$i\hbar \frac{\partial \Psi}{\partial t} = \Psi(\bar{H}' + \bar{H}_M) - (\bar{H}' + \bar{H}_M)\Psi$$

$$= \Big[\vec{\alpha}\vec{p} + \beta\Big(\frac{1+\tau_3}{2} M_N c^2 + \frac{1-\tau_3}{2} M_P c^2\Big)$$

$$+ \frac{4\pi g_1 c}{\varkappa^2}\left(\operatorname{div} U^{+}\cdot\tilde{Q} + \operatorname{div}\tilde{U}^{+}\cdot Q\right)$$

$$- \frac{g_1}{\varkappa}\left\{(\tilde{U}_{\alpha}\tilde{Q}) + (U_{\alpha}Q)\right\} \pm 4\pi g_2 c \left\{U^{+}\rho_2\vec{\sigma}\tilde{Q} + (\tilde{U}^{+}\rho_2\vec{\sigma}\,Q)\right\}$$

$$+ \frac{g_2}{\varkappa^2}\left\{\operatorname{curl}\tilde{U}\cdot\rho_3\vec{\sigma}\tilde{Q} + \operatorname{curl} U\,\rho_3\vec{\sigma}\,Q\right\}$$

$$+ \frac{4\pi}{\varkappa^2}\left\{g_1^2(\tilde{M}_0\cdot\tilde{Q} + g M_0 Q) + \frac{4\pi g_2^2}{\varkappa^2}(\quad)\right\}\Big]\Psi$$

$\Psi^{(1)}$ … $\Psi^{(5)}$, $\Psi^{(1)}$ … 0, 0, 0; $\vec{Q}$

$$\left.\begin{aligned} \Psi^{(1)}\cdot\{\tilde{\Psi}^{(1)}\Psi^{(5)} + \tilde{\Psi}^{(2)}\Psi^{(6)} + \cdots\} &= \Psi^{(5)} \\ &= 0, \end{aligned}\right\}$$

$\Psi^{(1)} \cdot \tilde{\Psi}Q\Psi \cdot \tilde{\Psi}Q\Psi$.

provided that $\vec{M}, M_0$, and $\vec{T}, \vec{S}$ in $\bar{H}'$ is given ~~expressed~~ by the simplest expressions (38) and (39)

$$\Psi^{(1)}\{\tilde{\Psi}^{(5)}\Psi^{(1)} + \cdots\} + \{\tilde{\Psi}^{(1)}\Psi^{(5)} + \cdots\} = \tilde{\Psi}^{(5)}\Psi^{(1)} \quad \Psi^{(5)}$$

$\tilde{\Psi}Q\Psi \cdot \tilde{M}_0'$

$\tilde{\xi}\,\rho_3\sigma$

$\rho\,(\tilde{\xi}\sigma)\rho_3\tilde{Q}$

§7. Deduction of ~~the~~ exchange Forces between the neutron and the proton.

The interaction between the neutron and the proton, which is caused by the virtual ~~absorpt~~ emission and absorption of the heavy quanta, can be calculated from the Hamiltonian (50) as second order effect by the straightforward application of the perturbation theory as in II, §3. Namely, we first transform the variables for the ~~U-field into~~ and ~~the heavy particle~~ into the normal coordinates, then express the unperturbed system with the Hamiltonian $\bar{H}_U + \bar{H}_M$ perturbation energy H' in terms of these coordinates and finally, after ~~per~~ ~~lengthy~~ ~~out~~ rather complicated calculations, we arrived at the required formulae. An alternative method, which is similar to that given in I, ~~however~~ and leads to the ~~same~~ final formulae ~~results~~ more quickly, will be mentioned in the following. We verified ~~that these~~ the results of two methods are ~~exactly~~ identical.

We consider ~~the~~ a heavy particles in ~~the~~ states ~~U etc~~

. . .

When the kinetic energies ~~differences of energies~~ of ~~the heavy particle~~ two particles respectively are both small compared with m_0c^2, $\rho_1^{(j)}\vec{\sigma}^{(j)}$, $\rho_2^{(j)}\vec{\sigma}^{(j)}$ are negligible ~~and~~ $\rho_3^{(j)}\vec{\sigma}^{(j)} \simeq \vec{\sigma}^{(j)}$ is approximately equal to $\vec{\sigma}^{(j)}$. †

Moreover, the terms Moreover, the terms in (49) . . . .

~~We consider a heavy particle~~ with the coordinates $\vec{r}_1$, the ~~momentum~~ $\vec{p}_j$, the spin matrices ρ . .

In this case ~~The~~ U-field ~~due to the~~ at the point $\vec{r}_2$, due to the presence of the heavy particle $\vec{r}_1$ is given by the solution of

† In this approximation, ~~Thus~~, the wave equations (53) reduce to .

$$i\hbar\frac{\partial\Psi}{\partial t} = [\vec{\alpha}\vec{p} + \beta(\qquad) + \frac{4\pi g_1 c}{H'\kappa}(\mathrm{div}\ \cdots) + \frac{g_2}{\kappa^2}\{\mathrm{curl}\,\tilde{U}\cdot\vec{\sigma}\,\tilde{Q} + \cdots\} + \cdots]\Psi\} \quad (54)\ (\cancel{53})$$

$H' =$.

~~the field~~ originated by the presence of the heavy particles,

Moreover, . . .

so that the equations (49) becomes approximately

$$\cancel{\frac{1}{c^2}\frac{\partial^2 U}{\partial t^2}} - \Delta U + \kappa^2 U = 4\pi g_2 \mathrm{curl}\, S$$

$$-\Delta \tilde{U}^{+} + \kappa^2 \tilde{U}^{+} = \frac{g_1}{\kappa c}\, \mathrm{grad}\, M_0 \qquad (56)$$

where ~~M_0, S are given~~

$$M_0 = \tilde{\Psi}\tilde{Q}\Psi, \quad \vec{S} = \tilde{\Psi}\vec{\sigma}\tilde{Q}\Psi. \qquad (\cancel{57})$$

~~Now, the U-field~~ Integrating (56), we obtain the U-field at $\vec{r}_1$ originated by the heavy particle at r_2 ~~with the quantized wave functions~~

$$U(\vec{r}_1) = -g_2\, \mathrm{curl}_1\, S(\vec{r}_2)\, \delta(\vec{r}_1, \vec{r}_2)\, \frac{e^{-\kappa r}}{r}$$

$$\tilde{U}^{+}(\vec{r}_1) = \frac{g_1}{4\pi\kappa c}\, \mathrm{grad}\, M_0(\vec{r}_2)\, \delta(\vec{r}_1, \vec{r}_2)\, \frac{e^{-\kappa r}}{r} \quad \text{where } \vec{r} = \vec{r}_1 - \vec{r}_2. \qquad (56)$$

Inserting this into (55), ~~Thus, the term V (55)~~ the ~~out~~ potential energy ~~at r_1~~ due the interaction of two particles becomes

$$M_{12} = \frac{g_1^2}{\kappa^2} \cancel{\mathrm{grad}\, M}\, (\tilde{Q}_1 Q_2 + Q_1 \tilde{Q}_2)\, \mathrm{div}\, \mathrm{grad}\, \frac{e^{-\kappa r}}{r}$$

$$- \frac{g_2^2}{\kappa^2} (\tilde{Q}_1 Q_2 + \ldots)\, \vec{\sigma}^{(1)} \mathrm{curl}\, \mathrm{curl}\left(\vec{\sigma}^{(2)} \frac{e^{-\kappa r}}{r}\right)$$

$$+ \frac{4\pi g_1^2}{\kappa^2} (\tilde{Q}_2 Q_1 + Q_2 \tilde{Q}_1)\, \delta(\vec{r}) + \frac{4\pi g_2^2}{\kappa^2} (\ldots + \ldots)(\vec{\sigma}^{(1)} \vec{\sigma}^{(2)})\, \delta(\vec{r}), \qquad (57)$$

~~which reduces to the simple form~~

By ~~the help~~ using the relation

$$\Delta \frac{e^{-\kappa r}}{r} = \kappa^2 \frac{e^{-\kappa r}}{r} - 4\pi\delta(\vec{r}) \qquad (58)$$

the terms involving δ-function ~~cancel~~ compensate with one ~~another~~ each other, so that (57) reduces to the simple form

$$\left.\begin{aligned}\vec{U} &= \sum_k \sum_{j=1,2,3} (u_{jk} + \tilde{v}_{jk})\vec{e}_{jk}\, e^{i(\vec{k}\vec{r})} \\ \vec{U}^\dagger &= \sum_k \sum_{j=1,2,3} (u^\dagger_{jk} + \tilde{v}^\dagger_{jk})\vec{e}_{jk}\, e^{-i(\vec{k}\vec{r})}\end{aligned}\right\} \quad (15)$$

$$\left.\begin{aligned}\dot{u}_{jk} &= 4\pi c^2(\kappa^2 + k^2\delta_{j1})\tilde{u}^\dagger_{jk} \\ \dot{\tilde{u}}^\dagger_{jk} &= \frac{1}{4\pi\kappa^2}(k^2\delta_{j1} - k^2 - \kappa^2)u_{jk}\end{aligned}\right\}$$

$$\left.\begin{aligned}\dot{u}_{jk} &= 4\pi\kappa^2c^2\tilde{u}^\dagger_{jk} + 4\pi k^2c^2\delta_{j1}\tilde{u}^\dagger_{1k} \\ \dot{\tilde{u}}^\dagger_{jk} &= -\frac{k^2+\kappa^2}{4\pi\kappa^2}u_{jk} + \frac{k^2}{4\pi\kappa^2}\delta_{j1}u_{1k} \\ \dot{\tilde{v}}_{jk} &= 4\pi\kappa^2c^2 v^\dagger_{jk} + 4\pi k^2c^2\delta_{j1}v^\dagger_{1k} \\ \dot{v}^\dagger_{jk} &= \cancel{4} -\frac{k^2+\kappa^2}{4\pi\kappa^2}\tilde{v}_{jk} + \frac{k^2}{4\pi\kappa^2}\delta_{j1}\tilde{v}_{1k}\end{aligned}\right\} \quad (16)$$

$$\left.\begin{aligned}&u_{jk}u^\dagger_{lk'} - u^\dagger_{lk'}u_{jk} = \frac{i\hbar}{2}\delta_{jl}\delta(\vec{k},\vec{k}') \\ &u_{jk}u_{lk'} - u_{lk'}u_{jk} = 0 \\ &\text{etc.}\end{aligned}\right\} \quad (17)$$

$$\begin{aligned}\bar{H}_U = \sum_k \sum_{j=1,2,3} \Big\{ & 4\pi\kappa^2c^2(\tilde{u}^\dagger_{jk} + \tilde{v}^\dagger_{jk})(u^\dagger_{jk} + v^\dagger_{jk}) + 4\pi k^2c^2\delta_{j1}(\tilde{u}^\dagger_{jk} + v^\dagger_{jk}) \\ & (u^\dagger_{jk} + v^\dagger_{jk}) \\ & + \frac{k^2}{4\pi\kappa^2}(1-\delta_{j1})(\tilde{u}_{jk} + v_{jk})(u_{jk} + \tilde{v}_{jk}) + \frac{1}{4\pi}(\ )(\ )\end{aligned}$$

$$\left.\begin{aligned}= \sum\sum\Big\{ & 4\pi c^2(\kappa^2 + k^2\delta_{j1})(\tilde{u}^\dagger_{jk} + v^\dagger_{jk})(u^\dagger_{jk} + \tilde{v}^\dagger_{jk}) \\ & + \frac{1}{4\pi\kappa^2}(k^2 + \kappa^2 - k^2\delta_{j1})(\tilde{u}_{jk} + v_{jk})(u_{jk} + \tilde{v}_{jk})\Big\}\end{aligned}\right\} (18)$$

$$-ik_0c\,\tilde{u}^\dagger_{jk} = \left(-\frac{k^2+\kappa^2}{4\pi\kappa^2} + \frac{k^2\delta_{j1}}{4\pi\kappa^2}\right)u_{jk} \qquad (\tilde{u}_{jk} - v_{jk})$$

$$\begin{aligned}H_U &= \sum\sum 4\pi c^2 \frac{(\kappa^2 + k^2\delta_{j1})\left(\frac{k_0^2 \mp k^2\delta_{j1}}{4\pi\kappa^2}\right)^2(u_{jk} - \tilde{v}_{jk})}{k_0^2c^2} \\ &\quad + \frac{1}{4\pi\kappa^2}(k^2 + \kappa^2 - k^2\delta_{j1})(\tilde{u}_{jk} + v_{jk})(u_{jk} + \tilde{v}_{jk}) \\ &= \sum\sum\frac{(k_0^2 - k^2\delta_{j1})}{4\pi\kappa^2}(u_{jk}\tilde{u}_{jk} + \tilde{u}_{jk}u_{jk}\ \cancel{+\ k_0}\ \tilde{v}_{jk}v_{jk} \\ &\qquad + v_{jk}\tilde{v}_{jk})\end{aligned}$$

By using the relations (16) or

$$-ik_0 c\, u^{\dagger}_{jk} = \frac{k_0^2 - k^2\delta_{j1}}{4\pi\kappa^2}\tilde{u}_{jk}$$

The commutation relations (17) become

$$N_{jk} = \frac{(k_0^2 - k^2\delta_{j1})}{2\pi\kappa^2 \cdot k_0\hbar c}\tilde{u}_{jk}u_{jk} \qquad (19)$$

$$M_{jk} = \frac{(k_0^2 - k^2\delta_{j1})}{2\pi\kappa^2 k_0\hbar c}\tilde{v}_{jk}v_{jk}$$

$$-ik_0 c\, u^{\dagger}_{jk} = \frac{k_0^2 - k^2\delta_{j1}}{4\pi\kappa^2}u_{jk}$$

k_0

$$u_{jk}\tilde{u}_{jk} - \tilde{u}_{jk}u_{jk} = \frac{\hbar c}{2i}\left(\frac{2\pi\kappa^2}{k_0^2 - k^2\delta_{j1}}\right)\delta_{jk} \text{ etc.,}$$

so that the variables

are all commutative with one another . . . .

The Hamiltonian[48] now takes the form

(20)

The variable . . .

$$U^{\dagger}U - UU^{\dagger} \;\cancel{\propto}\; \sum(\tilde{u}_{jk} - v_{jk})(u_{jk} + \tilde{v}_{jk})$$
$$- (u_{jk} + \tilde{v}_{jk})(\tilde{u}_{jk} - v_{jk})$$
$$= (\tilde{u}_{jk}u_{jk} - u_{jk}\tilde{u}_{jk}) + (\tilde{v}_{jk}v_{jk} - v_{jk}\tilde{v}_{jk})$$

$$\cancel{\tilde{U}^{\dagger}U - U\tilde{U}^{\dagger} \propto \sum(u_{jk} - \tilde{v}_{jk})(u_{jk} + \tilde{v}_{jk})} \quad (\tilde{u}_{jk} + v_{jk})$$

$$\tilde{U}U - U\tilde{U} \quad \cancel{- (u_{jk} + v_{jk})(u_{jk} - \tilde{v}_{jk})}$$

$$\tilde{U}^{\dagger}U^{\dagger} - U^{\dagger}\tilde{U}^{\dagger} = \quad \propto \sum(\tilde{u}_{jk} + v_{jk})(u_{jk} + \tilde{v}_{jk}) - (u_{jk} + \tilde{v}_{jk})$$